Orbital Energy Based Foot Placement during Gait in a 3D Linear Inverted Pendulum Model with Foot.

M. Vlutters¹, E.H.F. Van Asseldonk¹, L. Wang¹, H. van der Kooij^{1,2}

1)Lab. of Biomechanical Engineering, University of Twente, 2) Lab. of Biomechanical Engineering, Delft University of Technology

(m.vlutters@utwente.nl)

1 Motivation) Appropriate foot placement in both anterioposterior (AP) and medio-lateral (ML) direction is crucial for maintaining balance during human gait. Research suggest that foot placement provides the overall balance control, whereas ankle strategies are used for fine-tuning [1]. In order to realize a flexible and adaptive gait in bipedal robots, an online, state dependent prediction of a foot placement location and time are required.

2 State of the Art) Various balance concepts have been proposed, such as the widely used zero-moment point (ZMP) [2] or the easy to compute Capture Point (CP) [3]. The former is often limited adaptive due to the use of predefined ZMP trajectories. The latter does not provide direct information on how to keep walking, nor on a suitable time of stepping. Furthermore, many balancing principles are often simulated in 2D only, while in 3D the AP and ML directions are linked in time and therefore must move synchronized for straight gait.

3 Approach) The single stance phase of gait can be modeled with the linear inverted pendulum model (LIPM) [4], which is here extended with a finite sized stance foot. The ankle joint is considered the origin ($[0,0]^T$). The center of mass (COM) and center of pressure (COP) positions in the horizontal plane are $\mathbf{r}_{COM} = [\mathbf{x}_{COM} \ \mathbf{y}_{COM}]^T$ and $\mathbf{r}_{COP} = [\mathbf{x}_{COP} \ \mathbf{y}_{COP}]^T$ respectively. The COP is assumed controllable within the stance foot. By assuming that equal but opposing vertical forces act on the COM and COP, and by using a small angle approximation, the equations of motion can be reduced to:

In which $\omega_0 = \sqrt{z_0/g}$, g is the earth's gravitational constant and z_0 is the (constant) pendulum height.

The sum of the model's potential and kinetic energies, known as orbital energy (E_{orb}) [5], equals:

$$E_{orb} = 0.5 (\dot{r}_{COM}^2 - \omega_0^2 (r_{COM} - r_{PCOP})^2)$$
[II]

with \mathbf{r}_{PCOP} any possible \mathbf{r}_{COP} location. The pendulum's true \mathbf{E}_{orb} is found for $\mathbf{r}_{PCOP} = \mathbf{r}_{COP}$, and is constant as long as \mathbf{r}_{COP} does not move.

The solutions to differential equation [I] can be substituted into [II]. Using the current pendulum state (\mathbf{r}_{COM} , $\dot{\mathbf{r}}_{COM}$ and \mathbf{r}_{COP}), a remaining swing time (t_{rem}) and a desired \mathbf{E}_{orb} , [II] can be solved for \mathbf{r}_{PCOP} to yield a stepping location after t_{rem} seconds. A desired \mathbf{E}_{orb} can be obtained from a desired gait consisting of a reference step length, width and swing time.

Modulation of \mathbf{r}_{COP} within the stance foot can be used to ensure synchronized AP and ML movement by steering the COM towards the correct velocities. Additional adjustments to t_{rem} can be made if this modulation would be insufficient. Variable t_{rem} is constrained by a state dependent upper bound and a fixed lower bound. When the desired gait has been obtained, modulation of \mathbf{r}_{COP} within the foot will be no longer required and step length, width and time will remain constant. **4 Results)** Simulations show that moving \mathbf{r}_{COP} within the foot can help adjust COM movement during swing and assure better recovery from perturbations. Figure 1 shows a simulation of gait using a reference step length, width and swing time of 0.55 m, 0.10 m and 0.50 s respectively. At t=0, $\mathbf{r}_{COM} = [0,0]^T$, $\mathbf{r}_{COM} = [0,0]^T$, $\mathbf{r}_{COP} = [0,0]^T$. Height z_0 was taken 1m. At t=1.80s (arrow) a perturbation was applied by instantly shifting the COM 0.05 m to the left. During the simulation

r_{COP} could instantly change position within the foot. After simulation start, **r**_{COP} shifts backward and to the left to initiate COM movement, while t_{rem} is increased to give the COM sufficient time to accelerate. The desired gait is obtained after foot placement (foot 2). When the ML perturbation is applied, **r**_{COP} responds by shifting in both AP and ML direction (foot 3) in order to reestablish synchronized pendulum movement. This requires a temporarily increase in swing time as well. With subsequent foot placement (foot 4) the desired gait is obtained again.



Figure 1) 3D LIPM simulation. Reference parameters; 0.55m, 0.10m, 0.50s for step length, width and swing time respectively. Initial COM and COP are in $[0,0]^T$. Initial COM velocity is $[0,0]^T$. A 0.05m COM shift to the left is applied at t=1.80 as perturbation. Top) COM velocity over time. Mid) COM position in horizontal plane. Bottom) COP position within foot. Black squares indicate the foot origin and correspond with those in the third plot.

5 Best possible outcome) Model extension by incorporation of a double support phase and additional constraints to realize more realistic behavior ideally leads to an accurate online prediction of a foot placement location and time.

References)

1.Hof et al., (2007), Gait and Posture

- 2. Vukobratovic and Juricic, (1969), IEEE Trans. on Biomed Eng 3. Pratt et. al., (2006), Proceedings 6th IEEE-RAS Intern. Conference 4. Kaiita and Tani, (1991). Proceedings IEEE Intern. Conference
- 5.Kajita et al, (1992), IEEE Trans. on Robotics and Automation