CMU Mathematical Sciences
21-120: Differential and Integral Calculus
Prerequisite Waiver Exam Topic List

Intended use of this document: This document provides a list of objectives that students are expected to know and demonstrate mastery of on the 21-120 Prerequisite Waiver Exam. This is a comprehensive list, and all questions on the exam will be represented by one or a combination of several of these objectives.

Note: the use of any and all technology, including calculators, is prohibited on the exam (without approved disability accommodations).

- Be able to find one-sided and two-sided limits of functions from their graphs.
- Be able to use various algebraic techniques to calculate one or two-sided limits at a point or at plus/minus infinity. Be able to identify limits by using the Squeeze Theorem.
- Be able to identify where a given function is continuous or discontinuous, and classify any discontinuities.
- Be able to use the Intermediate Value Theorem to show that there exists a solution to a given equation.
- Be able to identify all vertical and horizontal asymptotes of a function.
- Be able to use the limit definition of the derivative to calculate the derivative of a function at a point or as another function.
- Be able to find an equation of the line tangent to a given differentiable function at a point.
- Be able to interpret the quantitative and qualitative meaning of the derivative of a function in terms of the function's behavior (increasing, decreasing, constant, etc.).
- Be able to use basic differentiation rules (power, product, quotient) to compute derivatives of functions such as polynomial functions, rational functions, and root functions.
- Be able to use the facts $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$ to find other limits.
- Be able to compute derivatives of trigonometric, inverse trigonometric, exponential, and logarithmic functions.
- Be able to correctly apply the Chain Rule for derivatives of composed functions.
- Be able to use implicit differentiation to find the derivative of an implicitly-defined function.
- Be able to use the Inverse Function Theorem to find the derivative of an inverse function.
- Be able to use logarithmic differentiation to compute the derivatives of complicated functions, including those of the form $(f(x))^{g(x)}$.
- Be able to set up and solve problems involving related rates of change.

- Be able to construct the linear approximation to a given function at a point and use it to estimate the function's value at a nearby point.
- Be able to find the differential dy given $y = f(x)$ and use it to estimate the error and relative error in calculations based on given tolerances for differentials.
- Be able to find all extreme values, both global and local, of a given function over its domain or a closed interval. Be able to find all critical numbers of a function.
- Be able to apply Rolle's Theorem and the Mean Value Theorem to find values in a given interval where a function's average rate of change over the interval is equal to its instantaneous rate of change. Be able to use the the Mean Value Theorem to answer questions about how much or little a function may change over an interval. Be able to use Rolle's Theorem or the Mean Value Theorem to verify certain aspects of a function's behavior.
- Be able to use derivatives to determine where a given function is increasing, decreasing, concave up, concave down, and the locations of the function's critical and inflection points. Be able to apply the First Derivative Test, the Second Derivative Test, and the Concavity Test.
- Be able to apply L'Hospital's Rule to indeterminate forms of the type $0/0$ or ∞/∞.
- Be able to rewrite expressions yielding the indeterminate forms $0 \cdot \infty$ and $\infty - \infty$ so that L'Hospital's Rule can be applied.
- Be able to rewrite expressions yielding the exponential indeterminate forms 0^0, ∞^0, and 1^∞ so that L'Hospital's Rule can be applied.
- Be able to set up and solve applied optimization problems, including those that require elimination of one or more variables via problem constraints or problems that require the construction of a demand function.
- Be able to find antiderivatives and solve initial value problems.
- Be able to manipulate sigma notation and calculate basic Riemann sums used in approximating definite integrals. Be able to compute approximate areas using left or right endpoint approximations. Be able to find the net area between a curve and the x-axis when a graph of the curve is given and the areas are known.
- Be able to evaluate definite integrals when formulas for the areas are known.
- Be able to apply the properties of definite integrals, including the comparison properties, to various problems involving integrals.
- Be able to apply the first part of the Fundamental Theorem of Calculus to compute derivatives of certain functions defined as integrals.
- Be able to evaluate indefinite and definite integrals using the second part of the Fundamental Theorem of Calculus (i.e., by finding antiderivatives and evaluating them at the limits of integration).
- Be able to use the technique of u-substitution to evaluate indefinite and definite integrals.
- Be able to interpret the quantitative and qualitative meaning of the integral of a function in terms of the function's meaning in applied problems.

- Be able to use the technique of Integration by Parts to evaluate definite and indefinite integrals.