mathematical sciences news 2019

Carnegie Mellon University

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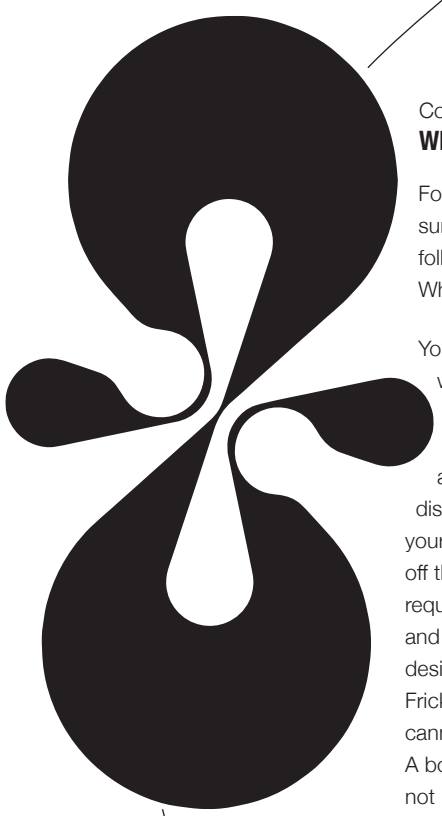


Cover Image

What Can You Draw?

For his research project during the summer of 2019, Fei Peng raised the following deceptively simple question: What can you draw?

Your canvas is the plane $\mathbb{R}^2$ — colored white to begin with — and you are given a pencil, which produces a black unit disk wherever it meets the canvas, and an eraser, which produces a white unit disk. There are no further restrictions on your artistic freedom. You may raise the tool off the canvas, that is, there is no continuity requirement for the centers of disks you draw and you can switch tools as many times as desired. Peng and his research mentor Florian Frick showed the main result is that drawability cannot be characterized by local obstructions. A bounded set can be locally drawable, while not being drawable. The figure presented here is an example of such a set; the boundary has curvature less than one, but the set is not drawable in the large.

Letter from Mathematics Department Head, Tom Bohman

Undergraduate research has become a hallmark of a Carnegie Mellon University education, with students citing that it has helped to prepare them for their futures in academia and the workforce. Mathematics hasn't traditionally been seen as a field where students do meaningful research; over the course of the last few years, the department has made a concerted effort to change that.

In this issue of the Department of Mathematical Sciences Newsletter, we highlight undergraduate research in the department and the programs we have developed to provide research experiences for our students. Our main interest is in summer programs that allow students to focus on a project without the distractions of coursework. With the institution of these programs, the number of students engaged in summer research has grown over the last four years from a handful to more than 60 working on high-quality, open-ended mathematics projects. This is an extraordinary program that very few, if any, other mathematics departments can match.

In addition to exposing students to research, the summer programs, which include the Summer Undergraduate Research Fellowships (SURF) and the Mathematical Finance Summer Undergraduate Research Program (MFSURP), give students the opportunity to be mentored by our renowned faculty. The mathematical finance program, which has grown dramatically in the last few years, is administered by Bill Hrusa. The SURF program involves a large fraction of the faculty, including Florian Frick, who recently joined us as an assistant professor and was awarded a Sloan Research Fellowship this year (see the articles on pages 14 and 16 for more on Professor Frick and his research).

The summer research programs have been made possible through the support of our alumni, including David Simmons, Larry Jennings, David and Jacqui Martin, and Elizabeth and Konstatin Andreev. To learn more about the research our students engage in, see page 28 for details on the 2019 SURF projects and page 10 for an article on the evolution of the summer programs. Additionally, the image on the cover of this issue is drawn from the SURF project of Fei Peng, a junior mentored by Professor Frick.

Of course, there have been many other developments in the department. In this issue you can also read about our new faculty, faculty awards (including Po-Shen Loh's receipt of the Presidential Early Career Award in Science in Engineering) and student awards.

I hope that the many alumni of the Department of Mathematical Sciences have a chance to reconnect with the department by visiting cmu.edu/math

Let us know what's new with you!

Ian Tice Wins Julius Ashkin Award

Assistant Professor Ian Tice was awarded the Mellon College of Science's 2019 Julius Ashkin Teaching Award for his devotion and effectiveness in teaching.

Tice is an inspiring educator, both challenging and guiding his students to become astute mathematicians equipped with inquisitive minds. Tice has crafted hundreds of homework problems and written over 1,300 pages of notes for his courses.

While his homework problems are known for taking hours to complete, by the end, students understand step-by-step how to assemble a proof for a full result; this, Tice says, helps students synthesize mathematical arguments and trains them to think like mathematicians.

Tice knows he demands a lot of his students, but his door is always open. Students will spend hours in his office, be it to get help on a problem, to revisit a concept from lecture that they didn't understand or to talk more about their interests in math.

"Professor Tice is someone who very clearly loves teaching," wrote former student Jose Arrillaga. Arrillaga spoke with Tice about his difficulty understanding some results in convergent series, "He made sure to put an additional exercise on that week's problem set, which helped immensely in clarifying my uncertainties," he said.

Additionally, Tice is a strong supporter of undergraduate math research. His experience as a research advisor inspired him to apply for a National Science Foundation (NSF) CAREER grant. NSF awarded him $38,500 to fund summer research for 11 students over the next five years. One of Tice's favorite activities is organizing and supervising the Department of Mathematical Sciences' poster competition at Carnegie Mellon's annual undergraduate research symposium, Meeting of the Minds.

Beyond teaching, Tice has been a driving force in developing and modernizing parts of the undergraduate and graduate curriculum in the department.

He completely revamped the undergraduate computational and applied mathematics concentration, helped reform the graduate curriculum in analysis and partial differential equations (PDE) and revised courses in the math honors program. He has developed four new courses, restructured the department's mathematical modeling and graduate PDE courses and took on teaching a number of these courses, both new and old.

"Ian is an enthusiastic and devoted teacher who has demonstrated the ability to create excellent courses, inspire students and train some of the best undergraduates," wrote Professor Dejan Slepčev in supporting Tice's nomination.

Giovanni Leoni Wins Richard Moore Award

Professor Giovanni Leoni was honored with the Mellon College of Science's 2019 Richard Moore Award. This award is presented every year to a faculty member in the college who is making substantial and sustained contributions to the educational mission of the college.

Leoni created the department's Undergraduate Curriculum Committee, which has completely transformed and strengthened the undergraduate program over the course of the last decade. Since its inception, Leoni has served as chair of the committee with the goal of providing excellent mathematical education to all Carnegie Mellon University students, regardless of major or college. The committee's main achievements include a complete redesign of the introductory course sequence for math and computer science students and a redesign of the honors course sequence for mathematics majors.

Leoni has worked with faculty across the university to tailor the mathematical sciences program to suit students from all majors. His efforts have also helped grow and strengthen the mathematics major at Carnegie Mellon; each cohort has nearly 100 math majors, which is three times more than the number of students before the new curriculum was implemented.

Another focus of Leoni's has been to equip postdoctoral students with the tools to effectively teach upper-level mathematics courses. In the summer of 2017, he implemented an orientation program for all incoming postdocs to instruct them on how to write syllabi in line with Faculty Senate guidelines, how to conduct student assessments, how to work with teaching assistants and how to use Canvas, the university's learning management system. Leoni also provided constructive feedback for postdocs through "mock" classes.

