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Editor-in-Chief
Tom Bohman

Writer
Amy Pavlak Laird

Contributing Writers
Jocelyn Duffy
Julianne Mattera
Emily Payne
Wes Pegden

Graphic Design and Photography
Carnegie Mellon University
Marketing & Communications

Carnegie Mellon University
Department of Mathematical Sciences
Wean Hall 6113
Pittsburgh, PA 15213

math.cmu.edu

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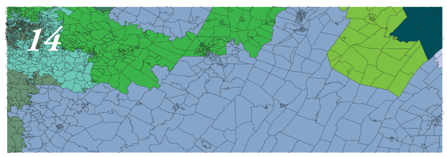
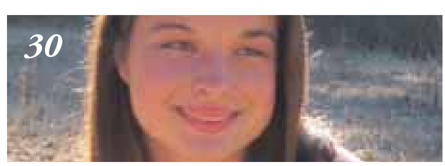




About the cover

The William Lowell Putnam Mathematical Competition is the preeminent mathematics competition for undergraduate college students in the United States and Canada. Every year thousands of students participate in the competition, and the top five scorers are named Putnam Fellows. Three Carnegie Mellon University students achieved Putnam Fellow status on the 2016 exam. They received the medallions that appear on the cover of this issue of the newsletter.

Inside Cover: This edition's inside cover is graced by images of the Crab Nebula, a supernova remnant. The beautiful finger-like tendrils evident in the nebula are a byproduct of the Rayleigh-Taylor fluid instability, which occurs when a low density fluid experiences a net acceleration into a high density fluid. Fluid instabilities of this sort play a key role in many other important phenomena, from the swirling patterns observed in Jupiter's red spot, to weather prediction on Earth, to the design of fusion reactors. Assistant Professor Ian Tice's research is concerned with the nonlinear analysis of the equations modeling fluid instabilities, which present a number of deep mathematical challenges.

Image courtesy of NASA.

Letter from Mathematics Department Head, Tom Bohman

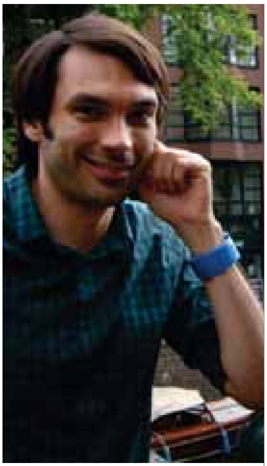
Regular readers of this newsletter will know that the Department of Mathematical Sciences has had some impressive developments in recent years. This trend certainly continued in 2017. Robust interest in mathematics at Carnegie Mellon is persisting, and our undergraduate students continue to impress us, including in dramatic fashion this year. For the first time, Carnegie Mellon won the Putnam competition, marking the sixth year in a row that our Putnam team placed in the top five. All three members of the CMU Putnam team were named Putnam Fellows, and 44 CMU students placed in the top 500.

Our faculty also continue to impress. The inside cover of this issue is drawn from a Hubble Space Telescope image of a Crab Nebula. The tendrils in a Crab Nebula are produced by the acceleration of one fluid into another, a phenomenon involving boundaries between fluids. Assistant Professor Ian Tice, who studies such phenomena, was awarded an NSF CAREER grant this year (page 4). This is the fourth NSF CAREER grant awarded to Math Department faculty in the last five years.

The feature story in this issue discusses a novel Markov Chain algorithm developed by Mathematical Sciences faculty Alan Frieze and Wes Pegden in collaboration with Maria Chikina of the University of Pittsburgh. This algorithm is applied to a very large collection of mathematical objects and quickly determines whether or not a given element of the collection is an outlier with respect to some specified statistic. The researchers originally were motivated by political gerrymandering and a desire to test whether or not a given districting is an outlier with respect to some measure of fairness in districting. And the mathematical basis for the algorithm is quite nice. This is an excellent example of our department’s outward-looking research and the way in which our mathematical work can be enriched by questions that come from other disciplines.

I hope that the many alumni of the Department of Mathematical Sciences have a chance to reconnect with the department by visiting math.cmu.edu/alumni and letting us know what's new with you!

The department continues to look to partner with friends and alums as we build on our recent successes and meet the growing demand for mathematics at Carnegie Mellon. Through generous alumni support, we recently initiated endowed funds that are helping us tackle key challenges, which include meeting the high demand for research and capstone projects in mathematics and expanding the very successful Bachelor of Science in Computational Finance program. While these endowed funds are already having an impact, they are not fully financed. There is still much more to be done.



Ian Tice Receives NSF CAREER Award

What do the swirling patterns observed in Jupiter's red spot have in common with weather prediction on Earth? Moving interfaces in multiphase fluid flow underlie both of these phenomena. While understanding moving interfaces in fluids can be complex, Assistant Professor Ian Tice is up to the challenge. And the National Science Foundation (NSF) has taken notice. Earlier this year, Tice received a Faculty Early Career Development (CAREER) award, the NSF's most prestigious award in support of junior faculty who exemplify the role of teacher-scholars.

The five-year grant will support Tice's work on the nonlinear analysis of the partial differential equations (PDEs) modeling moving interfaces in multiphase fluid flow. He's focusing on several specific models of viscous fluid flow: contact line dynamics, surfactant-driven flows, and gaseous stars and related models in astrophysics, such as the Rayleigh-Taylor fluid instability that leads to the beautiful finger-like tendrils evident in the Crab Nebula pictured above and on the inside cover of this issue of Mathematical Sciences News.

Under the grant, Tice will also develop an undergraduate collaborative reading and research program focused on fluid flow, assist with course development and mentor graduate researchers. In recent years, Tice has been teaching the Math Studies analysis courses, which are part of our honors course sequence in mathematics. He has developed a set of notes — 700 total pages — for this two-course sequence that are very highly regarded by faculty and students alike.

Sebastien Vasey Wins Student Research Award

Doctoral student Sebastien Vasey won the Guy C. Berry Graduate Student Research Award, which recognizes excellence in research by Mellon College of Science graduate students. Vasey conducts research in model theory, focusing on classification theory for abstract elementary classes (AECs). Working under the direction of Professor Rami Grossberg, Vasey has solved cases of one of the most important problems in model theory — Shelah's Categoricity Conjecture. In the late 1970s, Saharon Shelah conjectured that Morley's theorem, one of the central results in the model theory of first-order logic, extends in a natural way to all AECs. He believed that an AEC that is categorical in one high-enough cardinal would be categorical in all high-enough cardinals. Vasey combined recent work of Shelah with an alternative strategy to Shelah's approach put forth by the Grossberg group to obtain a proof of Shelah's eventual categoricity conjecture for universal classes, a general family of AECs. The result is the best approximation of the conjecture to date and also produces machinery that is useful for other applications in model theory.

"Sebastien is already a major, leading figure in a field that very few people have managed to enter in the last 40 years," Grossberg said.

After earning his Ph.D. this past spring, Vasey joined the Department of Mathematics at Harvard University as a Benjamin Peirce Assistant Professor.

The Department Welcomes Three New Faculty Members

Johannes Muhle-Karbe

Associate Professor Johannes Muhle-Karbe joined the probability and mathematical finance group. His research uses tools from probability theory, optimal control and partial differential equations to study stochastic models for financial markets, particularly models with "frictions" such as transaction costs, price impact or asymmetric information. He studies how these market imperfections affect portfolio choice and asset prices.

This spring, he will teach a Ph.D.-level course in mathematical finance that will focus on his research interests. Next fall, he will begin teaching the Continuous-Time Finance and Stochastic Calculus courses in the Master of Science in Computational Finance (MSCF) program, a program that he is particularly excited to join.

"CMU has an exceptional history of interdisciplinary collaboration. This is a great opportunity to interact with outstanding researchers from finance, statistics and other areas, and will hopefully lead to exciting new research," said Muhle-Karbe.

Muhle-Karbe earned his bachelor's and doctoral degrees in mathematics from the Technical University of Munich and completed a post-doctoral fellowship at the University of Vienna. Prior to coming to CMU, he held faculty positions at ETH Zürich and the University of Michigan.

Yu Gu

Assistant Professor Yu Gu also has joined the probability and mathematical finance group. Gu conducts research on stochastic equations and propagation of disorders. Specifically, he considers partial differential equations used in physics and tries to understand how the randomness and disorder found in the environment propagates into the equation's solutions.

"CMU has very strong groups working on applied analysis, probability and math finance. I'm excited to see what I can learn from these groups that I can apply to analyzing random equations," said Gu.

In the fall semester, Gu taught Numerical Methods to MSCF students. In future semesters, he'll teach courses in analysis and probability.

Gu earned his bachelor's degree in mathematics and physics from Tsinghua University in China, his master's degree in applied mathematics from Brown University

and his Ph.D. in applied mathematics from Columbia University. He comes to Carnegie Mellon from Stanford University, where he was a Szegö Assistant Professor.

