Controllability and viability: a look at robustness of walking

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1 Motivation

A good walking robot must be stable, whatever that means. Some questions about stability include: How is stability defined or measured? How can a robot be made stable? How can stability be 'increased' or maximized? There are no unique, widely accepted answers to these questions. One recently popular approach is based on Jerry Pratt's capture regions [1, 2]. For a given state of the robot, a capture region is all locations on the ground where the robot can step and then come to a complete stop in one step or in a specified number of steps. The capture region framework is a useful tool for developing a motion controller, since it says where you need to step in order to stop (hence, not fall down). If the robot has such ability in its current state, the state is called stable.

We try to approach the issue of robustness with a framework, similar to that of capture regions. We would like to understand how a person does, and a robot might, both not fall down and also achieve specific goals of locomotion, such as standing still, moving with a desired speed or in a desired direction. We study the avoidance of falls with the concept of *viability*, and the achieving of different locomotion tasks through *controllability*. We use the same concepts to understand robustness of a robot against external disturbances and noise.

2 Approach: viability and controllability

A given dynamical state of a robot is called n-step viable, if there is any way, within the limits of the actuator abilities and any other constraints, for the robot to take n steps without falling. We define V_n (the n-step viable region) to be the set of all points in the phase space that are n-step viable. For example, V_1 is all the states from which it is possible to make at least one step without falling. Viable regions form a nested sequence ($V_{n+1} \subset V_n$), the limit of which is V_∞, the set of all states for which falling is avoidable with some feasible control. If a robot is in a state outside of V_∞, then no control will be able to prevent it from falling.

We define controllable regions with respect to more specific goals. We may have a specific configuration, speed, location on the ground, or, in general, a region in the phase space that we desire to achieve. We call C_0, the zero-step controllable region, the set of all states satisfying the goal. Then C_n is all states in the phase space that can, with some achievable controls, lead to one of the goal states in C_0 in n or fewer steps. The sequence $\{C_n\}$ is nested, and the ∞-step controllable region C_∞ is the set of all points from which the biped can get to a desired state. If the desired state is the upright position with no velocity, the regions C_n correspond to Koolen's viable-capture basins [2], which they use as a way to approach capture regions.

In construction of the viable and controllable regions, we assume no disturbances occur during the motion, and all sensor inputs, if any, are perfect. It is possible, though, to take them into consideration if characteristics of the noise are known. For example, one might introduce an additional requirement that all goals and constraints at each step are met for any disturbances within the considered bounds. Also, in the same way, additional motion constraints can be used, such as limitations on the actuators, or the requirement to use a specific control law.

3 Results

To start with we use the Inverted Pendulum Model (IPM) in 2D. The IPM has a point mass at the hip and two rigid massless legs, and collisions are assumed plastic. Location of the step and push off impulse along the stance leg just before the collision are two control parameters of the system per each step. The number of dynamic variables is also two: angle and angular rate of the stance leg.

For this model of walking, we numerically find all viable V_n and a few of controllable regions C_n. We choose the target state to be a fixed velocity at midstance. We also find these regions for the case when the step duration is constrained to be not less than a fixed positive value - this is a proxy for leg swinging limitations of actuators. Based on the graphs of C_n and V_n we discover that for the 2D IPM, $V_\infty = C_\infty$. That is, being able to ever reach the target is equivalent to being able to not fall down. In fact, we argue that the same statement holds true for most any target and most any bipedal robot. A notable exception, suggested in [2], is passive walkers.

Two steps is almost everything. Another observation we make is *two steps is almost everything*. Based on the computed C_n regions, we find that the two-step controllable region C_2 builds up most of what is controllable (C_∞), and $C_2 = C_\infty$ for the case of constrained step duration. In other words, if you can reach the target at all, you can do it in two steps. The part of phase space that is controllable in three or more steps, but not in two steps, is small. This is in agreement with different treadmill experiments, e.g. ones presented in [3, 4, 5, 6]. This suggests that a walking controller has no need to plan more than two steps ahead.

4 Future Work

We plan to use the controllability and viability concepts for 3D models of walking, such as the IPM in 3D, and the Linear Inverted Pendulum model, to see if the results here are maintained. We also plan to use controllable and viable regions to design a controller of motion, or to stabilize an existing one (e.g. an energy optimal controller): the controller should keep the robot as much inside the controllable or viable regions as possible.

References

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