

# DATA-DRIVEN HIGH-DIMENSIONAL INVERSE PROBLEMS: A JOURNEY THROUGH STRONG GRAVITATIONAL LENSING DATA ANALYSIS

Laurence Perreault-Levasseur

# Modern Cosmology+Astrophysics



SKA



TMT



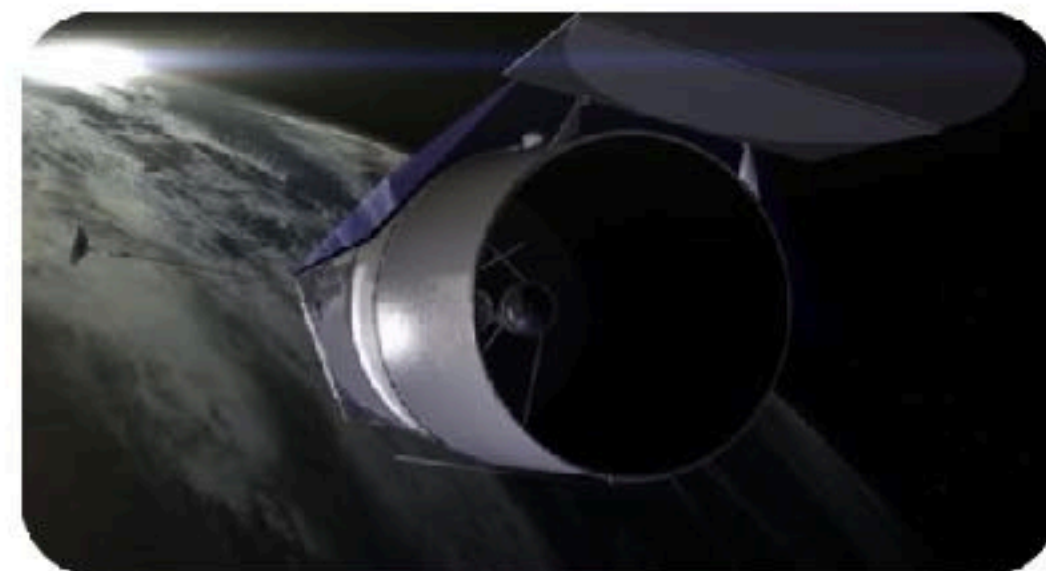
JWST



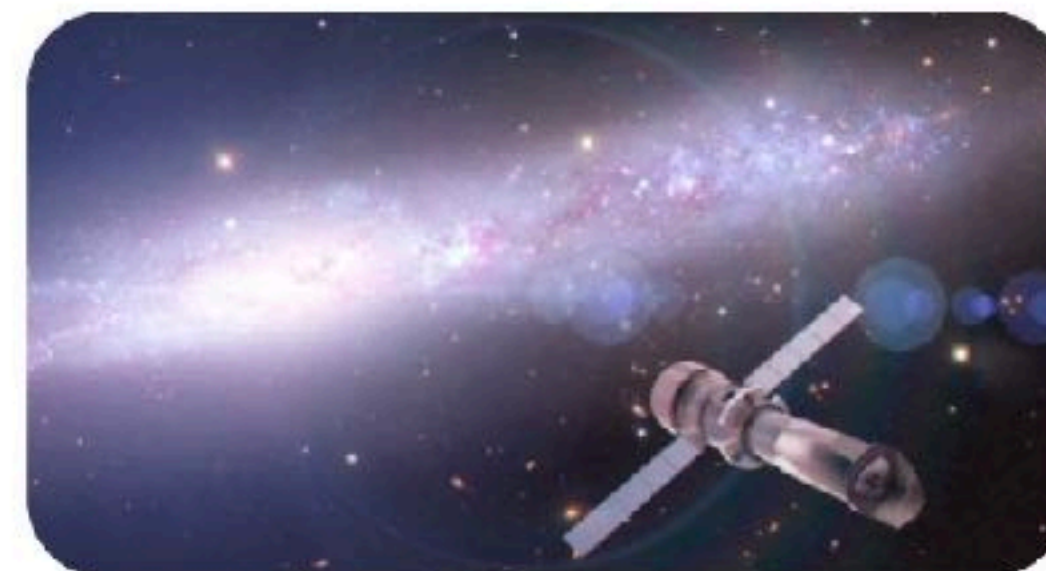
LSST



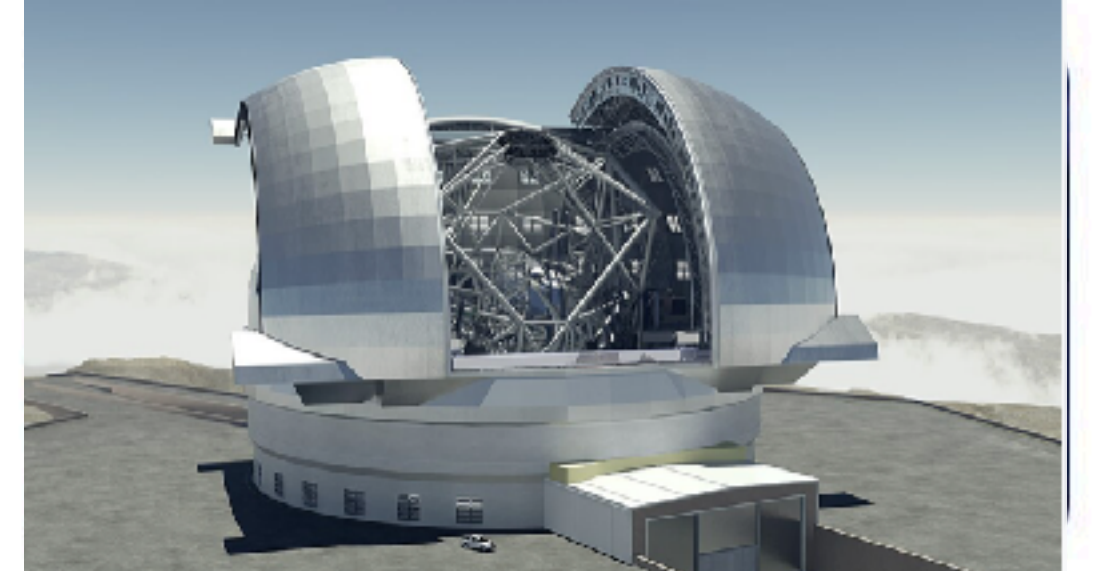
