CMU Mathematical Sciences
21-090 Sample Prerequisite Waiver Exam #2
Answer Key

1. Which of the following expressions is equal to

$$\frac{x^2-16}{x^2-2x-8}+\frac{x-1}{x-4}?$$

A. $\dfrac{2x^2-x-14}{(x-4)(x+2)}$

B. $\dfrac{2x^2-x+18}{(x-4)(x+2)}$

C. None of these choices

D. $\dfrac{2x^2+x-18}{x-4}$

E. $\dfrac{x^2+x-18}{(x-4)(x+2)}$

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2. Describe all real numbers, in interval notation, that satisfy the inequality

$$|4x^2-3|\geq 9.$$

A. $\left(-\infty,-\sqrt{\frac{3}{2}}\right]\cup\left[\sqrt{\frac{3}{2}},\infty\right)$

B. None of these choices

C. $(-\infty,-\sqrt{3}]\cup[\sqrt{3},\infty)$

D. $\left(-\infty,-\frac{3}{2}\right]\cup\left[\frac{3}{2},\infty\right)$

E. $[-\sqrt{3},\sqrt{3}]$

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3. Find the center and radius of the circle with equation

$$x^2+8x+y^2-6y+9=0.$$

A. Center: $(4,-3)$ Radius: 4

B. Center: $(-4,3)$ Radius: 16

C. None of these choices

D. Center: $(-4,3)$ ***Radius:*** 4

E. Center: $(4,-3)$ Radius: 16

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4. Describe all real numbers, in interval notation, that satisfy

$$\frac{x+2}{x^2-4x-5} > 0.$$

A. $(-\infty,-2)\cup(-1,5)$
B. $(-\infty,-1)\cup(5,\infty)$
C. $[-2,-1)\cup(5,\infty)$
D. $(-2,-1)\cup(5,\infty)$
E. None of these choices

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5. Give the domain of the function

$$f(x)=\frac{\sqrt{2^x-1}}{\ln(x^2-1)}.$$

A. $(-\infty,-1)\cup(1,\infty)$
B. $(1,\infty)$
C. $(1,\sqrt{2})\cup(\sqrt{2},\infty)$
D. $[0,\infty)$
E. None of these choices

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6. Find all real numbers x that satisfy

$$\ln(2x-3)-\ln(x-2)=\ln x.$$

A. There are no real solutions.
B. $x=1$
C. $x=3$
D. $x=1$ and $x=3$
E. None of these choices

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7. How many distinct real roots does the polynomial

$$p(x)=x(x^2-9)^2$$

have?

A. None of these choices
B. 2
C. 1
D. 3
E. 5

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8. Find the inverse function $f^{-1}(x)$ if

$$f(x)=\sqrt{\frac{3x+5}{x-2}}.$$

A. None of these choices

B. $f^{-1}(x)=\dfrac{2x+5}{x-3}$

C. $f^{-1}(x)=\dfrac{3x^2+5}{x^2-2}$

D. $f^{-1}(x)=\dfrac{2x^2+5}{x^2-3}$

E. $f^{-1}(x)=\sqrt{\dfrac{2x+5}{x-3}}$

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9. Simplify the expression

$$\frac{6x^4+10x^3+13x^2-5x}{2x^2-1}$$

using long division of polynomials.

A. $3x^2+5x+5+\dfrac{8}{2x^2-1}$

B. None of these choices

C. $3x^2+5x+8-\dfrac{8}{2x^2-1}$

D. $3x^2+5x+8+\dfrac{8}{2x^2-1}$

E. $3x^2+5x+8$

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10. Simplify

$$\log_4(64)-\log_2(32)+\log_8(4).$$

A. $-\dfrac{4}{3}$

B. $\dfrac{4}{3}$

C. $\dfrac{2}{3}$

D. None of these choices

E. -2

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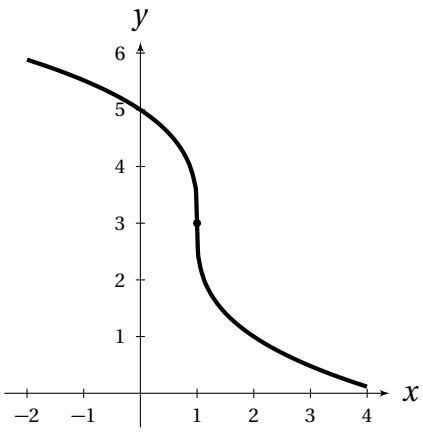
11. Which of the following functions is represented by the graph below?