Jason Howell joined the department as an associate teaching professor and as the director of undergraduate studies.

"I am excited to work with our undergraduate math majors, who are some of the most promising math students in the world. I hope to help guide them through their undergraduate experience here at CMU and prepare them for the next stage in their career and life," said Howell.

As director of undergraduate studies, Howell will oversee the department's undergraduate degree programs, advise all newly-declared mathematical sciences majors, serve on the Committee for Undergraduate Affairs and advise the Math Club. He will also teach the in the Propel junior seminar, part of the Mellon College of Science's new Core Education.

Howell conducts research in computational mathematics that focuses on numerical methods for partial differential equations and large linear algebra problems. He hopes to continue to work on interdisciplinary projects that span math and chemistry.

Howell comes to CMU from the College of Charleston where he was an assistant professor of mathematics. He earned his bachelor's degree from the College of Charleston and his master's degree and Ph.D. from Clemson University. No stranger to CMU, Howell was an NSF RTG Postdoctoral Associate in the Center for Nonlinear Analysis from 2007 to 2010.

Jason Howell

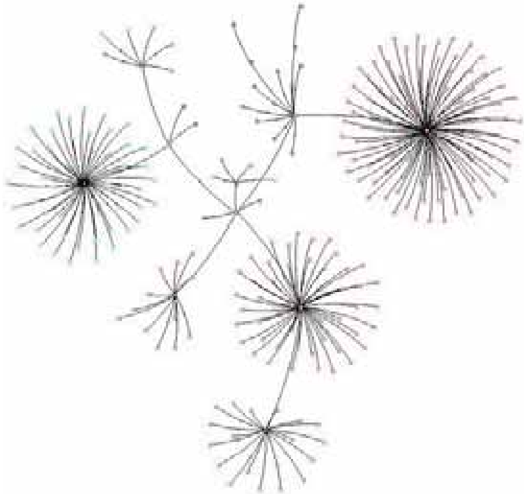
Irene Fonseca Named to Abel Prize Committee

The Norwegian Academy of Science and Letters has appointed Irene Fonseca to the Abel Committee, which is responsible for selecting the winner of the prestigious Abel Prize. The Abel Prize is the most important prize recognizing contributions to mathematics over the course of a career and is considered by many to be the equivalent of a Nobel Prize. Fonseca will serve a two-year term on the committee.

One of the world's leading researchers in the field of applied mathematics, Fonseca has a strong international presence in the mathematics community. She sits on committees and on the boards of several major international universities and research centers, and she served as president of the Society for Industrial and Applied Mathematics from 2013-2014. At Carnegie Mellon, she directs the university's Center for Nonlinear Analysis. Fonseca's research lies at the interface of applied analysis with materials and imaging sciences. In particular, her work focuses on the mathematical study of a variety of novel man-made materials, including ferroelectric, magnetic and magnetostrictive materials, shape memory alloys, composites and liquid crystals. She also studies the variational analysis of denoising, detexturing, inpainting and recolorization in computer vision.

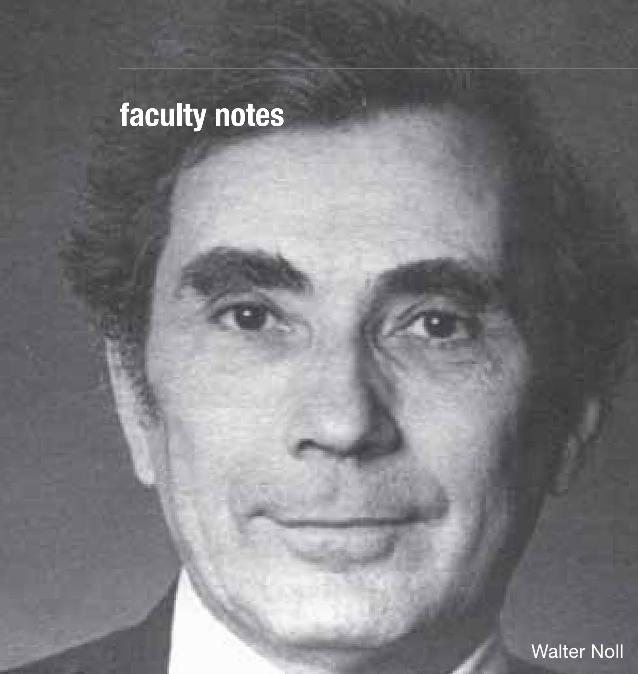
Top: The image above is the logo for the Abel Prize, which recognizes outstanding scientific work in the field of mathematics, including mathematical aspects of computer science, mathematical physics, probability, numerical analysis and scientific computing, statistics, and also applications of mathematics in the sciences.

Alan Frieze Named University Professor



Professor Alan Frieze received the elite distinction of being named University Professor, the highest academic accolade a faculty member can achieve at Carnegie Mellon. The rank of University Professor recognizes a faculty member for representing the intellectual leadership of Carnegie Mellon through their expertise and accomplishments in their respective fields of study.

Frieze is one of the founders of the field of random discrete structures and has remained at the forefront of that field for 35 years. His randomized algorithms have been used to solve important questions in computer science and his work has impacted diverse fields, including business and information networking. In 1991, he received the prestigious Fulkerson Prize for his work on computing the volume of a convex body. His contributions to graph theory led to a plenary address at the quadrennial International Congress of Mathematicians in 2014. At CMU, he co-founded the interdisciplinary Algorithms, Combinatorics and Optimization Ph.D. program, which currently has 21 Ph.D. students across three schools and colleges.



Walter Noll

In Memoriam: Respected Mathematicians, Friends and Passionate Educators

An unlikely pair though they were, Juan Schäffer and Walter Noll were not only colleagues at Carnegie Mellon but dear friends. It all started with World War II. Schäffer, born to a Jewish family in Vienna, Austria, escaped to Switzerland a few months before the war began. After short stays in Switzerland and France, the Schäffer family emigrated to Uruguay in 1939, never to return to their home country. Noll, on the other hand, stayed in Germany throughout the war, having been unwillingly drafted into the German Army in 1943 — his only goal at this time was survival.

Though their beginnings were strikingly disparate, their paths eventually converged at Carnegie Mellon, where their dedication to mathematics created a lifelong friendship. The pair shared a strong belief in improving the way mathematics is taught. This led to their collaboration in establishing the mathematical studies program, which remains an integral part of the Department of Mathematical Sciences' curriculum. Developed as an alternative to the standard education, the program leads students through multiple courses and semesters together, teaching courses in an integrated, sophisticated fashion rather than as isolated topics.

Walter Noll

Professor Emeritus of Mathematics Walter Noll died June 6. He was 92.

Noll was born in Berlin, Germany, and grew up with a propensity for physics and mathematics. After the conclusion of World War II, Noll studied mathematics at the Technical University of Berlin.

As living conditions improved post-war, Noll studied at the Sorbonne in Paris and later earned his Ph.D. in applied mathematics in 1954 from Indiana University.

He began working at the University of Southern California in 1955. A year later, he joined Carnegie Institute of Technology, now Carnegie Mellon, and spent the rest of his career with the university.

Best known for his work in thermodynamics and continuum mechanics, Noll helped develop the Coleman-Noll procedure. The procedure serves as an interpretation to the Second Law of Thermodynamics, which places restrictions on the materials that can occur in nature.

Noll was a fellow of the American Mathematical Society and a founding member of the Society for Natural Philosophy and held seven visiting professorships throughout his career.

He also co-authored "The Non-Linear Field Theories of Mechanics." First published in 1965 and reprinted in 2004, it has become the standard reference work in the field.

"Walter would define mathematics as the art of avoiding unnecessary computations," said William Hrusa, professor of mathematical sciences. "He absolutely wanted to foster independence and creative thinking on the part of his students."

In 1993, Noll retired from Carnegie Mellon, but he continued to teach, write and lecture at the university and around the world.

Twice widowed, Noll married Marilyn Smith Noll in 2000, and they spent their retirement traveling the world and snorkeling in the Great Barrier Reef. He is survived by his wife and children; three stepchildren; three grandchildren, seven step-grandchildren and nine step-great-grandchildren.

Juan Schäffer


Juan Schäffer

Juan Schäffer, professor of mathematics, died February 12. He was 87.

Schäffer earned his doctorates in electrical engineering from the Swiss Federal Institute of Technology and in mathematics from the University of Zürich in 1956. The following year, Schäffer began teaching engineering and mathematics at the Universidad de la República in Montevideo, Uruguay and received a Guggenheim Fellowship in 1959.

In 1968, he joined the Carnegie Mellon faculty as a professor of mathematics, a position he held until his death.

