# CMU Mathematical Sciences
# 21-120 Sample Prerequisite Waiver Exam #1
# Answer Key

1. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x\to 2^-} \frac{x^2-3x+2}{x^3-4x}.$$

A. 0

B. $-\infty$

C. $-\frac{1}{8}$

***D.*** $\frac{1}{8}$

E. None of these choices

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2. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x\to 4} \frac{4x-x^2}{2-\sqrt{x}}.$$

A. $-\frac{1}{16}$

***B.*** 16

C. None of these choices

D. $-16$

E. 0

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3. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x\to\infty} (\ln x)^{3/x}.$$

A. $\ln 3$

***B. None of these choices***

C. 3

D. $\infty$

E. 0

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4. Find the following limit, or state that the limit does not exist (DNE):

$$\lim_{x\to 0}\frac{x^2e^x}{\sin^2(3x)}.$$

A. 0

***B.*** $\frac{1}{9}$

C. $\infty$

D. None of these choices

E. $\frac{1}{3}$

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5. Find $f'(x)$ if $f(x)=x^{4/5}\left(\frac{4}{5}\right)^x$.

A. $x^{4/5}\left(\frac{4}{5}\right)^x\ln\left(\frac{5}{4}\right)+\frac{4}{5}x^{-1/5}\left(\frac{4}{5}\right)^x$

***B.*** $x^{4/5}\left(\frac{4}{5}\right)^x\ln\left(\frac{4}{5}\right)+\frac{4}{5}x^{-1/5}\left(\frac{4}{5}\right)^x$

C. None of these choices

D. $x^{4/5}\left(\frac{4}{5}\right)^x\ln\left(\frac{4}{5}\right)$

E. $\frac{4}{5}x^{-1/5}\left(\frac{4}{5}\right)^x$

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6. Find $f''(x)$ if $f(x)=\cot(2x^2+1)$.

***A.*** $-4\csc^2(2x^2+1)+32x^2\csc^2(2x^2+1)\cot(2x^2+1)$

B. $4x^2\csc^2(2x^2+1)\cot(2x^2+1)$

C. $-4\csc^2(2x^2+1)+16x^2\csc^2(2x^2+1)\cot(2x^2+1)$

D. None of these choices

E. $8x\csc^2(2x^2+1)$

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7. Find an equation of the line tangent to the curve

$$x^3+y^2=3xy$$

at the point $(2,2)$.

A. $y-2=2(x-2)$

B. None of these choices

C. $y-2=-2(x-2)$

D. $y-2=-3(x-2)$

***E.*** $y-2=3(x-2)$

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8. Find the absolute minimum value of

$$f(x)=\frac{2-\cos x}{\sin x}$$

on the interval $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$.

***A.*** $\sqrt{3}$

B. None of these choices

C. $-\sqrt{3}$

D. $\dfrac{1-\sqrt{3}}{2}$

E. $\sqrt{2}$

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9. A rocket, rising vertically, is tracked by a radar station that is on the ground 4 miles from the launchpad. How fast is the rocket rising when it is 3 miles high and its distance from the radar station is increasing at a rate of 1800 miles per hour?

A. 2000 miles per hour

B. 2400 miles per hour

C. 1800 miles per hour

D. None of these choices

***E.*** 3000 ***miles per hour***

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10. A rectangular plot of land is to be fenced using two kinds of fencing. Two opposite sides use heavy-duty fencing at \$3 per foot, and the remaining sides use standard fencing at \$2 per foot. What is the maximum area that can be fenced if the budget is \$6000?

A. 200,000 $\text{ft}^2$

B. 250,000 $\text{ft}^2$

***C. None of these choices***

D. 300,000 $\text{ft}^2$

E. 400,000 $\text{ft}^2$

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11. Evaluate $\displaystyle\int_4^9 5x^{-3/2}\,dx$.

A. $\dfrac{5}{18}$

B. $\dfrac{5}{2}\left(9^{-1/2}-4^{-1/2}\right)$

C. None of these choices

D. $\dfrac{5}{3}\left(9^{-3/2}-4^{-3/2}\right)$

***E.*** $\dfrac{5}{3}$

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12. Find $\dfrac{d}{dx}\displaystyle\int_{\tan x}^3 \frac{t^2}{1+t^2}\,dt$.

A. $-\dfrac{\tan^2 x}{1+\tan^2 x}$

B. $\dfrac{\tan^2 x}{1+\tan^2 x}$

C. None of these choices

***D.*** $-\dfrac{\tan^2 x}{1+\tan^2 x}\sec^2 x$

E. $\dfrac{\tan^2 x}{1+\tan^2 x}\sec^2 x$

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13. Let $F(x)=\displaystyle\int_0^x \frac{\sin t}{t^2+1}\,dt$. Find $F''(\pi)$.

A. $-\dfrac{1}{(\pi^2+1)^2}$

***B.*** $-\dfrac{1}{\pi^2+1}$

C. $\dfrac{1}{\pi^2+1}$

D. None of these choices

E. 0

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14. Evaluate $\displaystyle\int_{\sqrt{e}}^{e} \frac{\ln x}{x^2}\,dx$.

***A.*** $-\dfrac{2}{e}+\dfrac{3}{2\sqrt{e}}$

B. $\dfrac{1}{e}-\dfrac{1}{\sqrt{e}}$

C. $\dfrac{2}{e}-\dfrac{3}{2\sqrt{e}}$

D. None of these choices

E. $-\dfrac{1}{e}+\dfrac{1}{\sqrt{e}}$

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15. Evaluate $\displaystyle\int_{0}^{\pi} x(1+\cos x)\,dx$.

***A.*** $\dfrac{\pi^2}{2}-2$

B. $\pi$

C. 0

D. None of these choices

E. $\dfrac{\pi^2}{4}+\pi$

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