Modeling the Performance of Flattened Tubular Solid Oxide Fuel Cell

In this paper, we develop a computational model to predict the electrical performance of the flattened tubular solid oxide fuel cell (SOFC) designed by Siemens Westinghouse Corporation. This design is an improvement over the conventional cylindrical SOFC and allows higher power densities. We modeled the current transport in a cross section of the cell for a given cell operating voltage and local Nernst voltage. We solved the resulting system of simultaneous nonlinear equations using n-dimensional Newton-Raphson algorithm. The output gives the current density distribution and also the total current at the cross section, which is used to obtain the total cell current (and power) for the given voltage. The results of the model are in good agreement with the experimental performance reported in literature. [DOI: 10.1115/1.1842783]

Introduction

The Department of Energy’s SECA program set a target cost of $400/kW and power density of 0.35 W/cm² for Siemens Westinghouse’s cathode-supported sealless solid oxide fuel cells (SOFC) [1,2]. The present design of the tubular SOFC operates at a power density of 0.13 W/cm² (at 0.65 V) and it appears unlikely that this design would be able to meet the targets of both cost and performance. With this objective, a new design, high power density (HPD) was developed that has a flattened cathode with ribs [1,3]. The presence of ribs reduces the current path length in the cell and consequently the cell is likely to have a lower resistance and higher power density. In this paper we develop a computational model to predict the performance of the HPD and compare it with the tubular SOFC.

Problem Formulation

Figure 1 shows a cross section of the flattened SOFC, or HPD design of the SOFC of Siemens Westinghouse Corporation [1,3,4]. This cell has four ribs (or five channels). The cathode is made of doped lanthanum manganese, the electrolyte is composed of yttria-stabilized zirconia (YSZ) and the anode is nickel-doped zirconia. The cell has a length (L), which is normal to the plane of the paper.

We discretized the length of the cell into various cross-sectional elements (Fig. 1) and the thickness of each such element is \( \Delta x \) (perpendicular to plane of the paper). Air flows inside the tube and fuel flows on the outer surface of the tube. We assume that the fuel is partially re-formed before it enters the anode layer. Nernst voltage is generated at the site of the electrochemical reaction, which is at the interface between the electrolyte and the anode. The Nernst voltage at each cross section can be calculated if the local cell temperature and partial pressures of fuel and oxidant are known. At the cell exit, the fuel and oxidant concentrations can be obtained from the assumed value of the fuel utilization. The cell is symmetric about the vertical axis and therefore we have modeled one half of the cell. Figure 2 shows an equivalent resistor network diagram for current transport. This is based on the “distributed parameter” model developed by Sverdrup et al. [5].

We discretized the cell into \( N (=150) \) such elements in the electrode layers and \( M (=75) \) elements in the interconnect layer.

In Fig. 2, \( N_1 \) and \( N_3 \) correspond to the nodes where the ribs are attached to the cathode. \( N_0, N_2, \) and \( N_4 \) are the nodes where the current enters the cell from the interconnect of the previous cell. Similarly, \( M_1 \) and \( M_2 \) represent the nodes where the ribs are attached to the interconnect. The diagram is representative and shows only a few nodes (or differential elements) due to space constraint. The actual number of nodes (differential elements) used in computation is as follows: \( N_1=28; N_2=58; N_3=85; N_4=110; N_5=150; M_1=25; M_2=50; M=75 \).

Current enters the cell from the nickel-foils contacts, which are located mid-way between the ribs. It distributes along several possible pathways in the cathode, anode, and electrolyte and this distribution depends on the resistivity and thickness of the layers. We assume that current travels along the circumference in the cathode and anode and crosses the electrolyte as ionic current. Thus, for instance, a particular current path could travel along the anode to a particular point, cross over to the cathode through the electrolyte, and then travel through the rib to the interconnect and on to the next cell in the series. Another possible path could travel all the way to the end of the anode layer and then cross over to the cathode and the interconnect, thus bypassing the ribs. The current enters the interconnect from the ribs and from the gap between interconnect and anode layer. Since the interconnect has a finite thickness, there is a current distribution in the interconnect as well.

The current transport in the cell encounters resistance from the following sources: ohmic losses in the cathode, anode, and nickel contact; losses due to ionic current transport in the electrolyte; and activation and diffusion polarizations. These determine the terminal voltage of the cell.

In the following sections, we explain the governing equations used to solve the current density distribution in the cell.

