CMU Mathematical Sciences
21-259: Calculus in Three Dimensions
Prerequisite Waiver Exam Topic List

Intended use of this document: This document provides a list of objectives that students are expected to know and demonstrate mastery of on the 21-259 Prerequisite Waiver Exam. This is a comprehensive list, and all questions on the exam will be represented by one or a combination of several of these objectives.

Note: the use of any and all technology, including calculators, is prohibited on the exam (without approved disability accommodations).

- Be able to perform vector arithmetic in $\mathbb{R}^2$ and $\mathbb{R}^3$, including dot and cross products, and interpret these geometrically (angles, orthogonality, areas, and volumes).
- Be able to compute vector, scalar, and orthogonal projections.
- Be able to parametrize lines and curves in $\mathbb{R}^3$, convert between parametric and symmetric forms, and determine whether lines are parallel, intersecting, or skew.
- Be able to find equations of planes, compute angles between lines and planes, and compute distances between points, lines, and planes.
- Be able to identify quadric surfaces and cylinders from their equations and describe their cross-sections.
- Be able to compute derivatives and integrals of vector-valued functions (componentwise), including arc length, reparametrization by arc length, curvature, and the unit tangent, normal, and binormal vectors.
- Be able to interpret contour maps, including determining directions of increase and decrease and relating them to gradients and directional derivatives.
- Be able to evaluate limits and determine continuity of functions of several variables, including using algebraic simplification, the squeeze theorem, and path-based arguments.
- Be able to compute partial derivatives (including higher-order derivatives) and apply Clairaut's Theorem.
- Be able to apply the multivariable chain rule and perform implicit differentiation.
- Be able to compute directional derivatives and gradients, and interpret the gradient as the direction of greatest increase and as normal to level sets.
- Be able to find equations of tangent planes to graphs and level surfaces, and use linear approximations.
- Understand differentiability for functions of several variables, including its relationship to linear approximation.

- Be able to find and classify critical points and determine local extrema using the second derivative test.
- Be able to apply the Extreme Value Theorem to find absolute extrema on closed and bounded regions.
- Be able to use Lagrange multipliers to find extrema subject to constraints.
- Be able to compute double integrals over planar regions, including setting up iterated integrals and applying Fubini's Theorem to reverse the order of integration.
- Be able to evaluate double integrals in polar coordinates and perform change of variables using Jacobians.
- Be able to set up and compute triple integrals over solid regions using Cartesian, cylindrical, and spherical coordinates.
- Be able to use symmetry to simplify integrals and compute areas, volumes, mass, and centers of mass.
- Be able to interpret vector fields in two and three dimensions from their graphical representations and match vector fields with their corresponding arrow diagrams.
- Be able to compute divergence and curl, interpret their meanings, and use them to identify conservative and divergence-free vector fields; for conservative vector fields, be able to find potential functions when they exist.
- Be able to compute scalar and vector line integrals and interpret them (e.g., mass, work, circulation), including understanding how orientation affects vector line integrals.
- Be able to apply the Fundamental Theorem of Line Integrals.
- Be able to apply Green's Theorem to relate line integrals to double integrals.
- Be able to compute scalar and vector surface integrals and interpret them (e.g., surface area and flux), including how orientation affects flux.
- Be able to apply Stokes' Theorem and the Divergence Theorem to evaluate integrals by converting between line, surface, and volume integrals.