

CMU Mathematical Sciences
21-090 Sample Prerequisite Waiver Exam #1
Answer Key

1. Find the center and the radius of the circle with the equation

$$x^2 - 2x + y^2 + 4y - 11 = 0.$$

- A. None of these choices
 - B. Center: $(-1, 2)$ Radius: 4
 - C. Center: $(1, -2)$ Radius: 4**
 - D. Center: $(1, -2)$ Radius: $2\sqrt{2}$
 - E. Center: $(1, 2)$ Radius: 4
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2. Which of the following expressions is equal to

$$\frac{x^2 - 9}{x^2 + 4x + 3} - \frac{2x + 1}{x + 3} ?$$

- A. $\frac{x^2 - 3x - 10}{(x + 1)(x + 3)}$
 - B. $\frac{-x^2 - 3x + 10}{x^2 + 4x + 3}$
 - C. $\frac{-x^2 - 3x - 10}{(x + 1)(x + 3)}$**
 - D. $\frac{x^2 - 9 - (2x + 1)}{x^2 + 4x + 3}$
 - E. None of these choices
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3. Describe all real numbers, in interval notation, that satisfy the inequality $|2x - 5| < 3$.

- A. None of these choices
 - B. $(-1, 4)$
 - C. $(2, 4)$
 - D. $(1, 3)$
 - E. $(1, 4)$**
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4. Describe all real numbers, in interval notation, that satisfy

$$\frac{x-4}{x^2-3x-10} \geq 0.$$

- A. $(-\infty, -2) \cup (2, 5]$
 - B. None of these choices**
 - C. $(-\infty, -2) \cup (5, \infty)$
 - D. $(-\infty, -2) \cup (4, 5) \cup (5, \infty)$
 - E. $(-\infty, -2) \cup [4, \infty)$
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5. How many distinct real roots does the polynomial

$$p(x) = 2x^4 + 2x^2 - 12$$

have?

- A. None of these choices
 - B. 2**
 - C. 1
 - D. 3
 - E. 4
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6. Give the domain of the function

$$f(x) = \sqrt{5-x} + e^{1/(x-2)}.$$

- A. $(2, 5]$
 - B. $(-\infty, 5)$
 - C. $(-\infty, 2) \cup (2, 5]$**
 - D. $(-\infty, 5]$
 - E. None of these choices
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7. Find the domain and range of the function

$$f(x) = \log_3(x-2) + 1.$$

- A. Domain: $[2, \infty)$ Range: $(-\infty, \infty)$
 - B. Domain: $(2, \infty)$ Range: $(-\infty, \infty)$**
 - C. None of these choices
 - D. Domain: $(0, \infty)$ Range: $(2, \infty)$
 - E. Domain: $(2, \infty)$ Range: $(0, \infty)$
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8. Find the inverse function $f^{-1}(x)$ if