In addition to his focus on the broader educational and administrative aspects of the department, Leoni cares deeply about his students and dedicates his time to making sure they succeed. He holds recitations for courses that do not have a recitation and conducts research projects with students over the summer.

"Giovanni Leoni regularly steps in and takes charge of administrative details that have a very high impact on the education of students in mathematics courses," wrote Tom Bohman, Alexander M. Knaster Professor and head of the Department of Mathematical Sciences, in his nomination letter for Leoni. "He is a significant part of the great success of our undergraduate program and mathematics."

Po-Shen Loh Wins University's Top Teaching Award

Associate Professor Po-Shen Loh came to Carnegie Mellon in 2010 and quickly became known for his enthusiasm, effectiveness and overall excellence as an instructor. For these reasons, he received Carnegie Mellon University's 2019 William H. and Frances S. Ryan Award for Meritorious Teaching.

Tom Bohman, Alexander M. Knaster Professor and head of the Department of Mathematical Sciences, said Loh's most impressive contribution to teaching at CMU has been his restructuring and expansion of the Putnam Seminar, which prepares undergraduate students to participate in the premier Putnam Mathematical Competition. After Loh began teaching the class, enrollment more than tripled to about 200 students each year.

"He teaches it in the style of an 'inverted' or 'flipped' classroom, with student discussion of the problems and presentation of solutions taking up a part of each class," Bohman wrote in his nominating letter. "Rather than viewing this class as preparation for a competition, Po re-envisioned it as a problem-guided tour of mathematics, which develops core problem-solving skills that enhance the ability to learn and use higher-level mathematics."

Loh holds seminar sessions each day and tailors each class to the students' "presumed knowledge," Bohman said.

Loh's efforts have paid dividends as CMU placed first in the competition in 2016 and for the last seven consecutive years has had the second-most students from schools in the U.S. and Canada place among the top 500 individuals. In 2016, three students earned the Putnam Fellow designation by finishing among the top five individuals.

Bohman also noted in his nomination letter the course Discrete Mathematics, which has earned a "sterling reputation" under Loh's teaching.

"As a department head, I wish that I could assign him to more courses," Bohman said.

Loh also is the lead coach for the U.S. International Mathematical Olympiad team of high school students. In 2015, he brought the team's training program to CMU, where the team prepares for the competition. The U.S. has won four world championships in the last five years.

CMU junior Deborah Blank commended Loh for his collaborative teaching approach.

"Professor Loh would ask the students for ideas or intuitions on how to solve the problem...which fostered an inclusive classroom environment," she said. "He made me feel that I mattered as an individual and a teammate, and instilled in us a hunger to do our best and to be excited about competition math."

Senior Nina Edwards said she has never seen a teaching style quite like Loh's. She praised his dedication to students, creativity in designing problems and his "remarkably engaging and dynamic style of lecturing."

Junior Theodore Li said Loh was a "math evangelist" who makes math fun.

"Dr. Loh shares his enjoyment and passion for math and can rekindle any jaded student's love for mathematics, which he would consider more of a success for his class than our constant top placings," Li said.

Po-Shen Loh Honored with Presidential Early Career Award

Po-Shen Loh has received the prestigious Presidential Early Career Award for Scientists and Engineers (PECASE). The PECASE is the highest honor bestowed by the United States government to outstanding scientists and engineers who are beginning their independent research careers and who show exceptional promise for leadership in science and technology.

A global ambassador for mathematics, Loh travels the world giving talks to diverse audiences, from fellow mathematicians to high school students.

Loh's research focuses on the intersection of discrete systems, probability and computer science. He recently attended the World Economic Forum's Annual Meeting of New Champions in Dalian, China, as a member of the Forum's Young Scientists community.

Established in 1996, the PECASE acknowledges the contributions scientists and engineers have made to the advancement of science, technology, engineering and mathematics education, and to community service as demonstrated by scientific leadership, public education and community outreach. The awards are conferred annually at the White House following recommendations from participating departments and agencies.

Math at CMU Welcomes Two New Faculty Members

David Offner

David Offner joined the Department of Mathematical Sciences as an associate teaching professor in fall 2019. He received his Ph.D. from Carnegie Mellon and was most recently an associate professor at Westminster College in New Wilmington, Pennsylvania. Offner also previously served as a Shelly Visiting Associate Professor at Carnegie Mellon.

"I am interested in the structural properties of graphs," Offner said about his research in the field of combinatorics. Graphs can be thought of as informal "networks" with nodes and connections, he notes, and his research looks at questions such as how the large version of these networks can be decomposed into copies of smaller sub-networks, or how many of those connections in a network must be severed to remove all the copies of a sub-network within it.

"Much of my work is related to questions in theoretical computer science and optimization that are also beautiful and challenging pure mathematics problems in their own right," Offner said. "There are a number of easy to state conjectures in this field that nonetheless have proven hard to understand and will require new insights to solve."

Several years ago, Offner organized a research collaboration to study the topic of polychromatic colorings of graphs. In a paper published last year in the Journal of Combinatorics, Offner and his collaborators specifically looked at these colorings on hypercubes, geometric figures in four or more dimensions that are analogous to regular cubes in the three-dimensional world.

Offner has an impressive record as a mentor of undergraduate research projects. He has advised over 30 students who completed projects in the last few years, and many of these students went on to present their work at professional meetings or publish them in mathematics journals.

Outside of his research, Offner enjoys spending time with his wife and two children and coaching and watching sports of all kinds.

"I am excited to work with the talented and hard-working students at Carnegie Mellon and to interact and collaborate with brilliant colleagues," Offner said.

Martin Larsson

Martin Larsson joined the Department of Mathematical Sciences as an associate professor in fall 2019. A native of Sweden, Larsson received his Ph.D. from Cornell University and was most recently an assistant professor at the Swiss Federal Institute of Technology in Zürich.

"My research is in mathematical finance," Larsson said. "It's about developing the mathematics needed to model phenomena that we see in financial markets and economic systems more broadly."

Larsson said he found himself attracted to this area of mathematics because it blends theory with applications. "I get to talk to all kinds of people, from pure mathematicians to finance academics to bankers and regulators," Larsson said, noting that his research touches on everything from interest rates to energy prices to models for volatility.

"Lots of interesting things pop up when you start to think about individuals that interact, often in a highly strategic manner, in an environment with lots of noise and randomness," Larsson said. "And for the record: it's not about predicting the stock market!"

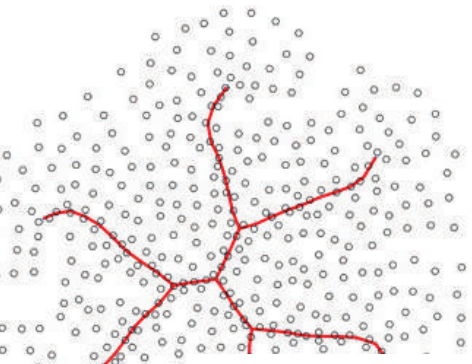
An example of Larsson's work is a study published in October 2019 in The Annals of Applied Probability titled "affine Volterra processes." In it, Larsson and his

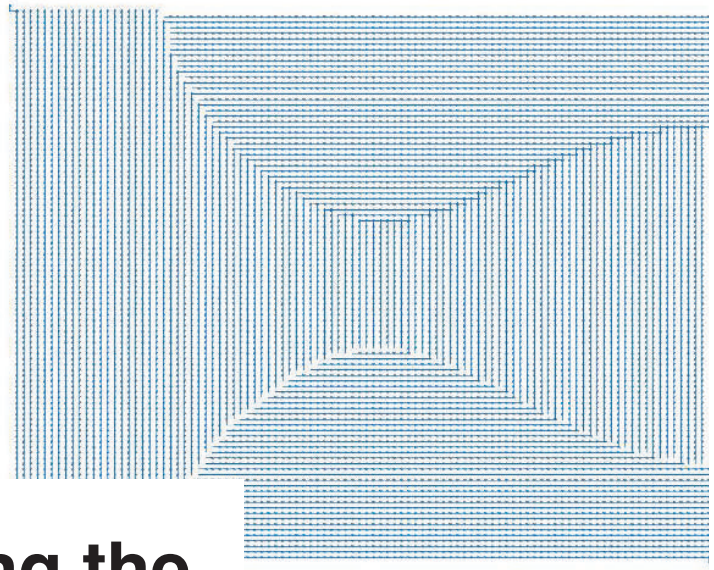
collaborators developed a theory for a class of models that have become increasingly important for describing volatility, thus making them more practical to work with.

