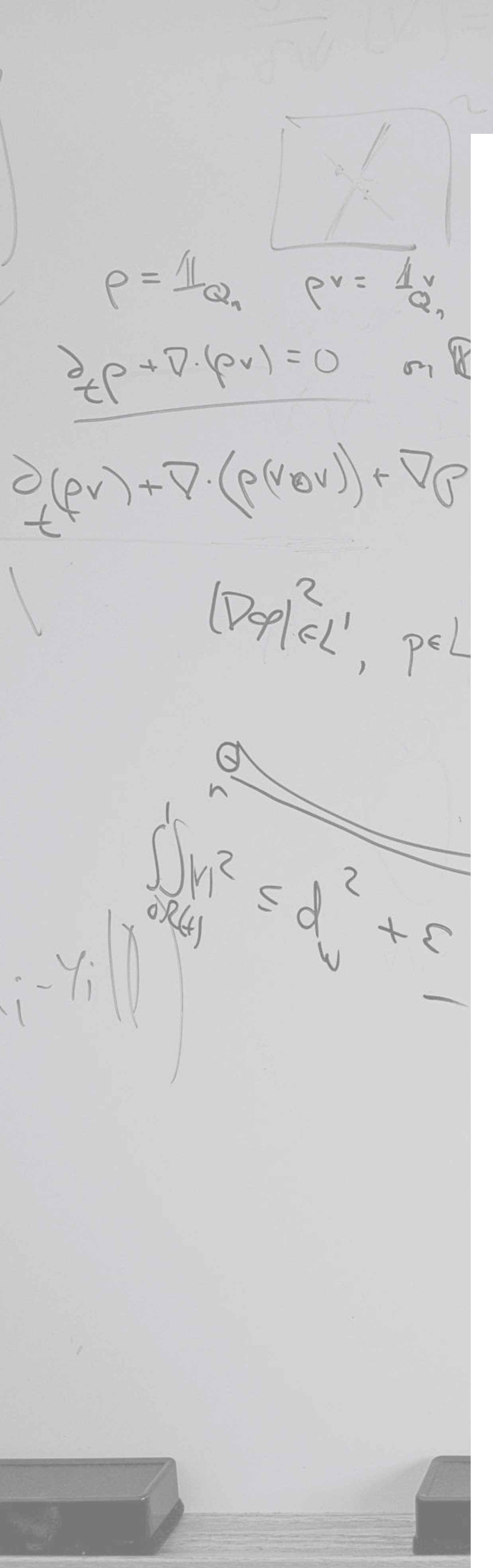
mathematical sciences news 2015

Carnegie Mellon University



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2015

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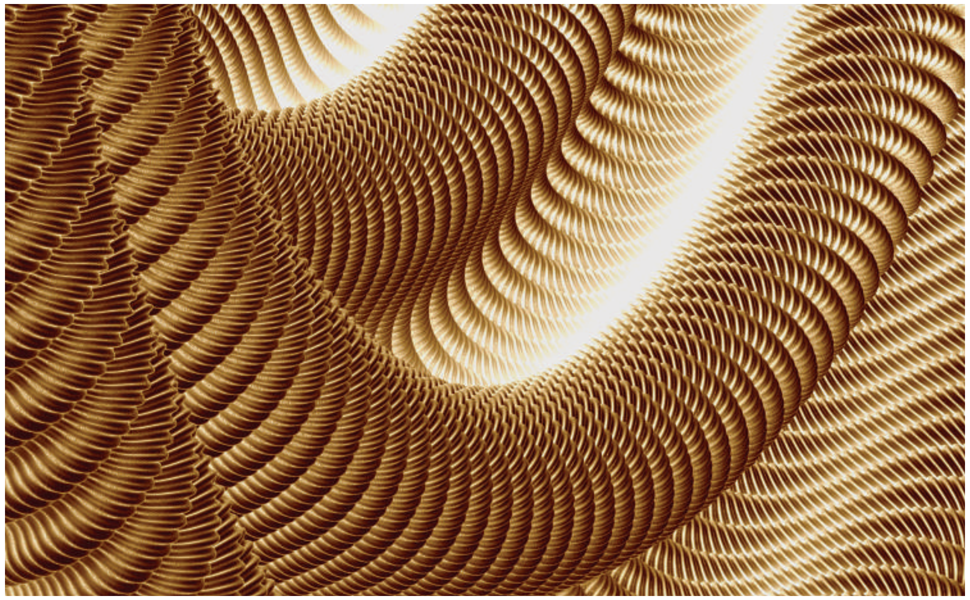
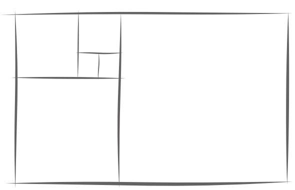


About the cover

The image on the cover is a visualization of an isometric embedding of a flat torus into 3-dimensional space. The proof of the existence of such embeddings is one of the celebrated mathematical contributions of CMU alumnus John Nash. The visualization on the cover was recently produced by the Hevea Project. See **math.univ-lyon1.fr/~borrelli/Hevea/Presse/index-en.html**. Further discussion can also be found on page 13.

A visual proof of the following fact can also be found on the cover. If F_n is the n th Fibonacci number then

$$F_n F_{n+1} = F_1^2 + F_2^2 + \cdots + F_n^2$$



Letter from Mathematics Department Head, Tom Bohman

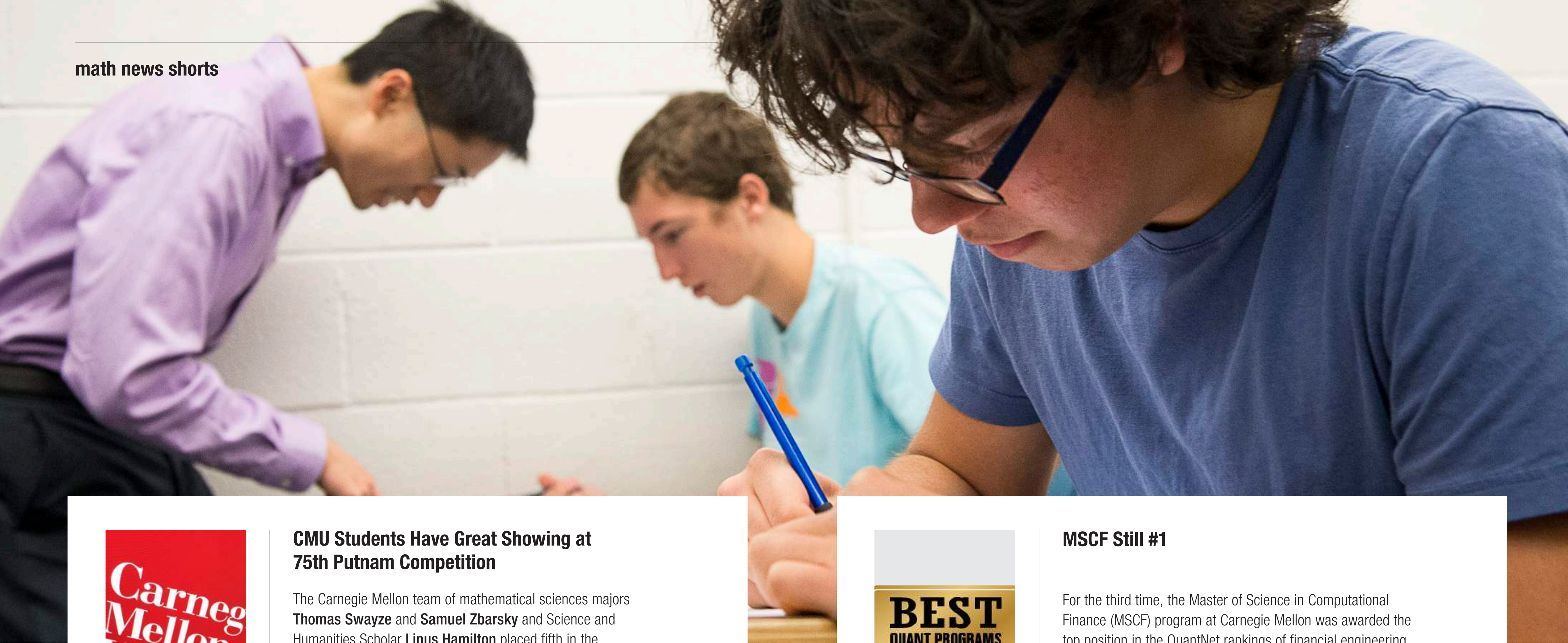
May 2015 saw the passing of one of our most famous mathematics alumni — John Forbes Nash Jr. — who graduated from Carnegie Tech in 1948 with bachelor's and master's degrees in mathematics. He went on to complete a Ph.D. in mathematics at Princeton. His dissertation introduced the game theoretic notion now known as the Nash equilibrium, work for which he was awarded the Nobel prize in Economics in 1994. Nash also solved some of the most famous mathematics problems of the day, including Hilbert's 19th problem and the problem of isometric realization of abstract Riemannian manifolds in Euclidean space. Nash was awarded the 2015 Abel Prize for these seminal contributions that had an enormous impact on the fields of nonlinear partial differential equations and geometric analysis. This issue of the Mathematical Sciences Newsletter includes retrospectives on John Nash's life and his remarkable mathematics.

