CMU Mathematical Sciences
21-259 Sample Prerequisite Waiver Exam Booklet #1
Answer Key

1. Determine the area of the triangle in $\mathbb{R}^3$ with vertices $A=(1,0,2)$, $B=(3,4,1)$, and $C=(2,5,3)$.

   A. $\dfrac{3\sqrt{14}}{2}$

   B. $\dfrac{3\sqrt{7}}{2}$

   C. $\dfrac{9\sqrt{14}}{2}$

   D. None of these choices

   E. $3\sqrt{14}$

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2. Two lines ℓ_1 and ℓ_2 are given by

$$\ell_1: \quad \mathbf{r}_1(t)=\langle 4t-1, 2t+3, -3t+5\rangle,$$
$$\ell_2: \quad \mathbf{r}_2(s)=\langle 3s+2, -s+4, -6s+2\rangle.$$

   Determine the point of intersection of ℓ_1 and ℓ_2.

   A. $\left(-\frac{1}{5}, \frac{21}{5}, \frac{16}{5}\right)$

   B. None of these choices

   C. The lines do not intersect

   D. $\left(\frac{7}{5}, \frac{21}{5}, \frac{16}{5}\right)$

   E. $\left(\frac{7}{5}, \frac{21}{5}, -\frac{16}{5}\right)$

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3. Let C be the curve parametrized by $\mathbf{r}(t) = \langle 2\sqrt{2}t, e^{2t}, e^{-2t}\rangle$ for $-\infty < t < \infty$. Determine the curvature of C at the point $(0,1,1)$.

   A. None of these choices

   B. $\sqrt{2}$

   C. $\dfrac{\sqrt{2}}{4}$

   D. $\dfrac{\sqrt{2}}{2}$

   E. $\dfrac{1}{4}$

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4. Determine the value of C, if any, such that f is continuous at $(0,0)$.

$$f(x,y)=\begin{cases}\dfrac{1-\cos(x^2+y^2)}{(x^2+y^2)^2} & \text{if } (x,y)\neq(0,0),\\ C & \text{if } (x,y)=(0,0).\end{cases}$$

A. None of these choices

B. 1

C. $\frac{1}{2}$

D. f cannot be made continuous at $(0,0)$.

E. 0

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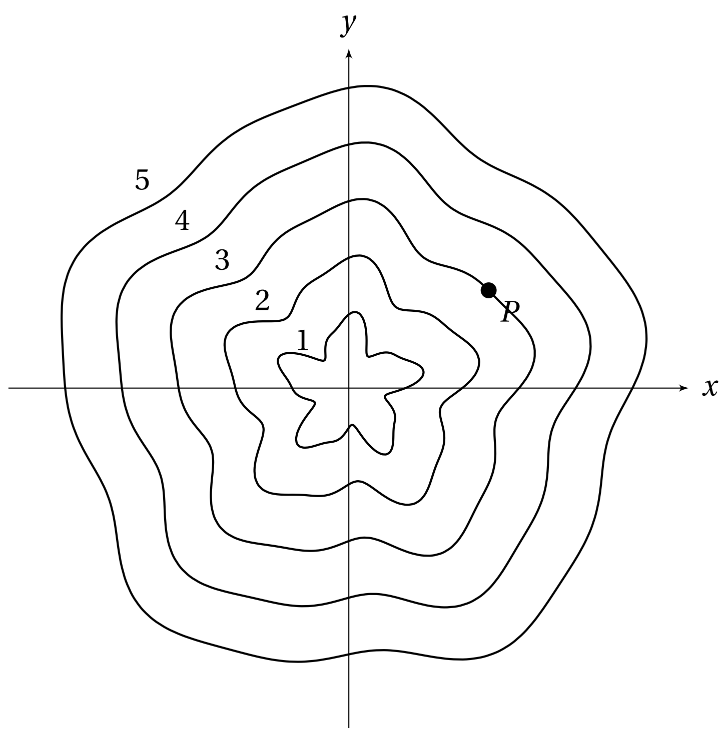
5. The contour map for a function $z=f(x,y)$ is given below. At the point labeled P, which of the following vectors points in the direction of ∇f?



A. $\langle -1,1\rangle$

B. $\langle 1,1\rangle$

C. $\langle 1,-1\rangle$

D. None of these choices

E. $\langle -1,-1\rangle$

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6. Find the equation of the tangent plane to the surface $e^{xy}+z^2=2$ at the point $(0,1,-1)$.