ALMA



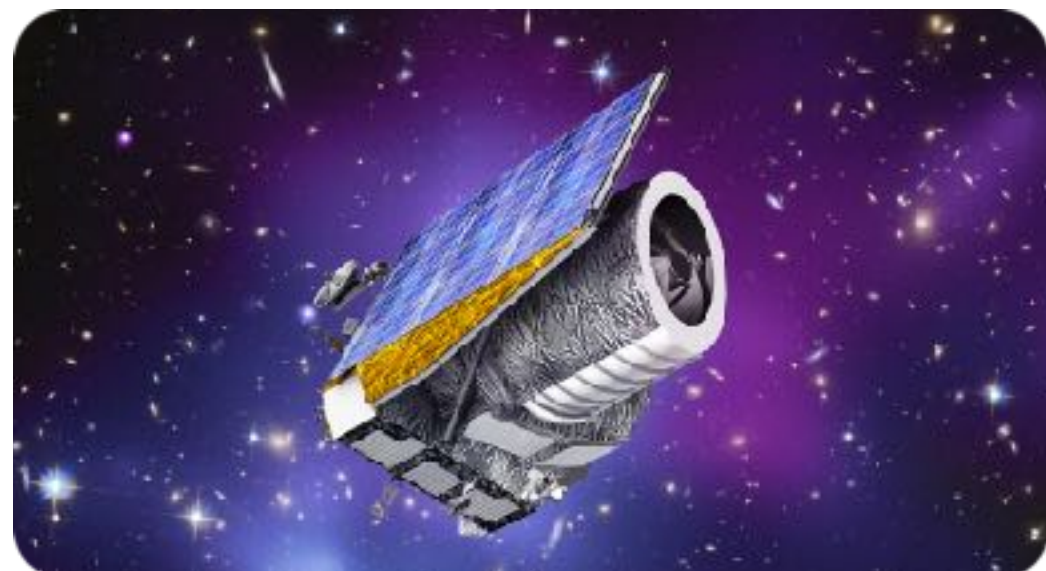
ROMAN



ATHENA



ELT



Euclid



Simons Observatory



CHIME



SPHEREX

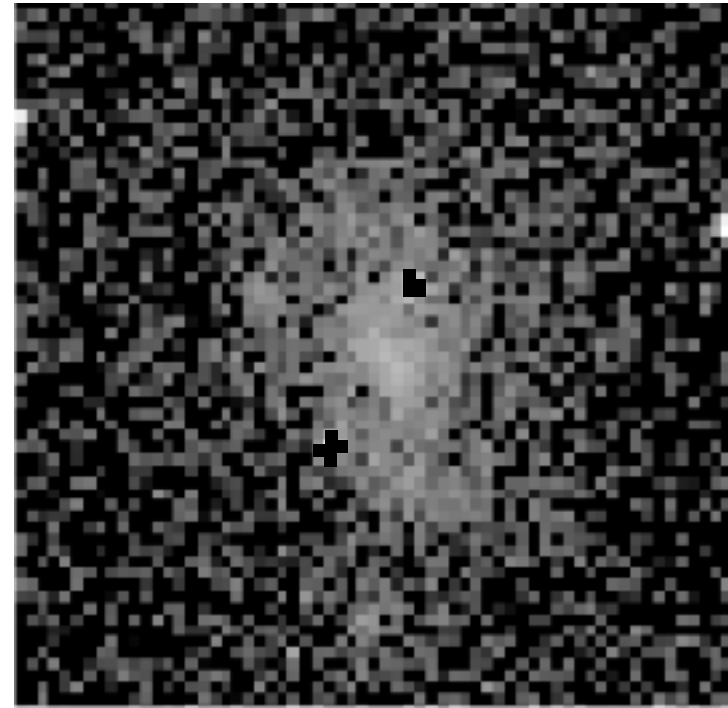
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Simple/low resolution/low SNR data -> simple model might be sufficient

