CMU Mathematical Sciences
21-090 Sample Prerequisite Waiver Exam #2

Instructions:

- **On the Answer Sheet, clearly print your name in the Name field and your Andrew ID in the ID field. Darken the appropriate circle in the Version field (see the header above).**
- **Do not open the exam until told to do so.**
- **Read all problem statements carefully.**
- **No books, notes, calculators, or other electronic devices, including phones, earbuds/headphones, or tehcnology-enabled eyewear. Make sure your phone will not ring during the exam. Any use of technology is considered an Academic Integrity Violation.**
- **If you would like to leave the exam room before turning in your exam,**
  1. **bring your Exam Booklet and Answer Sheet to the front of the classroom and leave them on the table.**
  2. **demonstrate to the exam proctor that you are not taking any technology with you as you leave the room.**
- **The back side of each page, and the last page, can be used for scratch work. Do not put any scratch work on the Answer Sheet.**
- **If you need any additional paper for scratch work during the exam, please raise your hand.**
- **Time limit: 90 minutes.**
- **You may leave when you are finished if you finish before time is up.**
- **You must turn in ALL materials when you leave, including any scratch paper you were given.**
- **You must show your CMU ID card when you turn in your exam, Answer Sheet, and scratch paper.**
- **You must answer 16 of 23 questions correctly to earn the prerequisite waiver for 21-090.**

For each item, select the appropriate choice and darken the appropriate circle on your answer sheet completely. "None Of These" should be selected when none of the given choices is the correct answer.

Note: all angles are given in radian measure.

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1. Which of the following expressions is equal to

$$\frac{x^2-16}{x^2-2x-8}+\frac{x-1}{x-4}?$$

A. $\dfrac{2x^2-x-14}{(x-4)(x+2)}$

B. $\dfrac{2x^2-x+18}{(x-4)(x+2)}$

C. None of these choices

D. $\dfrac{2x^2+x-18}{x-4}$

E. $\dfrac{x^2+x-18}{(x-4)(x+2)}$

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2. Describe all real numbers, in interval notation, that satisfy the inequality

$$|4x^2-3|\geq 9.$$

A. $\left(-\infty,-\sqrt{\frac{3}{2}}\right]\cup\left[\sqrt{\frac{3}{2}},\infty\right)$

B. None of these choices

C. $(-\infty,-\sqrt{3}]\cup[\sqrt{3},\infty)$

D. $\left(-\infty,-\frac{3}{2}\right]\cup\left[\frac{3}{2},\infty\right)$

E. $[-\sqrt{3},\sqrt{3}]$

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3. Find the center and radius of the circle with equation

$$x^2+8x+y^2-6y+9=0.$$

A. Center: $(4,-3)$ Radius: 4

B. Center: $(-4,3)$ Radius: 16

C. None of these choices

D. Center: $(-4,3)$ Radius: 4

E. Center: $(4,-3)$ Radius: 16

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4. Describe all real numbers, in interval notation, that satisfy

$$\frac{x+2}{x^2-4x-5} > 0.$$

A. $(-\infty,-2)\cup(-1,5)$
B. $(-\infty,-1)\cup(5,\infty)$
C. $[-2,-1)\cup(5,\infty)$
D. $(-2,-1)\cup(5,\infty)$
E. None of these choices

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5. Give the domain of the function

$$f(x)=\frac{\sqrt{2^x-1}}{\ln(x^2-1)}.$$

A. $(-\infty,-1)\cup(1,\infty)$
B. $(1,\infty)$
C. $(1,\sqrt{2})\cup(\sqrt{2},\infty)$
D. $[0,\infty)$
E. None of these choices

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6. Find all real numbers x that satisfy

$$\ln(2x-3)-\ln(x-2)=\ln x.$$

A. There are no real solutions.
B. $x=1$
C. $x=3$
D. $x=1$ and $x=3$
E. None of these choices

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7. How many distinct real roots does the polynomial

$$p(x)=x(x^2-9)^2$$

have?

A. None of these choices
B. 2
C. 1
D. 3
E. 5

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8. Find the inverse function $f^{-1}(x)$ if

$$f(x)=\sqrt{\frac{3x+5}{x-2}}.$$

A. None of these choices
B. $f^{-1}(x)=\dfrac{2x+5}{x-3}$
C. $f^{-1}(x)=\dfrac{3x^2+5}{x^2-2}$
D. $f^{-1}(x)=\dfrac{2x^2+5}{x^2-3}$
E. $f^{-1}(x)=\sqrt{\dfrac{2x+5}{x-3}}$

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9. Simplify the expression

$$\frac{6x^4+10x^3+13x^2-5x}{2x^2-1}$$

using long division of polynomials.

A. $3x^2+5x+5+\dfrac{8}{2x^2-1}$
B. None of these choices
C. $3x^2+5x+8-\dfrac{8}{2x^2-1}$
D. $3x^2+5x+8+\dfrac{8}{2x^2-1}$
E. $3x^2+5x+8$

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10. Simplify

$$\log_4(64)-\log_2(32)+\log_8(4).$$

A. $-\dfrac{4}{3}$
B. $\dfrac{4}{3}$
C. $\dfrac{2}{3}$
D. None of these choices
E. -2

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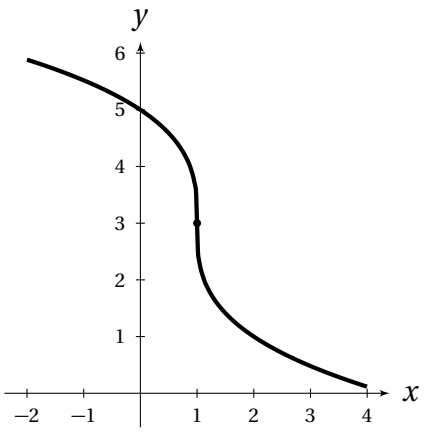
11. Which of the following functions is represented by the graph below?



