

BASIC EXAMINATION
PROBABILITY
FALL 2015

Time allowed: 120 minutes.

1. Obtain the proofs of the 1st and 2nd Borel-Cantelli lemmas from the convergence theorems of sums of independent random variables.
2. Let (Y_n) be IID RVs taking values 1 and -1 with equal probabilities. Will $\sum_n Y_n/2^n$ converge almost surely? If yes, then compute the distribution of the limit?
3. Let (X_n) be independent nonnegative RVs in \mathcal{L}_1 . We know that

$$\lim_{n \rightarrow \infty} \prod_{k=1}^n \mathbb{E}[X_k] = a < \infty.$$

Will the sequence $\prod_{k=1}^n X_k$, $n \geq 1$, converge in distribution?

4. Let (X_n) be a symmetric random walk on integers starting at 0. For an integer $a > 0$ define the hitting time

$$\tau = \min\{n \geq 0 : |X_n| = a\}.$$

Compute $\text{Var}(e^{-\tau}) = \mathbb{E}[(e^{-\tau} - \mathbb{E}[e^{-\tau}])^2]$.

5. A monkey types a capital letter at random, out of 26 possible letters. Compute the expected time of getting the word “HI”.