Figure 3 shows such a differential element for modeling the current transport between anode and cathode layers.

Here, \( I_i(k) \) is defined as the anode current per unit cell length (A/m) leaving the kth node. Similarly, \( I_i(k) \) is defined as the cathode current per unit cell length (A/m) leaving the kth node. \( I_k(k) \) is the current per unit cell length (A/m) flowing in the kth branch. \( R_a \) and \( R_c \) are the differential resistances of the anode and cathode, respectively.
Here, \( \rho_a \) and \( \rho_c \) are the resistivity of anode and cathode, respectively; \( \delta_a \) and \( \delta_c \) are the thicknesses of the anode and cathode layers and \( \Delta l \) is the width of the differential element. \( E_r \) is the reversible Nernst voltage, which is assumed to be a constant at a particular cross section. \( R_e \) represents the resistance of the electrolyte layer and \( R_p \) is the sum of activation and concentration polarizations.

**Kirchhoff’s Current Law for Anode Nodes.** Equation (2) is the current balance at each node on the anode surface.

\[
I_a(k) = I_a(k-1) - I_b(k)
\]  

Equations (3)–(5) are the current balance for the nodes located at the points where current enters the cell from interconnect of the lower adjoining cell. Here, part of the current travels along the anode, while the remaining enters the electrolyte in the branch.

\[
I_a(1) = I_1 - I_b(1)
\]

\[
I_a(N_2) = I_a(N_2-1) + I_2 - I_b(N_2)
\]

\[
I_a(N_4) = I_a(N_4-1) + I_3 - I_b(N_4)
\]

**Kirchhoff’s Current Law for Cathode Nodes.** Equation (6) is the current balance equation for each cathode node. Current enters each cathode node from the electrolyte and from the adjacent node and leaves to the next node in series.

\[
I_c(N) = I_a(N) - I_b(N)
\]
Kirchoff’s Voltage Law for Differential Element

Kirchoff’s voltage balance is satisfied in each of these differential elements (Eq. (10)). Here, \( E \) is the ideal Nernst voltage at the cross section; \( \eta_{act} \) and \( \eta_{conc} \) are the activation and polarization losses respectively; \( i \) is the current density in the branch; \( R_a \) and \( R_c \) are the resistances of the differential anode and cathode elements, respectively. The resistivity of the anode and cathode are taken from literature [6].

\[
E_r - \eta_{conc}(k) - \eta_{act}(k) - i(k)\rho_s \delta_s - I_r(k)R_r \Delta x
\]

\[
= E_i - \eta_{conc}(k+1) - \eta_{act}(k+1) - i(k+1)\rho_s \delta_s
\]

\[
- I_r(k+1)R_r \Delta x
\]

The concentration polarization is obtained from the following expression:

\[
\eta_{conc}(k) = -\frac{RT}{nF} \ln \left(1 - \frac{i(k)}{i_0}\right) \tag{11}
\]

Activation polarization is obtained from the Butler–Volmer equation:

\[
i = i_0 \left[ \exp \left( \frac{\beta Z F \eta_{act}}{RT} \right) - \exp \left( - (1 - \beta) \frac{Z F \eta_{act}}{RT} \right) \right] \tag{12}
\]

However, this is an implicit equation for the current density and therefore, we use the approximate form:

\[
\eta_{act}(k) = \frac{RT}{\beta nF} \ln \left( \frac{i(k)}{i_0} \right), \text{ if } i \ll i_0 \tag{13a}
\]

\[
\eta_{conc}(k) = \frac{RT}{nF} \ln \left( \frac{i(k)}{i_0} \right), \text{ if } i \gg i_0 \tag{13b}
\]

Kirchoff’s Current Law for Interconnect

Current enters interconnect from the ribs \( I_{l1} \) and \( I_{l2} \) and as terminal cathode current \( I_h \). We discretized the interconnect layer into 75 elements and assumed that the current travels in the vertical direction \( I_h \) and horizontal direction \( I_v \). Equation (14) is the current balance for a node in the interconnect layer.

\[
I_h(k) = I_h(k-1) + I_h(k) \tag{14}
\]

Equations (15) and (16) are valid for a node located at the intersection of the ribs and interconnect. Here, current enters the node from the previous node and from the rib.

\[
I_h(M_1 - 1) + I_v = I_h(M_1) + I_h(M_2) \tag{15}
\]

\[
I_h(M_2 - 1) + I_v = I_h(M_2) + I_h(M_2) \tag{16}
\]