$$f(x) = \frac{5x+2}{x-1}.$$

A. $f^{-1}(x) = \frac{x-2}{x-5}$

B. $f^{-1}(x) = \frac{x+2}{5-x}$

C. $f^{-1}(x) = \frac{x+2}{x-5}$

D. None of these choices

E. $f^{-1}(x) = \frac{5x+2}{x-1}$

9. Find all points of intersection of the line $y = 3x - 2$ and the parabola $y = x^2 - 4x + 6$.

A. $(1, -1)$ and $(8, 22)$

B. $(1, 1)$ and $(3, 7)$

C. The curves do not intersect

D. $(1, 1)$ and $(8, 22)$

E. None of these choices

10. Simplify

$$\log_5(25) + \log_{25}(5) - \log_5(125).$$

A. None of these choices

B. -1

C. $-\frac{1}{2}$

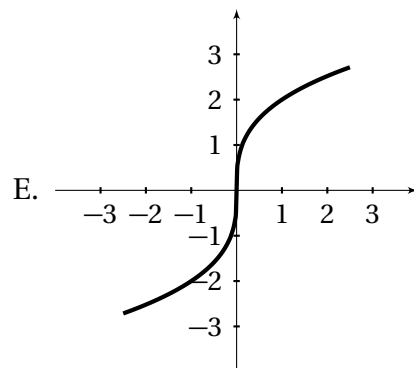
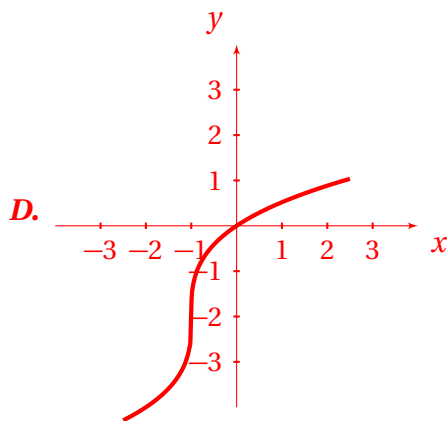
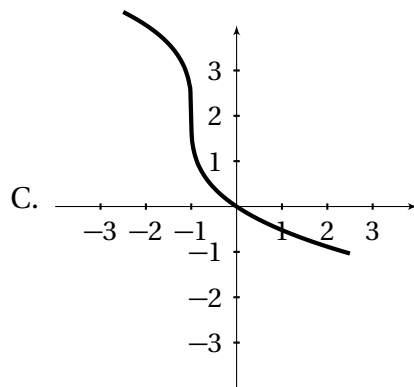
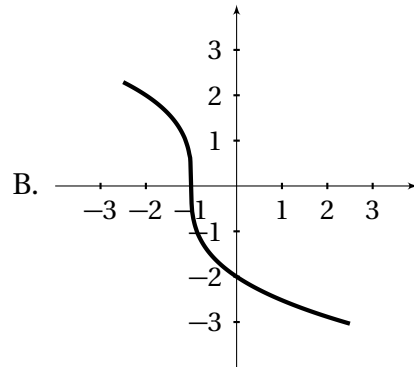
D. $\frac{1}{2}$

E. 1

11. Which of the following graphs represent the function

$$y = 2(x + 1)^{1/3} - 2?$$

A. None of these choices



12. Solve the equation, writing your answer using only natural logarithms:

$$3^{2x-1} = e^{x+2}.$$

A. $x = \frac{2 + \ln 3}{2 \ln 3 - 1}$

B. $x = \frac{\ln 3 - 2}{2 \ln 3 - 1}$

C. None of these choices

D. $x = \frac{2 - \ln 3}{2 \ln 3 - 1}$

E. $x = \frac{2 + \ln 3}{\ln 3 - 2}$

13. Which of the following expressions is equal to

$$\frac{4^{n+1} - 2^{2n}}{2^{n+1}}?$$

A. 2^{n-1}

B. $3 \cdot 2^n$

C. $3 \cdot 2^{n-1}$

D. None of these choices

E. 4^{n-1}

14. For the function

$$f(x) = \frac{x^2 - 2x + 3}{x^2 - 1},$$

describe all real values of x for which $f(x) = 0$.