Many aspects of John Nash's storied career are reflected in the current life of the Department. We have a major effort in the study of partial differential equations within the Center for Nonlinear Analysis, which has a rich history of contributions to the field. The Department also has a significant research effort in the mathematics of finance, which, like game theory, is an area in which the application of mathematics impacts economics. We have educational programs in the mathematics of finance at the B.S. and Ph.D. levels, and we play a major role in CMU's interdisciplinary Master of Science in Computational Finance program, which continues to garner the number one ranking (see pg. 5).

The parallel with Nash also extends to the realm of our undergraduate program. Today we have a very active Math Club, just as the university did 70 years ago when Nash was a member (see images on page 10). Nash has something else in common with current students — very strong showings on the Putnam exam (see pg. 4).

The demand for mathematics courses and the mathematics major continues to grow. Our success in drawing students to the mathematics degree continues to bring many challenges to the Department. Through the generous contributions of several alumni, we recently initiated an endowed **Innovation Fund for the Mathematical Sciences** that is helping the Department meet these challenges. This fund is already supporting undergraduate research and capstone experiences in mathematics, but there is still much more to be done. In the future we hope to use this fund to support the Math Club and to assist in faculty recruiting and retention.

I hope that many alumni of the Department of Mathematical Sciences have a chance to reconnect with the Department by visiting **math.cmu.edu/alumni**. Let us know what's new with you!



CMU Students Have Great Showing at 75th Putnam Competition

The Carnegie Mellon team of mathematical sciences majors **Thomas Swayze** and **Samuel Zbarsky** and Science and Humanities Scholar **Linus Hamilton** placed fifth in the Mathematical Association of America's 75th William Lowell Putnam Competition, the premier mathematics contest for undergraduate students. Additionally, CMU had 55 students who scored among the top 507, the second most of any university.

Number of students in the top 507:

MIT	82
Carnegie Mellon	55
Harvard	32
Stanford	19
Berkeley	16
Princeton	15
Yale	15

Ten of the 55 CMU students who placed in the top 507, including the three members of the official team, are part of the university's Knaster-McWilliams Scholars program, which has been funded through the generosity of a physics alumnus and a mathematics and electrical engineering alumnus. It is one of only a few scholarship-supported programs in the country that also is paired with an honors program that features increased access to faculty and early research opportunities.

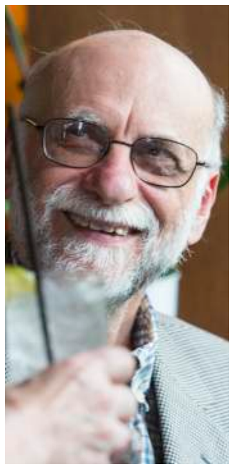
MSCF Still #1

For the third time, the Master of Science in Computational Finance (MSCF) program at Carnegie Mellon was awarded the top position in the QuantNet rankings of financial engineering programs. In addition to the #1 ranking for 2015, MSCF shared the #3 spot for financial engineering programs with the best employment outcomes. More than 50 of the world's largest financial services firms recruit from the MSCF program.

"Carnegie Mellon created the first professional degree in quantitative finance more than 20 years ago, and we have invested substantial resources toward the goal of making ours the best quantitative finance degree in the world," said Steve Shreve, the Orion Hoch University Professor of Mathematical Sciences and one of the founders of the CMU MSCF program.

> *"It is gratifying to see that even though scores of similar programs have since been created at many other elite universities, our number one standing continues."*

MSCF is a joint program between the Department of Mathematical Sciences, the Tepper School of Business, the Dietrich College of Humanities and Social Sciences' Department of Statistics, and the Heinz College. The Department of Mathematical Sciences plays a critical role in the MSCF program, teaching a third of the curriculum and providing leadership in policy matters.



FriezeFest 2015

More than 150 probabilists, discrete mathematicians and theoretical computer scientists gathered on Carnegie Mellon's campus July 27-31 for the 17th International Conference on Random Structures and Algorithms. On the first day of the conference, participants attended FriezeFest 2015, a day of talks in honor of the 70th birthday of Alan Frieze.

A professor of mathematical sciences, Frieze has been a leading force in probabilistic combinatorics and randomized algorithms. His research has created key, versatile tools used by scientists to advance both fields. His polynomial-time algorithm for approximating the volume of a convex body (joint work with M. Dyer and R. Kannan) has had a lasting impact on theoretical computer science. Another of Frieze's major contributions (also joint work with R. Kannan) is a weak version of the Szemerédi regularity lemma, which has become a critical tool in combinatorics.

At the 2014 International Congress of Mathematicians, Frieze delivered an invited plenary address, "Random Structures and Algorithms." Frieze is an AMS, SIAM and Simons Fellow. In 1991, he (along with M. Dyer and R. Kannan) received the Fulkerson Prize in Discrete Mathematics from the American Mathematical Society and the Mathematical Programming Society.

Methods of Mathematical Finance

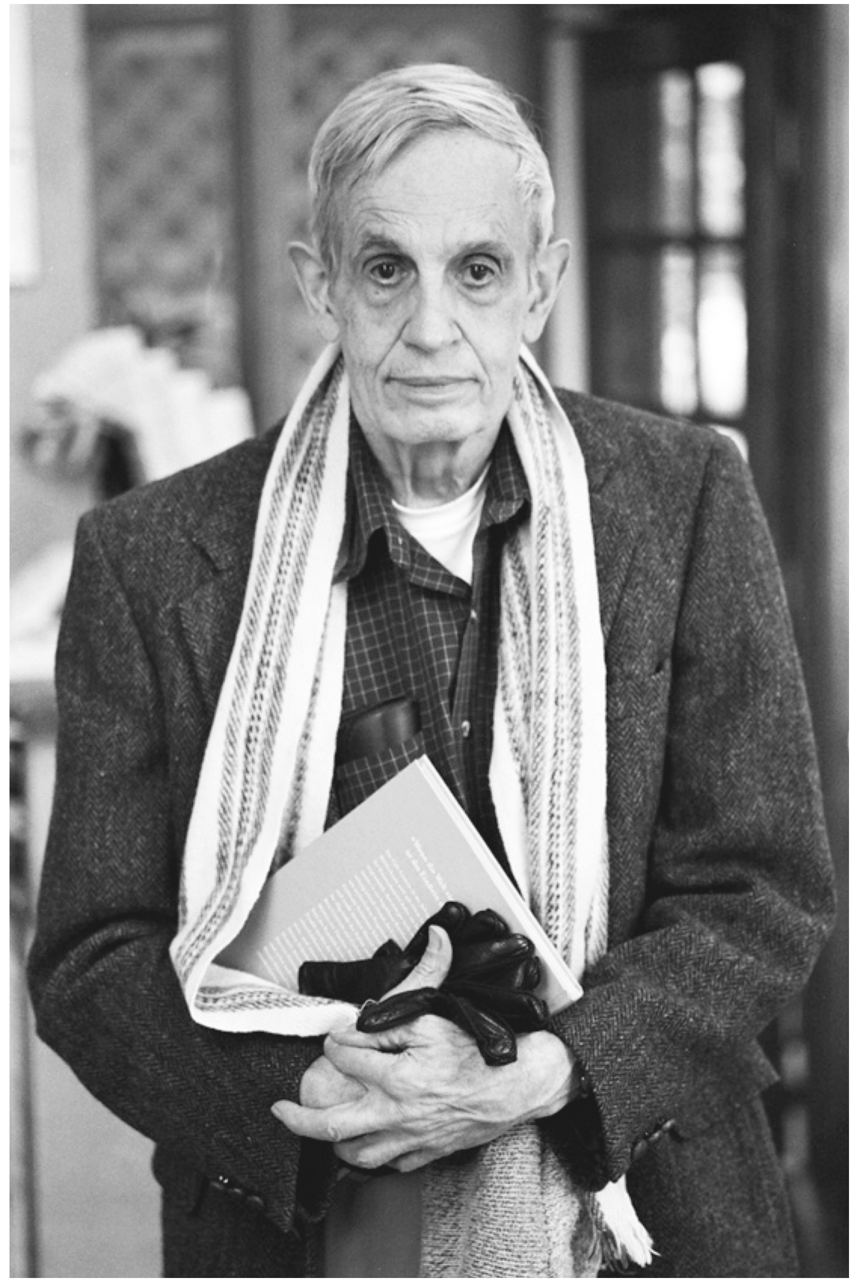
Mathematical finance researchers working at universities and financial services companies around the world came together in early June for a five-day conference in honor of Steve Shreve's 65th birthday. Topics ranged from option pricing and dual optimal martingale measures to the proof of the fundamental theorem of asset pricing.

