Department of Mathematical Sciences
Basic Exam in General Topology
January 2022

Instructions: Answer each of the 5 questions.

1. If $\{X_\alpha\}_{\alpha\in J}$ is a collection of non-empty topological spaces and $A_\alpha \subset X_\alpha$ for each $\alpha \in J$, show that $A = \prod A_\alpha$ is dense in $X = \prod X_\alpha$ if and only if A_α is dense in X_α.

2. Prove or disprove the following:

   (a) If $A \subset X$ is path connected then $\bar{A}$ is path connected.

   (b) If X and Y are path connected then the product $X \times Y$ is path connected.

3. (a) If Z is a totally bounded subset of a metric space (X, d) show that $\bar{Z}$ is also totally bounded.

   (b) Show that every compact metric space (X, d) has a countable basis.

4. Construct compactifications of $(0, \infty)$ and $\mathbb{R}$ for which the function $f(x) = \ln(x)$ has a continuous extension.

5. Let X be a compact topological space and (Y, d) be a metric space. If $\mathcal{F}$ is a subset of $C(X, Y)$ which is equicontinuous, show that its closure in the sup-metric is also equicontinuous.