A. $x = 1$

B. $x = -1$

C. ***There are no real solutions.***

D. None of these choices

E. $x = \pm\sqrt{3}$

15. Write the expression as the logarithm of a single quantity:

$$\frac{1}{2}(\ln 2x - 2 \ln x) + 3 \ln x.$$

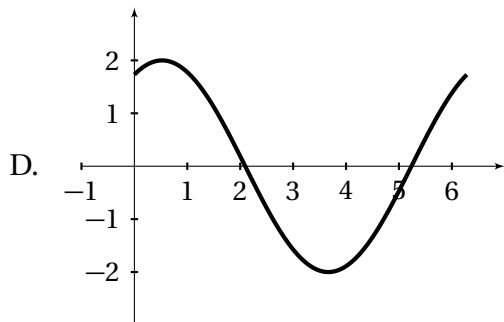
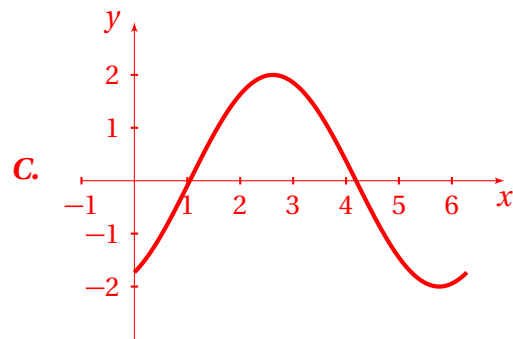
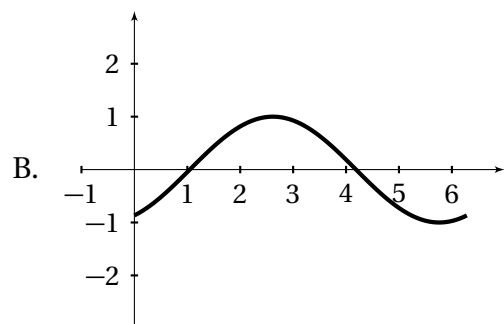
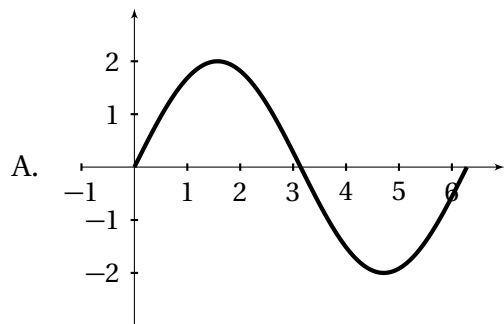
- A. $\ln(\sqrt{2} x^{5/2})$
 - B. $\ln(\sqrt{2} x^3)$
 - C. $\ln(\sqrt{2} x^{1/2})$
 - D. None of these choices
 - E. $\ln(2x^{5/2})$
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16. Evaluate $\tan\left(\frac{47\pi}{6}\right)$.

- A. $\sqrt{3}$
 - B. $\frac{1}{\sqrt{3}}$
 - C. $-\frac{1}{\sqrt{3}}$
 - D. None of these choices
 - E. $-\sqrt{3}$
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17. Which of the following graphs represent the function

$$y = 2 \sin\left(x - \frac{\pi}{3}\right)?$$



E. None of these choices

18. Evaluate $\sin\left(\cos^{-1}\left(\frac{-1}{2}\right)\right)$.

- A. $\frac{1}{2}$
 - B. $-\frac{1}{2}$
 - C. None of these choices
 - D. $\frac{\sqrt{3}}{2}$**
 - E. $-\frac{\sqrt{3}}{2}$
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19. Evaluate $\cot\left(\sec^{-1}\left(\frac{8}{3}\right)\right)$.

- A. None of these choices
 - B. $\frac{\sqrt{55}}{3}$
 - C. $\frac{8}{\sqrt{55}}$
 - D. $\frac{3}{\sqrt{55}}$**
 - E. $\frac{\sqrt{55}}{8}$
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20. Find an algebraic expression equivalent to $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$.

- A. $\frac{\sqrt{x^2+2}}{\sqrt{2}}$
 - B. $\frac{\sqrt{2}}{\sqrt{x^2+2}}$
 - C. $\frac{\sqrt{x^2+2}}{x}$**
 - D. $\frac{x}{\sqrt{x^2+2}}$
 - E. None of these choices
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21. Find all solutions in the interval $[0, 2\pi)$ to the equation

$$1 + \cos x = \sin x.$$

- A. $x = \frac{\pi}{2}, \frac{3\pi}{2}$.
- B. None of these choices
- C. $x = \pi, \frac{3\pi}{2}$.
- D. $x = \frac{\pi}{2}, \pi$.**
- E. $x = \frac{\pi}{2}$ only.
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22. Describe all real solutions to the equation

$$2 \cos(3x) - 1 = 0.$$

- A. None of these choices
- B. $x = \frac{\pi}{3} + 2k\pi$, where k is any integer.
- C. $x = \frac{\pi}{6} + k\pi$, where k is any integer.
- D. $x = \frac{\pi}{9} + \frac{2k\pi}{3}$ or $x = -\frac{\pi}{9} + \frac{2k\pi}{3}$, where k is any integer.**
- E. $x = \frac{\pi}{9} + k\pi$, where k is any integer.
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23. Simplify the expression

$$\sin^2 x \cot x + \sin^3 x \sec x.$$

- A. None of these choices
- B. $\frac{\sin^2 x}{\cos x}$
- C. $\cos x$
- D. $\tan x$**
- E. $\sin^2 x$
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