Shreve, the Orion Hoch Professor of Mathematical Sciences, is internationally recognized for his role in laying the foundations for the modern mathematical theory of optimal portfolio construction in the presence of market uncertainty and his other work in mathematics applied to finance, including the development of models for pricing exotic derivative securities and convertible bonds. In addition to his research, Shreve helped found CMU's highly regarded bachelor's, master's and doctoral programs in computational and mathematical finance. Several of the programs' alumni, including Karel Janeček (see story on page 16), attended the conference to honor Shreve.

In 2013, Shreve was named a University Professor, the highest academic accolade a faculty member can achieve at CMU. In 2000, he received CMU's highest education award, the Robert E. Doherty Award for Sustained Contributions to Excellence in Education. Shreve is a fellow of the Institute of Mathematical Statistics.

Alumnus and Nobel Laureate John Nash Receives the Abel Prize

Written By Amy Pavlak Laird



John Forbes Nash, Jr. by Peter Badge

▲ *John Nash with the Carnegie Tech Math Club. 1947*

◄ *John Nash, graduate school portrait. 1950. Courtesy of Princeton University Library.*

When John F. Nash Jr. was in elementary school, he was doing arithmetic with larger numbers than his classmates. "I would have several digits and they would have maybe two or three digits," Nash recalled. In high school he was reading "Men of Mathematics" by E.T. Bell, and was proving Fermat's little theorem.

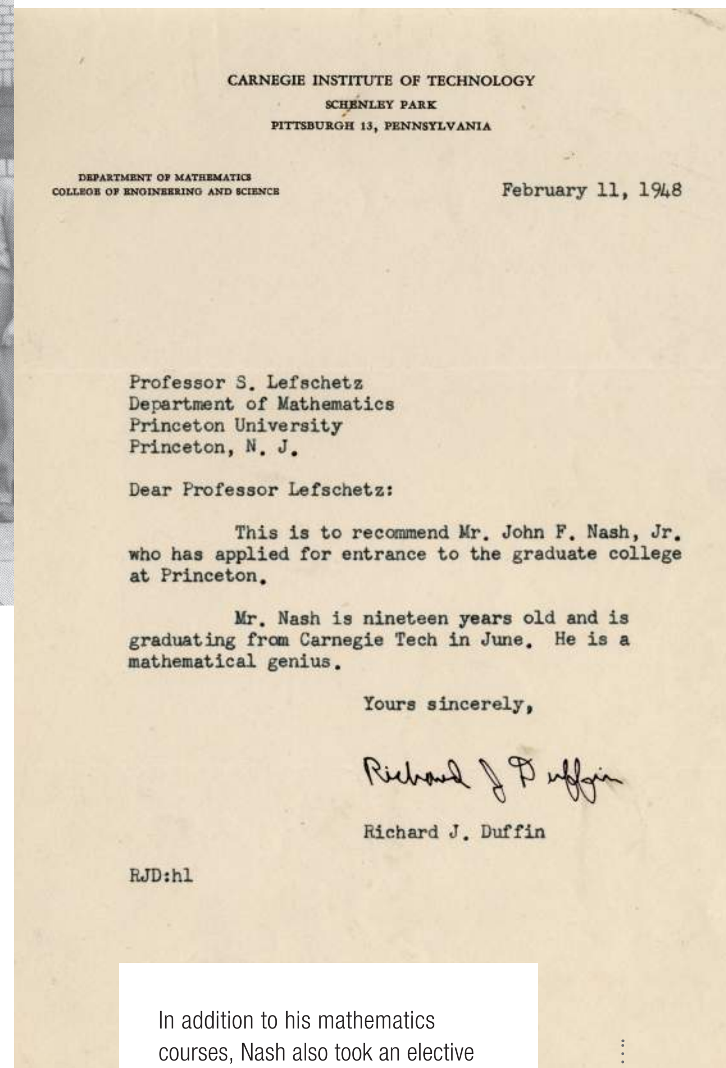
Even though his mathematical ability was evident early on, when Nash set foot on Carnegie Mellon's campus (then Carnegie Tech), he was there as a chemical engineering major. He soon found that neither chemical engineering nor chemistry suited his interests. Noticing his natural talent in mathematics, the faculty in the Department of Mathematics assured him that he could have a good career as a mathematician and encouraged him to switch his major.

There can be no doubt that the Carnegie Tech math faculty members were right about Nash and his future. Nash reached the pinnacle of the mathematical field in March 2015, receiving the Abel Prize — the most important prize honoring contributions to mathematics over the course of a career — for his work on partial differential equations. Less than a week after he received the award in a ceremony in Norway, Nash and his wife died in a car accident in New Jersey. He was 86.

Nash excelled as an undergraduate mathematics student at Carnegie Tech. He published articles on number theory and topology in The Carnegie Technical, a student-run magazine produced by students in the College of Engineering and Science. And he participated in the William Lowell Putnam Mathematical Competition in 1946, winning an honorable mention, and in 1947, placing among the top 10.

Nash's classmates definitely recognized his talents. Raoul Bott, who earned his Ph.D. from Carnegie Tech in 1949, recalled taking Professor Richard Duffin's course on Hilbert spaces with Nash, who was an undergrad at the time. "We were reading von Neumann's book on quantum mechanics, which developed Hilbert spaces at the same time," Bott said. "And it soon became clear that Nash was ahead of all of us in understanding the subtleties of infinite-dimensional phenomena."

Duffin, who had a long and influential career as a member of the faculty of the Mathematical Sciences Department, also was aware of Nash's ability. In a letter recommending Nash for graduate studies at Princeton University, Duffin didn't mince words, "He is a mathematical genius."



CARNEGIE INSTITUTE OF TECHNOLOGY
SCHENLEY PARK
PITTSBURGH 13, PENNSYLVANIA

DEPARTMENT OF MATHEMATICS
COLLEGE OF ENGINEERING AND SCIENCE

February 11, 1948

Professor S. Lefschetz
Department of Mathematics
Princeton University
Princeton, N. J.

Dear Professor Lefschetz:

This is to recommend Mr. John F. Nash, Jr. who has applied for entrance to the graduate college at Princeton.

Mr. Nash is nineteen years old and is graduating from Carnegie Tech in June. He is a mathematical genius.

Yours sincerely,

Richard J Duffin

Richard J. Duffin

RJD:hl

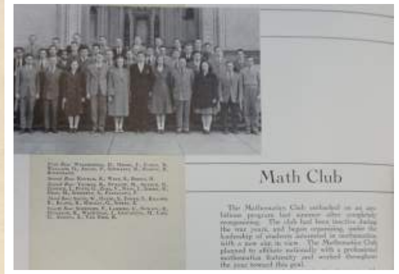
In addition to his mathematics courses, Nash also took an elective class in international economics. That exposure to economic ideas and problems was the spark that eventually led to Nash's interest in game theory.

Nash graduated from Carnegie Tech with bachelor's and master's degrees in mathematics in 1948. He went on to earn his doctorate at Princeton and was a member of the mathematics faculty at the Massachusetts Institute of Technology from 1951-58. He later returned to join the faculty at Princeton as a senior research mathematician.