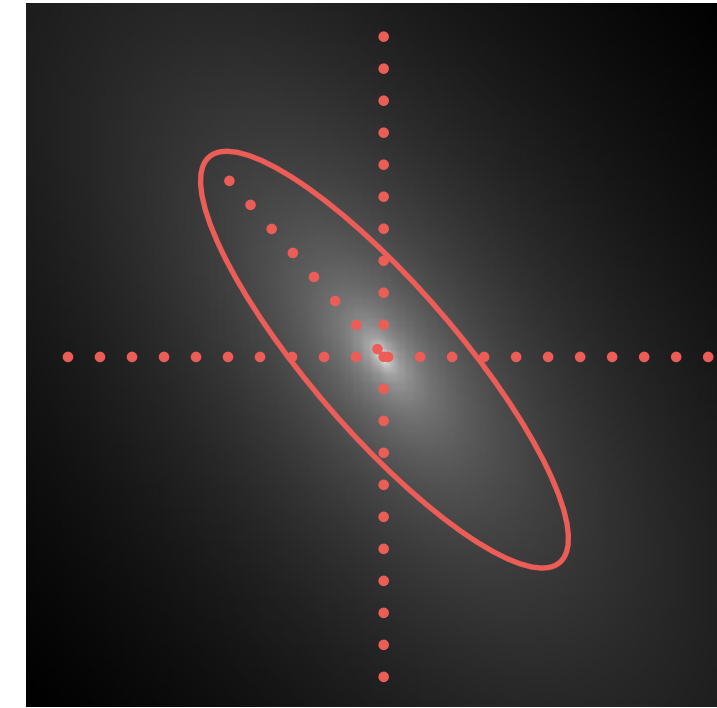
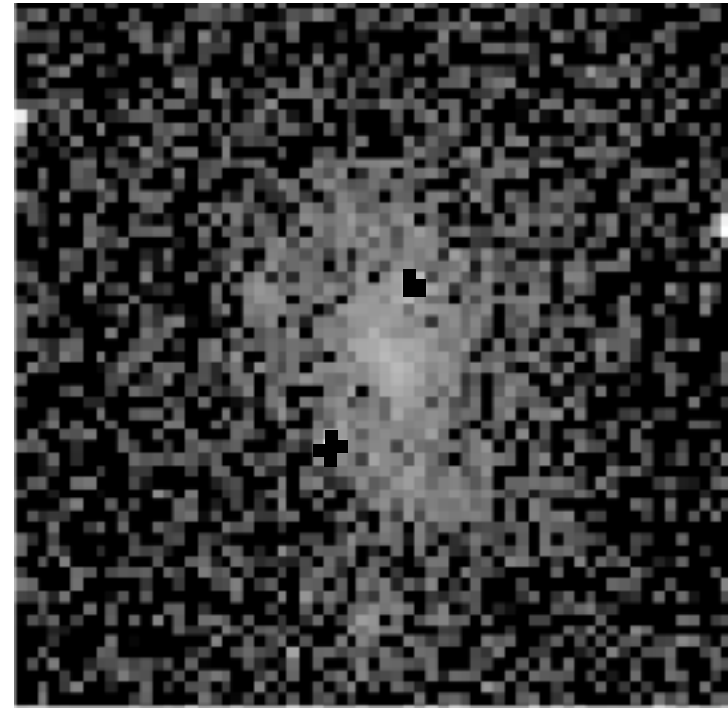
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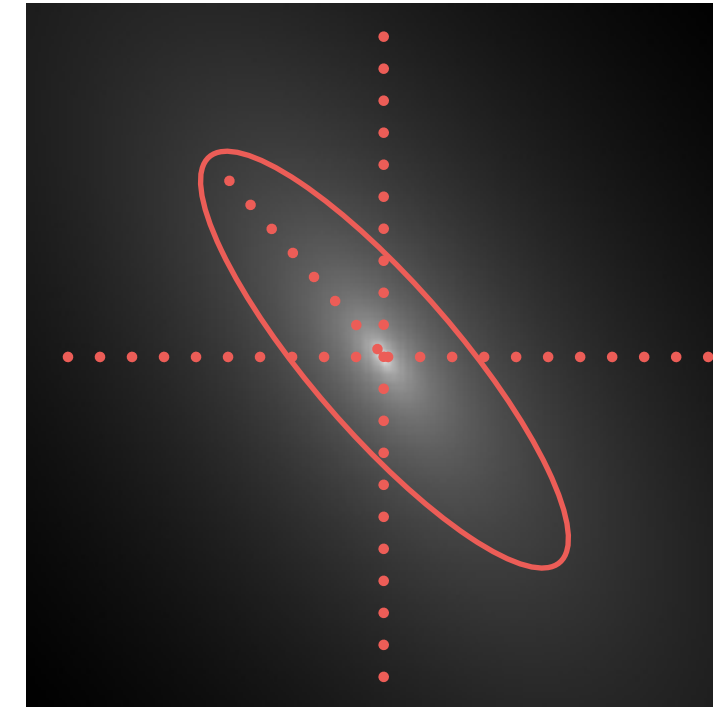
