Actuated SLIP Model: Partial Feedback Linearization and Two-Part Control Strategy

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Introduction

Biological data suggest that legs regulate energy production and removal via muscle activation: in this work we consider the **active SLIP model**, an energetically non-conservative version of the SLIP model with series actuation.

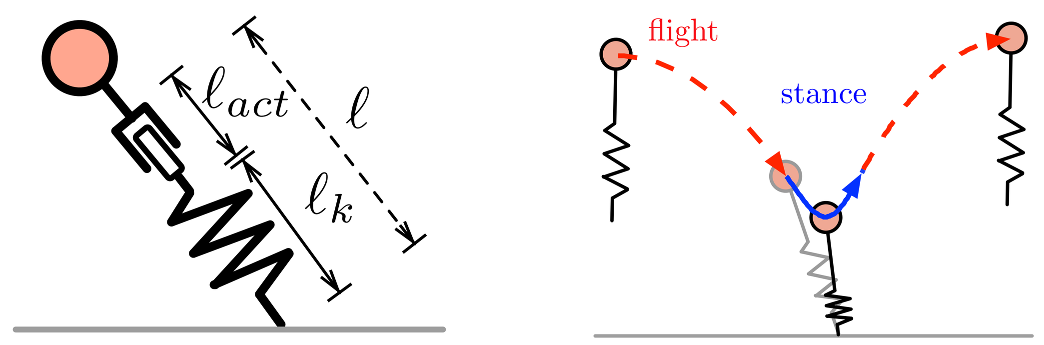
We propose a **strategy for actuator displacement** in order to:

- add/remove energy from the system,
- analytically solve part of its dynamics,
- online computation of actuator displacement and leg positioning to drive the system to a desired state, even in the presence of terrain perturbation

Active SLIP model

Passive SLIP model:

- Point mass, M, attached to a massless spring leg, with length ℓ and spring stiffness constant k.
- Piston-like actuator ℓ_{act} added in series with the spring
- Running dynamics consist of two phases: the flight phase and the stance phase



- Equations of motion during stance not analytically solvable:

$$\ddot{\ell}(t) = -\frac{k}{M}(\ell(t) - \ell_0 - \ell_{act}(t)) - g\sin\theta + \ell\dot{\theta}^2, \qquad (1)$$

$$\ddot{\theta} = -2\frac{\dot{\ell}}{\ell}\dot{\theta} - \frac{g}{\ell}\cos\theta. \qquad (2)$$

Approximating the stance phase dynamics through partial feedback linearization

Divide total actuator displacement in two parts: $\ell_{act} = \ell_{nl} + \ell_c$, such as:

- $\ell_{nl}(t)$, has the purpose of cancelling the nonlinear terms in (1):

$$\ell_{nl}(t) = \frac{M}{k}[g\sin\theta(t) - \ell(t)\dot{\theta}(t)^2].$$

- We drive the second term, ℓ_c, to a constant value $\bar{\ell}_c$, moving with constant velocity.
- We are then able to solve analytically the e.o.m of the leg length $\ell(t)$, and we use an approximation for the dynamics of the angle $\theta(t)$.

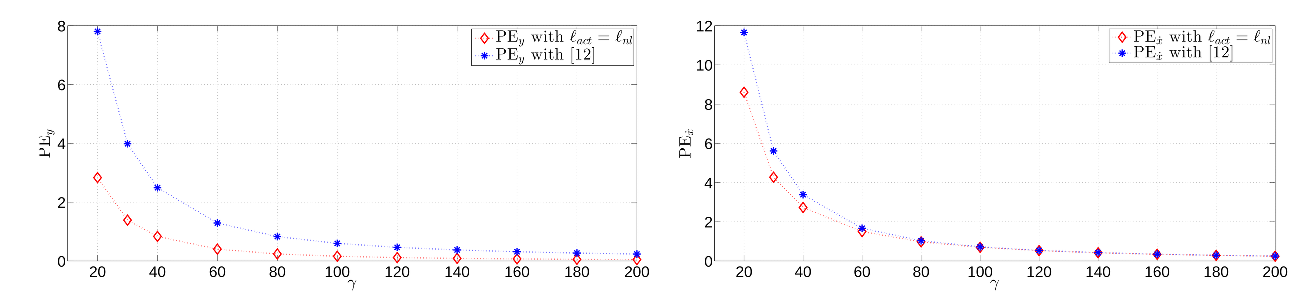
Error Reduction

What are the benefits of having a partial analytical solution to the stance phase?