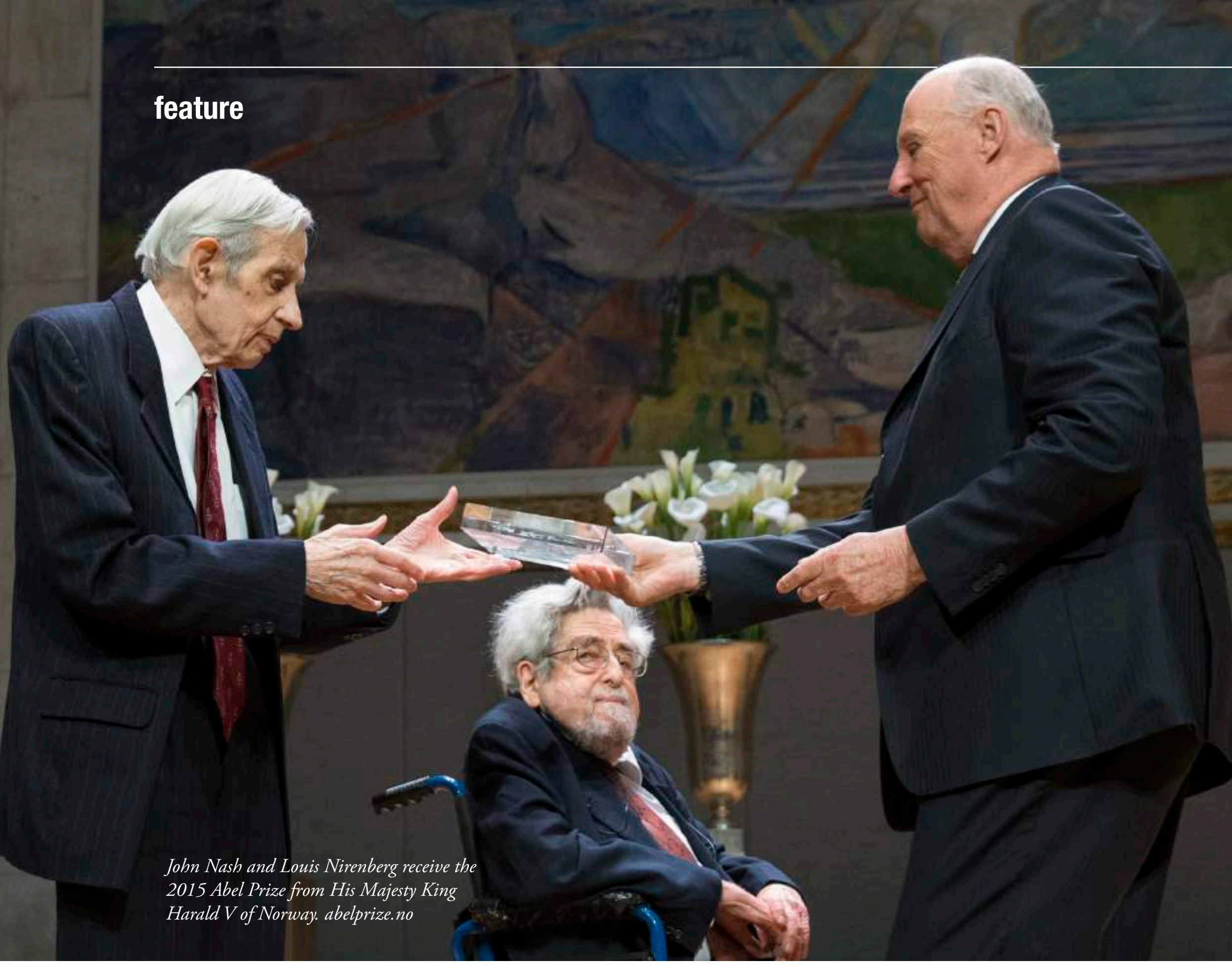
◄ *Richard J. Duffin's letter recommending John Nash for graduate study at Princeton. Courtesy of Princeton University Library.*

▼ *1947 Carnegie Tech Math Club.*

▼ *John Nash received an honorary degree in economics from Carnegie Mellon in 1999.*



"He is a mathematical genius."



John Nash and Louis Nirenberg receive the 2015 Abel Prize from His Majesty King Harald V of Norway. abelprize.no

In 1994, Nash won the Nobel Memorial Prize in Economic Sciences for his work on game theory, which was the subject of his 1950 Princeton doctoral thesis. But, according to Nash's biographer Sylvia Nasar, many "mathematicians joke that he got his Nobel for his most trivial work." Many mathematicians regard his research on geometry and partial differential equations as his most important and deepest work, and they were thrilled when that work was finally recognized with the 2015 Abel Prize. Nash received the award with Louis Nirenberg (pictured above) for "striking and seminal contributions to the theory of nonlinear partial differential equations and its applications to geometric analysis."

"Their breakthroughs have developed into versatile and robust techniques that have become essential tools for the study of nonlinear partial differential equations," wrote the Abel committee. "Their impact can be felt in all branches of the theory."

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The Work of John Nash

by David Kinderlehrer

In economics and the social sciences, Nash's eminence owes to the influence of his work on non-competitive games, for which he shared the Nobel Prize in Economics in 1994. The tremendous influence of his spectacular work on partial differential equations and Riemannian geometry is the basis for his receipt of the 2015 Abel Prize, shared with Louis Nirenberg. The simple statements of these accomplishments belie their difficulty and profundity.

Consider the question of smoothness of solutions of parabolic equations. Contemporary functional analytic and topological approaches to solving nonlinear equations and systems of equations provide the solution as an element of a function space, or its dual space. A major challenge is to decide when this item is actually a smooth function in the ordinary, now called 'classical,' sense. These new functional and topological methods evolved over the 20th century and were foreshadowed in the statement of the *Hilbert Nineteenth Problem* in 1900. They give rise to functions u that are nominally solutions to equations like:

(1) $$\frac{\partial u}{\partial t} = \sum_{i,j=1,\ldots,n} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u}{\partial x_j}\right), \quad \text{in } \Omega,$$

where in favorable situations A is a positive definite symmetric matrix:

(2) $$A = (a_{ij}),\ a_{ij} = a_{ji},\ \sum_{i,j=1..n} a_{ij}\xi_i\xi_j \geqq \lambda|\xi|^2 \text{ , for some } \lambda > 0,$$

and, importantly, the coefficients a_{ij} are assumed only bounded (and measurable) but not more regular than that, and $\Omega \subset R^n$. Nash proved (1958) that a bounded solution of (1) is Hölder continuous, that is:

(3) $$|u(x,t) - u(y,s)| \leqq C(|x-y|^{\mu} + |t-s|^{\mu/2}), \text{ for } x,y \in \Omega \text{ and } t,s > 0,$$

where $C;\mu > 0$ depend only on the matrix A. Even this may appear to be mysterious so we must keep in mind, in an oversimplfied description here, that the coefficients a_{ij} are typically functions of u. So the new smoothness of u translates to smoothness of the matrix A and from there the Schauder Theory (1930s) renders the solution smooth in the classical sense.

The Work of John Nash, continued

In more general situations, the work of Louis Nirenberg, Nash's co-recipient, and his collaboration Agmon-Douglis-Nirenberg is invoked. The result is a companion to, and is nearly simultaneous with, work of Ennio De Giorgi (1928-1996). A subsequent method was found by Jürgen Moser (1928-1999). Together, the circle of results is under the rubric de Giorgi-Nash-Moser. It is important to note that there were many significant contributions prior to the de Giorgi-Nash-Moser, in particular by C.B. Morrey Jr. The smoothness result fails for systems. A counter example due to de Giorgi, Enrico Giusti and Mario Miranda was found only much later, in 1968. The importance of Nash's work was instantly recognized and, indeed, the Math Review item was written by Morrey. It is part of the everyday equipment of scientists working in elliptic and parabolic partial differential equations.

Although the smoothness questions for (1) date to the early 20th century, the isometric embedding of manifolds in Euclidean space emerges from Riemann's idea of an abstract manifold presented in his Habilitationsschrift in Göttingen under Gauss,1862. Hence his concept of a manifold, which we call a Riemannian manifold, and its geometry may be introduced intrinsically, unrelated to an ambient space. But are these equivalent to manifolds defined extrinsically, with metrics induced by the ordinary Euclidean metric, for example, as we think of the two dimensional sphere or torus in three dimensions? This is the problem of isometric embedding. As the illustrations and their captions show, when the ambient space has low dimension, the embedded manifold may be quite rough. To formulate the problem, consider an n dimensional compact and smooth manifold M with metric $g = (g_{ij})$, $1 \leqq i, j \leqq n$ with some local coordinates $x = (x^1, \dots, x^n)$ and consider an embedding into R^m, $m > n$. To solve, then, are the $\frac{1}{2} n(n+1)$ equations:

$$\nabla u^i \cdot \nabla u^j = \sum_{k=1,\dots,n} \frac{\partial u^i}{\partial x^k}\frac{\partial u^j}{\partial x^k} = g_{ij} \tag{4}$$

for the m components of the unknown embedding:

$$u : M \to R^m. \tag{5}$$

There are more unknowns than equations provided $m > \frac{1}{2} n(n+1)$. Nash proved two seminal results. In its improved version by Kuiper, the first states that any smooth short embedding of M into R^m may be uniformly approximated by C^1 isometric embeddings when $m \geqq n+1$. An astonishing consequence of this, for example, is that the standard 2–sphere S^2 admits an isometric embedding into an arbitrarily small ball in R^3, even though in this situation there are the same number of equations as unknowns. Early local results are due to Schläfli, and in the 20th century, to Janet and Cartan. In the second result, Nash shows that for m sufficiently large, there is a C^k, $k \geq 3$, isometric embedding of M into R^m m independent of k. These results have been developed and exploited by many mathematicians, including Moser, cited above, and, more recently Gromov (h principle), Müller and Šverák (wild solutions of elliptic systems), and De Lellis and Székelyhidi (Euler Equations). Moreover, the Nash methods lay the foundation for the `Nash-Moser' implicit function theory and the Kolmogorov-Arnold-Moser (KAM) Theory of Hamiltonian systems.