Throughout his academic career, Schäffer published two research monographs, two textbooks and more than 80 research papers. His research focused on functional analysis and differential equations and on geometry in the setting of infinite dimensional spaces. Schäffer's research in these areas had significant impact, and a particular class of infinite-dimensional spaces important in this work now carries his name. He also supervised three doctoral dissertations.

"Juan contributed greatly to the development of the mathematical studies and honors programs in the department in the early 1970s. These programs remain central parts of the undergraduate curriculum," said Russell Walker, teaching professor and acting director of the undergraduate program in the Department of Mathematical Sciences.

Schäffer also had a passion for working with high school students in the Pennsylvania Governor's School for the Sciences.

Outside of teaching, Schäffer served several terms on the Faculty Senate and two terms as chair of the Faculty Organization, helping to develop several key faculty governance policies. He also served as associate dean of the Mellon College of Science from 1986-1993 and as acting dean in 1991.

History was another great passion of Schäffer's. He developed a history of mathematics course for MCS and frequently contributed to archontology.org, a site that records the history of world leaders. Schäffer contributed to his own history as well. In an oral history interview with the United States Holocaust Memorial Museum Oral History Branch, he told the story of his family and his life, from his escape from Nazi-occupied Austria to his career as a mathematician.

He is survived by his son, Alejandro Schäffer, and grandson, Daniel.

Several students interested in the mathematics of finance completed research projects in the summer of 2017 under the direction of Professor Bill Hrusa. These students are pictured above together with Professor Hrusa and Rebecca Doerge, the dean of the Mellon College of Science.

Quant Club Draws Large Crowds in Its Inaugural Year

Inspired by hackathons, a popular event in the field of computer science, Carnegie Mellon's Quant Club held the first-ever Quantathon on a chilly Saturday in February. One hundred fifty students showed up at 9 a.m. to participate in the open-ended problem-solving competition.

"This competition was a great opportunity for students to apply their quantitative skills such as mathematics, probability, statistical analysis and so on in the field of finance. Knowledge in finance was not required, and students with all backgrounds were welcome to participate," said Sijie Wei, a computational finance, statistics and machine learning major and Quant Club member who initiated the idea of the Quantathon.

Steve Shreve, the Quant Club's faculty adviser, created a problem that challenged participants to develop a betting strategy to reach a goal. Teams of students received the problem at 9 a.m. and submitted summaries of their progress at 2 p.m. Based on these summaries, the judges (Shreve, William Hrusa and Po-Shen Loh) chose six teams to present their final results beginning at 5 p.m. At the end of the competition, the judges selected the first, second and third place winners based on their presentations and answer to the problem — with the first-place team finding the optimal betting strategy.

The first-place team was comprised of three Carnegie Mellon sophomores: Raymond Hogenson, a mathematics major;

Patrick Lin, a computer science and mathematics major; and David Zeng, a computer science major.

“The most amazing thing was the number of people who showed up on a Saturday morning. There were 38 teams of three to four students,” Shreve said. “The Quant Club plastered campus with posters, made announcements in relevant classes and announced it through their extensive mailing list. It was great to see their hard work pay off.”

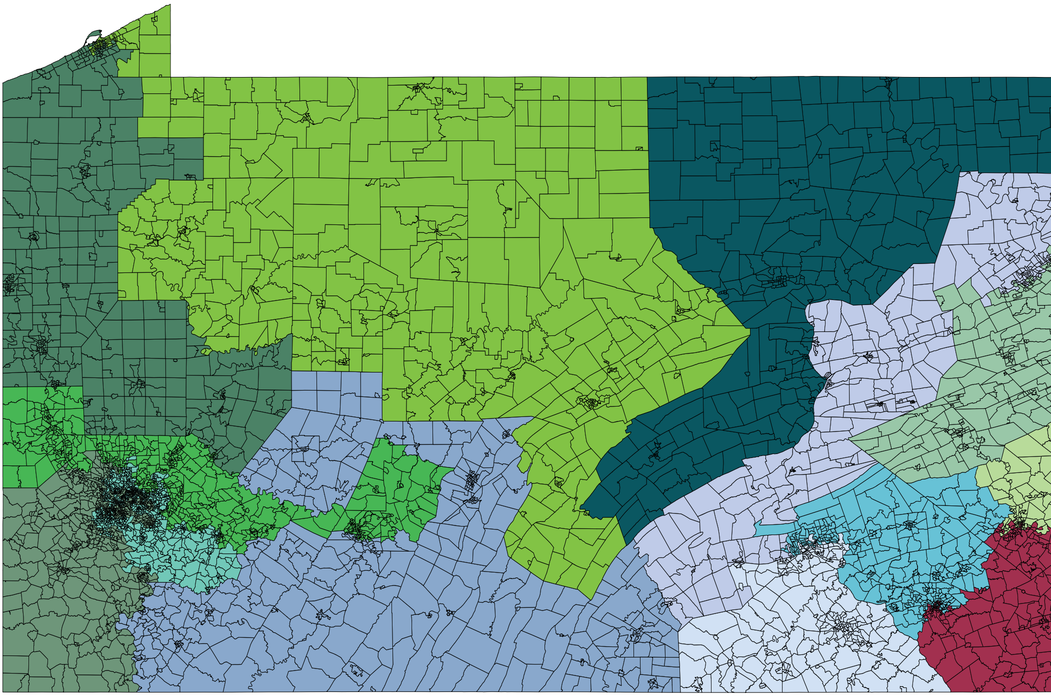
“I cannot say enough good things about this group of students. They are really exciting to work with,” he added.

Initiated in 2016 by a group of students in the undergraduate computational finance program, which is administered by the Department of Mathematical Sciences, the Quant Club brings together students interested in quantitative finance and financial mathematics. Their goal is to prepare students for their future careers and foster relevant undergraduate research.

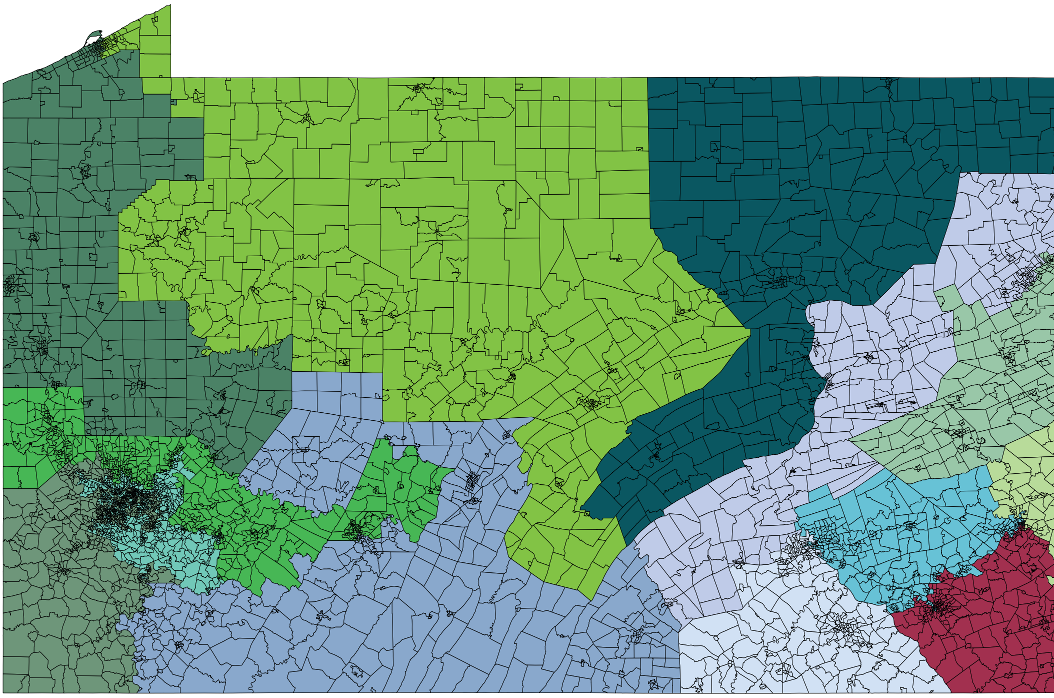
Since its formation, the Quant Club has held a number of successful events, including quant talks, informational sessions about the computational finance degree program, networking events with people from the finance industry and with Carnegie Mellon alumni, and mock interviews that prepare students for the interview process. Students have also led research projects with other students and faculty members.

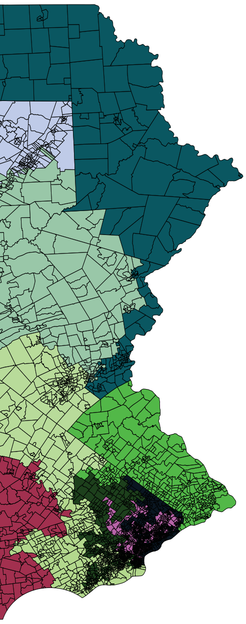
Top: Quant Club officers April Li and Cissy Shi.