   A. $x+2z=-2$

   B. $x-z=1$

   C. $2x-z=2$

   D. None of these choices

   E. $x-2z=2$

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7. Let $f(x,y)=3x^2y-2x^3+2y^3-24y$. Which of the following statements is true?

   A. f has exactly 2 local maxima and 2 saddle points.

   B. f has exactly 2 local minima and 2 local maxima.

   C. f has exactly 2 local minima and 2 saddle points.

   D. f has exactly 1 local minima and 3 saddle points.

   E. None of the above statements are true.

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8. Find the maximum value of $f(x,y)=xy+14$ subject to the constraint $x^2+y^2=18$.

   A. 14

   B. 23

   C. None of these choices

   D. 9

   E. 18

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9. Let $f(x,y)$ be a continuous function. Which of the following iterated integrals is equal to

$$\int_0^9\int_{\sqrt{x}}^3 f(x,y)\,dy\,dx?$$

   A. $\displaystyle\int_0^3\int_{y^2}^9 f(x,y)\,dx\,dy$

   B. $\displaystyle\int_0^3\int_0^{\sqrt{y}} f(x,y)\,dx\,dy$

   C. None of these choices

   D. $\displaystyle\int_0^3\int_0^{y^2} f(x,y)\,dx\,dy$

   E. $\displaystyle\int_0^9\int_0^{y^2} f(x,y)\,dx\,dy$

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10. Let D be the parallelogram with vertices $(0,0)$, $(2,1)$, $(3,3)$, and $(1,2)$. Evaluate $\displaystyle\iint_D e^{3(x-y)}\,dA$.

   A. $e^3 - e^{-3}$

   B. $\dfrac{e^3 + e^{-3}}{3}$

   C. $\dfrac{e^3 - e^{-3}}{3}$

   D. None of these choices

   E. $\dfrac{e^3 + e^{-3} - 2}{3}$

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11. Let S be the part of the paraboloid $z = 9 - x^2 - y^2$ above the xy-plane, oriented upward, and let $\mathbf{F}(x, y, z) = \langle x,\ y,\ 2\rangle$. Calculate the flux of $\mathbf{F}$ across S.

   A. 18π

   B. None of these choices

   C. 48π

   D. 81π

   E. 99π

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12. Let C be the curve parametrized by

$$\mathbf{r}(t) = \left\langle \sin(\pi t),\ t^2 + t,\ e^{\sin(\pi t)} \right\rangle, \quad 0 \le t \le 1,$$

and let $\mathbf{F}(x, y, z) = \langle 2x,\ z,\ y\rangle$. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$.

   A. None of these choices

   B. $2e$

   C. 2

   D. $1 + 2e$

   E. 0

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13. Let D be the region in quadrant I bounded by the coordinate axes and the circles $x^2+y^2=1$ and $x^2+y^2=4$. Let
$$\mathbf{F}(x,y)=\left\langle e^{\sin x}-\sqrt{x^2+y^2},\ \ln(x^2+y^2)\right\rangle.$$
Evaluate the circulation of $\mathbf{F}$ around the boundary of D, oriented counterclockwise.

A. $\frac{3}{2}$

B. $\frac{7}{2}$

C. 6

D. None of these choices

E. 2

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14. Let C be the closed curve in the yz-plane consisting of the line segment from $(0,1,0)$ to $(0,0,0)$, the line segment from $(0,0,0)$ to $(0,0,1)$, and the quarter circle $y^2+z^2=1$ from $(0,0,1)$ back to $(0,1,0)$. Let
$$\mathbf{F}(x,y,z)=\langle e^x+y,\ \sin(y^2)+z,\ x+z^3\rangle.$$
Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$.

A. $-\frac{\pi}{2}$

B. $\frac{\pi}{2}$

C. None of these choices

D. $\frac{\pi}{4}$

E. $-\frac{\pi}{4}$

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15. Let S be the closed surface consisting of the portion of the sphere $x^2+y^2+z^2=4$ in the first octant, together with the three quarter-disks in the coordinate planes $x=0$, $y=0$, and $z=0$, oriented outward. Let
$$\mathbf{F}(x,y,z)=\left\langle x^3+y\ln(z^2+1),\ xz^3+y^3,\ \sin(x)+z^3\right\rangle.$$
Find the flux of $\mathbf{F}$ across S.

A. $\frac{384\pi}{5}$

B. $\frac{48\pi}{5}$

C. $\frac{16\pi}{5}$

D. $\frac{96\pi}{5}$

E. None of these choices