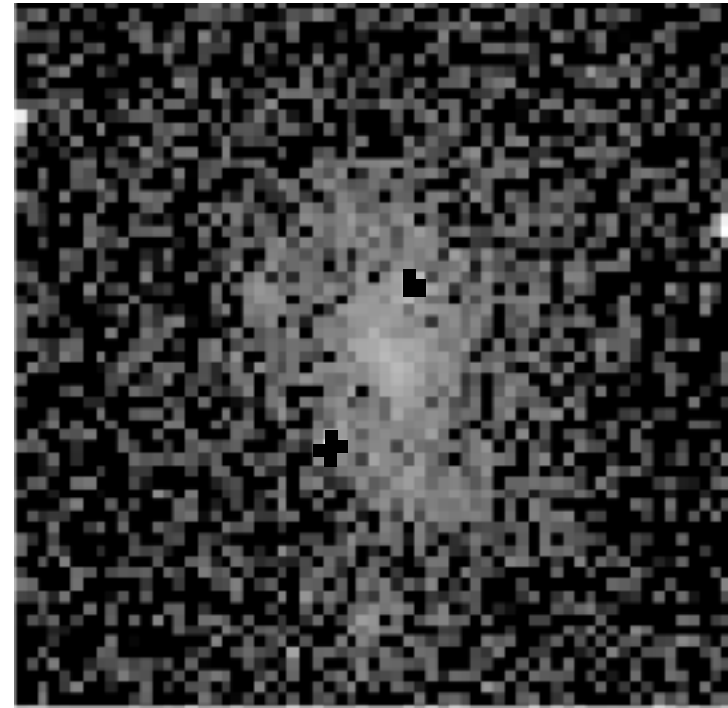
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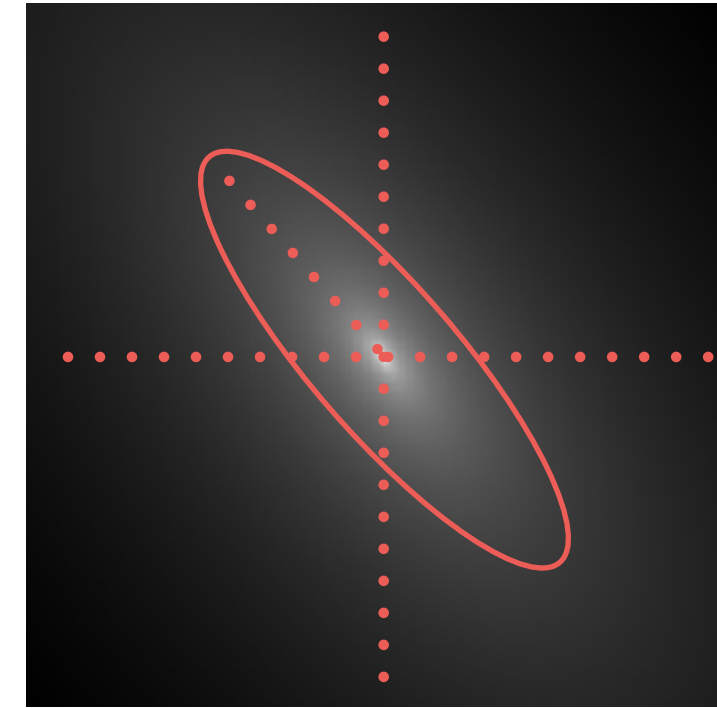
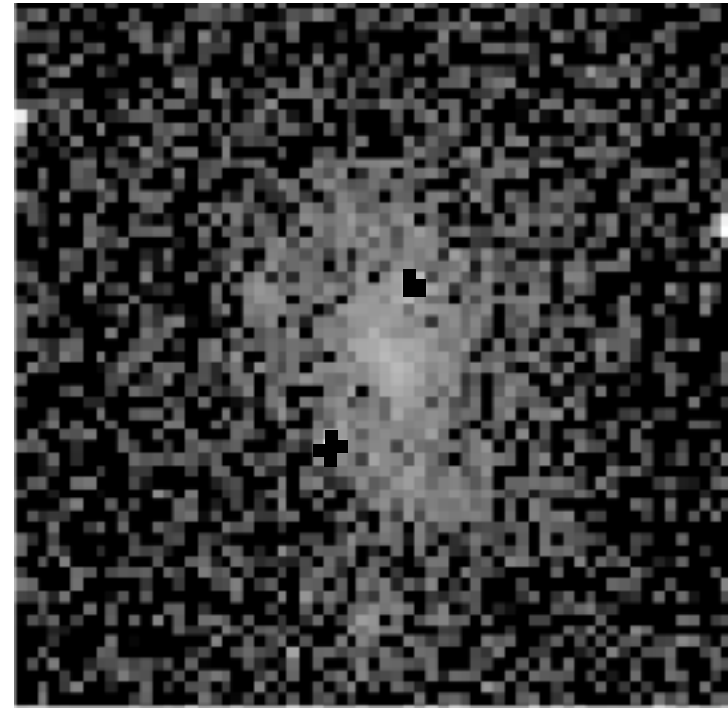
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