Equation (17) is Kirchoff’s voltage balance in each of the differential elements in the interconnect layer. \( R_h \) and \( R_v \) are the resistances of the horizontal and vertical elements, respectively.

\[
I_h(k)R_h + I_v(k + 1)R_v = I_h(k)R_v \tag{17}
\]

Equations (21) and (22) are the Kirchoff’s voltage law for the loops enclosed by the ribs, cathode layer and the interconnect layer.

\[
\sum_{k=1}^{N} I_h(k)R_h + \sum_{k=1}^{M_2-1} I_h(k)R_h - I_hR_v = 0 \tag{18}
\]

\[
\sum_{k=1}^{N} I_h(k)R_h + I_vR_v + \sum_{k=1}^{M_2-1} I_h(k)R_h - I_hR_v = 0 \tag{19}
\]

Boundary Conditions

The above system of equations is subject to three boundary conditions. First condition is that the cell is operated at a specified voltage \( V_{op} \).

Secondly, the total current entering the cell is \( I_0 \) and hence the current in the half-cell is \( I_0/2 \). \( I_0 \) is still not known and it is determined as an output

\[
I_1 + I_2 + I_3 = I_0/2 \tag{20}
\]

Finally, we assume that the points where the current enters the cell from the lower Nickel-felt contact points are at the same potential. Thus, the points represented by nodes \( N_1, N_2, \) and \( N_3 \) in Fig. 2 are equipotential. This can be expressed mathematically in Equations (21) and (22).

\[
\sum_{k=1}^{N} I_h(k)R_h = 0 \tag{21}
\]

\[
\sum_{k=1}^{M_1} I_h(k)R_h = 0 \tag{22}
\]

Solution Procedure

The above procedure results in \( 3N + 2M + 3 \) equations at each cross section, of which \((N-1)\) equations are nonlinear. Figure 4 shows the iterative procedure used to solve the above system of equations.

At each cross section, we first obtained the Nernst voltage. Strictly speaking, the Nernst voltage distribution along the cell length is required to precisely obtain the total current (and hence power). The Nernst voltage at the any cross section can be obtained using the local partial pressures of the fuel and oxidant and the temperature \([7]\). We assume the cell to be isothermal and as a first approximation assume that the Nernst voltage varies linearly between the values at cell inlet \((x=0)\) and cell exit \((x=L)\). The inlet and exit Nernst voltages were assumed to be 0.90 and 0.77 V, respectively, for a cell operating at 1 atm with internally reformed natural gas with a fuel utilization of 85%. These calculations are based on calculations carried out in an earlier work \([7]\).

We start with the inlet cross section \((x=0)\) and make an assumption for the total cell current \( I_0 \). We then solved the above system of equations in MATLAB using an \( n \)-dimensional Newton–Raphson algorithm to obtain the current density distribution for the assumed value of \( I_0 \) and Nernst voltage \( E_v \). Consequently, we obtain the voltage \( V \) across the cell and verify whether this voltage matches with the specified cell operating voltage \( V_{op} \). If not, the cross sectional current \( I_0 \) is iteratively varied until the voltages match. This procedure specifies the current (and hence power) at the inlet cross section \((x=0)\). We then go over to the next cross sectional element and repeat the above procedure. The total current generated in the cell (corresponding to a voltage \( V_{op} \)) is obtained by summing the currents \((I_0)\) produced by all the
cross sectional elements. This procedure gives one point on the \( V-I \) curve. We then varied the cell operating voltage \( (V_{op}) \) within a given range to construct the \( V-I \) curve. The Appendix lists the values of various variables and parameters used in the above equations.

**Results**

Figure 5 shows the \( V-I \) characteristics of the cell. From the figure, a voltage of 0.65 V corresponds to a current density of 320 mA/cm\(^2\) and a power density of 0.21 W/cm\(^2\). These results match quite closely with the experimental results reported by Siemens Westinghouse for the HPD design.