We compute the error reduction as a percentage of the distance to the desired apex state:

$$PE_y = 100 \parallel y_{real} - y_{appr} \parallel / \ell_0, \quad PE_{\dot{x}} = 100\tau \parallel \dot{x}_{real} - \dot{x}_{appr} \parallel / \ell_0,$$

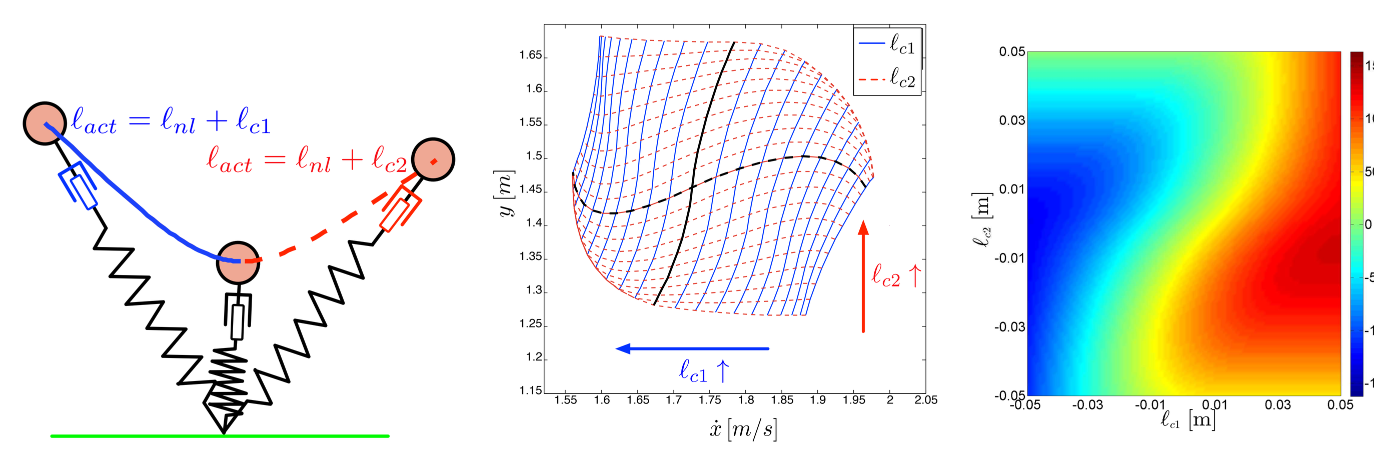
and as a function of the *relative spring stiffness* $\gamma = \frac{k\ell_0}{Mg}$.



Quadrupes and bipeds: $\gamma \simeq 10 - 20$ per leg.

Choice of Actuator Constant Value

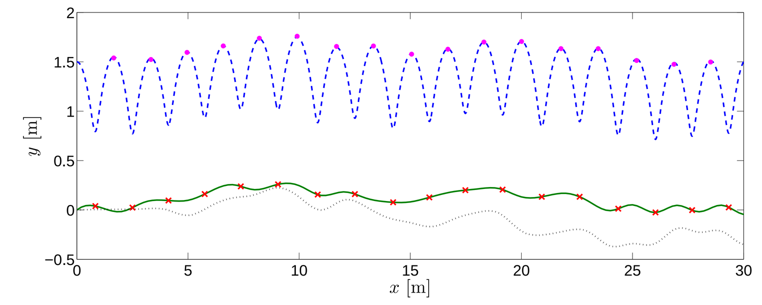
- Divide the stance phase in two parts, separated by the point of maximal leg compression
- Chose two constant values for ℓ_c: one for the first part, ℓ_{c1}, and one for the second part ℓ_{c2}, of the stance phase.



- Using optimization algorithm, chose best actuator values and touch-down angle to reach a desired apex state.
- In presence of **unexpected sensing error on terrain:** upon touching the ground, recompute the actuator value for the second part of the stance phase based on the new terrain information.