A. $y=-2\sqrt[3]{x+1}+3$

B. $y=-2\sqrt[3]{x-1}-3$

C. $y=-2\sqrt[3]{x-1}+3$

D. None of these choices

E. $y=2\sqrt[3]{x-1}+3$

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12. Solve the equation, writing your answer using only natural logarithms:

$$5^{x-2}=3e^{2x}.$$

A. None of these choices

B. $x=\dfrac{\ln 3+2\ln 5}{\ln 5-2}$

C. $x=\dfrac{\ln 3+2\ln 5}{2-\ln 5}$

D. $x=\dfrac{\ln 3+\ln 5}{\ln 5-2}$

E. $x=\dfrac{\ln 3-2\ln 5}{\ln 5-2}$

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13. Which of the following expressions is equal to

$$\frac{3^{n+2}-3^n}{3^{n-1}}?$$

A. 8

B. $8\cdot 3^n$

C. 24

D. 18

E. None of these choices

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14. Find all real zeros of

$$x^3+9x^2+24x+20.$$

A. $x=-5$ ***and*** $x=-2$

B. $x=2$ and $x=5$

C. $x=-5$, $x=-2$, and $x=2$

D. $x=-4$ and $x=-5$

E. None of these choices

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15. Write the expression as the logarithm of a single quantity:

$$2\ln(x+1)-\frac{1}{2}\ln x+\ln 3.$$

A. $\ln\left(3(x+1)^2\sqrt{x}\right)$

B. None of these choices

C. $\ln\left(\dfrac{3(x+1)^2}{\sqrt{x}}\right)$

D. $\ln\left(\dfrac{3(x+1)}{\sqrt{x}}\right)$

E. $\ln\left(\dfrac{(x+1)^2}{3\sqrt{x}}\right)$

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16. Evaluate

$$\sin\left(\frac{53\pi}{6}\right).$$

A. $\frac{\sqrt{3}}{2}$

B. $-\frac{1}{2}$

C. None of these choices

D. $\frac{1}{2}$

E. $-\frac{\sqrt{3}}{2}$

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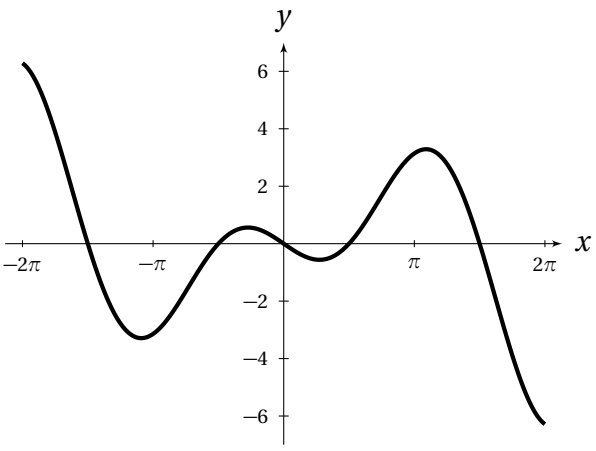
17. Which of the following functions is represented by the graph below?



A. $y = x\cos(\pi - x)$

B. None of these choices

C. $y = \cos(\pi - x)$

D. $y = -x\sin x$

E. $y = x\cos x$

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18. Evaluate

$$\cos^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right).$$

A. $\frac{5\pi}{6}$
B. $\frac{\pi}{6}$
C. $\frac{2\pi}{3}$
D. None of these choices
E. $\frac{\pi}{3}$

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19. Evaluate

$$\sec\left(\tan^{-1}\left(\frac{7}{4}\right)\right).$$

A. None of these choices
B. $\frac{7}{\sqrt{65}}$
C. $\frac{4}{\sqrt{65}}$
D. $\frac{\sqrt{65}}{4}$
E. $\frac{\sqrt{65}}{7}$

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20. Find an algebraic expression equivalent to

$$\csc(\arctan(x+1)).$$

A. $\frac{\sqrt{(x+1)^2+1}}{x+1}$
B. $\frac{\sqrt{(x+1)^2+1}}{1}$
C. $\frac{x+1}{\sqrt{(x+1)^2+1}}$
D. None of these choices
E. $\frac{\sqrt{x^2+1}}{x+1}$

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21. Find all solutions in the interval $[0, 2\pi)$ to the equation

$$2\sin^2 x - \sin x = 0.$$

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}$
B. None of these choices
C. $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$
D. $x = 0, \pi$
E. $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

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22. Describe all real solutions to the equation

$$\tan^2(2x) - 3 = 0.$$

A. $x = \frac{\pi}{3} + \frac{k\pi}{2}$ or $x = -\frac{\pi}{3} + \frac{k\pi}{2}$, where k is any integer.
B. None of these choices
C. $x = \frac{\pi}{6} + k\pi$ or $x = -\frac{\pi}{6} + k\pi$, where k is any integer.
D. $x = \frac{\pi}{12} + \frac{k\pi}{2}$ or $x = -\frac{\pi}{12} + \frac{k\pi}{2}$, where k is any integer.
E. $x = \frac{\pi}{6} + \frac{k\pi}{2}$ ***or*** $x = -\frac{\pi}{6} + \frac{k\pi}{2}$***, where*** k ***is any integer.***

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23. Simplify the expression

$$\cos^2 x \tan x + \sin x \cos x.$$

A. $\sin(2x)$
B. $\cos x$
C. None of these choices
D. $\sin^2 x$
E. $\sin x$