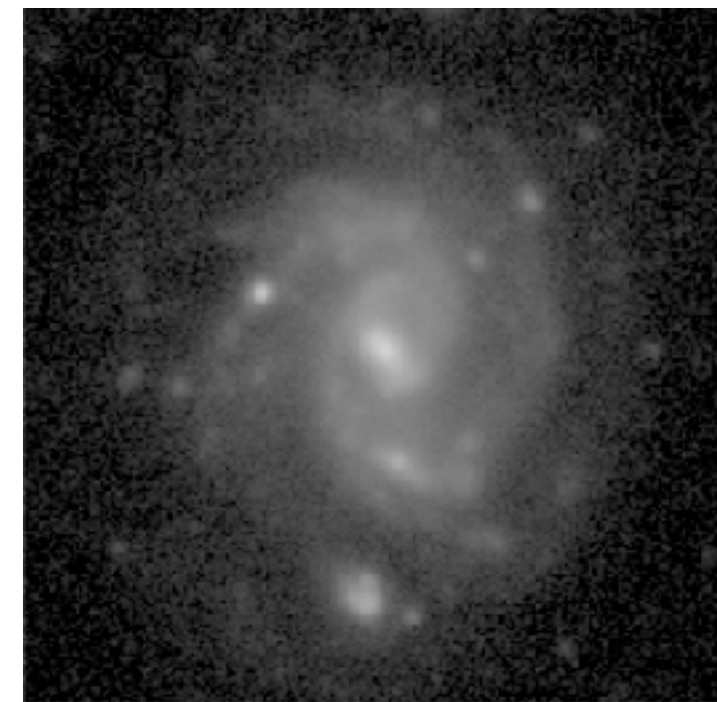
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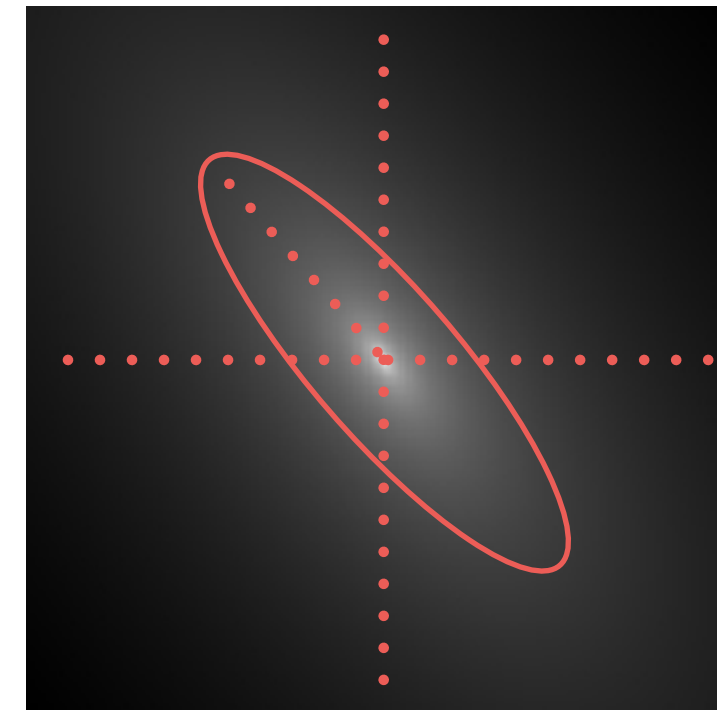
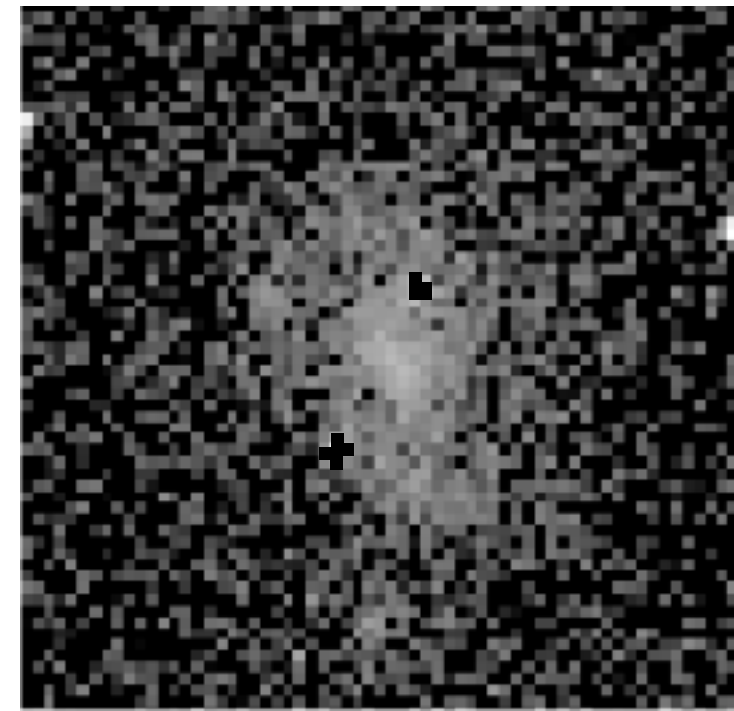


