

**CMU Mathematical Sciences**  
**21-120 Sample Prerequisite Waiver Exam #2**  
**Answer Key**

1. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x \rightarrow 2^-} \frac{x^2 - 9}{x^2 - 5x + 6}.$$

- A. DNE
  - B. 0
  - C.  $\infty$
  - D. None of these choices
  - E.  $-\infty$**
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2. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x \rightarrow -\infty} \frac{5x^3 - 3x + 1}{\sqrt{4x^6 + x^2}}.$$

- A. None of these choices
  - B.  $\frac{5}{2}$
  - C.  $-\infty$
  - D. 0
  - E.  $-\frac{5}{2}$**
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3. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{8x \cos(7x) \sec(2x)}.$$

- A.  $\frac{3}{8}$**
  - B. None of these choices
  - C.  $\frac{3}{112}$
  - D.  $\frac{3}{28}$
  - E. 0
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4. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x \rightarrow 0^+} (1 - e^x) \ln x.$$

- A. None of these choices
  - B.  $\infty$
  - C. 0**
  - D. DNE
  - E.  $-\infty$
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5. Find  $f'(x)$  if  $f(x) = 4x^{2/3} \cos^{-1}(3x)$ .

- A.  $\frac{8 \cos^{-1}(3x)}{3x^{1/3}} - \frac{12x^{2/3}}{\sqrt{1-9x^2}}$**
  - B. None of these choices
  - C.  $\frac{8 \cos^{-1}(3x)}{3x^{1/3}} - \frac{12x^{2/3}}{1+9x^2}$
  - D.  $\frac{8 \cos^{-1}(3x)}{3x^{1/3}} - \frac{4x^{2/3}}{\sqrt{1-9x^2}}$
  - E.  $\frac{8 \cos^{-1}(3x)}{3x^{1/3}} - \frac{4x^{2/3}}{|x|\sqrt{1-9x^2}}$
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6. Find  $f'(x)$  if

$$f(x) = \log_2 \left( \frac{x^2 + 1}{\sqrt{x-1}} \right).$$

- A.  $\ln 2 \left( \frac{2x}{x^2 + 1} - \frac{1}{2(x-1)} \right)$
  - B.  $\frac{1}{\ln 2} \left( \frac{2x}{x^2 + 1} - \frac{1}{2(x-1)} \right)$**
  - C.  $\frac{2x}{x^2 + 1} - \frac{1}{2(x-1)}$
  - D.  $\frac{1}{\ln 2} \left( \frac{2x}{x^2 + 1} + \frac{1}{2(x-1)} \right)$
  - E. None of these choices
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7. Find an equation of the line tangent to the curve

$$x^2y + 3xy^2 = 14$$

at the point  $(-7, 2)$ .

- A.  $y - 2 = -\frac{16}{35}(x + 7)$
  - B.  $y - 2 = \frac{16}{7}(x + 7)$
  - C.  $y - 2 = -\frac{35}{16}(x + 7)$
  - D.  $y - 2 = -\frac{16}{7}(x + 7)$
  - E. None of these choices
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8. Find the absolute minimum value of

$$f(x) = x^3 - 3x^2 - 9x + 4$$

on the interval  $[-2, 4]$ .

- A. 2
  - B. -16
  - C. **None of these choices**
  - D. 9
  - E. -25
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9. Suppose that a liquid is filtered through a filter in the shape of an inverted right circular cone that is 16 cm high and has a radius of 4 cm at the top. Suppose also that the liquid is draining from the filter at a constant rate of  $2 \text{ cm}^3/\text{min}$ . At what rate is the depth of the liquid changing at the instant when the liquid in the filter is 8 cm deep? Recall that the volume of a right circular cone

is  $V = \frac{1}{3}\pi r^2 h$ .

- A.  $-\frac{2}{\pi} \text{ cm/min}$
  - B. None of these choices
  - C.  **$-\frac{1}{2\pi} \text{ cm/min}$**
  - D.  $-\frac{1}{\pi} \text{ cm/min}$
  - E.  $-\frac{1}{4\pi} \text{ cm/min}$
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10. A closed rectangular crate is required to have volume  $4000 \text{ cm}^3$ . The base is square, and it costs twice as much per square centimeter for the top and bottom as it does for the four sides. Find the dimensions of the crate of least cost.

A. Base side length 10 cm and height 40 cm.

B. None of these choices

**C. Base side length  $10\sqrt[3]{2}$  cm and height  $20\sqrt[3]{2}$  cm.**

D. Base side length 20 cm and height 10 cm.

E. Base side length  $10\sqrt{2}$  cm and height 20 cm.

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11. Evaluate

$$\int_1^8 \frac{3}{4x^{5/3}} dx.$$

A.  $\frac{9}{8}$

**B.  $\frac{27}{32}$**

C.  $\frac{9}{32}$

D. None of these choices

E.  $\frac{3}{8}$

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12. Evaluate

$$\int_0^{\sqrt{\ln 2}} x e^{x^2} dx.$$

A.  $\frac{e^{\sqrt{\ln 2}} - 1}{2}$

**B.  $\frac{1}{2}$**

C. 1

D.  $\frac{\ln 2}{2}$

E. None of these choices

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13. Find

$$\frac{d}{dx} \int_1^{x^2} \frac{\sin t}{t} dt.$$

- A.  $\frac{\sin x}{x}$   
B.  $2 \sin(x^2)$   
C.  $\frac{x^2 \cos x^2 - \sin(x^2)}{x^2}$   
**D. None of these choices**  
E.  $\frac{\sin(x^2)}{x^2}$
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14. Evaluate

$$\int_{\pi/6}^{\pi/2} (x+2) \sin x dx.$$

- A. None of these choices  
B.  $1 + \sqrt{3} + \frac{\pi\sqrt{3}}{12}$   
C.  $\frac{1}{2} + \frac{\pi\sqrt{3}}{12}$   
D.  $\frac{1}{2} - \frac{\pi\sqrt{3}}{12}$   
**E.  $\frac{1}{2} + \sqrt{3} + \frac{\pi\sqrt{3}}{12}$**
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15. Solve the initial value problem

$$\frac{dy}{dt} = \sec^2 t - \sin t, \quad y\left(\frac{\pi}{4}\right) = 2.$$

- A.  $y = \sec t + \cos t + 1 - \frac{\sqrt{2}}{2}$   
**B.  $y = \tan t + \cos t + 1 - \frac{\sqrt{2}}{2}$**   
C.  $y = \tan t - \cos t + 1 + \frac{\sqrt{2}}{2}$   
D. None of these choices  
E.  $y = \tan t + \cos t + \frac{\sqrt{2}}{2}$
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