The ease with which we may understand Nash's results disguises their place as innovative and fundamental contributions to contemporary functional analysis and partial differential equations. This is, after all, why he was awarded the Abel Prize. In their time, Riemann and Hilbert, too, would have understood the statements of Nash's theorems, but could not have comprehended the proofs. We shall be ever thankful for what he gave us as we shall ever mourn the genius cut short by tragic illness.

Writer's note: I am indebted to J.M. Ball, G.-Q. Chen, and M. Slemrod for their help in preparing this narrative.

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▲ Torus

Mathematically, a flat torus is a 2-dimensional square where the top and the bottom edges are identified, as are the left and the right edges. Does there exist a way to realize this object into 3-dimensional space in such a way that the top and bottom edges, as well as the left and right edges, are glued together, and we bend but do not stretch the torus? It is hard to imagine doing this with paper since it appears that one would need to stretch it after the top and bottom edges have been glued. However, the famed embedding theorem by Nash and Kuiper implies that it is possible to do so by making a series of corrugations in the paper (no stretching!). The images here and on the cover, by Borrelli, Jabrane, Lazarus and Thibert, illustrate how this is done.

For a more detailed account of the mathematics, see their web page at **bit.ly/1xVV6bm**

Alumnus Karel Janeček Gives Voters A Better Way to Decide

Written By Amy Pavlak Laird

Alumnus Karel Janeček earned a Ph.D. in computational finance in 2004 and went on to found RSJ, one of the world's top market makers on international derivative exchanges. He's recently turned his attention to revitalizing the democratic process with his innovative voting system, Democracy 2.1.

Karel Janeček was upset. The latest political scandal in his native Czech Republic had reached an all-time low. The mayor of Prague was allegedly colluding with a Czech businessman to use the city's budget to influence sales of city and state property, fix office appointments and give expensive gifts to officials.

"I was so disappointed in the Prague mayor and these people in the Prague council," recalls Janeček, who has always felt a need to do something about the things he doesn't like. As the founder of RSJ, a top market maker of derivative exchanges, he was in a position to affect change in Prague and around the country. In 2011 he created the Anti-Corruption Endowment to provide financial and moral support for whistleblowers who expose corruption in government.

While the Anti-Corruption Endowment was successful in bringing cases of corruption to light, Janeček started to realize something: "The core of the problem is not corruption — corruption is a consequence," he says. "The problem is the people in power."

True to his nature, he started thinking about to how change things for the better. He focused on the Czech Republic's voting system, which, like many other European countries', enables the parties to hide corrupted and other wicked individuals behind closed party lists. This system also very often forces voters to vote for the "lesser evil" option.

But what if voters had more options? Using some simple math and basic logic, Janeček created an innovative voting system that gives voters that and more. Called Democracy 2.1, voters get multiple votes — and sometimes a minus vote.

"Mathematics teaches one to think logically, …so as not to be manipulated"

Let's say voters were at the ballot boxes to elect two members of the city council. Twelve candidates are vying for two seats. Using Democracy 2.1 (D21), each voter would have four votes, allowing a Democrat, for example, to vote for her party's two candidates while also giving her the freedom to cast her remaining votes for other candidates whose ideas she also likes. She also may have the opportunity to cast a minus vote for a candidate that she absolutely does not want to sit on the city council. The effect of multiple votes supports consensual and democratic candidates, and makes the selection of leaders more just.

"The most important ingredient, dare I say the revolutionary idea of D21, is the effect of multiple votes," Janeček says. According to D21's website, offering multiple votes almost doubles voters' satisfaction with the final choice because there is a higher probability that one of the options someone voted for will be chosen as one of the winners.

Although Janeček's initial motivation for developing D21 was to revamp the political election process, he's found that D21 has widespread appeal.

"I have been so surprised at the universal applicability of D21 not just for politics but for any situation where people make decisions out of many options."

One of the most exciting uses of D21 so far has been in participatory budgeting, where community members directly decide how to spend part of a public budget. Earlier this year, residents of several New York City districts voted to decide how they wanted the city to spend over $31 million of the budget. They were offered a number of choices in each district, which ranged from buying an air conditioner for a school cafeteria to making specific streets greener. D21 consulted in cooperation with Stanford University on developing a digital ballot for the vote and implemented it in five districts. The D21 voting algorithm was tested in two of them. The voters' results revealed which projects had the most consensus and which were the most divisive. The D21 team is continuing its collaboration with New York City for next year's vote.

"The amazing thing is that it's not just about voting — it's also about feedback," says Janeček, who was present for the community vote in New York City. "The city council can learn so much more from people. This is an amazing thing, this information flow. That I didn't expect."

D21 is also being implemented in elementary schools and companies to query student and employee opinions, respectively. On the political front, Janeček and his D21 team are currently working as consultants to the electoral parliamentary commission of the Tunisian parliament, which will soon (in a few years time) have its first regional elections.

While D21 keeps Janeček busy — he recently received a research fellowship from Cambridge University to work on D21 — he's still involved with RSJ, serving as chairman of its supervisory board. He also is engaged with his many philanthropic endeavors, including the Anti-Corruption Endowment, the Karel Janeček Foundation, the Foundation Aid Fund, and Neuron, which supports scientific research in the Czech Republic.

Janeček points to his education as a mathematician — and his experience at Carnegie Mellon — as the root of his success.

"Mathematical thinking is extremely valuable. Mathematics teaches one to think logically, to argue, to be able to understand others and the logic of their arguments so as not to be manipulated," Janeček says. "Of course, on top of that, the education I got at CMU has influenced my life in a major way. It combined deep mathematics with practical applications — finance and modeling of derivatives — that can be very nicely applied in practice in finance and investment. I am certain that, if it wasn't for CMU, RSJ wouldn't have been as successful."

D21

Learn more about the D21 global project at

democracy21.com

Po-Shen Loh Receives NSF CAREER Award

Po-Shen Loh likes interesting problems, and he's always on the lookout for ones that pique his interest. Thanks to an award from the National Science Foundation (NSF), his hunt for these sorts of problems continues.

Earlier this year, Loh received a Faculty Early Career Development (CAREER) award, the NSF's most prestigious award in support of junior faculty who exemplify the role of teacher-scholars through outstanding research, excellent education and the integration of education and research.

"The CAREER award is allowing me to continue hunting the world for elegant, deeply connected, challenging problems that will advance the state of human knowledge in some area of mathematics," says Loh, associate professor of mathematical sciences.

Loh, whose research lies at the intersection of combinatorics and probability theory, recently discovered a problem that met all of his requirements — and then some.

Let's say you have some triples of whole numbers, all of which are less than or equal to some number N:

1 2 4

3 3 1

4 4 1

1 5 5

When you compare any two rows (not necessarily immediately one after another) by looking at the numbers, you can see that at least half of the columns grew from the earlier row to the latter row. For example, when you compare the rows "3 3 1" and "1 5 5", the second and third columns grew from the earlier row to the latter row.

Loh's question is: What's the longest sequence of rows that you can write with this rule for each particular N?

Solving that question has turned out to be a challenge for Loh and for other mathematicians who have attempted it. Even though the problem is simple to state, it is related to fundamental questions in discrete math, including Ramsey theory and the Szemerédi regularity lemma.

"I find it very appealing to suddenly discover something which looks like it should be child's play but turns out to be very hard,"says Loh, who adds that the question interests him on many levels. "I love these sorts of questions because you can tell these to the most enthusiastic and inexperienced person and they can understand it. And that will help them understand the mystique in math."

