Spreading processes with population heterogeneity over multi-layer networks

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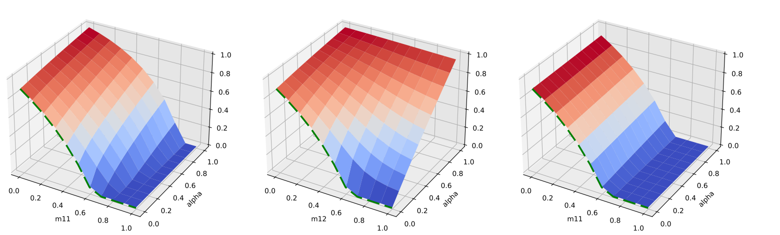
Modeling spreading processes over complex networks has been receiving increasing attention. For example, bond percolation models considering population heterogeneity have been used to derive insights into disease spread and misinformation control. However, most works on spreading processes with population heterogeneity only concentrate on single-layer contact networks. To study how the course of a spreading process changes due to *multiple* layers of contact networks (e.g., neighborhood vs. schools or Twitter vs. Facebook) while considering population heterogeneity from a principled, mathematical lens, we propose the *Multi-layer Mask model* based on SIR dynamics. We derive analytical expressions for three fundamental epidemiological quantities: the probability of emergence, the epidemic threshold, and the expected epidemic size. Analytical results are shown to be in near-perfect agreement with the numerical results obtained through extensive simulations. These results reveal the impact of the structure of the multi-layer contact network, viral transmission dynamics, and population heterogeneity on the final state of the spreading process. For an illustrative demonstration of the analytical results of our model, we present the work under the school opening problem.

Specifically, let $\mathbb{C}$ stand for the community contact network of individuals on the node set $\mathcal{N}$. Let $\mathbb{S}$ represent the school contact network with the assumption that each node in $\mathcal{N}$ is a member of the school network $\mathbb{S}$ with probability $\alpha \in (0, 1]$, independently from each other. The baseline viral transmission probabilities are represented by T_c and T_s for layer $\mathbb{C}$ and $\mathbb{S}$, respectively. We split the entire population into 4 non-overlapping categories based on their layer-dependent mask-wearing habits: masked on $\mathbb{C}$ and masked on $\mathbb{S}$, not-masked on $\mathbb{C}$ and masked on on $\mathbb{S}$, masked on $\mathbb{C}$ and not-masked on $\mathbb{S}$, and not-masked on $\mathbb{C}$ and not-masked on $\mathbb{S}$. $\boldsymbol{m} = [m_{11}, m_{21}, m_{12}, m_{22}]$ represents the proportion of the above 4 categories. Based on the single-layer *Mask model* proposed by [2], we characterize the mask protection with inward and outward efficiency (ϵ_{in} and ϵ_{out}).

Utilizing the model, we investigate how the structure of the multilayer network, viral transmission dynamics, and the distribution of the different types of masks within the population impact the aforementioned quantities. Considering the underlying regional factors, communities have potentially different percentages of the school participation rate (α), and local authorities may enforce different mask-wearing policies ($\boldsymbol{m}$). As an example analytical case, we compare three school-opening policies and observe the impact of $\boldsymbol{m}$ and α on PE. Figure 1(a) provides a scenario where nodes' mask-wearing habits remain consistent (layer-independent) if we open up layer-$\mathbb{S}$ ($m_{12} = m_{21} = 0$, $m_{11} + m_{22} = 1$). Figure 1(b) explores the situation where nodes do not have to wear masks at school due to educational purposes ($m_{11} = m_{21} = 0$, $m_{12} + m_{22} = 1$). Figure 1(c) mimics the scenario where mask-wearing is mandated in school ($m_{12} = m_{22} = 0$ and $m_{11} + m_{21} = 1$). As the baseline, we mark out the PE values obtained from *Mask model* [2] for the layer-$\mathbb{C}$-only setting in green dashed lines in Figure 1(a) to 1(c). We can see that as α increases, compared to the baseline, PE in Figure 1(a) and 1(b) increases. However, α has a relatively smaller impact on PE in Figure 1(c) where mask-wearing is mandated in $\mathbb{S}$. Meanwhile, Figure 1(a) to Figure 1(c) present different mitigation effects in reducing PE as mask wearers increase in the population. These observations reveal the trade-off among multiple factors that lead to an epidemic. Different policies result in different mitigation effects.



(a) Layer-independent mask-wearing (b) No masks at school (c) Masks mandated at school

Figure 1: *PE of the three policies on random networks described in [1].* $T_c = 0.5$, $T_s = 0.4$, $\boldsymbol{\epsilon}_{out} = \boldsymbol{\epsilon}_{in} = [0.8, 0]$. *The mean degree is 5 and 8 for* $\mathbb{C}$ *and* $\mathbb{S}$.

We also see that it is possible to identify a *safe* condition, i.e., conditions where the epidemic will not happen (i.e., PE equals zero) for a given parameter setup with proper mitigation policies in place.

[1] Y. Tian and O. Yağan, *Spreading processes with population heterogeneity over multi-layer networks*. http://arxiv.org/abs/2211.07479
[2] Y. Tian, A. Sridhar, C. W. Wu, S. A. Levin, Kathleen M. Carley, H. V. Poor, and O. Yağan, *Role of Masks in Mitigating Viral Spread on Networks*, to appear in Phys. Rev. E. Available: http://arxiv.org/abs/2110.04398