

CMU Mathematical Sciences
21-259 Sample Prerequisite Waiver Exam Booklet #1

Instructions:

- **On the Answer Sheet, clearly print your name in the Name field and your Andrew ID in the ID field. Darken the appropriate circle in the Version field (see the header above).**
- **Do not open the exam until told to do so.**
- **Read all problem statements carefully.**
- **No books, notes, calculators, or other electronic devices, including phones, earbuds/headphones, or technology-enabled eyewear. Make sure your phone will not ring during the exam. Any use of technology is considered an Academic Integrity Violation.**
- **If you would like to leave the exam room before turning in your exam,**
 1. **bring your Exam Booklet and Answer Sheet to the front of the classroom and leave them on the table.**
 2. **demonstrate to the exam proctor that you are not taking any technology with you as you leave the room.**
- **The back side of each page, and the last page, can be used for scratch work. Do not put any scratch work on the Answer Sheet.**
- **If you need any additional paper for scratch work during the exam, please raise your hand.**
- **Time limit: 90 minutes.**
- **You may leave when you are finished if you finish before time is up.**
- **You must turn in ALL materials when you leave, including any scratch paper you were given.**
- **You must show your CMU ID card when you turn in your exam, Answer Sheet, and scratch paper.**
- **You must answer 10 of 15 questions correctly to earn the prerequisite waiver for 21-259.**

For each item, select the appropriate choice and darken the appropriate circle on your answer sheet completely. “None Of These” should be selected when none of the given choices is the correct answer.

Note: all angles are given in radian measure.

1. Determine the area of the triangle in \mathbb{R}^3 with vertices $A = (1, 0, 2)$, $B = (3, 4, 1)$, and $C = (2, 5, 3)$.

A. $\frac{3\sqrt{14}}{2}$

B. $\frac{3\sqrt{7}}{2}$

C. $\frac{9\sqrt{14}}{2}$

D. None of these choices

E. $3\sqrt{14}$

2. Two lines ℓ_1 and ℓ_2 are given by

$$\ell_1: \mathbf{r}_1(t) = \langle 4t - 1, 2t + 3, -3t + 5 \rangle,$$

$$\ell_2: \mathbf{r}_2(s) = \langle 3s + 2, -s + 4, -6s + 2 \rangle.$$

Determine the point of intersection of ℓ_1 and ℓ_2 .

A. $\left(-\frac{1}{5}, \frac{21}{5}, \frac{16}{5}\right)$

B. None of these choices

C. The lines do not intersect

D. $\left(\frac{7}{5}, \frac{21}{5}, \frac{16}{5}\right)$

E. $\left(\frac{7}{5}, \frac{21}{5}, -\frac{16}{5}\right)$

3. Let C be the curve parametrized by $\mathbf{r}(t) = \langle 2\sqrt{2}t, e^{2t}, e^{-2t} \rangle$ for $-\infty < t < \infty$. Determine the curvature of C at the point $(0, 1, 1)$.

A. None of these choices

B. $\sqrt{2}$

C. $\frac{\sqrt{2}}{4}$

D. $\frac{\sqrt{2}}{2}$

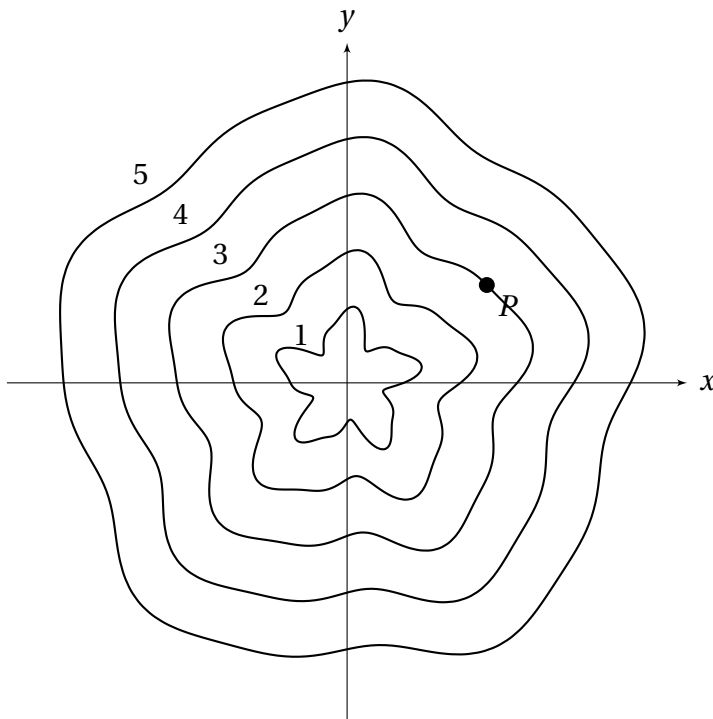
E. $\frac{1}{4}$

4. Determine the value of C , if any, such that f is continuous at $(0,0)$.

$$f(x, y) = \begin{cases} \frac{1 - \cos(x^2 + y^2)}{(x^2 + y^2)^2} & \text{if } (x, y) \neq (0, 0), \\ C & \text{if } (x, y) = (0, 0). \end{cases}$$

- A. None of these choices
B. 1
C. $\frac{1}{2}$
D. f cannot be made continuous at $(0,0)$.
E. 0
-

5. The contour map for a function $z = f(x, y)$ is given below. At the point labeled P , which of the following vectors points in the direction of ∇f ?