> *"I find it very appealing to suddenly discover something which looks like it should be child's play but turns out to be very hard."*

Loh, a self-described math enthusiast and evangelist, routinely finds ways to bridge the worlds of research mathematicians with students and the general public who are enthusiastic about math and science. He travels the world giving talks to everyone from graduate students and fellow mathematicians to high school students and visitors to the National Museum of Mathematics in New York City. In addition, he coaches the U.S. International Mathematical Olympiad team, which finished in first place at the 2015 competition in Thailand.

At Carnegie Mellon, Loh has developed an innovative problem-solving seminar series for undergraduate students, inspired by the annual Putnam exam. This past year, 205 CMU students participated in the Putnam Competition, with the team bringing home a fifth place finish (see story on page 4).

The $400,000 CAREER award will help support Loh's efforts to bring the joy of mathematics to students of all ages, and to continue his hunt for interesting problems.

Boris Bukh Receives Sloan Research Fellowship

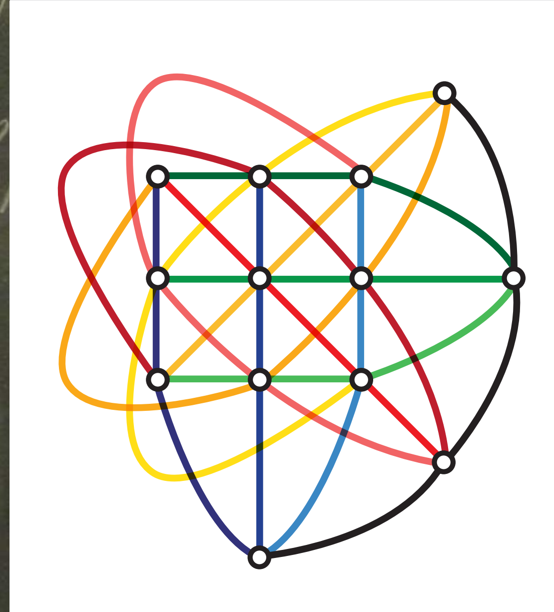
Boris Bukh, an assistant professor of mathematical sciences, has received a 2015 Sloan Research Fellowship. He was among 126 early-career scientists and scholars to receive the fellowship, which seeks to stimulate fundamental research by early-career scientists and scholars of outstanding promise.

Professor Bukh's research is the area of extremal combinatorics, an area of discrete mathematics that focuses on questions of the form: What is the largest (or smallest) object in the collection of objects on a fixed ground set that has some desirable property? It sometimes happens that the optimal objects are produced by beautiful geometric constructions. This can be quite surprising when the original problem has nothing directly to do with geometry. The quest to find and understand these objects is one part of Bukh's research program.

Recently, Bukh has been working on solving a particular aspect of the Turán problem, which is the most fundamental question in extremal graph theory. The general Turán problem asks: For any graph H, what is the maximum number of edges in a graph on n vertices that does not contain the graph H as a subgraph? (Recall that a graph is a network that consists of a collection of nodes, or vertices, together with a collection of edges that join pairs of vertices.)

It turns out that this question is particularly challenging when H is a complete bipartite graph. The complete bipartite $K_{k,k}$ has 2k vertices divided into two parts containing k vertices each and the collection of all k^2 edges that join a vertex from one part with a vertex from the other part. The problem of determining the maximum number of edges in a graph on n vertices that contains no copy of $K_{k,k}$ is still widely open. It has been solved only for $K_{2,2}$ and $K_{3,3}$. In each of these cases, the optimal graph comes from a geometric construction. For $K_{2,2}$, the construction comes from the arrangement of points and lines in **a finite projective plane**. The construction for $K_{3,3}$ is generated by collections of spheres and points in discrete 3-space. The discrete mathematics community has long wondered whether or not the optimal graphs for the Turán problem for $K_{k,k}$ might be generated by some higher dimensional generalizations of these objects.

In joint work with Pavle Blagojević and Roman Karasev, Bukh looked for 4-dimensional 'shapes' that would help solve the Turán problem for $K_{4,4}$-free graphs in the same way that lines and spheres did for $K_{2,2}$-free and $K_{3,3}$-free, respectively. They proved that no such shapes exist. The proof uses equivariant algebraic topology. Informally, the reason behind the nonexistence is that such a hypothetical shape would have to be too symmetrical. The Turán problem of determining the maximum number of edges in a graph on n vertices with no copy of $K_{4,4}$ remains widely open.

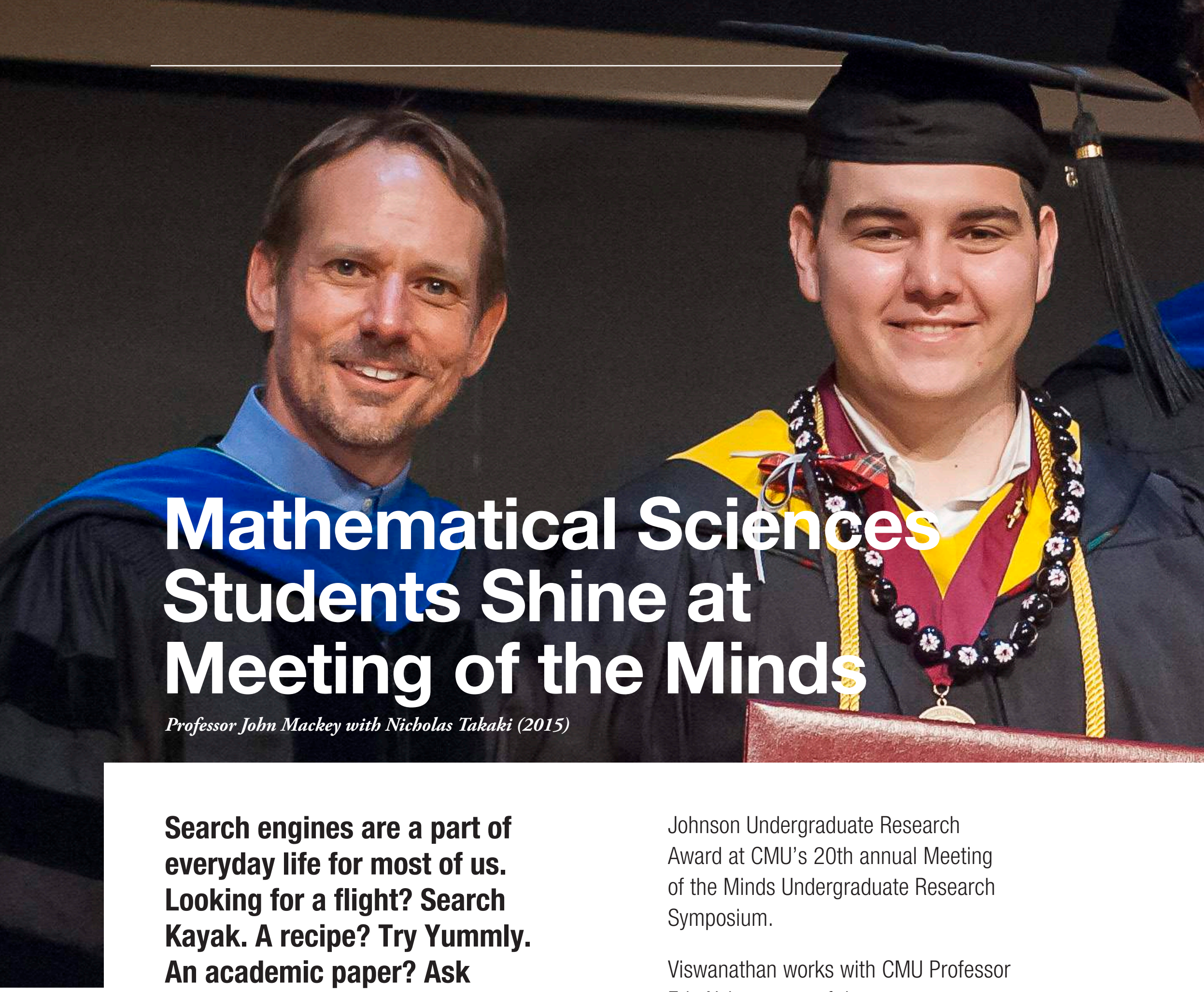


A Finite Projective Plane

This particular finite geometry consists of a set of 13 points and a set of 13 lines, where each line consists of four points. Each line in the plane here has been rendered in a different color. This collection of points and lines has the property that any two lines intersect in exactly one point.

This geometry can be used to construct a graph on 26 vertices that has 52 edges and no copy of the complete bipartite graph $K_{2,2}$. Each point and each line in the geometry corresponds to a vertex in the graph, and we join the vertex corresponding to a point p with the vertex corresponding to the line l if p is a point in l.

