CMU Mathematical Sciences
21-120 Sample Prerequisite Waiver Exam #2

Instructions:

- **On the Answer Sheet, clearly print your name in the Name field and your Andrew ID in the ID field. Darken the appropriate circle in the Version field (see the header above).**
- **Do not open the exam until told to do so.**
- **Read all problem statements carefully.**
- **No books, notes, calculators, or other electronic devices, including phones, earbuds/headphones, or tehcnology-enabled eyewear. Make sure your phone will not ring during the exam. Any use of technology is considered an Academic Integrity Violation.**
- **If you would like to leave the exam room before turning in your exam,**
  1. **bring your Exam Booklet and Answer Sheet to the front of the classroom and leave them on the table.**
  2. **demonstrate to the exam proctor that you are not taking any technology with you as you leave the room.**
- **The back side of each page, and the last page, can be used for scratch work. Do not put any scratch work on the Answer Sheet.**
- **If you need any additional paper for scratch work during the exam, please raise your hand.**
- **Time limit: 90 minutes.**
- **You may leave when you are finished if you finish before time is up.**
- **You must turn in ALL materials when you leave, including any scratch paper you were given.**
- **You must show your CMU ID card when you turn in your exam, Answer Sheet, and scratch paper.**
- **You must answer 10 of 15 questions correctly to earn the prerequisite waiver for 21-120.**

For each item, select the appropriate choice and darken the appropriate circle on your answer sheet completely. "None Of These" should be selected when none of the given choices is the correct answer.

Note: all angles are given in radian measure.

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1. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x\to 2^-} \frac{x^2-9}{x^2-5x+6}.$$

A. DNE
B. 0
C. ∞
D. None of these choices
E. $-\infty$

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2. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x\to -\infty} \frac{5x^3-3x+1}{\sqrt{4x^6+x^2}}.$$

A. None of these choices
B. $\frac{5}{2}$
C. $-\infty$
D. 0
E. $-\frac{5}{2}$

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3. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x\to 0} \frac{\sin(3x)}{8x\cos(7x)\sec(2x)}.$$

A. $\frac{3}{8}$
B. None of these choices
C. $\frac{3}{112}$
D. $\frac{3}{28}$
E. 0

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4. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x\to 0^+}(1-e^x)\ln x.$$

A. None of these choices
B. ∞
C. 0
D. DNE
E. $-\infty$

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5. Find $f'(x)$ if $f(x)=4x^{2/3}\cos^{-1}(3x)$.

A. $\dfrac{8\cos^{-1}(3x)}{3x^{1/3}}-\dfrac{12x^{2/3}}{\sqrt{1-9x^2}}$
B. None of these choices
C. $\dfrac{8\cos^{-1}(3x)}{3x^{1/3}}-\dfrac{12x^{2/3}}{1+9x^2}$
D. $\dfrac{8\cos^{-1}(3x)}{3x^{1/3}}-\dfrac{4x^{2/3}}{\sqrt{1-9x^2}}$
E. $\dfrac{8\cos^{-1}(3x)}{3x^{1/3}}-\dfrac{4x^{2/3}}{|x|\sqrt{1-9x^2}}$

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6. Find $f'(x)$ if

$$f(x)=\log_2\left(\frac{x^2+1}{\sqrt{x-1}}\right).$$

A. $\ln 2\left(\dfrac{2x}{x^2+1}-\dfrac{1}{2(x-1)}\right)$
B. $\dfrac{1}{\ln 2}\left(\dfrac{2x}{x^2+1}-\dfrac{1}{2(x-1)}\right)$
C. $\dfrac{2x}{x^2+1}-\dfrac{1}{2(x-1)}$
D. $\dfrac{1}{\ln 2}\left(\dfrac{2x}{x^2+1}+\dfrac{1}{2(x-1)}\right)$
E. None of these choices

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7. Find an equation of the line tangent to the curve

$$x^2y + 3xy^2 = 14$$

at the point $(-7, 2)$.