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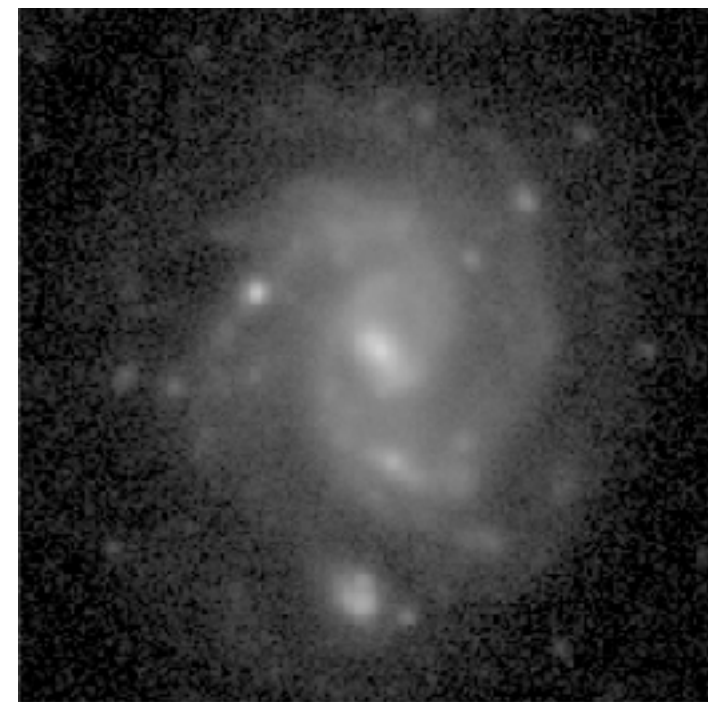