Applications: Hopping on Rough Terrain

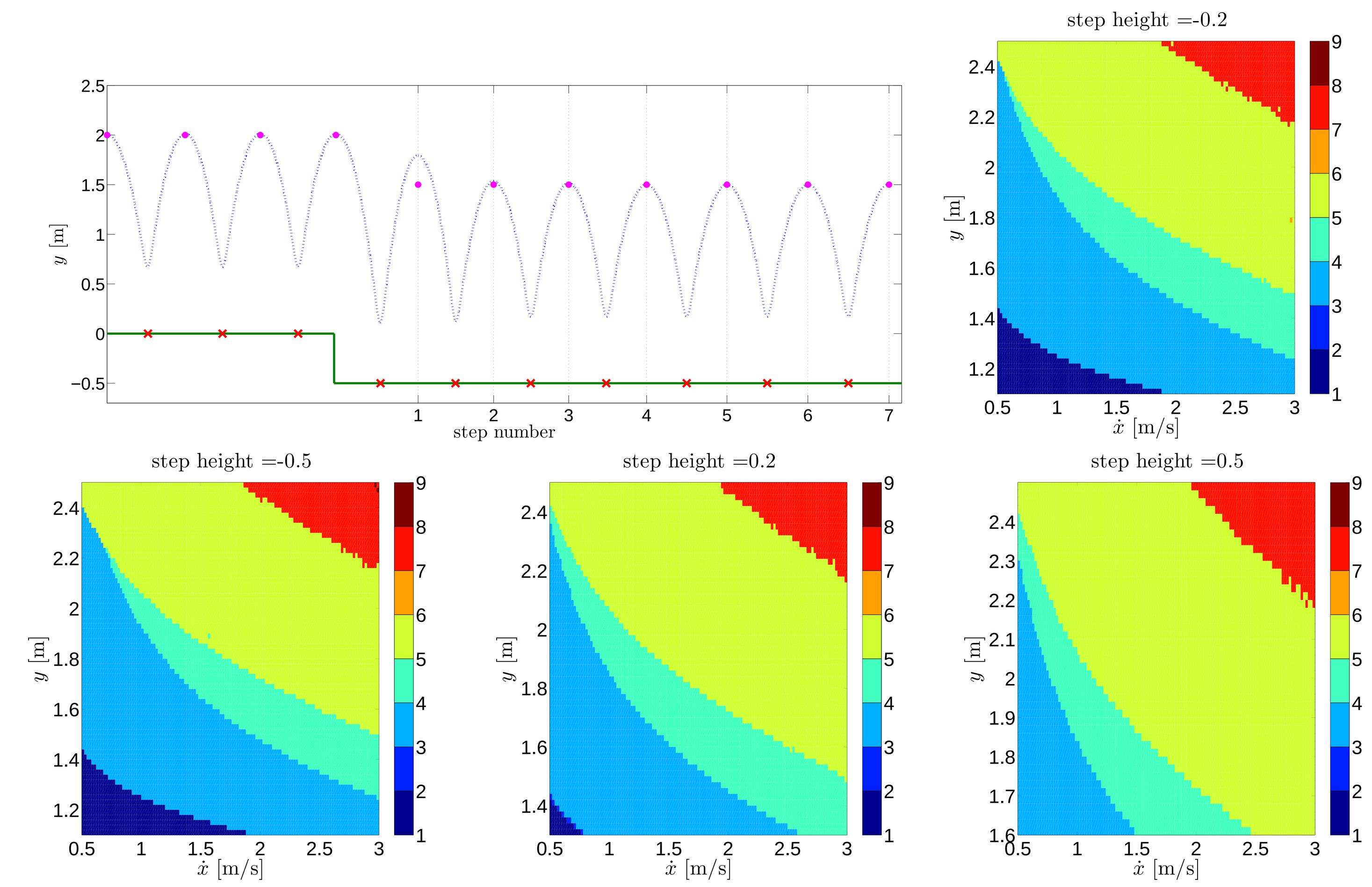
- Goal: maintain the same forward velocity and the same distance from the terrain with respect to the last step.
- Assume that the terrain measurements are faulty



$$J = \sqrt{PE_y^2 + PE_{\dot{x}}^2}$$

Applications: Recovery from perturbations

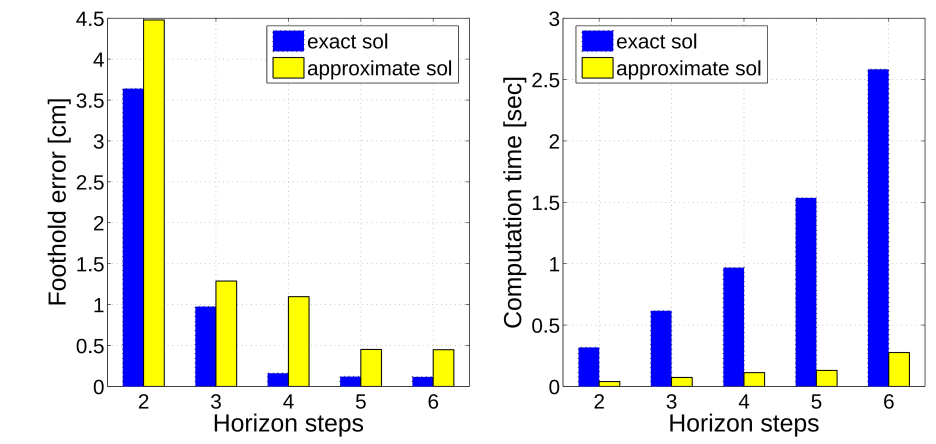
We test the recovery capabilities of our controller for unexpected (positive or negative) perturbations on the terrain height of up to 50% of the leg length.



Foothold Placement Control

- Terrain where only a specific set of N footholds is allowed: **minimizing the distance between the landing points and the desired footholds.**
- The longer the horizon N, the better the performance . The horizon length affects computational time.
  Compute optimal path via approximation reduces computational time $\longrightarrow$ possible to extend the planning horizon. But the approximation, as such, carries an error.
- **Trade-off between horizon length/computation time, and foothold error.**

average time during flight: $t \simeq 0.5sec$



References and Acknowledgments

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