The power density of the flattened SOFC design is significantly higher than that of the tubular SOFC, which has a power density of \(~0.13\ W/cm^2\) and a current density of 214 mA/cm\(^2\) at an operating voltage of 0.65 V \([3,7]\). The presence of ribs allows for shorter current paths in the anode and cathode and, therefore, the Ohmic losses are lower. This can be seen in Fig. 6. For an operating voltage of 0.72 V, the total current entering the cell \( (I_0) \) is 280 A; therefore, the power output of the cell is 201 W. Most of the current in the cell flows through the four ribs (206 A), the balance (74 A) travels until the end of the cathode layer and through the gap to the interconnect. The current flowing in each of the four ribs is roughly equal. Thus, the flattened design makes higher current (and power) densities possible.

Figure 7 shows the calculated current densities at cell inlet \( (x=0) \) and cell exit \( (x=L) \). The current densities are higher at the inlet cross section since the Nernst voltage is 0.90 compared to 0.77 V at cell exit. The current densities at the cell inlet approach the limiting current density at certain points in the cross section. The current density is highest close to the location of the ribs \( (N_1 \text{ and } N_3) \) and at the points where current enters from the adjoining cell \( (N_0, N_2, \text{ and } N_4) \).

The shape of the current density distribution curve depends on the relative resistances of the anode and cathode layers, or the ratio of resistivity to thickness \( (\rho/\delta) \). The resistivity of the anode at the average cell temperature of 900°C is \( 9.42 \times 10^{-6} \) \( \Omega \) m, while that of cathode is \( 1.17 \times 10^{-4} \) \( \Omega \) m \([6]\), and the thickness of the anode is 120 \( \mu \)m versus 1800 \( \mu \)m for cathode. Thus, the ratio \( \rho/\delta \) for the anode (0.0785) is slightly higher than that of the cathode (0.0585), and consequently the current shows a preference for traveling along the cathode. Thus, the current density is

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**Fig. 4** Iterative algorithm for solving the finite difference procedure

**Fig. 5** \( V-I \) Characteristics of the HPD design of SOFC
Fig. 6 Total current flowing in the cell and the distribution in ribs and branches at an operating voltage of 0.72 V.

Fig. 7 Current density distribution at two cross sections: Cell inlet (x=0) and cell exit (x=L). The current density is highest at the points where current enters the cell from interconnect (N_0, N_2, and N_4).

Fig. 8 Anode currents per unit cell length (A/m) flowing on two cross sections: cell inlet (x=0) and exit (x=L).
high close to the point where current enters the cell from the interconnect of the previous cell. A thinner cathode would imply that $p_l/d$ for the cathode would be closer to that of the anode, in which case the current density distribution curve would be flatter and more uniform.

Figure 8 shows the current per unit cell length ($A/m$) flowing on the anode surface at the inlet and exit cross sections. The anode current is highest at the point where the current enters the cell from the interconnect. As current begins to cross the electrolyte and flows towards the cathode, the anode surface current begins to decline and finally reaches zero. Thereafter, it begins to increase in the opposite direction and reaches a maximum corresponding to $N_2$ (location of the second interconnect). The current distribution between $N_0$ and $N_2$ satisfies the condition that $N_0$ and $N_2$ are equipotential (Eq. (21)). A similar result is observed between $N_2$ and $N_4$ (Eq. (22)). After $N_4$, the anode current declines and reaches almost zero at the end of the anode surface.

The cathode currents per unit cell length ($A/m$) at the inlet and exit cross sections are shown in Fig. 9. It is highest at points

![Graph of cathode currents per unit cell length](image1)

**Fig. 9** Cathode currents per unit cell length ($A/m$) flowing on two cross sections: cell inlet ($x=0$) and exit ($x=L$).

![Graph of anode and cathode surface voltage](image2)

**Fig. 10** Anode and cathode surface voltage at the inlet cross section. The voltage at points $N_0$, $N_2$, and $N_4$ is taken to be 0 as a base.
corresponding to the location of ribs ($N_1$ and $N_3$). It reaches a minimum (close to zero) at the locations of interconnects ($N_0$, $N_2$, and $N_4$). The cathode current after $N_3$ keeps increasing and finally equals the terminal current ($I_t$), which crosses the gap into the interconnect. The purpose of the ribs is to shorten the current lengths in the cell and to ensure that most of the current passes through the ribs. Therefore, the cell dimensions ought to be selected so as to minimize the terminal current ($I_t$) since that contributes to the voltage losses in the cathode and also in the fuel electrode gap.