Bottom: Jillian Wang at the Quant Club organized mock interview sessions in the fall of 2017, which helped prepare students for job interviews. This student-organized event brought nine experienced finance-industry professionals (from firms including Goldman Sachs, JPMorgan Chase, Morgan Stanley and Citigroup) to campus. Thirty-six students participated and 72 mock interviews were held.



Our method demonstrates the current districting of Pennsylvania is an outlier, in the sense that it is more biased than the overwhelming majority of geometrically similar districtings. The districting above is the current Congressional districting of Pennsylvania, and the districting below is an example of an alternative districting of Pennsylvania preserving the same counties as the present districting, and with similar overall district geometry (as measured by total district perimeter). This example was produced by roughly 750 billion steps of our Markov Chain. Furthermore, the districting below is more fair under various accepted measures of partisan bias.

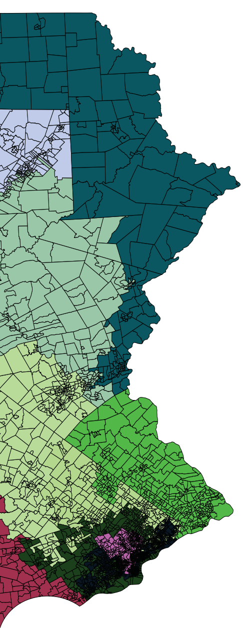




Can Math Solve the Gerrymandering Problem?

CMU mathematicians' work on detecting bias in congressional district maps makes its debut before the U.S. Supreme Court

In early October, the U.S. Supreme Court heard oral arguments in Gill v. Whitford, a case about partisan gerrymandering in Wisconsin. The highly anticipated case could transform the way congressional and legislative district lines are drawn. And mathematicians — including Carnegie Mellon's Alan Frieze and Wesley Pegden — are weighing in. Frieze and Pegden's work is cited in an amicus brief filed on behalf of the plaintiffs that addresses a question at the heart of the case: Is there a way to determine, in an unbiased, practical way, that extreme partisan gerrymandering has actually occurred?



Politicians have been gerrymandering — drawing congressional districts to favor one party or candidate over another — since the early 1800s, and the Supreme Court has considered its constitutionality many times. In 2004, the high court upheld a lower court's ruling that the partisan gerrymander in question was not unconstitutional, in part because no standards existed for adjudicating partisan gerrymandering claims. But Justice Anthony Kennedy posited that such a standard might one day exist. Frieze and Pegden's work could provide that standard.

In a paper published in the *Proceedings of the National Academy of Sciences*, Frieze, Pegden and the University of Pittsburgh's Maria Chikina used a Markov chain (see page 18) to rigorously demonstrate bias in the congressional districting maps of the state of Pennsylvania.

"The idea is, if you want to evaluate whether a particular division of a state is fair, a natural thing to try to do is to compare it to a typical districting — in other words, a random one," Pegden said. "Our method takes the actual layout of where people live, their political affiliation, and tests in a rigorous way whether the current districting is much worse than other typical districtings of the same state."

They began with a current map of Pennsylvania's congressional districts, and then ran the chain, which changed the map in random steps — wiggling little municipalities here and moving little groups of people across boundaries there. The shapes of the districts slowly changed while keeping a roughly equal population in each. The mathematicians observed that their simulation's randomly "redrawn" districts are much fairer that the initial district map they started with.

"Out of all the trillion districtings that we saw in this sequence of random steps, none of them were as bad as the very first one. What that shows is that this initial districting is much worse than other random districtings," Pegden said, adding, "There is no way that this map could have been produced by an unbiased process."

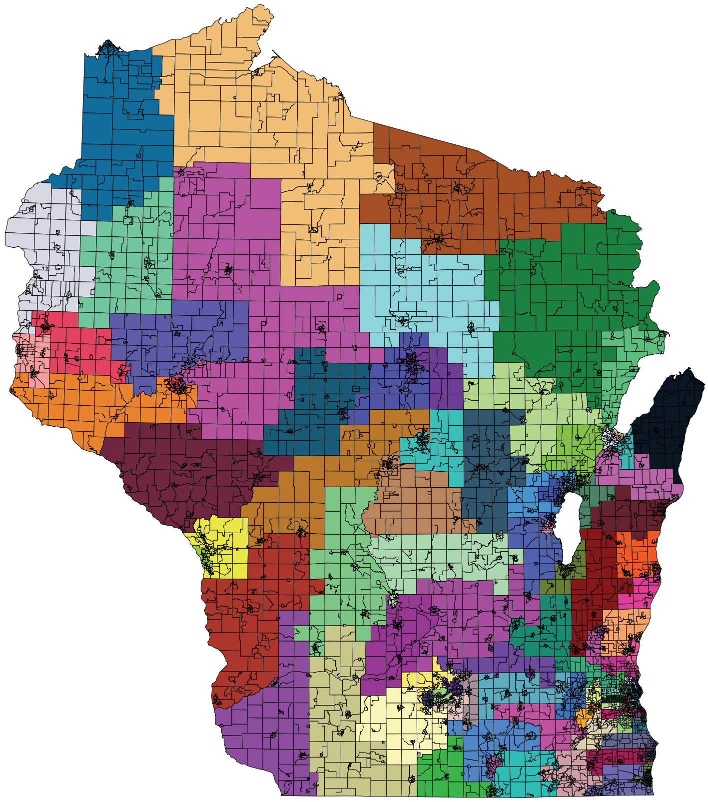
In June, Frieze and Pegden's method became part of a lawsuit filed by the League of Women Voters of Pennsylvania. The lawsuit is asking that the state's congressional district map be thrown out.

Shortly after that, Pegden heard from Nicholas Stephanopoulos, one of the attorneys for the plaintiff in the Wisconsin case. Stephanopoulos was interested in seeing what their analysis could say about Wisconsin. Pegden got to work. "To be doing some math on a deadline because the Supreme Court is going to hear a case? You don't usually have that," he joked.

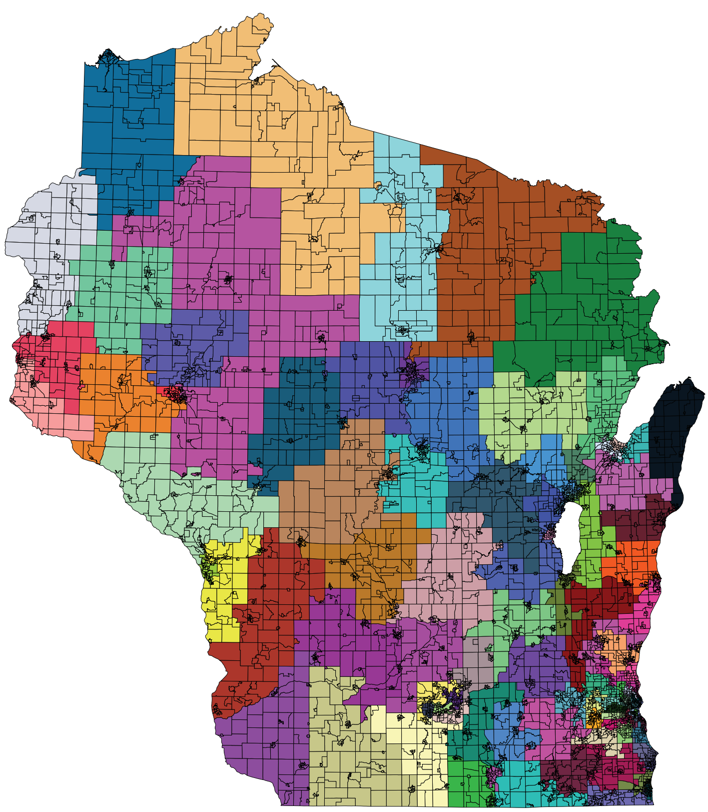
Pegden downloaded the Wisconsin voter data and ran the simulation. It turned out that the Wisconsin result was, in some sense, even cleaner than Pennsylvania's. "In Wisconsin, they did an amazing job gerrymandering. It's a beauty of extreme gerrymandering. It's really unbelievable."

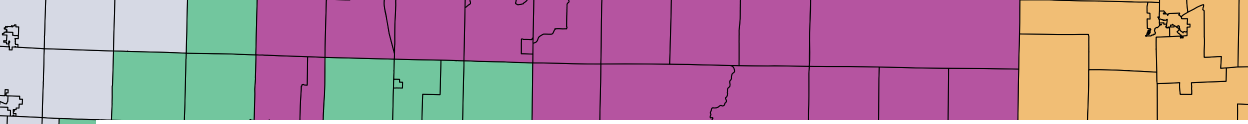
Will the Supreme Court justices agree?
We'll find out next spring.