"These models are tricky to analyze because they lack various nice features that people are used to working with," Larsson said. "We show that despite this, concrete computations are still possible in these more challenging models."

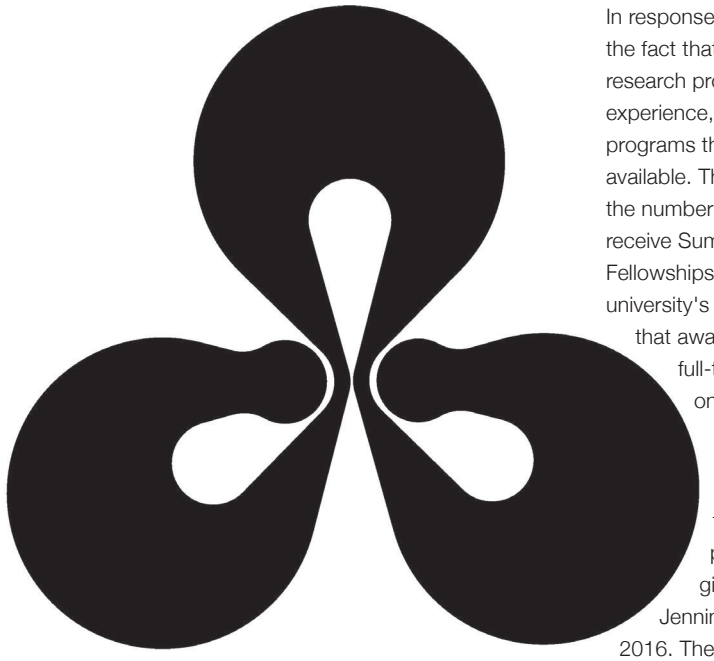
"Carnegie Mellon exerts a strong pull on anyone doing research in mathematical finance," Larsson said of what drew him to join the faculty here. Besides its research, Larsson also described Carnegie Mellon's bachelor's and master's programs in computational finance as "trailblazers" in his field. "Being part of that is very exciting."

Outside of his work, Larsson enjoys being outdoors with his wife and is looking forward to exploring the hiking opportunities around Pittsburgh, as well as its breweries, coffee shops and restaurants. "So far we've only begun to scratch the surface!" he said.





Expanding the Boundaries on Summer Undergraduate Research



Images. Summer Undergraduate Research Fellowship work of the last few years.

Since 2015 there has been a dramatic increase in the number of undergraduates pursuing mathematics research on campus in the summer months.

A traditional view among mathematicians is that mathematics is not well suited to undergraduate research. Students generally need several years of course work before they can understand the questions that are most interesting to mathematicians. However, there is growing demand for such experiences from undergraduates who are considering moving on to graduate study. Furthermore, research experiences can be key in helping students secure jobs and internships.

In response to this growing demand and the fact that a successful undergraduate research project can be a transformative experience, the department is developing programs that make these experiences more available. This began with an effort to expand the number of mathematics students who receive Summer Undergraduate Research Fellowships (SURF), a program under the university's Undergraduate Research Office that awards students stipends to pursue full-time summer research on campus.

Recognizing the value of this program, the department started to directly fund math students pursuing SURFs, leveraging gifts from alumni, including Larry Jennings and David Simmons, in 2016. The size of this program grew from a previous norm of one or two mathematics SURFs each summer to a total of 14 mathematics SURFs in the summer of 2019 (see page 29 for details on the SURF projects from 2019).

Furthermore, the quality of these projects has improved as the faculty has become more engaged with students' research. Almost half of the permanent faculty have advised at least one SURF project in one of the last three summers. "This is special in itself," said Department Head Tom Bohman, who created an incentive program for faculty who mentor summer research. The department also introduced the David Simmons competition for undergraduate research in mathematics and appointed Irina Gheorghiciuc to the position of director of undergraduate research in mathematics. In this role she helps match students who have an interest in research with faculty mentors.

SURF projects rely on one-on-one advising and being able to provide close faculty contact has had a direct effect on students' academic growth. One such student, senior Varun Gudibanda, worked with Assistant Professor Franziska Weber last summer on numerical analysis of liquid crystals to produce numerical schemes for the system and simulate the system in MATLAB. "My summer research helped me find confidence in my plans to attend graduate school and beyond," said Gudibanda. He also worked on a numerical analysis project with Associate Teaching Professor Jason Howell in the summer of 2018 and now plans to

obtain a Ph.D. in numerical analysis, with the goal of eventually providing numerical frameworks for exciting real-world problems such as Formula 1 race car aerodynamics.

As the SURF program grew and improved, it became clear that there was a need for a separate program focused exclusively on the mathematics of finance. Professor Bill Hrusa launched the Mathematical Finance Summer Undergraduate Research Program (MFSURP) in the summer of 2017.

Twelve students participated in the first MFSURP. This year, that number increased to a staggering 38 students.

MFSURP is an early research experience for first-years and sophomores to work on mathematical problems that are relevant to finance. Normally, students with an interest in finance look to secure an internship by their junior year. Having done research on a topic in finance has made a big difference for those students come internship recruiting season.

Following her first-year, current junior Lily Qiao and her partner Laura Li conducted research on portfolio optimization with order-size impact. They won the Young Researcher Award at the 2019 Simmons Competition.

"This is one of the most important topics in finance right now," said Qiao.

Current financial theory assumes that the price of a security doesn't depend on how many shares an investor buys. Qiao and Li developed simple models for order size impact and used them to explore the effect on the structure of optimal portfolios.

Qiao says that internship recruiters were impressed by her research, and she credits MFSURP with giving her the confidence to connect with people currently working in finance.

"I'm really grateful that Dr. Hrusa offers this undergraduate research experience," added Qiao, who secured an internship with Bank of America for next summer over a year in advance.

For sophomore Minty Xia, she appreciates that MFSURP gives students a chance to work on the same problems that people in the industry are currently grappling with.

This summer, Xia's group studied utility indifference pricing. In certain situations, a given security may have more than one price that is consistent with the basic theoretical framework. The idea of utility indifference pricing is to single out a price that is consistent with investors' risk preferences. Xia's group analyzed the limits of utility pricing to see how current models would behave when order sizes get large.

"We were doing things that we knew would be meaningful in a real-world context. That was the most meaningful part to me," Xia said.

She also believes the program has practical outcomes. Xia will intern with the Bank of America next summer, an impressive feat for a sophomore. "Most of my interviewers asked about my research because they would say it's really rare for a first-year student to have a research experience," she said.