One way to create this geometry is to consider a 3-dimensional vector space V over the 3-element field. Each 1-dimensional subspace gives a point in the geometry and each 2-dimensional subspace gives a line in the geometry.



Mathematical Sciences Students Shine at Meeting of the Minds

Professor John Mackey with Nicholas Takaki (2015)

David Mehrle (2015)

Jeremy Meza (2015)

Search engines are a part of everyday life for most of us. Looking for a flight? Search Kayak. A recipe? Try Yummly. An academic paper? Ask Google Scholar. If you're a biomedical researcher, you might use PubMed or Quertle® as your search engine. Thanks to research by senior math major Vijay Viswanathan, your queries, no matter what biomedical search engine you use, might one day be a little more effective.

Since a query is just a few words, it doesn't give you much meaning of what a user is looking for. Given a search log consisting of all of a user's queries, Viswanathan is deriving a list of the natural questions the user is asking, which provides a deeper representation of the user's information need. For his project, "Modeling the Relationship Between Queries and Questions in Biomedical Text," Viswanathan won the Johnson & Johnson Undergraduate Research Award at CMU's 20th annual Meeting of the Minds Undergraduate Research Symposium.

Viswanathan works with CMU Professor Eric Nyberg, one of the co-creators of IBM's Watson, the computer that famously beat Jeopardy! champions in a 2011 on-air showdown. "Dr. Nyberg's group has provided great exposure to active research," Viswanathan said. "I've learned a lot about information retrieval and natural language processing in an applied, systems-oriented environment."

Viswanathan was one of 18 students from the Department of Mathematical Sciences who participated in the Meeting of the Minds on May 6. Their research projects covered topics in mathematics, computer science and statistics.

For the first time, through the generosity of alumnus David Simmons (S 1986, TPR 1986), students competed in the new Mathematics Poster Competition, which is open to any student or team of students who have completed a project that has significant mathematical content. Its purpose is to encourage undergraduate projects and research in mathematics, and to educate the CMU community about the wide range of opportunities in mathematics. This year's first-prize winner was Mathematical Sciences and Computer Science double major Philip Garrison for his poster, "Good Graph Hunting."

Faculty and advisors have seen an increasing number of students interested in pursuing undergraduate research and capstone projects in mathematics. The Department is helping to support these undergraduate research projects through the Department's Innovation Fund.

Mathematical Sciences Students Participating in Meeting of the Minds 2015:

First Place - Mathematics Poster
Philip Garrison (2016)
Good Graph Hunting

Honorable Mention - Mathematics Poster
David Mehrle (2015)
Tomer Reiter (2015)
Newman's Conjecture in Function Fields

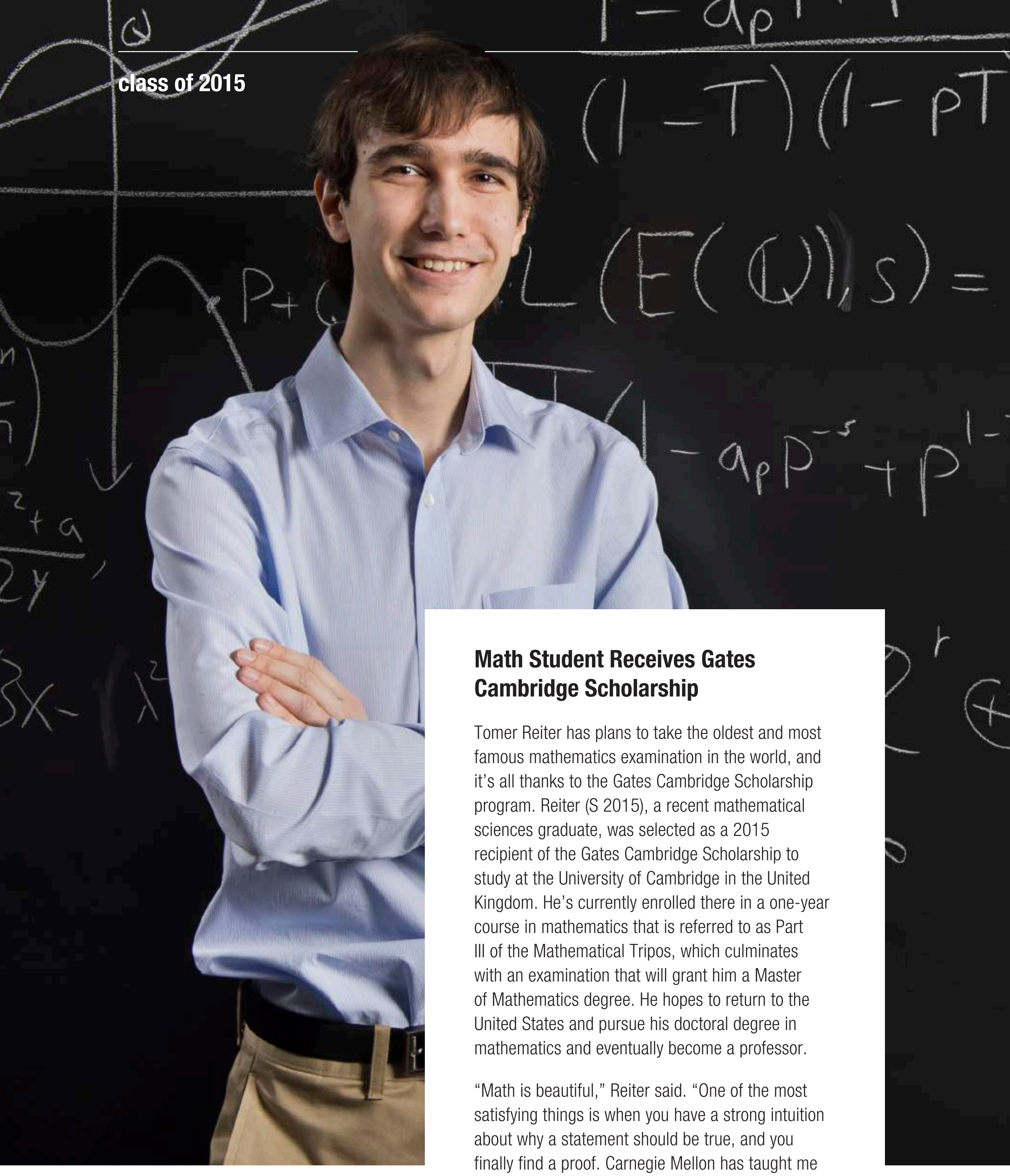
Nicholas Takaki (2015)
Image Segmentation with Dynamic Artifacts Detection and Bias Correction

Jeremy Meza (2015)
Explicit Bounds for Pseudospectra of Matrices and Operators

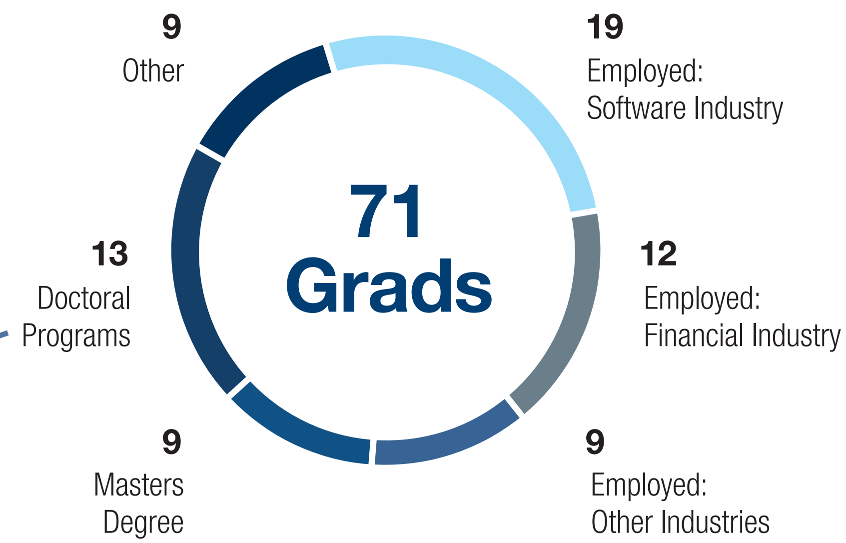
Johnson & Johnson Undergraduate Research Award
Vijay Viswanathan (2016)
Modeling the Relationship Between Queries and Questions in Biomedical Text

First Place - Statistics Poster
Eric Alpert (2016)
Classifying Kepler Objects of Interest