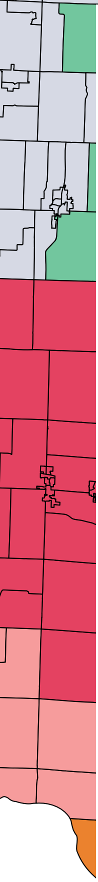
At right, above is the present Assembly districting of Wisconsin, and below that is an alternative map produced by roughly 1 trillion steps of our Markov chain. Note that superficially, the districtings are similar; our test works by showing that the current districting is politically unusual among the districtings with similar geometric properties.



“In Wisconsin ... It’s a beauty of extreme gerrymandering.”







Detecting Gerrymandering with Markov Chains

by Wes Pegden

In past and ongoing legal cases concerning the constitutionality of gerrymandering political districtings, a key question for courts has been the extent to which it is possible to rigorously evaluate claims of gerrymandering in a practical way.

Take, for example, the 2012 elections for congressional seats in Pennsylvania. Democrats won more than 50 percent of the total vote for Congressional seats in Pennsylvania, but won only five out of the 18 seats at stake. To Democrats, this is clear grounds to call foul. But wait! Republicans can (and do) point out that the distribution of Democratic voters itself is likely to cost the Democrats Congressional seats, without any nefarious districting efforts by Republicans. Cities tend to be strong bastions of support for the Democratic party: the current Congressman for Pennsylvania's 2nd district (in Philadelphia) won his general election with more than 90 percent of the vote. On the other hand, no Congressional district in the country is so solidly Republican. Democratic voters really are more concentrated than Republican voters are, which really does create a greater potential for Democrats to waste more votes in districts where they have strong supermajorities, and thus win fewer seats in Congress than Republicans would at the same overall level of support.

The question, then, is how to tell when the districting of a state is responsible for bias toward one political party, rather than just the state's existing political geography. How can we tell when the bias of a particular political districting is really atypical among the space of valid districtings of the same state? To rephrase this mathematically, how can we tell that a districting is more biased than a random valid districting of the same state?

First, we need to have a notion of what constitutes a valid districting of the state. Certainly, districts should be contiguous, and nearly equal in population. There should also be geometric constraints on the districts (for example, ensuring that the square of the perimeter of a district is not much larger than its area) to forbid fractal-like districts. Other requirements for valid districtings may include not splitting many counties, and respecting other natural requirements considered by legislatures.

Let's imagine that we have a bag full of hypothetical "valid" districtings of a state satisfying all of our requirements. Now we just need to pull a lot of random districtings out of our bag and compare them to the actual districting of our state. We would conclude that our districting is gerrymandered if it is more biased toward one political party than the vast majority of the random samples. (For you statisticians: if it's more biased than 99.9999995% of random districtings, it would be gerrymandered at a p-value of .000000005.)

The problem is: How do we pull things out of this bag of districts? In other words, how do we generate random geometric partitions of a region (a state) satisfying various constraints? One natural way to try to draw samples from a weird distribution like this is to use Markov chains, which are used for sampling in areas as disparate as protein folding and statistical mechanics.

Essentially, a Markov chain is a process that takes a random walk along a set of possibilities by making a sequence of small changes. Formally speaking, it is a sequence of random variables

$$X_0, X_1, X_2, X_3, \ldots$$

taking values on some state-space Σ, such that, for each i, and for any states $\sigma_0, \ldots, \sigma_i \in \Sigma$, the conditional probability

$$\Pr(X_i = \sigma_i \mid X_0 = \sigma_0, X_1 = \sigma_1, \ldots, X_{i-1} = \sigma_{i-1})$$

depends only on the choice of i, σ_i, and σ_{i-1}. In other words, the process is a random walk in the sense that it has no memory; where it ends up at step i depends only on where it was at step $i-1$.

In the case of political districtings, a state is divided into thousands of little geographic regions (precincts) used by the census. We can make a small random change to a districting by selecting a precinct on the boundary of two districts, and switching which district it belongs to, so long as the result still satisfies our requirements on valid districtings. The magic of Markov chains is that, under relatively mild assumptions, there are nice theorems that tell you that no matter where you start a Markov chain, after you run it for long enough, it will essentially be at a random configuration, drawn from a *stationary distribution* of the Markov chain; this is a distribution π such that

$$X_0 \sim \pi \Longrightarrow (\forall i) \;\; X_i \sim \pi$$

In other words, the stationary distribution π has the property that drawing a random sample according to π and then running the Markov chain for any number of steps still results in a sample drawn according to π.

In the case of political districtings, this means that we just need to run the chain long enough, and it will eventually spit out random valid districtings of our state according to the stationary distribution of the chain, which is just what we need to make our statistical claims of gerrymandering! We should note that it is possible to carefully construct our chain so that π is uniform over all valid districtings of our state.

The problem is that, though these nice theorems do tell us that our Markov chain will generate random districtings eventually, they unfortunately tell us nothing about how long we have to run it before this is the case. Thus, we can't actually run our chain for a long time and then claim rigorously to have shown that a districting is gerrymandered. For all we know, we may just not have run the Markov chain long enough to see truly random districtings for comparison. This problem actually pervades many scientific applications of Markov chains. Frequently, they are used to generate random samples when there is not enough theory to guarantee that samples being generated are actually random; this can and does lead to disagreement among different research groups about when a result is valid and when it's not. And there have been situations (e.g., simulations of the Potts model from statistical physics) in which practitioners have developed modified Markov chains that they believed, based on evidence from simulations, achieved faster convergence, but later found that the chains still had exponential mixing times in many settings.

In our work with Maria Chikina and Alan Frieze, we proved a new general theorem about Markov chains that avoids this problem when the reason we are using a Markov chain to generate random samples is (as in our application) to demonstrate that a specific element of our space is an outlier. Specifically, we proved the following:

Theorem 1. Let $M = X_0, X_1, \ldots$ be a reversible Markov chain with a stationary distribution π, and suppose the states of M have real-valued labels. If $X_0 \sim \pi$, then for any fixed k, the probability that the label of X_0 is an *ε-outlier* from among the list of labels observed in the trajectory $X_0, X_1, X_2, \ldots, X_k$ is at most $\sqrt{2\varepsilon}$.

Here, being an *ε-outlier* means that the label of the state X_0 is smaller (or larger) than all but an ε fraction of the labels seen on the trajectory; in our application to redistricting, the label of a districting can be the number of seats Republicans would have won in a hypothetical election with the districting in question. The requirement that the Markov chain is reversible essentially means that the random changes the Markov chain makes when taking its random walk can be undone just as easily as they can be done; this is a common feature of Markov chains used for sampling.

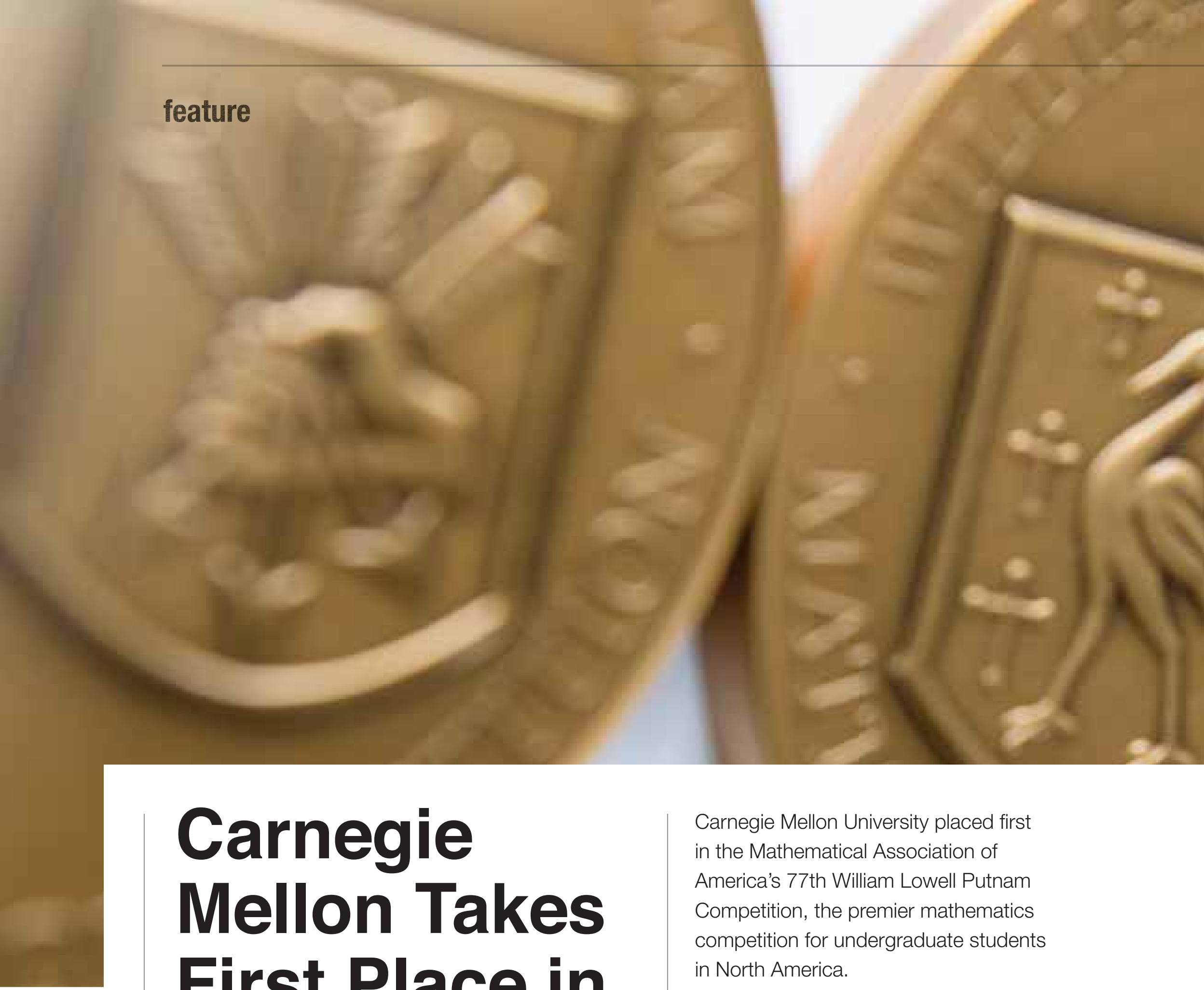
Theorem 1 leads us to the following new statistical test for reversible Markov chains:

