CMU Mathematical Sciences
21-090: Precalculus
Prerequisite Waiver Exam Topic List

Intended use of this document: This document provides a list of objectives that students are expected to know and demonstrate mastery of on the 21-090 Prerequisite Waiver Exam. This is a comprehensive list, and all questions on the exam will be represented by one or a combination of several of these objectives.

Note: the use of any and all technology, including calculators, is prohibited on the exam (without approved disability accommodations).

- Know the definition of the set of real numbers and its representation on the real number line. Be able to describe open, closed, and mixed intervals on the real line using interval notation and set-builder notation. Know the definition of the absolute value function and how it affects the solution sets of inequalities involving real numbers.
- Be able to simplify and expand algebraic expressions involving exponents, roots, and radicals.
- Be able to expand and factor polynomials, including the difference of squares, a perfect square, the sum or difference of two cubes, etc. Be able to use the Binomial Theorem to expand expressions of the form $(a+b)^n$ where n is a positive integer. Be able to factor by grouping and perform long division of polynomials. Be able to complete the square for quadratic and linear terms of a polynomial. Be able to solve real inequalities involving polynomials.
- Be able to simplify, multiply, divide, and combine rational expressions (i.e. ratios of polynomials) using common denominators, factoring and canceling, and other algebraic techniques.
- Be able to find real solutions to polynomial equations and inequalities by factoring, use of the quadratic formula, and other algebraic techniques.
- Know the definition of a function. Be able to identify when a particular curve represents a function. Be able to identify the domain and range of a function from its algebraic description, as sets or intervals of real numbers. Know how to interpret piecewise-defined functions.
- Be able to find and identify the x- and y-intercepts of a function based on its algebraic description or its graph.
- Be able to compose functions algebraically. Understand how function composition affects domains and ranges.
- Know the definition of an inverse function and the conditions required for a function to have an inverse. Be able to find the graph of an inverse functions from the graph of the function, when the inverse exists. Be able to solve for the algebraic description of an inverse function when one exists.
- Know how vertical and horizontal shifts of graphs of functions correspond to changes to the algebraic description of the function. Know how graph contractions and dilations, with respect to both the horizontal and vertical axes, correspond to changes to the algebraic description of the function.

- Be able to identify and calculate reflections of the graph of a function across coordinate axes. Be able to determine if a function is even, odd, or neither both graphically and algebraically.
- Be able to identify and roughly sketch graphs of the following types of functions: constant functions, linear functions, quadratic polynomials, and curves of the form $y = x^n$ where n is a rational number. Be able to find and identify horizontal and vertical asymptotes of functions from their graphs and their algebraic descriptions.
- Be able to evaluate and simplify expressions involving exponential and logarithmic functions for any positive real number base. Know the properties of exponents and logarithms, including how to change the base of an exponential or logarithmic expression. Be able to identify and sketch graphs of exponential and logarithmic functions, including finding asymptotes.
- Be able to describe solution sets of equations and inequalities involving exponential and logarithmic functions.
- Know how to describe an angle and convert between radian and degree measure.
- Know the definition of the six fundamental trigonometric functions (sine, cosine, tangent, secant, cosecant, cotangent), including their domains and ranges, and how the sine and cosine functions are defined via the unit circle. Know how to evaluate any trigonometric function at any angle whose radian measure is an integer multiple of $\pi/4$ or $\pi/6$. Be able to construct a right triangle to evaluate trigonometric functions or find angles, including in simple applications.
- Know the fundamental trigonometric identities (Pythagorean identities, half-angle identities, double-angle identities, and cofunction identities) listed at the end of this document. Be able to describe the odd/even properties of any trigonometric function. Be able to use the fundamental identities to simplify trigonometric expressions and evaluate trigonometric functions.
- Know the definitions of the six fundamental inverse trigonometric functions (arcsine, arccosine, arctangent, arcsecant, arccosecant, arccotangent), including their standard domains and ranges. Be able to evaluate inverse trigonometric functions at any standard input in their domain (standard input means a standard output of a trigonometric function) or by using right triangles. Be able to evaluate and simplify compositions of trig and inverse trig functions.
- Be able to identify and sketch graphs of the six fundamental trigonometric functions and the arcsine, arccosine, and arctangent functions. Know how shifts and scalings affect the shapes of trigonometric and inverse trigonometric graphs.
- Be able to describe solution sets of equations and inequalities involving trigonometric functions, including those that have periodic solutions. Be able to describe solution sets of equations and inequalities involving inverse trigonometric functions.

Fundamental Trigonometric Identities

- **Pythagorean Identities:**
  - $\sin^2(x) + \cos^2(x) = 1$
  - $\tan^2(x) + 1 = \sec^2(x)$
  - $1 + \cot^2(x) = \csc^2(x)$
- **Double Angle Identities:**
  - $\sin(2x) = 2\sin x \cos x$
  - $\cos(2x) = \cos^2 x - \sin^2 x$
  - $\cos(2x) = 1 - 2\sin^2 x$
- **Half Angle Identities:**
  - $\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$
  - $\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$
- **Cofunction Identities:**
  - $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$
  - $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$
  - $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$
  - $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$
  - $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$
  - $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$