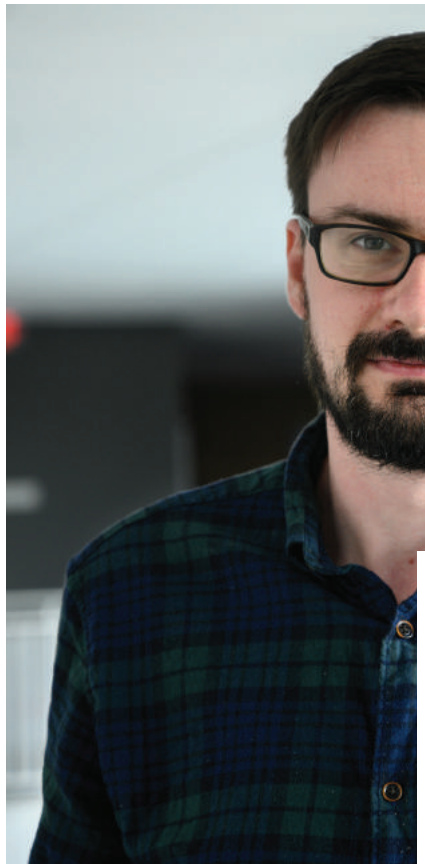
Most recently, the program added a weekly market watch seminar series, where students discuss what's going on in current financial markets, which also gave Xia a leg up.

"Before going to my final interviews, I had to prepare for market knowledge. Having been through that experience during MFSURP was really helpful," she said.

As it's grown, MFSURP has attracted corporate sponsorship from Principal Financial and Goldman Sachs and also garners support from the CMU Provost's Office.

These last four years have shown that not only are students highly motivated to pursue research but that it has a strong impact on their futures. The department continues to raise endowed funds to support the growing number of students undertaking summer research. The Martin Fund for Undergraduate Research supports mathematics SURF students and the Steven Shreve Fund for Mathematics supports the MFSURP.

Visit math.cmu.edu/giving for details.



Frick's Fellowship

Assistant Professor Florian Frick was among 126 recipients of 2019 Sloan Research Fellowships, which honor early career scholars.

"Sloan Research Fellows are the best young scientists working today," said Adam F. Falk, president of the Alfred P. Sloan Foundation in announcing the fellowships in February. "Sloan fellows stand out for their creativity, for their hard work, for the importance of the issues they tackle and the energy and innovation with which they tackle them. To be a Sloan fellow is to be in the vanguard of 21st century science."

The Sloan Fellowships are open to scholars in eight scientific and technical fields — chemistry, computer science, economics, mathematics, computational and evolutionary molecular biology, neuroscience, ocean sciences and physics. Winners receive a two-year, $70,000 fellowship to further their research.

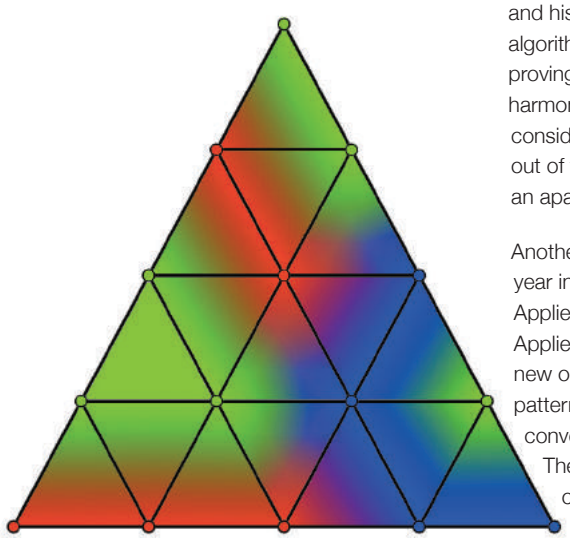


Figure. Sperner PL Map

For Frick, who has collaborators scattered around the United States and Europe, the fellowship will allow him to invite them to Carnegie Mellon to develop new ideas and further their work on common projects.

"Over the next two years, I hope to expand the reach of topological methods and find numerous new problem areas that can benefit from a topological viewpoint," Frick said of his plans for the fellowship. He noted that these geometric and topological methods can yield fruitful results in other branches of mathematics and their applications.

Frick describes his research as working "at the triple-point of combinatorics, topology and geometry." An example of Frick's research is his work to determine whether it's possible to find four points that form a square within a curve in a plane. Another area of mathematics he's interested in is one that has been relatable to many people at some point in their lives: fair rent division.

In a paper published this year in The American Mathematical Monthly, Frick and his collaborators introduced an algorithm based on Sperner's lemma proving that one can achieve "rental harmony" even while only taking into consideration the preferences of two out of three of the roommates sharing an apartment.

Another study of Frick published this year in the Society for Industrial and Applied Mathematics Journal on Applied Algebra and Geometry found new obstructions for intersection patterns to arise from arrangements of convex open sets in Euclidean space. These obstructions completely classify intersection patterns for a more flexible topological relaxation of arrangements of convex open sets.

This question has applications to neuroscience, particularly for studying cofiring patterns of so-called "place cells" in the brain that fire when an organism is in a certain place in its environment.

A native of Germany, Frick came to Carnegie Mellon in 2018 from Cornell University, where he was a H.C. Wang assistant professor. During that time, he also spent a semester as a postdoctoral fellow at the Mathematical Sciences Research Institute in California. He earned his Ph.D. in 2015 from Technische Universität Berlin, from which he also obtained his bachelor's and master's degrees.

Avoiding Coincidences

by Florian Frick

The complete graph on five vertices is non-planar that is, for any drawing in the plane, two edges share a point outside of a common vertex. The real projective plane $\mathbb{R}P^2$ does not embed into $\mathbb{R}^3$, that is, any continuous map $f : \mathbb{R}P^2 \to \mathbb{R}^3$ has a double point. In fact, any immersion of $\mathbb{R}P^2$ into $\mathbb{R}^3$ has a triple point. This article discusses some fragments of the story of developing a theory of non-embeddability and extensions to higher multiplicity coincidences (such as the existence of triple points, quadruple points, etc.) from the viewpoint of discrete geometry.

For real numbers $x_0 \leq x_1 \leq \ldots \leq x_{2n}$ the set $\bigcap_{i=0}^{n} [x_i, x_{2n-i}]$ consists only of the point x_n – the *median* of the x_i, which leaves precisely n of the x_i to either side of it. Bryan Birch, of Birch and Swinnerton-Dyer fame, showed in a 1959 paper that one may observe a similar phenomenon for tuples of real numbers: Any $3n$ points in the plane $\mathbb{R}^2$ can be split into n triples of points such that the corresponding n triangles all have a point in common.

For higher dimensions, Birch asked whether any $(d+1)n$ points X in $\mathbb{R}^d$ determine n vertex-disjoint simplices of dimension d that all contain a common point of intersection. Any such point of intersection c would be a *center point* for X: Any half-space containing c also contains at least a fraction of $\frac{1}{d+1}$ of the points of X, as it must contain at least one vertex per simplex. For $d = 1$ this recovers the notion of median. Birch already recognized that one can be more economical about the number of points. He conjectured that any $(n-1)(d+1)+1$ points in $\mathbb{R}^d$ may be partitioned into n sets $X_1, \ldots, X_n$ whose convex hulls contain a common point and confirmed this conjecture for $d = 2$. It is easily checked that for a sufficiently generic set of $(n-1)(d+1)$ points in $\mathbb{R}^d$ even the affine hulls of any partition $X_1, \ldots, X_n$ into n parts have empty intersection, and thus this result would be optimal.