The $\sqrt{\varepsilon}$ test: Observe a trajectory $\sigma_0, \sigma_1, \sigma_2 \ldots, \sigma_k$ from the state σ_0, for any fixed k. The event that the label of σ_0 is an ε-outlier among the labels of $\sigma_0, \ldots, \sigma_k$ is significant at $p = \sqrt{2\varepsilon}$, under the null-hypothesis that $\sigma_0 \sim \pi$.

For our redistricting problem, this means that if we start our Markov chain from the current districting of our state, run it for as long as we want (since k is arbitrary here) and then observe that the current districting of our state is more biased than 99.9999995% of the districtings encountered by the Markov chain along the way, then this is significant at

$$p = \sqrt{2 \cdot .000000005} = .0001$$

which, while not as good as significance at $p = .000000005$, is still good enough for government work.



Carnegie Mellon Takes First Place in 2016 Putnam Mathematics Competition

Three Students Named Putnam Fellows for the First Time in University History

Carnegie Mellon University placed first in the Mathematical Association of America's 77th William Lowell Putnam Competition, the premier mathematics competition for undergraduate students in North America.

Three Mathematical Sciences students, junior Joshua Brakensiek and seniors Thomas Swayze and Samuel Zbarsky, scored among the top five of all students, earning them the distinction of being named Putnam Fellows. Forty-four Carnegie Mellon students placed in the top 517, the second most of any university.

This is the sixth consecutive year that Carnegie Mellon's team has placed in the top five of the competition and the first time they have placed first. Prior to the current streak, the university had placed in the top five three other times – in 1946, 1949 and 1987. It's also the first time since 1990 that all three members of any university's Putnam team have been named Putnam Fellows.

"The meteoric rise of the Carnegie Mellon Putnam team reflects a culture of bold innovation at the university. Here

we encourage fresh ideas that challenge paradigms and turn the impossible into the possible," said Po-Shen Loh, associate professor of mathematical sciences and the team's coach.

The university's current success is the result of a great deal of hard work by the students who took the six-hour exam and a concerted effort by the university to create an environment where students can enthusiastically engage in math and problem-solving, take classes in top-ranked programs including math, finance, computer science and statistics, and receive support and mentorship from the research university's award-winning faculty.

Loh brings students from across the university's schools and colleges who plan to take the Putnam exam together each week for relaxed meetings where they

This is the first time since 1990 that all three members of any school's Putnam team were named Putnam Fellows. This was last achieved by Harvard in 1990.

work together to solve problems and socialize. Loh said this extended Putnam team contains some of the most creative analytical thinkers on campus, and he believes this collaboration between students from different disciplines makes all of the students who take the exam stronger and better prepared. The 175 Carnegie Mellon students who took the exam had diverse majors, including mathematical sciences, computer science, statistics, engineering, business, psychology and music.

Among the 44 students who placed in the top 517 were sophomore mathematical sciences major David Altizio, senior mathematical sciences major William Christerson, junior computer science major Jacob Imola, senior mathematical sciences major Ray Li and sophomore mathematical sciences and computer science major Victor Xu, who placed in the top 93, earning them honorable mentions.

Brakensiek, Swayze and Zbarsky, who received $1,000 for the first-place finish and $2,500 for being Putnam Fellows, are part of the Knaster-McWilliams Scholars Program. Funded by two Carnegie Mellon alumni, the program is one of only a few scholarship-supported programs in the country that is paired with an honors program and offers increased access to faculty and research opportunities. The Department of Mathematical Sciences received $25,000 for the first-place finish.

Putnam Statistics

3-Person Team Results

1. CMU
2. Princeton
3. Harvard
4. MIT
5. Stanford

Number of Students in Top 517

98 – MIT

44 – CMU

30 – Princeton

26 – Waterloo

24 – Harvard

22 – Berkeley

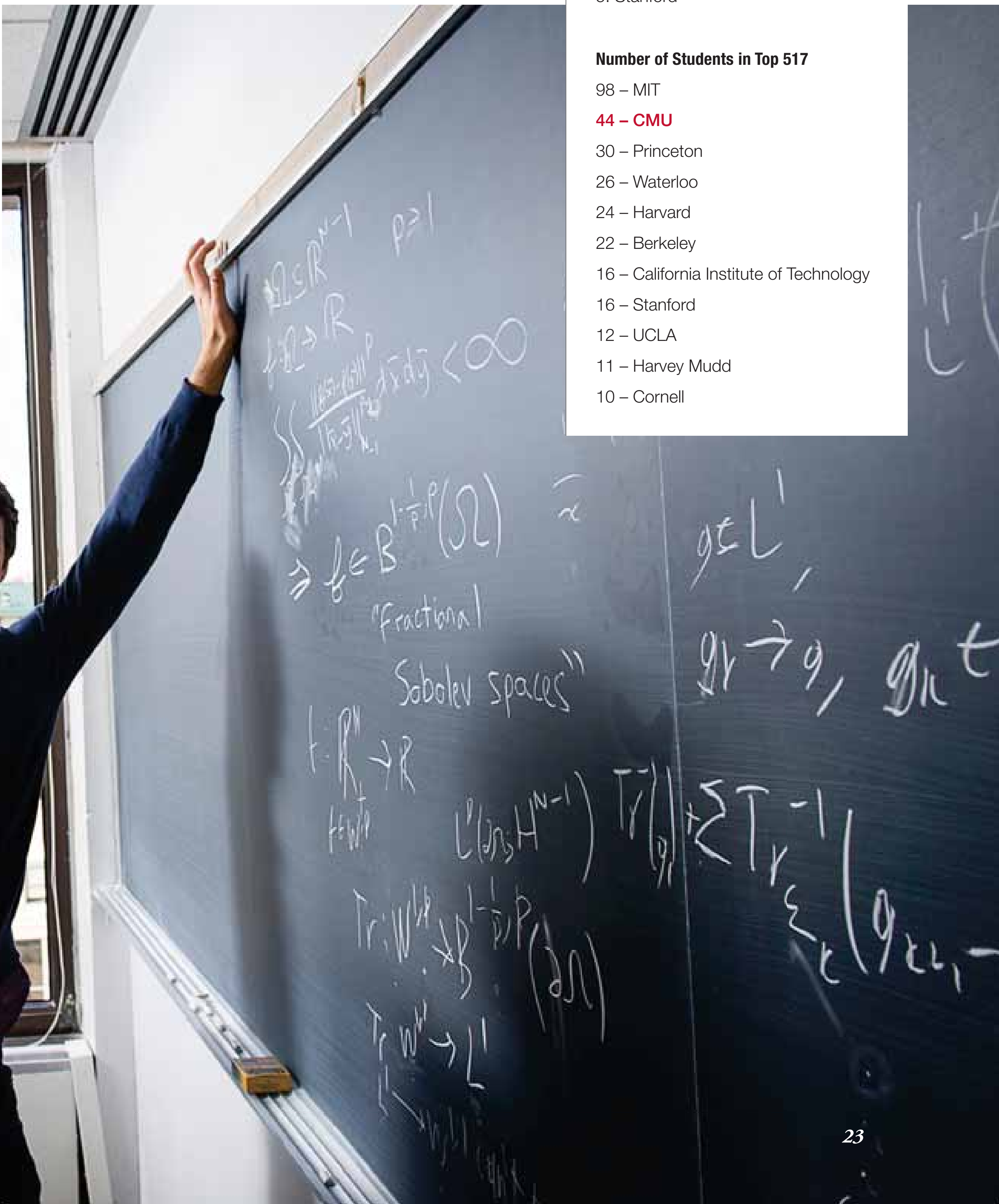
16 – California Institute of Technology

16 – Stanford

12 – UCLA

11 – Harvey Mudd

10 – Cornell



Left to right: Sam Zbarsky, Josh Brakensiek and Thomas Swayze

Alumnus and CMU Trustee Larry Jennings Donates $2 Million to Support SAMS

Carnegie Mellon's Summer Academy for Mathematics and Science (SAMS) marked its 17th year with its largest class ever. This past summer, 128 high school students spent six weeks on CMU's campus taking rigorous, college-preparatory STEM courses. SAMS, which focuses on providing an opportunity for students historically underrepresented in STEM fields, has hosted more than 1,200 students from across the United States, the District of Columbia, Puerto Rico and the Virgin Islands since its inception.