A. $y - 2 = -\dfrac{16}{35}(x+7)$

B. $y - 2 = \dfrac{16}{7}(x+7)$

C. $y - 2 = -\dfrac{35}{16}(x+7)$

D. $y - 2 = -\dfrac{16}{7}(x+7)$

E. None of these choices

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8. Find the absolute minimum value of

$$f(x) = x^3 - 3x^2 - 9x + 4$$

on the interval $[-2, 4]$.

A. 2

B. -16

C. None of these choices

D. 9

E. -25

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9. Suppose that a liquid is filtered through a filter in the shape of an inverted right circular cone that is 16 cm high and has a radius of 4 cm at the top. Suppose also that the liquid is draining from the filter at a constant rate of 2 cm^3/min. At what rate is the depth of the liquid changing at the instant when the liquid in the filter is 8 cm deep? Recall that the volume of a right circular cone is $V = \dfrac{1}{3}\pi r^2 h$.

A. $-\dfrac{2}{\pi}$ cm/min

B. None of these choices

C. $-\dfrac{1}{2\pi}$ cm/min

D. $-\dfrac{1}{\pi}$ cm/min

E. $-\dfrac{1}{4\pi}$ cm/min

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10. A closed rectangular crate is required to have volume 4000 cm^3. The base is square, and it costs twice as much per square centimeter for the top and bottom as it does for the four sides. Find the dimensions of the crate of least cost.

    A. Base side length 10 cm and height 40 cm.

    B. None of these choices

    C. Base side length $10\sqrt[3]{2}$ cm and height $20\sqrt[3]{2}$ cm.

    D. Base side length 20 cm and height 10 cm.

    E. Base side length $10\sqrt{2}$ cm and height 20 cm.

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11. Evaluate

$$\int_1^8 \frac{3}{4x^{5/3}}\,dx.$$

    A. $\frac{9}{8}$

    B. $\frac{27}{32}$

    C. $\frac{9}{32}$

    D. None of these choices

    E. $\frac{3}{8}$

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12. Evaluate

$$\int_0^{\sqrt{\ln 2}} xe^{x^2}\,dx.$$

    A. $\frac{e^{\sqrt{\ln 2}}-1}{2}$

    B. $\frac{1}{2}$

    C. 1

    D. $\frac{\ln 2}{2}$

    E. None of these choices

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13. Find

$$\frac{d}{dx}\int_1^{x^2} \frac{\sin t}{t}\,dt.$$

A. $\dfrac{\sin x}{x}$

B. $2\sin(x^2)$

C. $\dfrac{x^2\cos x^2 - \sin(x^2)}{x^2}$

D. None of these choices

E. $\dfrac{\sin(x^2)}{x^2}$

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14. Evaluate

$$\int_{\pi/6}^{\pi/2} (x+2)\sin x\,dx.$$

A. None of these choices

B. $1+\sqrt{3}+\dfrac{\pi\sqrt{3}}{12}$

C. $\dfrac{1}{2}+\dfrac{\pi\sqrt{3}}{12}$

D. $\dfrac{1}{2}-\dfrac{\pi\sqrt{3}}{12}$

E. $\dfrac{1}{2}+\sqrt{3}+\dfrac{\pi\sqrt{3}}{12}$

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15. Solve the initial value problem

$$\frac{dy}{dt} = \sec^2 t - \sin t, \qquad y\left(\frac{\pi}{4}\right) = 2.$$

A. $y = \sec t + \cos t + 1 - \dfrac{\sqrt{2}}{2}$

B. $y = \tan t + \cos t + 1 - \dfrac{\sqrt{2}}{2}$

C. $y = \tan t - \cos t + 1 + \dfrac{\sqrt{2}}{2}$

D. None of these choices

E. $y = \tan t + \cos t + \dfrac{\sqrt{2}}{2}$

SCRATCH PAPER: You may detach this page from the exam, but this and all other scratch paper you use must be turned in when you leave. Raise your hand if you need any additional scratch paper during the exam.