# From Anchors to Templates: Exact and Approximate Reduction in Models of Legged Locomotion

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## I. Motivation

Animals exhibit low-dimensional dynamical variation during steady-state locomotion, effectively collapsing a large number (hundreds or thousands) of mechanical degrees-of-freedom to a behavior that can be reproduced by mathematical model with a small number (tens) of parameters and state variables [1]. The mechanisms underlying this empirically-observed reduction are partially understood: muscles activate synergistically; neural pattern generators synchronize; gaits and physiology possess symmetries. Yet few mathematical tools exist that can establish the observed order of reduction.

## II. State-of-the-Art

The model reduction problem is challenging since the dynamics of legged locomotion are generally (i) nonlinear due to pendulum-like pivoting over stance limbs, and (ii) discontinuous (or *hybrid*) owing to intermittent contact between limbs and substrate. Existing model reduction techniques applicable to nonlinear hybrid dynamics are generalizations of tools developed for smooth dynamical systems; examples include the Averaging Theorem, Noether's Theorem, and Feedback Linearization. These approaches impose assumptions on a system's dynamics that are difficult to verify for a given physical legged locomotor.

## III. Our Approach

Exploiting a phenomena unique to hybrid systems, we provide conditions under which hybrid dynamical models of legged locomotion reduce to smooth dynamical systems near periodic orbits. The technique is agnostic to the precise structure of the dynamics.

## IV. Results

Our theory can be used to construct a family of polypedal horizontal-plane models that reduce to a common template, the *Lateral Leg Spring*, and study the structural stability of this reduction. The original result in [2] provided a condition under which a hybrid system *exactly* reduces to a smooth dynamical system near a periodic orbit. We have extended this result to provide a generic condition guaranteeing that a hybrid system *approximately* reduces to a smooth system near an exponentially stable periodic orbit.

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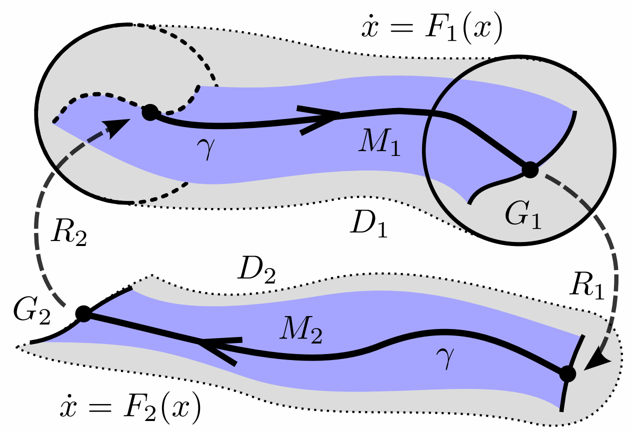


Fig. 1. Illustration of our reduction result in a model with two modes: near a periodic orbit $\gamma$, an invariant subsystem $M_j$ that attracts trajectories superexponentially emerges solely from the interaction between the dynamics of continuous "flow" and discrete "jumps".

## V. Best Outcomes

Our work provides a novel avenue through which model reduction may be formalized, discovered, and engineered in models for legged locomotion. Since the proposed approach imposes mild assumptions on a system's dynamics, we suspect it may apply to a broad class of physical systems.

We are actively pursuing the following extensions:

(a) anchored models for gaits in animals and robots with varying morphology, scale, and material composition that provably reduce to a common template;

(b) convincing evidence that animal evolution or robot controllers can exploit the proposed reduction technique to simplify control or enhance performance;

(c) generalization of a suite of tools from the theory of smooth control systems—for stabilization, estimation, adaptation, verification, etc.—to the hybrid setting using the smooth subsystem provided by our results.

## VI. Acknowledgments

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## References

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[2] S. Burden, S. Revzen, and S. S. Sastry, "Dimension reduction near periodic orbits of hybrid systems," in *Proceedings of the 50th IEEE Conference on Decision and Control*, pp. 6116–6121, 2011.