CMU Mathematical Sciences
21-122 Sample Prerequisite Waiver Exam Booklet #2

Instructions:

- **On the Answer Sheet, clearly print your name in the Name field and your Andrew ID in the ID field. Darken the appropriate circle in the Version field (see the header above).**
- **Do not open the exam until told to do so.**
- **Read all problem statements carefully.**
- **No books, notes, calculators, or other electronic devices, including phones, earbuds/headphones, or tehcnology-enabled eyewear. Make sure your phone will not ring during the exam. Any use of technology is considered an Academic Integrity Violation.**
- **If you would like to leave the exam room before turning in your exam,**
  1. **bring your Exam Booklet and Answer Sheet to the front of the classroom and leave them on the table.**
  2. **demonstrate to the exam proctor that you are not taking any technology with you as you leave the room.**
- **The back side of each page, and the last page, can be used for scratch work. Do not put any scratch work on the Answer Sheet.**
- **If you need any additional paper for scratch work during the exam, please raise your hand.**
- **Time limit: 90 minutes.**
- **You may leave when you are finished if you finish before time is up.**
- **You must turn in ALL materials when you leave, including any scratch paper you were given.**
- **You must show your CMU ID card when you turn in your exam, Answer Sheet, and scratch paper.**
- **You must answer 10 of 15 questions correctly to earn the prerequisite waiver for 21-122.**

For each item, select the appropriate choice and darken the appropriate circle on your answer sheet completely. "None Of These" should be selected when none of the given choices is the correct answer.

Note: all angles are given in radian measure.

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1. Evaluate $\int_0^1 x e^{2x}\,dx$.

   A. $\frac{2e^2+1}{4}$

   B. $\frac{e^2+1}{4}$

   C. None of these choices

   D. $\frac{e^2-1}{4}$

   E. $\frac{e^2}{2}$

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2. Evaluate $\int_0^{\pi/4} \tan^3(x)\sec(x)\,dx$.

   A. $\frac{2+\sqrt{2}}{3}$

   B. $\frac{2-\sqrt{2}}{3}$

   C. $\frac{1}{3}$

   D. None of these choices

   E. $\frac{\sqrt{2}-1}{3}$

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3. Evaluate $\int_0^3 \frac{x^2}{\sqrt{9-x^2}}\,dx$.

   A. $\frac{3\pi}{4}$

   B. $\frac{9\pi}{2}$

   C. $\frac{9\pi}{4}$

   D. None of these choices

   E. 9

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4. Evaluate $\displaystyle\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$.

   A. None of these choices

   B. $3\ln 2+2\ln 3$

   C. $\ln 12$

   D. $\ln 18$

   E. $\ln 6$

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5. Determine whether the improper integral $\displaystyle\int_0^1 3x^{-5/4}\,dx$ converges or diverges. If it converges, evaluate it.

   A. None of these choices

   B. $\dfrac{12}{5}$

   C. 12

   D. 3

   E. Divergent

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6. Determine whether the improper integral $\displaystyle\int_2^\infty \frac{x+1}{\sqrt{x^5-2}}\,dx$ converges or diverges.

   A. Divergent

   B. Convergent

   C. None of these choices

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7. Find the length of the curve $x(t)=e^t+e^{-t}$, $y(t)=2t$, $0\le t\le \ln 2$.

   A. $\ln 2$

   B. $2\ln 2$

   C. None of these choices

   D. $\dfrac{5}{2}$

   E. $\dfrac{3}{2}$

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8. Find $\frac{dy}{dx}$ for the parametric curve $x = \ln t$, $y = t^3 + 2t$, $t > 0$.

   A. $\frac{3t^2+2}{t}$

   B. None of these choices

   C. $\frac{1}{t(3t^2+2)}$

   D. $3t^2+2$

   E. $3t^3+2t$

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9. Solve the initial value problem $\frac{dy}{dx} = x(1+y^2)$, $y(2) = \sqrt{3}$.

   A. $y = \tan\left(\frac{x^2}{2} + \frac{\pi}{6} - 2\right)$

   B. $y = \tan\left(\frac{x^2}{2} + \frac{\pi}{3}\right)$

   C. $y = \tan\left(\frac{x^2}{2} + \frac{\pi}{3} - 2\right)$

   D. $y = \tan\left(\frac{x^2}{2} - \frac{\pi}{3} - 2\right)$

   E. None of these choices

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10. Find the sum of the series $\sum_{n=3}^{\infty} \frac{5}{2^n}$.

    A. $\frac{5}{8}$

    B. $\frac{5}{2}$

    C. $\frac{5}{4}$

    D. None of these choices

    E. $\frac{10}{3}$

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11. Determine whether the series $\sum_{n=1}^{\infty} \frac{3n+1}{n^3+4}$ converges or diverges.

    A. Divergent

    B. None of these choices

    C. Convergent

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12. Determine whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{n+1}{n^3+1}$ is absolutely convergent, conditionally convergent, or divergent.

    A. None of these choices
    B. Conditionally convergent
    C. Divergent
    D. Absolutely convergent

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13. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n2^n}$.

    A. Radius 2, interval $(-3,1]$
    B. Radius 2, interval $[-3,1)$
    C. Radius 2, interval $(-3,1)$
    D. Radius $\frac{1}{2}$, interval $\left(-\frac{3}{2},-\frac{1}{2}\right)$
    E. None of these choices

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14. Find the first three nonzero terms of the power series for $\frac{3x}{1-2x^2}$.

    A. $3x+6x^3+12x^5$
    B. $3x-6x^3+12x^5$
    C. $3x+6x^2+12x^4$
    D. None of these choices
    E. $3+6x^2+12x^4$

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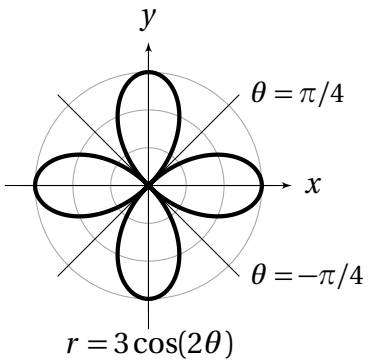
15. Find the area enclosed by one petal of the polar curve $r = 3\cos(2\theta)$.



A. $\frac{3\pi}{8}$

B. None of these choices

C. $\frac{9\pi}{4}$

D. $\frac{9\pi}{2}$

E. $\frac{9\pi}{8}$

SCRATCH PAPER: You may detach this page from the exam, but this and all other scratch paper you use must be turned in when you leave. Raise your hand if you need any additional scratch paper during the exam.