A. $y=-2\sqrt[3]{x+1}+3$
B. $y=-2\sqrt[3]{x-1}-3$
C. $y=-2\sqrt[3]{x-1}+3$
D. None of these choices
E. $y=2\sqrt[3]{x-1}+3$

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12. Solve the equation, writing your answer using only natural logarithms:

$$5^{x-2}=3e^{2x}.$$

A. None of these choices
B. $x=\dfrac{\ln 3+2\ln 5}{\ln 5-2}$
C. $x=\dfrac{\ln 3+2\ln 5}{2-\ln 5}$
D. $x=\dfrac{\ln 3+\ln 5}{\ln 5-2}$
E. $x=\dfrac{\ln 3-2\ln 5}{\ln 5-2}$

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13. Which of the following expressions is equal to

$$\frac{3^{n+2}-3^n}{3^{n-1}}?$$

A. 8

B. $8\cdot 3^n$

C. 24

D. 18

E. None of these choices

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14. Find all real zeros of

$$x^3+9x^2+24x+20.$$

A. $x=-5$ and $x=-2$

B. $x=2$ and $x=5$

C. $x=-5$, $x=-2$, and $x=2$

D. $x=-4$ and $x=-5$

E. None of these choices

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15. Write the expression as the logarithm of a single quantity:

$$2\ln(x+1)-\frac{1}{2}\ln x+\ln 3.$$

A. $\ln\left(3(x+1)^2\sqrt{x}\right)$

B. None of these choices

C. $\ln\left(\dfrac{3(x+1)^2}{\sqrt{x}}\right)$

D. $\ln\left(\dfrac{3(x+1)}{\sqrt{x}}\right)$

E. $\ln\left(\dfrac{(x+1)^2}{3\sqrt{x}}\right)$

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16. Evaluate

$$\sin\left(\frac{53\pi}{6}\right).$$

A. $\dfrac{\sqrt{3}}{2}$

B. $-\dfrac{1}{2}$

C. None of these choices

D. $\dfrac{1}{2}$

E. $-\dfrac{\sqrt{3}}{2}$

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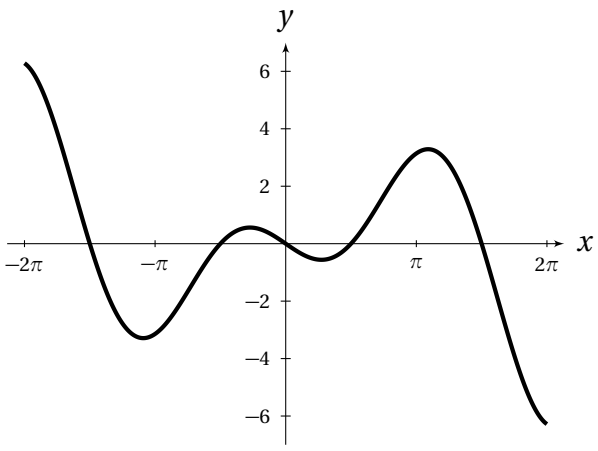
17. Which of the following functions is represented by the graph below?



A. $y = x\cos(\pi - x)$

B. None of these choices

C. $y = \cos(\pi - x)$

D. $y = -x\sin x$

E. $y = x\cos x$

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18. Evaluate

$$\cos^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right).$$

A. $\frac{5\pi}{6}$
B. $\frac{\pi}{6}$
C. $\frac{2\pi}{3}$
D. None of these choices
E. $\frac{\pi}{3}$

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19. Evaluate

$$\sec\left(\tan^{-1}\left(\frac{7}{4}\right)\right).$$

A. None of these choices
B. $\frac{7}{\sqrt{65}}$
C. $\frac{4}{\sqrt{65}}$
D. $\frac{\sqrt{65}}{4}$
E. $\frac{\sqrt{65}}{7}$

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20. Find an algebraic expression equivalent to

$$\csc(\arctan(x+1)).$$

A. $\frac{\sqrt{(x+1)^2+1}}{x+1}$
B. $\frac{\sqrt{(x+1)^2+1}}{1}$
C. $\frac{x+1}{\sqrt{(x+1)^2+1}}$
D. None of these choices
E. $\frac{\sqrt{x^2+1}}{x+1}$

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21. Find all solutions in the interval $[0, 2\pi)$ to the equation

$$2\sin^2 x - \sin x = 0.$$

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}$
B. None of these choices
C. $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$
D. $x = 0, \pi$
E. $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

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22. Describe all real solutions to the equation

$$\tan^2(2x) - 3 = 0.$$

A. $x = \frac{\pi}{3} + \frac{k\pi}{2}$ or $x = -\frac{\pi}{3} + \frac{k\pi}{2}$, where k is any integer.
B. None of these choices
C. $x = \frac{\pi}{6} + k\pi$ or $x = -\frac{\pi}{6} + k\pi$, where k is any integer.
D. $x = \frac{\pi}{12} + \frac{k\pi}{2}$ or $x = -\frac{\pi}{12} + \frac{k\pi}{2}$, where k is any integer.
E. $x = \frac{\pi}{6} + \frac{k\pi}{2}$ or $x = -\frac{\pi}{6} + \frac{k\pi}{2}$, where k is any integer.

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23. Simplify the expression

$$\cos^2 x \tan x + \sin x \cos x.$$

A. $\sin(2x)$
B. $\cos x$
C. None of these choices
D. $\sin^2 x$
E. $\sin x$

SCRATCH PAPER: You may detach this page from the exam, but this and all other scratch paper you use must be turned in when you leave. Raise your hand if you need any additional scratch paper during the exam.