

CMU Mathematical Sciences
21-122 Sample Prerequisite Waiver Exam Booklet #2
Answer Key

1. Evaluate $\int_0^1 x e^{2x} dx$.

A. $\frac{2e^2 + 1}{4}$

B. $\frac{e^2 + 1}{4}$

C. None of these choices

D. $\frac{e^2 - 1}{4}$

E. $\frac{e^2}{2}$

2. Evaluate $\int_0^{\pi/4} \tan^3(x) \sec(x) dx$.

A. $\frac{2 + \sqrt{2}}{3}$

B. $\frac{2 - \sqrt{2}}{3}$

C. $\frac{1}{3}$

D. None of these choices

E. $\frac{\sqrt{2} - 1}{3}$

3. Evaluate $\int_0^3 \frac{x^2}{\sqrt{9 - x^2}} dx$.

A. $\frac{3\pi}{4}$

B. $\frac{9\pi}{2}$

C. $\frac{9\pi}{4}$

D. None of these choices

E. 9

4. Evaluate $\int_0^1 \frac{5x+8}{x^2+3x+2} dx$.

- A. None of these choices
 - B. $3\ln 2 + 2\ln 3$
 - C. $\ln 12$
 - D. $\ln 18$**
 - E. $\ln 6$
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5. Determine whether the improper integral $\int_0^1 3x^{-5/4} dx$ converges or diverges. If it converges, evaluate it.

- A. None of these choices
 - B. $\frac{12}{5}$
 - C. 12
 - D. 3
 - E. Divergent**
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6. Determine whether the improper integral $\int_2^\infty \frac{x+1}{\sqrt{x^5-2}} dx$ converges or diverges.

- A. Divergent
 - B. Convergent**
 - C. None of these choices
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7. Find the length of the curve $x(t) = e^t + e^{-t}$, $y(t) = 2t$, $0 \leq t \leq \ln 2$.

- A. $\ln 2$
 - B. $2\ln 2$
 - C. None of these choices
 - D. $\frac{5}{2}$
 - E. $\frac{3}{2}$**
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8. Find $\frac{dy}{dx}$ for the parametric curve $x = \ln t$, $y = t^3 + 2t$, $t > 0$.

A. $\frac{3t^2 + 2}{t}$

B. None of these choices

C. $\frac{1}{t(3t^2 + 2)}$

D. $3t^2 + 2$

E. $3t^3 + 2t$

9. Solve the initial value problem $\frac{dy}{dx} = x(1 + y^2)$, $y(2) = \sqrt{3}$.

A. $y = \tan\left(\frac{x^2}{2} + \frac{\pi}{6} - 2\right)$

B. $y = \tan\left(\frac{x^2}{2} + \frac{\pi}{3}\right)$

C. $y = \tan\left(\frac{x^2}{2} + \frac{\pi}{3} - 2\right)$

D. $y = \tan\left(\frac{x^2}{2} - \frac{\pi}{3} - 2\right)$

E. None of these choices

10. Find the sum of the series $\sum_{n=3}^{\infty} \frac{5}{2^n}$.

A. $\frac{5}{8}$

B. $\frac{5}{2}$

C. $\frac{5}{4}$

D. None of these choices

E. $\frac{10}{3}$

11. Determine whether the series $\sum_{n=1}^{\infty} \frac{3n+1}{n^3+4}$ converges or diverges.

A. Divergent

B. None of these choices

C. Convergent

12. Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^3+1}$ is absolutely convergent, conditionally convergent, or divergent.

- A. None of these choices
 - B. Conditionally convergent
 - C. Divergent
 - D. Absolutely convergent**
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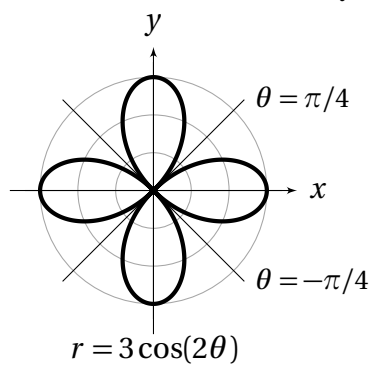
13. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n2^n}$.

- A. Radius 2, interval $(-3, 1]$
 - B. Radius 2, interval $[-3, 1)$**
 - C. Radius 2, interval $(-3, 1)$
 - D. Radius $\frac{1}{2}$, interval $(-\frac{3}{2}, -\frac{1}{2})$
 - E. None of these choices
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14. Find the first three nonzero terms of the power series for $\frac{3x}{1-2x^2}$.

- A. $3x + 6x^3 + 12x^5$**
 - B. $3x - 6x^3 + 12x^5$
 - C. $3x + 6x^2 + 12x^4$
 - D. None of these choices
 - E. $3 + 6x^2 + 12x^4$
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15. Find the area enclosed by one petal of the polar curve $r = 3 \cos(2\theta)$.



- A. $\frac{3\pi}{8}$
 - B. None of these choices
 - C. $\frac{9\pi}{4}$
 - D. $\frac{9\pi}{2}$
 - E. $\frac{9\pi}{8}$**
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