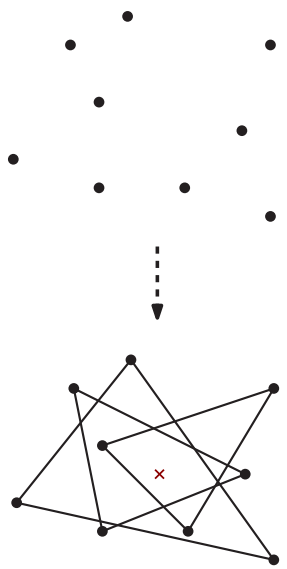


Figure 1. Any nine points in the plane can be split into three triangles that surround a common point — the red cross.

Two years after the publication of Birch's paper, Norwegian graduate student Helge Tverberg proved the same result, unaware of the earlier work by Birch. After learning about Birch's theorem for the planar case and the corresponding conjecture for higher dimensions, Tverberg visited Birch in Manchester in 1964. Tverberg would later describe the weather in Manchester during his visit as "bitterly cold," and unable to sleep for the last night of his visit, since the heater in the hotel had gone off, the solution to Birch's conjecture finally dawned on Tverberg. Since then Birch's conjecture has been known as Tverberg's theorem and is now considered a central result in discrete geometry.

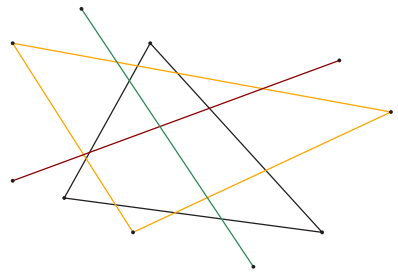


Figure 2. Ten points in the plane and a color-coded partition into four parts with intersecting convex hulls.

The $n = 2$ case of Tverberg's theorem, that any $d+2$ points in $\mathbb{R}^d$ can be split into two sets, whose convex hulls intersect, had already been established decades earlier by Johann Radon in 1921. The planar case of Radon's result states that for any four points in $\mathbb{R}^2$ either one point is contained in the triangle spanned by the other three or the four points come in two pairs such that the segments connecting the pairs intersect. Another way of saying this is that for any straight-line drawing of a tetrahedron in the plane two vertex-disjoint faces must intersect. It is natural to ask whether the drawing needs to consist of only straight lines, or whether the overlap of vertex-disjoint faces is a topological feature that holds more generally for any continuous drawing. Thus, in 1976, Imre Bárány conjectured that for any continuous map from the $(n-1)(d+1)$ – dimensional simplex to $\mathbb{R}^d$ there are n pairwise disjoint faces whose images have a common point of intersection. The case of an affine map exactly corresponds to Tverberg's theorem, since the image of a face under an affine map is the convex hull of its vertices.

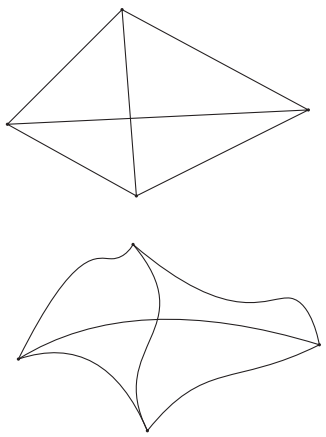


Figure 3. A linear and a continuous drawing of a tetrahedron in the plane.

Bárány's conjecture became known as the topological Tverberg conjecture and was soon after — in 1981— shown to hold for n a prime. Özaydin later showed the case of n a power of a prime. This dependence on divisibility conditions of the intersection multiplicity n should come as a surprise. It stems from the following construction: If $f : X \to \mathbb{R}^d$ avoids n-fold points of coincidence, then it induces a map from n-tuples of pairwise distinct points in X, the *configuration space* of X, to $(\mathbb{R}^d)^n$ that avoids the diagonal, that is, the subspace of $(\mathbb{R}^d)^n$, where all n factors are equal. Moreover, this map respects the symmetries of permuting coordinates. Obstructions for the existence of such *equivariant* maps depend on whether n is a prime power or not, since the symmetries have a different structure depending on the number of distinct prime divisors of n. More precisely, an elementary abelian group of symmetries acts transitively on coordinates if and only if n is a prime power.

That the topological Tverberg conjecture could only be settled in the case of n a prime power was believed to be an artifact of the proof method, which relies on lower bounds for the equivariant topology of the associated configuration space. Recent (2015) counterexamples to the topological Tverberg conjecture for every n with at least two distinct prime divisors thus were a surprise to many. The main ingredients for these counterexamples are an extension of the Whitney trick, which removes pairs of double points to construct an embedding, to higher multiplicity intersections due to Isaac Mabillard and Uli Wagner, and the "constraint method" of Pavle Blagojevic, Günter Ziegler, and the author. The former provides a way to transform vanishing obstructions for the existence of equivariant maps from the configuration space of X into maps $X \to \mathbb{R}^d$ that avoid n–fold points — at least as long as X has codimension $\frac{d}{n}$ in $\mathbb{R}^d$. The latter can be used to lift maps with no n-fold points in positive codimension to the case, where X may have dimension much larger than d— the case of interest for the topological Tverberg conjecture.

The constraint method proves results about the existence of n-fold points in a generalized fashion, encapsulating graph planarity, the embeddability of manifolds into $\mathbb{R}^d$, and their higher multiplicity generalizations. Perhaps surprisingly, it also exposes chromatic numbers of uniform hypergraphs as a more rigid version of the same theory.

Many seemingly simple questions remain unsolved. For instance, given three red, three green, three blue and one yellow point in the plane, can they always be split into four sets without repeated colors whose convex hulls all have a point in common?

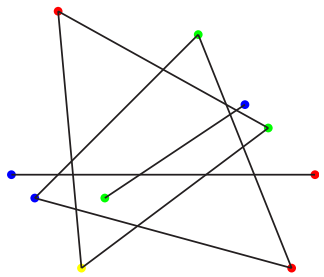


Figure 4. Three red, three green, three blue and one yellow point in the plane and a colorful 4-fold Tverberg partition.

Egon Balas

University Professor of Industrial Administration and Applied Mathematics and Thomas Lord Professor of Operations Research, died on March 18, 2019. He was 96.

"A beloved member of the CMU faculty for more than half a century, Egon Balas was a preeminent and legendary scholar who was enormously influential in the fields of operations research and applied mathematics," said Farnam Jahanian, president of Carnegie Mellon University. "Throughout his long and distinguished career as a researcher and teacher, he applied bold, focused and independent thinking to solve complex problems and also demonstrated a profound sense of humility, character and good humor. His extraordinary life and legacy will continue to serve as an inspiration to the entire CMU community."

Balas was born into a Hungarian-Jewish family in Cluj, Romania, in 1922. After high school, he wanted to continue his studies in physics but was blocked by anti-Semitic laws. Determined to fight Nazism, he joined the underground Hungarian Communist Party and was arrested by Fascist Hungarian authorities in 1944. He was sentenced to 14 years of hard labor but escaped during transport to Germany. On his return home to Cluj, he found that all of his immediate family had been killed along with most Jews who lived in the town.