And now, thanks to a historic gift from Math alumnus and CMU Trustee Larry E. Jennings Jr., the program can continue to transform students' lives for many years to come. He and Katherine Jennings donated $2 million to the program — $1.5 million to create an endowed fund and an additional $500,000 for current use.

"The program that eventually became SAMS was the beginning of my connection to Carnegie Mellon, a connection that I have cherished for almost 40 years. Through the program, I launched my education and my career. I also made lifelong friends, some who joined me at CMU, and others who went to other great institutions," Jennings said. "We all share a common connection in this extraordinary summer program that was our launch pad and network of support. I support SAMS to make such a rich experience possible for other young people who are eager to seize this opportunity to study at CMU."

Ty Walton, director of SAMS and the Carnegie Mellon Advising Resource Center, which is now part of the Center for Student Diversity and Inclusion, said the Jennings family's gift, and a gift from the Hopper-Dean Foundation that funded a group of students in the program, were the impetus behind the largest class.

"With a larger class, we're able to reach more students, challenge them and help them discover the wonder and excitement of science, math and engineering," Walton said. "That exposure allows more students to decide that's an area they love and want to pursue."

Walton said SAMS' curriculum has expanded to include enhanced computer programming seminars. This year, the program included biology, chemistry, two sections of physics and four levels of math.

Larry Jennings (left) and Interim President Farnam Jahanian

Jennings has been a longtime advocate of SAMS and friend to the Mathematical Sciences Department. He has served on the department's external advisory boards, he hosts events for fellow math alumni, and the Jennings family supports the Jennings Global Mathematics Scholarships and the Summer Undergraduate Research Fellowships for Math.

"I am proud to support educational opportunities in math and science for diverse students with the potential to change and improve our world," Jennings said.

Jennings earned his bachelor's degree in mathematical sciences and economics in 1984 and his MBA from the Tepper School of Business in 1987. He is senior managing director and one of the founders of ValStone Partners, a private equity firm with offices in Baltimore and Birmingham, Michigan.

Math Alumnus David Simmons Recognized for Exceptional Business Leadership

During his senior year at Carnegie Mellon, David Simmons plastered his walls with rejection letters from failed job interviews. It proved a pivotal moment for Simmons — he may have been knocked down, but he wasn't out. Flash forward 30 years, and he's receiving the prestigious Louis V. Gerstner, Jr. award for exceptional business leadership for his work as CEO of Pharmaceutical Product Development (PPD), a company with 20,000 employees and offices in 47 countries.

"It's a good thing I was resilient," Simmons said. "There was a lot of potential packed into that clueless 21-year-old."

Simmons came to Carnegie Mellon with an interest in and an aptitude for science and math. He majored in applied mathematics, later adding an additional major in industrial management. And while he may have been inexperienced when it came to the finer points of the job interview, Simmons was no slouch. He excelled at his coursework, making the dean's list for three straight years after a rocky start his freshman year. Those quantitative abilities weren't lost on Wheeling-Nisshin Steel, who sent Simmons a job offer — not a rejection letter.

After earning his bachelor's degree, Simmons joined the West Virginia-based company, designing their business software systems. After 10 years in the steel industry, he made the switch to pharmaceuticals when Pfizer offered him the opportunity to run its EMEA technology group. Over the next 15 years, Simmons climbed the pharmaceutical giant's corporate ladder, eventually becoming president and general manager of the emerging markets and established products business units. He was responsible for approximately $19 billion of Pfizer's revenue base. In June 2012, Simmons left Pfizer to become chairman and CEO of PPD, a clinical research organization that runs trials for all of the top biopharmaceutical companies.

"The joy of seeing a new medicine approved by the FDA and of improving patients' health is incredibly satisfying," Simmons said. He also enjoys the complexity of the business. At any one time, PPD is conducting clinical research for more than a thousand studies around the world.

It's in this space that Simmons taps into his Carnegie Mellon math training.

"The complexity of problems I was forced to confront by earning a math degree at CMU gave me the discipline and confidence to deconstruct incredibly complex challenges. When that happens, real insights emerge and I can make quality decisions to move PPD forward," he said.

He certainly has made good decisions for PPD. Under his leadership, PPD has doubled its workforce from 11,000 to 20,000 and its profits from $350 million in 2011 to over $700 million in 2017. The company accelerated its transformation into a dynamic, global contract research organization widely regarded as one of the most respected and innovative in its industry, according to a press release from The Carlyle Group, which presented Simmons with the Louis V. Gerstner, Jr. Award for management excellence this past September. The award is named for Gerstner, the legendary chairman and CEO of IBM, who has a say in who wins the award. To Simmons, Gerstner is a tough critic so winning the award was a big deal. "It's the most valuable recognition I've ever received."

Summer Undergraduate Research in Mathematics

As a first-year student, Michael Spoerl had his first introduction to the field of real analysis through a vector analysis course with Giovanni Leoni, professor of mathematical sciences, and knew he had found his niche. His subsequent analysis classes are where Spoerl first learned of the theory of Sobolev spaces, which provides an important framework through which to study and find solutions to partial differential equations. Though he'd never heard of the term before, Spoerl became fascinated by the concept.

After receiving a Summer Undergraduate Research Fellowship (SURF) this past summer, Spoerl worked under Leoni, his research advisor, researching and refining the conditions that determine if a function is in a Sobolev space.

Spoerl spent the summer combing the current research, reading papers and solving problems with known Sobolev space solutions. After getting a feel for these problems, Spoerl began solving open problems that the papers hadn't solved and coming up with new conditions to see whether a function is in a Sobolev space and whether the function is constant.

"I really liked it because I didn't have to show up at a lab at a set time every day;
I could work wherever I wanted, whenever I wanted, which happened to be just about all the time," said Spoerl.

Spoerl was drawn to the SURF program, he said, because it felt like a natural extension of his classes, allowing him to dive deeper into his interests and open the door to deciding his post-graduate path. Inspired by his undergraduate experiences, Spoerl plans to pursue his Ph.D. in an analysis-related field and continue his work on Sobolev spaces.

Facilitating research experiences like Spoerl's is a priority for the Department of Mathematical Sciences. The SURF program enables undergraduate students to pursue engaging and in-depth research during the summer months, and the experience can be transformative. The department is raising endowed funds to ensure the SURF program becomes a permanent fixture for mathematical sciences' students. David Martin (S 1987) and his wife Jacqui are matching gifts that are made to the endowed Martin Summer Undergraduate Research Fund to further support the department's endeavor. See math.cmu.edu/giving for details.

