Stability Analysis of Discontinuous Systems with Impacts and Friction via Sums-of-Squares

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1 Motivation

Many critical tasks in robotics, particularly locomotion, involve collisions between a rigid body and the environment or between multiple bodies. Sums-of-squares (SOS) based methods for numerical computation of Lyapunov certificates are a powerful tool for analyzing the stability of continuous nonlinear systems, which can play an important role in motion planning and control design. By extending these techniques to discontinuous systems, particularly rigid bodies with Coulomb friction undergoing inelastic impact events, we can generate Lyapunov certificates for stability, positive invariance, and reachability.

2 State of the Art

In the controls community, SOS has been widely applied, particularly in automating Lyapunov analysis of polynomial dynamical systems [1]. For smooth nonlinear systems, techniques for stability verification can be applied to incremental motion planning and control design strategies [4].

One formalism for modeling non-smooth systems is that of hybrid automata. A number of numerical techniques have been presented for addressing verification, stability, and control design of general hybrid automata [5]. However, for a general rigid body with m possible contact points, there are 2^m potential hybrid modes.

3 Our Approach

Our central observation is that the complementarity framework for modeling such systems[3] is directly compatible with SOS optimization. With this framework, and the notions of measure differential inclusions, we can generate Lyapunov certificates by directly reasoning over the set of admissible states and feasible contact forces, without suffering from the combinatorial explosion of modes resulting from distinct combinations of contacts.

4 Current Results

In our most recent work, we have applied these techniques to verifying the stability of equilibria on a number of robotics examples[2]. In particular, we perform smooth (non-hybrid) analysis of the region of attraction of the passive rimless wheel, while in double support.

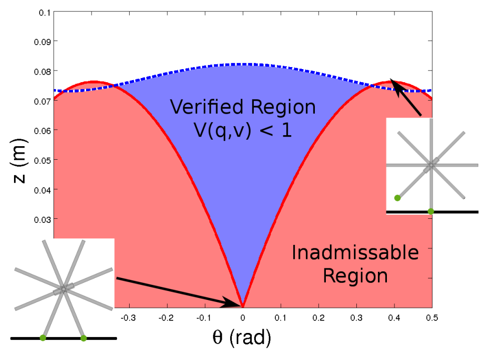

Figure 1: A slice of the rimless wheel state space is shown where all velocities are zero.

5 Best Possible Outcome

Going forward, we hope to continue this work to analyze trajectories and limit cycles of walking and running robots. We also plan to apply this method to synthesize controllers that the verified region through contact.

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References

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