Other Participants
David Bayani (2016)
Katherine Borst (2017)
William Croughan (2015)
Samantha Davies (2015)
Keenan Gao (2015)
Tian Guo (2016)
Akhil Prakash (2016)
Luke Serafin (2015)
Samuel Simon (2015)
Abigail Smith (2016)
Fan Yang (2015)



Mathematical Sciences Class of 2015



Ph.D. Destinations and Programs

University of California, Berkeley	Mathematics (2)
Carnegie Mellon University	Computer Science
Cornell University	Mathematics
Duke University	Mathematics
University of Illinois at Chicago	Mathematics
University of Illinois Urbana-Champaign	Mathematics
Johns Hopkins University	Physics & Astronomy
Maryland University	Mathematics
University of North Carolina, Chapel Hill	Statistics
University of Rochester	Optics
University of South Carolina	Mathematics
Stanford University	Computer Science

$

Median starting salary for graduates in industry:

$85,000

(of 29 salaries reported in the exit survey)

Math Student Receives Gates Cambridge Scholarship

Tomer Reiter has plans to take the oldest and most famous mathematics examination in the world, and it's all thanks to the Gates Cambridge Scholarship program. Reiter (S 2015), a recent mathematical sciences graduate, was selected as a 2015 recipient of the Gates Cambridge Scholarship to study at the University of Cambridge in the United Kingdom. He's currently enrolled there in a one-year course in mathematics that is referred to as Part III of the Mathematical Tripos, which culminates with an examination that will grant him a Master of Mathematics degree. He hopes to return to the United States and pursue his doctoral degree in mathematics and eventually become a professor.

"Math is beautiful," Reiter said. "One of the most satisfying things is when you have a strong intuition about why a statement should be true, and you finally find a proof. Carnegie Mellon has taught me how to tackle those difficult problems."

The Gates Cambridge Scholarship provides full support to students from outside the United Kingdom as they pursue a post-graduate degree at Cambridge University. There were 40 U.S. winners this year.

Innovation Fund for Mathematical Sciences

This endowment fund supports:

- **Undergraduate research and capstone projects in mathematics**
- **Math Club and other activities that enrich the CMU experience for math majors**
- **Mathematics faculty recruiting**

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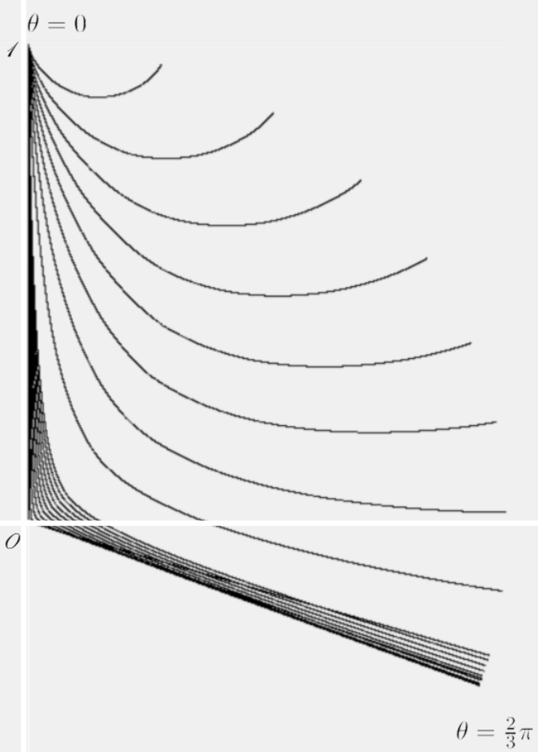
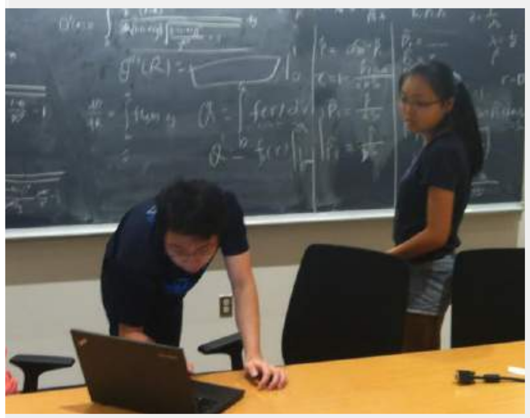
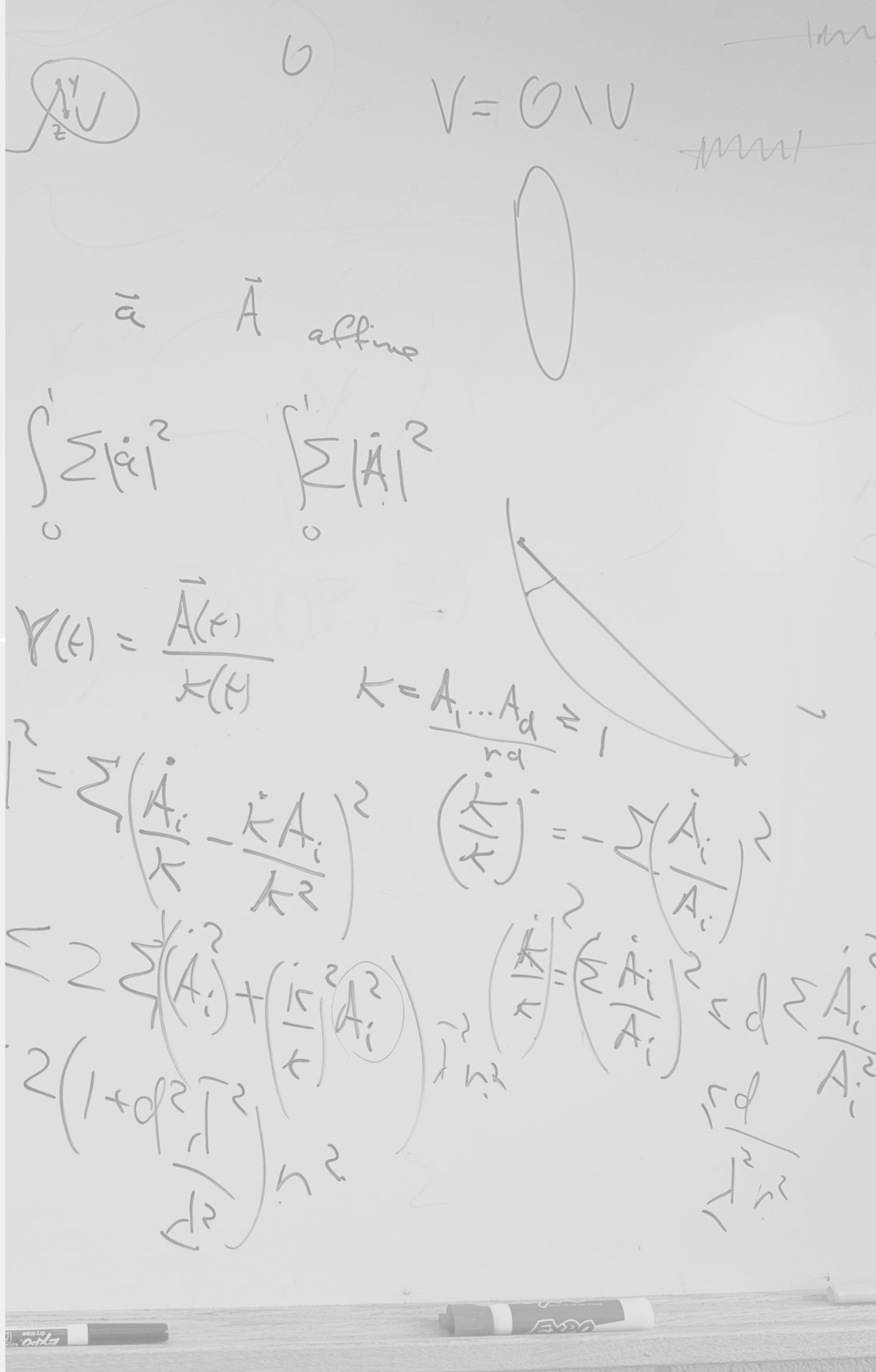




Undergraduate mathematics majors Keven Chionh and Laurie Jin spent the summer of 2015 working on a research project concerning paths of fastest transit in an inverse-square gravitational field. This is a variant of the classical brachistochrone problem posed by Johann Bernoulli in 1696. Keven and Laurie discovered some interesting new phenomena and are working during the academic year to provide careful mathematical justification of these results.

This project was supported by the Innovation Fund for Mathematical Sciences.

Carnegie Mellon University
Mathematical Sciences



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