- A. $\langle -1, 1 \rangle$
B. $\langle 1, 1 \rangle$
C. $\langle 1, -1 \rangle$
D. None of these choices
E. $\langle -1, -1 \rangle$
-

6. Find the equation of the tangent plane to the surface $e^{xy} + z^2 = 2$ at the point $(0, 1, -1)$.

- A. $x + 2z = -2$
 - B. $x - z = 1$
 - C. $2x - z = 2$
 - D. None of these choices
 - E. $x - 2z = 2$
-

7. Let $f(x, y) = 3x^2y - 2x^3 + 2y^3 - 24y$. Which of the following statements is true?

- A. f has exactly 2 local maxima and 2 saddle points.
 - B. f has exactly 2 local minima and 2 local maxima.
 - C. f has exactly 2 local minima and 2 saddle points.
 - D. f has exactly 1 local minima and 3 saddle points.
 - E. None of the above statements are true.
-

8. Find the maximum value of $f(x, y) = xy + 14$ subject to the constraint $x^2 + y^2 = 18$.

- A. 14
 - B. 23
 - C. None of these choices
 - D. 9
 - E. 18
-

9. Let $f(x, y)$ be a continuous function. Which of the following iterated integrals is equal to

$$\int_0^9 \int_{\sqrt{x}}^3 f(x, y) dy dx?$$

- A. $\int_0^3 \int_{y^2}^9 f(x, y) dx dy$
 - B. $\int_0^3 \int_0^{\sqrt{y}} f(x, y) dx dy$
 - C. None of these choices
 - D. $\int_0^3 \int_0^{y^2} f(x, y) dx dy$
 - E. $\int_0^9 \int_0^{y^2} f(x, y) dx dy$
-

10. Let D be the parallelogram with vertices $(0, 0)$, $(2, 1)$, $(3, 3)$, and $(1, 2)$. Evaluate $\iint_D e^{3(x-y)} dA$.

- A. $e^3 - e^{-3}$
 - B. $\frac{e^3 + e^{-3}}{3}$
 - C. $\frac{e^3 - e^{-3}}{3}$
 - D. None of these choices
 - E. $\frac{e^3 + e^{-3} - 2}{3}$
-

11. Let S be the part of the paraboloid $z = 9 - x^2 - y^2$ above the xy -plane, oriented upward, and let $\mathbf{F}(x, y, z) = \langle x, y, 2 \rangle$. Calculate the flux of \mathbf{F} across S .

- A. 18π
 - B. None of these choices
 - C. 48π
 - D. 81π
 - E. 99π
-

12. Let C be the curve parametrized by

$$\mathbf{r}(t) = \langle \sin(\pi t), t^2 + t, e^{\sin(\pi t)} \rangle, \quad 0 \leq t \leq 1,$$

and let $\mathbf{F}(x, y, z) = \langle 2x, z, y \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

- A. None of these choices
 - B. $2e$
 - C. 2
 - D. $1 + 2e$
 - E. 0
-

13. Let D be the region in quadrant I bounded by the coordinate axes and the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. Let

$$\mathbf{F}(x, y) = \langle e^{\sin x} - \sqrt{x^2 + y^2}, \ln(x^2 + y^2) \rangle.$$

Evaluate the circulation of \mathbf{F} around the boundary of D , oriented counterclockwise.

- A. $\frac{3}{2}$
 B. $\frac{7}{2}$
 C. 6
 D. None of these choices
 E. 2

14. Let C be the closed curve in the yz -plane consisting of the line segment from $(0, 1, 0)$ to $(0, 0, 0)$, the line segment from $(0, 0, 0)$ to $(0, 0, 1)$, and the quarter circle $y^2 + z^2 = 1$ from $(0, 0, 1)$ back to $(0, 1, 0)$. Let

$$\mathbf{F}(x, y, z) = \langle e^x + y, \sin(y^2) + z, x + z^3 \rangle.$$

Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

- A. $-\frac{\pi}{2}$
 B. $\frac{\pi}{2}$
 C. None of these choices
 D. $\frac{\pi}{4}$
 E. $-\frac{\pi}{4}$

15. Let S be the closed surface consisting of the portion of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant, together with the three quarter-disks in the coordinate planes $x = 0$, $y = 0$, and $z = 0$, oriented outward. Let

$$\mathbf{F}(x, y, z) = \langle x^3 + y \ln(z^2 + 1), xz^3 + y^3, \sin(x) + z^3 \rangle.$$

Find the flux of \mathbf{F} across S .

- A. $\frac{384\pi}{5}$
 B. $\frac{48\pi}{5}$
 C. $\frac{16\pi}{5}$
 D. $\frac{96\pi}{5}$
 E. None of these choices

SCRATCH PAPER: You may detach this page from the exam, but this and all other scratch paper you use must be turned in when you leave. Raise your hand if you need any additional scratch paper during the exam.