Still in the Communist Party, Balas taught himself economics and changed his birth name, Blatt, a common Jewish surname, to Balas in order to serve in the Romanian government as economics director in the Ministry of Foreign Affairs. During a power struggle in 1952, he was arrested by party leaders and put in solitary confinement for more than two years.

Released from prison in 1954, Balas became disenchanted with communism, especially after a trip with his wife, Edith, to the Soviet Union exposed economic conditions much worse than depicted in the state press. In a 2016 interview, Balas described the difficulty of his transition after spending decades trying to make economic sense of Marxism and socialism. But he also said this disillusionment helped spur his turn to mathematics. In 1959, at the age of 37, Balas immersed himself in the then-emerging field of linear programming, gaining recognition with a novel solution to a timber-harvesting problem. He called his solution the Additive Algorithm, similar to what is known as implicit numeration or constraint propagation today. He later earned Ph.D. degrees in economics (University of Brussels, 1967) and mathematics (University of Paris, 1968).

He was recruited to Carnegie Mellon in 1967, where he continued to develop new methods in the field of integer programming, in particular disjunctive programming. At CMU, he co-founded the Algorithms, Combinatorics and Optimization program, a joint Ph.D. program between the Mellon College of Science, Tepper School of Business and School of Computer Science.

In 1995, he earned the John von Neumann Prize from the Institute for Operations Research and the Management Sciences, which is considered the Nobel Prize of operations research. In 2001, he won the EURO Gold Medal, and in 2006, he was elected to the National Academy of Engineering and the IFORS Operational Research Hall of Fame.

Balas is survived by his wife of 70 years, Edith Balas, an emeritus professor of Art History at Carnegie Mellon; two children, Anna Balas and Vera Balas Koutsoyannis; three grandchildren; and four great-grandchildren.

Patsy J. "P.J." McCarthy

Longtime office manager and administrative assistant to the heads of the Department of Mathematical Sciences, died on Sept. 27, 2019. She was 68.

For the last 34 years, which her family described as "very good years," Patsy (P.J.) McCarthy could be found working at Carnegie Mellon, first in the College of Humanities & Social Sciences, then in the math department's Wean Hall offices. She came to CMU from Westinghouse Research, where she worked until the company's Harmar, Pennsylvania offices closed.

Having come from a staid corporate environment, McCarthy had to adapt to the different world of academic management. She quickly made the transition and enjoyed the setting. Her adaptability was further demonstrated by her success in working with four different department heads, William Williams, Jim Greenberg, Nic Nicolaides and Tom Bohman, all of whom had greatly differing personalities and styles.

"Part of P.J.'s job was to execute directives by the department head, but on occasion she did not do that right away because she knew the department head should reconsider what he was doing. By consciously waiting, she gave him a chance to do this," said Orion Hoch University Professor Steven. E. Shreve. "I don't think the heads ever knew that she did this intentionally. They were just relieved when they changed their mind to find out that she had not yet acted. An assistant like this is the most valuable kind."

All agreed that she was indispensable to the department. During the transitions between department heads, McCarthy was credited

with holding the department together.

"The secret of her success lay in a cheerful cynicism and great good humor," said Professor Emeritus Williams.

McCarthy was an efficient and dedicated administrator who kept the department running smoothly for many years. She managed the department's administrative staff. She was known for running a supportive workplace, with most staff under her supervision staying with the department for decades.

Her work contained numerous responsibilities, including managing correspondence, securing visas for new faculty, acting as a liaison to facilities management, coordinating documentation for the department's advisory board report and much, much more.

In addition to her work with the department, McCarthy was a history buff and enjoyed being a pen pal through Pocket Pals.

McCarthy is survived by her husband of 42 years, Barry McCarthy.

Richard A. Moore

Professor of mathematics and former department head, died on Oct. 20, 2018. He was 94.

Born in Ohio in 1924, Moore grew up in Cleveland, New York City and St. Louis. After graduating from high school in 1942, he attended Yale University for one semester before joining the infantry to serve in World War II from 1943-45. After completing his service, he returned to Washington University where he earned his bachelor's degree in 1948, master's degree in 1950 and doctorate in 1953. He then served as an instructor at the University of Nebraska and at Yale.

Moore came to Carnegie Mellon, then called the Carnegie Institute of Technology, in September 1956 as an assistant professor of mathematics and remained with the department for the next 30 years until his retirement as a professor. He served two terms as associate department head, and was department head from December 1971 to June 1975.

"The word that best described Dick was 'affable,'" said Professor Emeritus William Williams. "He was always low key, unflappable and pragmatic in dealing with both faculty and students. An ideal associate department head, he always found the way to implement policy and procedure smoothly with minimal irritation to all parties."

Moore was seen as an educational leader by the department, college and university. He planned most of the developments in the department's undergraduate curriculum that happened during his tenure and was involved in curriculum planning at the university level.

He received the university's William H. and Frances S. Ryan Award for Meritorious Teaching. In his nomination for this award, he was characterized as a "student's professor," referring to the extra commitment he gave to his students.

Moore also made significant contributions to graduate education at Carnegie Mellon, playing a key role in the development of the Doctor of Arts in Mathematics program, which was designed to prepare students to become teachers at the collegiate level.

As an administrator, he was known to efficiently handle the thankless details that kept the department running, such as scheduling classes, assigning instructors, advising students, supervising teaching assistants and coordinating with the rest of the university.

His commitment to education reached beyond Carnegie Mellon. He consulted with local school districts and area colleges and represented the university to groups such as the Mathematical Association of America, the National Council of Teachers of Mathematics, the Pennsylvania Council of Teachers of Mathematics and Mathematics Council of Western Pennsylvania.

In 1995, the Mellon College of Science established the Richard Moore Education Award to recognize faculty for their substantial and sustained contributions to the educational mission of the college.

Moore is survived by his wife of 69 years, Ruth Neuhoff Moore; three children, Peter A. Moore, Susan Moore McJunkin and Sarah M. Moore; four granddaughters; and six great grandchildren.

Lending Data to Loans

Alumni fuse technology and finance to make investing smarter and safer

At 22 years old, Hugh Edmundson took a risk he wasn't sure would pay off. He left his first post-graduate job as a trader at Morgan Stanley, moved to San Francisco and convinced his fellow computational finance classmate Abeer Agrawal to start their own company.

Five years later that company, Theorem, manages over $1 billion in assets for a variety of clients. It's safe to say that the risk is paying off.

Theorem is an investment management and technology company rolled into one — the company uses data science, machine learning and software engineering to invest in marketplace lending loans.

In marketplace lending, investors back personal consumer loans — such as debt consolidation, business or home improvement loans. Replacing the traditional role of a bank, companies like Prosper and Lending Club help connect investors directly to consumers to fund these peer-to-peer loans.

In this process, investors generally see a handful of simple metrics such as FICO scores, income and the interest rate of the loan to determine buying a loan. On the other hand, the lenders — i.e., Prosper and Lending Club — look at upwards of hundreds of different factors when pricing a loan.

"There's a huge information asymmetry between the originator of the loan and the person who is actually bearing the risk and buying the loan," said Edmundson. This asymmetry could lead to a problem if loan lenders over-originate loans or originate lower quality loans.

In coming up with the idea for Theorem, Edmundson saw an opportunity to move from understanding risk at the portfolio level (what investors see) to the loan level (what lenders see), but there was one missing piece — an ability to consider the multitude of factors that go into originating loans.

