

CMU Mathematical Sciences
21-122 Sample Prerequisite Waiver Exam Booklet #1
Answer Key

1. Evaluate $\int_0^1 x^2 e^{3x} dx$.

A. $\frac{5e^3 - 2}{27}$

B. $\frac{e^3}{3}$

C. $\frac{3e^3 - 2}{27}$

D. None of these choices

E. $\frac{5e^3 - 2}{54}$

2. Evaluate $\int_0^{\pi/3} \sin^3(x) \cos^2(x) dx$.

A. $\frac{17}{480}$

B. None of these choices

C. $\frac{17\sqrt{3}}{240}$

D. $\frac{47}{480}$

E. $\frac{2}{15}$

3. Evaluate $\int_3^6 \frac{dx}{x^2 \sqrt{x^2 - 9}}$.

A. None of these choices

B. $\frac{1}{9}$

C. $\frac{\sqrt{3}}{18}$

D. $\frac{\sqrt{3}}{9}$

E. $\frac{1}{18}$

4. Evaluate $\int_0^1 \frac{4x + 13}{x^2 + 3x - 10} dx$.

- A. None of these choices
 - B. $\ln 6 - \ln 5 + 3 \ln 2$
 - C. $\ln\left(\frac{3}{20}\right)$**
 - D. $\ln\left(\frac{6}{5}\right)$
 - E. $\ln\left(\frac{20}{3}\right)$
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5. Determine whether the improper integral $\int_1^{\infty} \frac{3}{x^{5/2}} dx$ converges or diverges. If it converges, evaluate it.

- A. $\frac{3}{2}$
 - B. 2**
 - C. Divergent
 - D. None of these choices
 - E. $\frac{2}{3}$
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6. Determine whether the improper integral $\int_1^{\infty} \frac{2 + \sin x}{x^2} dx$ converges or diverges.

- A. Convergent**
 - B. None of these choices
 - C. Divergent
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7. Find the length of the curve $x(t) = t^2$, $y(t) = \frac{2}{3}t^3$, $0 \leq t \leq 3$.

- A. $10^{3/2} + 2$
 - B. None of these choices
 - C. $10^{3/2}$
 - D. $\frac{2}{3}(10^{3/2} + 1)$
 - E. $\frac{2}{3}(10^{3/2} - 1)$**
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8. Find $\frac{dy}{dx}$ for the parametric curve $x = e^t$, $y = t^2 - 1$.
- A. $2t$
 - B. $\frac{2t}{e^t}$**
 - C. $2te^t$
 - D. $\frac{e^t}{2t}$
 - E. None of these choices
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9. Solve the initial value problem $\frac{dy}{dx} = xy^2$, $y(0) = 1$.
- A. None of these choices
 - B. $y = 1 + \frac{x^2}{2}$
 - C. $y = \frac{1}{1 + \frac{x^2}{2}}$
 - D. $y = \frac{1}{\frac{x^2}{2} - 1}$
 - E. $y = \frac{1}{1 - \frac{x^2}{2}}$**
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10. Determine whether the sequence $a_n = \frac{(n+1)|\cos(n^2)|}{n^3 - n}$ converges or diverges. If it converges, find the limit.
- A. 1
 - B. 0**
 - C. Divergent
 - D. None of these choices
 - E. $\frac{1}{2}$
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11. Determine whether the series $\sum_{n=1}^{\infty} \frac{n+2}{n^3+1}$ converges or diverges.

- A. Divergent
- B. None of these choices

C. Convergent

12. Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n+1)}{n+1}$ is absolutely convergent, conditionally convergent, or divergent.

- A. None of these choices
- B. Divergent

C. Conditionally convergent

- D. Absolutely convergent
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13. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$.

- A. Radius $\frac{1}{3}$, interval $(\frac{5}{3}, \frac{7}{3})$
- B. Radius 3, interval $(-1, 5]$
- C. Radius 3, interval $(-1, 5)$

D. Radius 3, interval $[-1, 5)$

- E. None of these choices
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14. Find the first three nonzero terms of the Maclaurin series for $x^2 \sin(3x)$.

A. $3x^3 - \frac{9}{2}x^5 + \frac{81}{40}x^7$

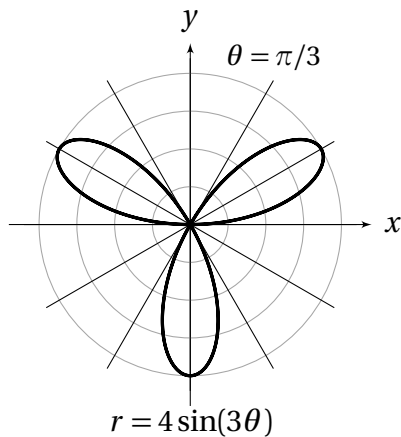
B. None of these choices

C. $3x - \frac{9}{2}x^3 + \frac{81}{40}x^5$

D. $3x^2 - \frac{9}{2}x^4 + \frac{81}{40}x^6$

E. $3x^3 + \frac{9}{2}x^5 + \frac{81}{40}x^7$

15. Find the area enclosed by one petal of the polar curve $r = 4 \sin(3\theta)$.



A. $\frac{4\pi}{3}$

B. 4π

C. $\frac{2\pi}{3}$

D. None of these choices

E. $\frac{8\pi}{3}$