Figure 10 shows the voltages on the cathode and anode surfaces at the inlet cross section (x=0). The points where the current enters the cell ($N_1$, $N_2$, and $N_4$) are equipotential (Eqs. 21 and 22) and their voltage is set to zero as reference. The anode surface voltage decreases from the base value (and is always less than zero) because of the Ohmic losses in the anode surface. The cathode voltage at any point is simply the sum of the anode voltage and the Nernst voltage at corresponding point less the activation and concentration polarization losses in the electrolyte. Thus, the cathode surface voltage is highest at points corresponding to $N_1$, $N_2$, and $N_4$ on the anode. This is because the anode voltage is highest at these points.

Finally, the effect of changing the rib width is shown in Fig. 11. Here, the rib width is 4000 μm versus the base case of 2000 μm. This decreases the resistance of the rib and therefore increases the current flow in each of the two of the two ribs by about 10%. For the same operating voltage of 0.72 V, the total current flowing in the cell is 295 A compared to 280 A in the base case: an increase of 5.4%. Therefore, the power of the cell increases to 211 W when the rib width is doubled.

Conclusions

In this paper we developed a computational model to predict the performance of the high power density SOFC. A resistor network was set up including the nonlinear activation and concentration polarizations and the resulting system of nonlinear equations was solved numerically to model the current transport in the cross section of the cell. The results are in good agreement with the experimental results quoted by Siemens Westinghouse for this design. The flattened SOFC gives a higher power density than the conventional tubular SOFC and the presence of ribs allows for shorter current paths in the cell resulting in lower losses.

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Appendix: Values of variables and parameters used in the model.

<table>
<thead>
<tr>
<th>Cell width ($W$)</th>
<th>10 cm</th>
<th>[1–3]</th>
</tr>
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<tbody>
<tr>
<td>Cell height ($H$)</td>
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<td>[1–3]</td>
</tr>
<tr>
<td>Cathode thickness ($\delta_c$)</td>
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<td>[1–3]</td>
</tr>
<tr>
<td>Anode thickness ($\delta_a$)</td>
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<td>[1–3]</td>
</tr>
<tr>
<td>Electrolyte thickness ($\delta_h$)</td>
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<td>[1–3]</td>
</tr>
<tr>
<td>Fuel electrode gap</td>
<td>5 mm</td>
<td>[1–3]</td>
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<tr>
<td>Interconnect height ($H_i$)</td>
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<td>[1–3]</td>
</tr>
<tr>
<td>Anode resistivity ($\rho_a$)</td>
<td>$e^{\exp(-6.03-1100/T)}$ Ω cm</td>
<td>[6]</td>
</tr>
<tr>
<td>Cathode resistivity ($\rho_c$)</td>
<td>$e^{\exp(-5.48+1210/T)}$ Ω cm</td>
<td>[6]</td>
</tr>
<tr>
<td>Electrolyte resistivity ($\rho_h$)</td>
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<td>[6]</td>
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<tr>
<td>Interconnect resistivity ($\rho_i$)</td>
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<td>Exchange current ($i_0$)</td>
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Nomenclature

- $E_r$: reversible Nernst voltage (V)
- $I$: current density (A/m²)
- $I_a$: anode current per unit depth (A/m)
- $I_c$: cathode current per unit depth (A/m)
- $I_0$: exchange current (A/m²)
- $I_t$: total current per unit depth at a cross section (A/m)
- $R_a$: anode resistance of a differential element per unit depth of cell
- $R_c$: cathode resistance of a differential element per unit depth of cell
- $R_e$: electrolyte resistance of a differential element per unit depth of cell
- $R_p$: activation and polarization losses of a differential element per unit depth of cell
- $T$: cell operating temperature
- $V_{op}$: cell operating voltage (V)
- $i_l$: limiting current (A/m²)
- $\delta_a$: anode thickness (microns)
- $\delta_c$: cathode thickness (microns)
- $\delta_h$: electrolyte thickness (microns)
- $\eta_{act}$: activation polarization (V)
- $\eta_{anode}$: Ohmic polarization in anode (V)
- $\eta_{cathode}$: Ohmic polarization in cathode (V)
- $\eta_{conc}$: concentration polarization (V)
- $\eta_{electrolyte}$: polarization in electrolyte (V)
- $\eta_{ohm}$: total Ohmic voltage losses (V)
\[ \rho_a = \text{anode resistivity (}\Omega \text{ m}) \]
\[ \rho_c = \text{cathode resistivity (}\Omega \text{ m}) \]
\[ \rho_e = \text{Electrolyte resistivity (}\Omega \text{ m}) \]

References