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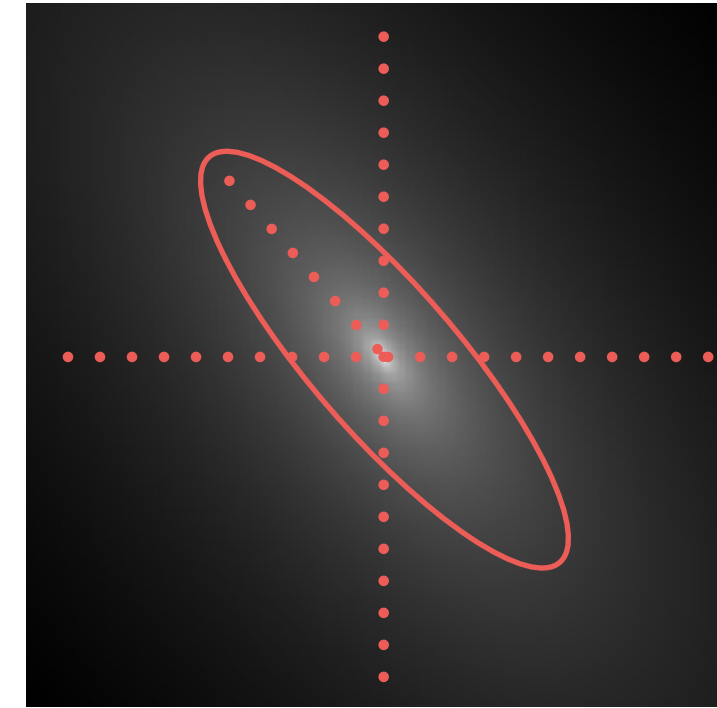
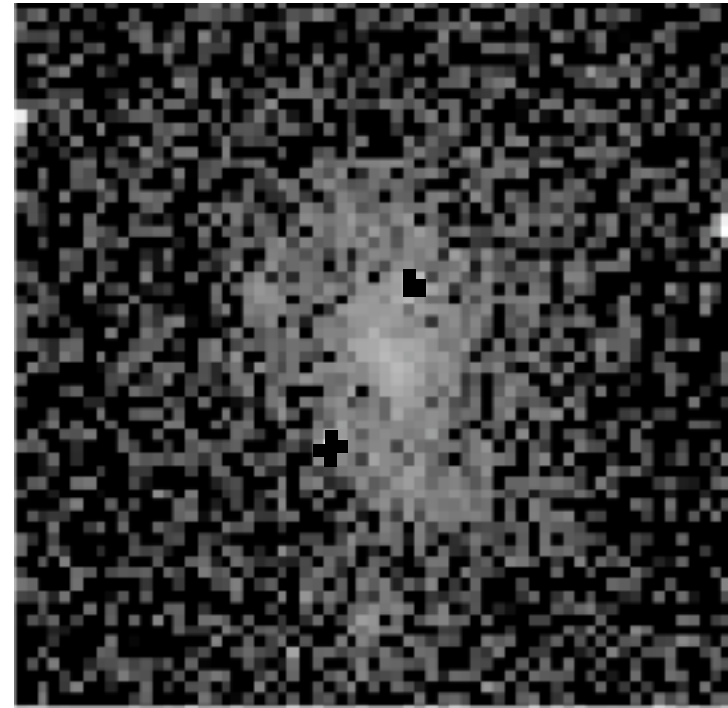