Enter Abeer Agrawal, Theorem's chief technology officer. Agrawal, who studied computational finance and electric and computer engineering at Carnegie Mellon, began his professional career at Google and then as a founding employee at an early stage startup, MobileSpan, which helped businesses access and secure their content on mobile devices. Coincidentally, around the time the company was acquired by Dropbox, Edmundson and Agrawal had reconnected.

In 2014, the pair joined Y Combinator, a seed accelerator that has helped launch companies such as Dropbox, Airbnb and Reddit. As part of the program, selected startups spend three months in Silicon Valley developing their company and preparing their pitch to investors.

With seed funding from Y Combinator and an investment fund of $50,000, from names such as Ron Conway and Max Levchin, Edmundson and Agrawal hit the ground running to build their company.

They started by hiring the best quantitative talent they could find. "We are a tech start-up more than anything else," said Agrawal. "Most of our team is engineers and researchers, and we are essentially using algorithms and machine learning to optimize the data and make our decisions."

What Theorem essentially does is fact-check marketplace lending loans through their own underwriting models to evaluate the pricing of loans and quality of investment for clients.

Their loan scoring technology analyzes more than 1,000 data points to identify the best loans, reduce default rates and build strong, diversified portfolios for their clients.

To prove their models work, Edmundson says, the company had to play high stakes show-and-tell by buying a bunch of loans and letting them ride for 18-24 months. One of the hardest challenges was building that institutional credibility. But, in just under two years, the company secured their first institutional client, a large university endowment, and within three years reached their first $100 million in assets.

As their clients and assets grew, so did the company's talent. In the last 18 months, Theorem has grown from a team of six to a team of over 30.

"We are the least smart people at the company," said Agrawal. He credits the quality of their team and the culture their team has built with Theorem's continued success.

"We're still hiring!" added Edmundson. Edmundson and Agrawal encourage anyone interested in what Theorem does to take a look at their careers page.

Giovanni Gravina Receives Graduate Student Teaching Award

Giovanni Gravina was honored with the Mellon College of Science's 2019 Hugh Young Graduate Student Teaching Award. A 2019 graduate, Gravina is now a postdoc at St. Charles University in Prague.

"I was very grateful for the recognition that I have received for my work and I was reminded of the enthusiasm needed to grow as a teacher," Gravina said of the honor.

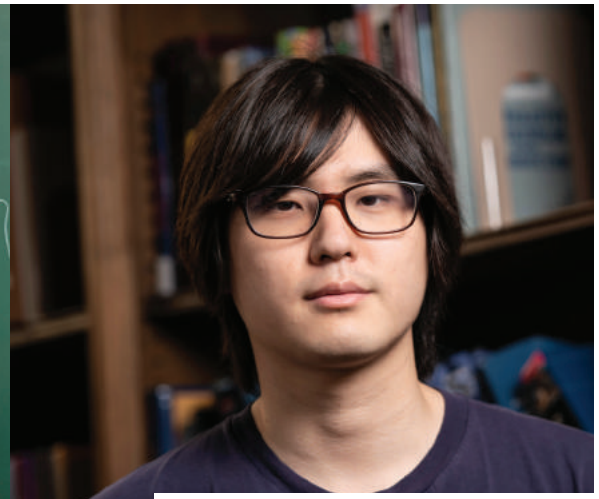
"Giovanni takes his teaching extremely seriously and our students really appreciate him," wrote Professor William Hrusa in nominating Gravina. "He has been an outstanding citizen around the department."

Hrusa noted that Gravina substituted for or assisted with a wide variety of undergraduate courses in his time at Carnegie Mellon, and earned the respect of the department's faculty. In 2016, for example, the department entrusted Gravina with assisting a professor having serious health issues because "we had complete faith in him to handle the class on his own if necessary."

"His office hours were always well attended, and our students often mentioned his name with a tone of camaraderie, as if Giovanni was someone they felt very comfortable with," Associate Teaching Professor Dana Mihai wrote about her experience teaching with Gravina in 2015. "Throughout the class, I came to rely on Giovanni with confidence that everything will be done well."

Several of Gravina's former students also wrote letters attesting to Gravina's teaching prowess. "His thoughtfulness, respect for students and genuine interest in helping us learn still stand out to me," wrote Meredith Wong about Gravina's teaching of her 2017 Differential Equations course. "I felt like he genuinely cared about my and other students' learning because he was so expressive in his teaching and was always willing to put in the extra time to make sure we understood things."

"He was always exceptionally prepared, well-spoken and attentive to the needs of the class," Newton Xie wrote about Gravina. "His instruction set a high bar for excellence and helped ease my transition academically during my first semester at the university."



Jung Joo Suh Recognized as Goldwater Scholar

Undergraduate student Jung Joo Suh was named a 2019 Goldwater Scholar by the Barry Goldwater Scholarship and Excellence in Education Foundation.

Suh, a native of South Korea who came to Carnegie Mellon from New Jersey, plans to enter a Ph.D. program in mathematics after graduation, with a focus on research in descriptive set theory and analysis.

"I've been interested in math for a long time," Suh noted, especially set theory, which studies well-behaved subsets of certain topological spaces. "These topological spaces can sometimes be visualized as a certain graph, and with the graph theoretic interpretation a lot of interesting questions can be asked."

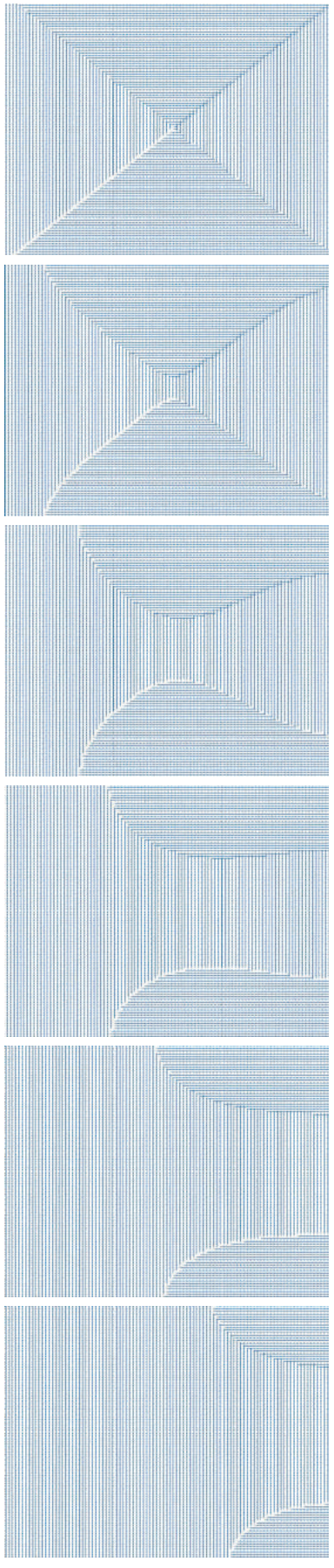
Under Associate Professor Clinton Conley, Suh has been focusing especially on clopen sets — a set that is both open and closed in a specific topological space.

Suh said he enjoys doing research, and that its challenges make discovering new ideas rewarding. "When you're stuck on a problem, you can't ask for help," Suh noted.

"Jung Joo is a wonderful representative of Carnegie Mellon's robust undergraduate mathematics program, where he has built deep foundations in several subfields and taken on ambitious research under the mentorship of Professor Clinton Conley," said Richelle Bernazzoli, assistant director of the Undergraduate Research Office. "This is the sort of intellectual boldness that the Goldwater Scholarship seeks out and nurtures. We are delighted to see Jung Joo's work recognized with this prestigious national award and we look forward to following his career as a mathematician."