Summer 2017 Math SURF Research Projects

Matthew Bowen
On the Sprague-Grundy Values of Auxiliary-Nim
Advisor: Wesley Pegden
(Image on left page)

William Cooperman
Large Equiangular Sets of Lines
Advisor: Boris Bukh

Xinhui Guo
Index Fund with PCA and Factor Model
Advisor: David Handron

Shuailin He and Huiwen Zhang
Building and Calibrating a Binomial Asset Pricing Model to Reflect Market Data
Advisor: David Handron

Raymond Hogenson
The Longest Common Subsequence in Shifted Words
Advisor: Boris Bukh

Serhan Kiliccote
A New Planar Graph with Lazy Cop Number Greater than Three
Advisor: David Offner

Nicholas Sieger
Generalized Cycle Double Covers
Advisor: Mary Radcliffe

Michael Spoerl
On Sufficient Conditions for a Measurable Function to be Constant
Advisor: Giovanni Leoni

Xiaorong (Sean) Zhang
Towards a Multicolor Version of Rado's Theorem
Advisor: Boris Bukh

Yisu Zhou and Guoyi Jiang
Multi-period Portfolio Utility Optimization via Withdrawal Optimization On Multi-period Binomial Model
Advisor: David Handron

Meeting of the Minds Poster Competition

First Place: David Simmons Prize for Undergraduate Research in Mathematics:
Sam Zbarsky
"*Energy Decay in a Fluid with a Free Boundary*"

Runner-up:
Ananya Kumar (CS major)
"Streaming Algorithms for Approximate Convex Hulls"

Honorable Mention:
Manuel Fernandez
"*Online Purchasing of Minimal Spanning Trees*"

Liuyu Jin
"*Existence, Uniqueness and Asymptotic Behavior of Solutions to a Linear System of PDEs with Weak Viscoelastic Damping*"

Tim Li
"*L^1 Regularization for Compact Support*"

Other Math Major Award Winners:

Christine Shen
Award for artistic excellence for "Word Play"

Benjamin Teo
Statistics poster competition, second place for "Determining the Structure of a Population of Neurons"

Jeffrey Phillips
Statistics poster competition, young researcher award for "An Extension of the Kelly Criterion for Scenarios with Discrete Bet Amounts"

Math Major Participants:

Jiaping Bian, 2018
Jingyi Chen, 2018
Zixuan Chen, 2018
Tianyu Gu, 2019
Ran Huan, 2019
Guoyi Jiang, 2018
Liuyu Jin, 2018
Timothy Li, 2018
Hang Liao, 2019
Cameron Montag, 2019
Jacob Neumann, 2019
Gidon Orelowitz, 2018
Jeffrey Phillips, 2018
Liyunshu Qian, 2017
Samuel Schneider, 2017
Moqing Shi, 2018
Abigail Smith, 2016
Benjamin Teo, 2017
Kyle Weaver, 2017
Yi Yang, 2018
Xingjian Yu, 2019
Samuel Zbarsky, 2017
Yisu Zhou, 2019

Alumna Dives into the World of Finance

After graduating this past May, Jackie Hudepohl hit the ground running in her first job as a quantitative trader at IMC Financial Markets in Chicago and hasn't stopped since.

Hudepohl works on the FICC desk trading options on cattle and hog futures, spending most of her time analyzing trades and considering ways to improve trading through the company's automated system.

"My job is both varied and competitive. No work day looks the same," she said. This is exactly what excites her most about the position. "Some days I'm focused on operational responsibilities — monitoring our trading system, manually hedging positions, etc. — while on other days I get to work on longer term quantitative modeling projects like improving a trading algorithm, statistically analyzing how to better parameterize a model."

As an undergraduate, Hudepohl majored in mathematical sciences, with a concentration in statistics, and minored in computational finance and computer science. She's proud to say that all of these elements are present in her current role.

"From an academic perspective, I get to use and develop the mathematical and technical concepts that I found interesting in college. My courses in computational finance are relevant as they provide a (simplified) framework for some of the models that we use at work. Also, most of what we do is very automated and involves large amounts of data, so I use a lot of what I learned in various computer science and statistics classes," said Hudepohl.

Initially, Hudepohl saw herself heading down a path in computer science, pursuing a graduate degree in a machine learning-related field. But she kept coming back to her studies in computational finance, which encompass all her interests in quantitative modeling, statistical analysis, practical applications and unsolved problems. With that, she decided trading was the avenue to pursue and hopes to one day be able to bring her real-world experience to teaching.

Her experience at Carnegie Mellon was instrumental in not only developing her interests but challenging her in each and every way, which has prepared her to do the same in her professional career. "CMU taught me how to think critically and attack a problem. Knowing how to make headway on a large problem space and isolating the most critical parts of a solution are skills I continue to use and refine on a daily basis," said Hudepohl.

Mathematical Sciences Class of 2017

22
Employed: Software Industry

19
Employed: Financial Industry

13
Employed: Other Industries

21
Other

18
Continuing Education

11 – Master's Degree

7 – Doctoral Programs

Carnegie Mellon – Machine Learning

Carnegie Mellon – Mathematics

Stanford – Computer Science

Princeton – Mathematics

UCLA – Mathematics

U Mass-Amherst – Computer Science

University of Surrey, UK – Hospitality and Tourism Management

Firms that hired more than one member of the class of 2017:

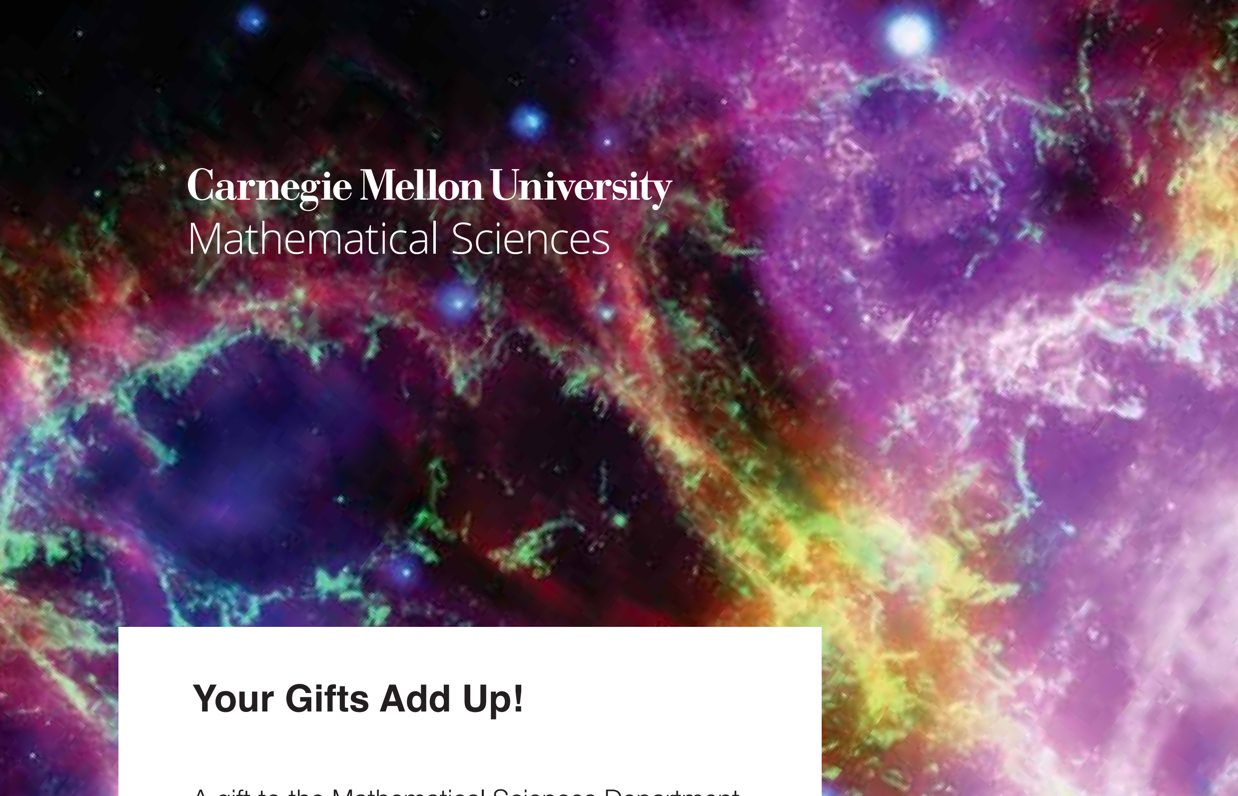
citi

Deutsche Bank

Goldman Sachs

Google

Median starting salary for graduates in industry: **$85,000** (of 36 salaries reported in the exit survey)



Carnegie Mellon University
Mathematical Sciences

Your Gifts Add Up!

A gift to the Mathematical Sciences Department expresses your love for mathematics and your commitment to our vision. We encourage you to direct your gift to one of three priority funds:

- **Math Summer Undergraduate Research Fund:** Gifts to this fund will enable students to undertake in-depth research during the summer months.
- **Computational Finance Undergraduate Program Endowment Fund:** Gifts to this fund will support expansion of the program, introduce a summer research option, expand support for the Quant Club, invest in curriculum updates and sponsor alumni events and travel to competitions.
- **Innovation Fund for Mathematics:** Proceeds from this endowed fund support activities that enrich the undergraduate experience for mathematics majors and faculty recruiting.

To make a gift, visit

math.cmu.edu/giving

For more information or to have a confidential discussion about your philanthropy, please contact the Mellon College of Science Development Staff:

LISA GEORGI
Director of Development
409A Mellon Institute
412-268-2019
lgeorgi@andrew.cmu.edu

Carnegie Mellon University
Mathematical Sciences

Wean Hall 6113
5000 Forbes Avenue
Pittsburgh, PA 15213

math.cmu.edu