Besides research, Suh has also spent time working as a peer tutor, grader and teaching assistant. He's excited about his Goldwater Scholarship, and he said applying for it has helped him organize his research into a coherent narrative. Overall, Suh believes being recognized as a Goldwater scholar will aid his goal of attending graduate school. "It shows that you're ready to be a researcher," Suh said.

He is one of 496 recipients of the scholarship, which recognizes second- and third-year college students intending to pursue research careers in mathematics, engineering and the natural sciences. A total of 1,223 students were nominated by 443 academic institutions for the award, which provides up to $7,500 per undergraduate year for tuition, mandatory fees, books, room and board.



SURF spotlight

Numerical Analysis of Liquid Crystals

Liquid crystals occur naturally in cell membranes and are perhaps most famous for their application in liquid crystal displays. Liquid crystals are an intermediate state between a liquid and a crystal. In a liquid, all the particles are randomly oriented while in a crystal, all the particles are completely aligned. In liquid crystals, particles are thought of as being "vaguely oriented" in that the particles in a small section of the liquid crystal will usually be pointing in the same direction. Physicists have produced a system of partial differential equations (PDEs) that models a system of liquid crystals and predicts their alignment in space and time. These equations are very nonlinear, which makes finding exact solutions nearly impossible.

In his SURF project Varun Gudibanda, now a senior, worked with Assistant Professor Franziska Weber on a numerical scheme that computes an approximation to the solution of these PDEs. The sequence of images shows the evolution of this numerical approximation. The small lines represent the orientation of the liquid crystal molecules. The orientations evolve according to a gradient flow with respect to a certain energy potential towards equilibrium. The energy potential is composed of an elastic part and a bulk part (thermotrophic). The simulation was done with a finite difference scheme that represents a discrete version of the continuous gradient flow and preserves the structure of the energy and its decay over time, roughly speaking.

Gudibanda is currently working on extending the scheme to more complicated energy potentials and adding additional physics, including fluid flow and external forces. He is also working on rigorously proving convergence of the numerical method.

Summer 2019 Math SURF Research Projects

Max Aires
Edge Density of Maximally Linkless Graphs
Advisor: Florian Frick

Zhijie Chen
Finite Difference Scheme for Solving the Allen-Cahn Equation Coupled with an Elliptic Equation
Advisor: Franziska Weber

Varun Gudibanda
Numerical Analysis of Liquid Crystals
Advisor: Franziska Weber
(Image sequence to the left)

Scott Harvey-Arnold
1D Free Boundary Problems
Advisor: Giovanni Leoni

Zhiyang He
New Upper Bound on Extremal Number for Even Cycles
Advisor: Boris Bukh

Philip Lamkin
Log-convexity of Moments of Averages
Advisor: Tomasz Tkocz

Yile Liu
Multilevel Monte Carlo
Advisor: Yu Gu

Hannah Milano
Learning Transport Velocity Fields from Data
Advisor: Hayden Schaeffer

Fei Peng
What Can You Draw?
Advisor: Florian Frick
(Image on cover)

Matthew Shi
Inequalities in Convex Geometry
Advisor: Raghavendra Venkatraman

Xiang Si
Minimizing Curve and Distribution of Average Network Commute Time Problem
Advisor: Dejan Slepčev

Noah Stevenson
Steady-State Stability Analysis of Periodic Micropolar Fluids
Advisor: Ian Tice

Jianming Wang
Horizontal-Vertical Geometry for Signal Comparison
Advisor: Dejan Slepčev

Yuepeng Yang
An Estimator of Point/Manifold-Manifold Distance Based on Local PCA
Advisor: Dejan Slepčev

Meeting of the Minds Poster Competition

David Simmons Prize for Undergraduate Research in Mathematics
Emily Zhu
Multicolor Ramsey Numbers for Small Hypergraphs

Honorable Mention
Alp Müyesser
Unavoidable Colorful Patterns in Graphs

Xinyu Wu
A Log-Sobolev Inequality for the Multislice, with Applications

Annie Xu
Traces for Homogeneous Sobolev Spaces in Infinite Cylindrical Domains

Young Research Award
Laura Li & Lily Qiao
Portfolio Optimization with Order Size Impact

Graduate Emily Zhu Receives Judith A. Resnik Award

Emily Zhu received the 2019 Judith A. Resnik Award, an award named after the Challenger astronaut and CMU alumna, during Commencement Weekend. The award recognizes an exceptional, senior woman graduating with an undergraduate technical course of study who will be pursuing graduate or professional training in a technical field.

Zhu graduated with a bachelor of science in mathematical sciences, with a concentration in discrete mathematics and logic, and a master of science in mathematical sciences. She earned the two degrees concurrently as part of her participation in the department's honors program.

"Emily's intellect is unsurpassed by other math majors in her cohort. She is incredibly talented mathematically and she has a work ethic to match," Jason Howell, director of undergraduate studies for the Department of Mathematical Sciences, said.

Zhu's research is in combinatorics. She completed work described as "cutting edge" by Howell for her honors thesis on multicolor Ramsey numbers for 3-uniform hypergraphs with 3 edges, advised by Alexander M. Knaster Professor and Department Head Tom Bohman.

Interest in her thesis topic developed in her first year when she took Bohman's course in discrete mathematics and continued to grow when she studied at the Budapest Semesters in Mathematics Program after her first year and through her later coursework.

After her sophomore year, Zhu studied at Clemson University as part of the Research Experience for Undergraduates program, where she worked on research in coding theory and contributed to a paper titled "Multicast Triangular Semilattice Network."

In addition to research, Zhu wanted to help others develop their skills and increase their appreciation of mathematics. She became a math tutor for other Carnegie Mellon students in her first year. She also volunteered with Western Pennsylvania American Regions Mathematics League team to mentor high school students for math competitions. She became a teaching assistant (TA) for her department in 2017 and served as a Head TA for Mathematical Concepts and Proofs.

"At CMU, I worked with a variety of people in math and gained a sense of how to explain things in an effective way," Zhu said.

While at Carnegie Mellon, Zhu was involved in Math Club and was an organizer for an undergraduate summer math seminar and Carnegie Mellon's Informatics and Mathematics Competition. She participated in designing and building the Math Club Booth for Spring Carnival three years in a row, winning Best Blitz Booth in 2018.

Zhu graduated with university and college honors and is pursuing doctoral studies at the University of California, San Diego.

"I will remember CMU's amazing network of supportive professors and my collaborative peer group," Zhu said.

Image. Emily Zhu with Dr. Amy Burkert, vice provost for education.

Class of 2019 Destinations

Firms that hired more than one member of the class:

Microsoft
Goldman Sachs
APT A Mastercard Company
citi
Bank of America
ROIVANT SCIENCES
Google

Doctoral Programs:

Berlin Mathematics School
Dartmouth, CS
Illinois, Math
Tennessee, Math
McGill, Math
Penn, Math
Pitt, Physics
UCSD, Math

Franziska Weber
Assistant Professor,
Department of Mathematical Sciences

WE CAN
MAKE
A SCIENTIFIC REVOLUTION
POSSIBLE
together.

MAKE
POSSIBLE

THE CAMPAIGN FOR
CARNEGIE MELLON UNIVERSITY

makepossible.cmu.edu/mcs

For more information on the Mellon College of Science priorities, please contact Nancy Felix at nfelix@andrew.cmu.edu or 412-268-6442.

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