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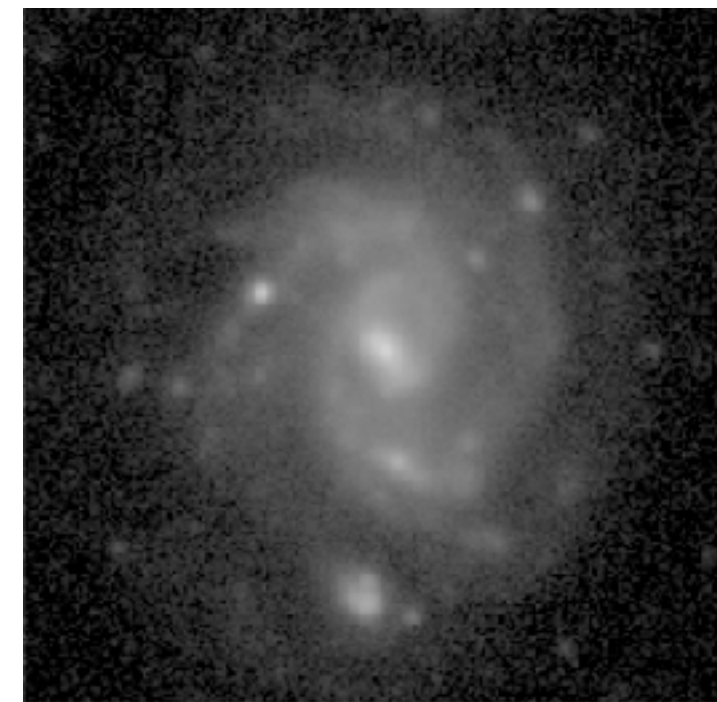


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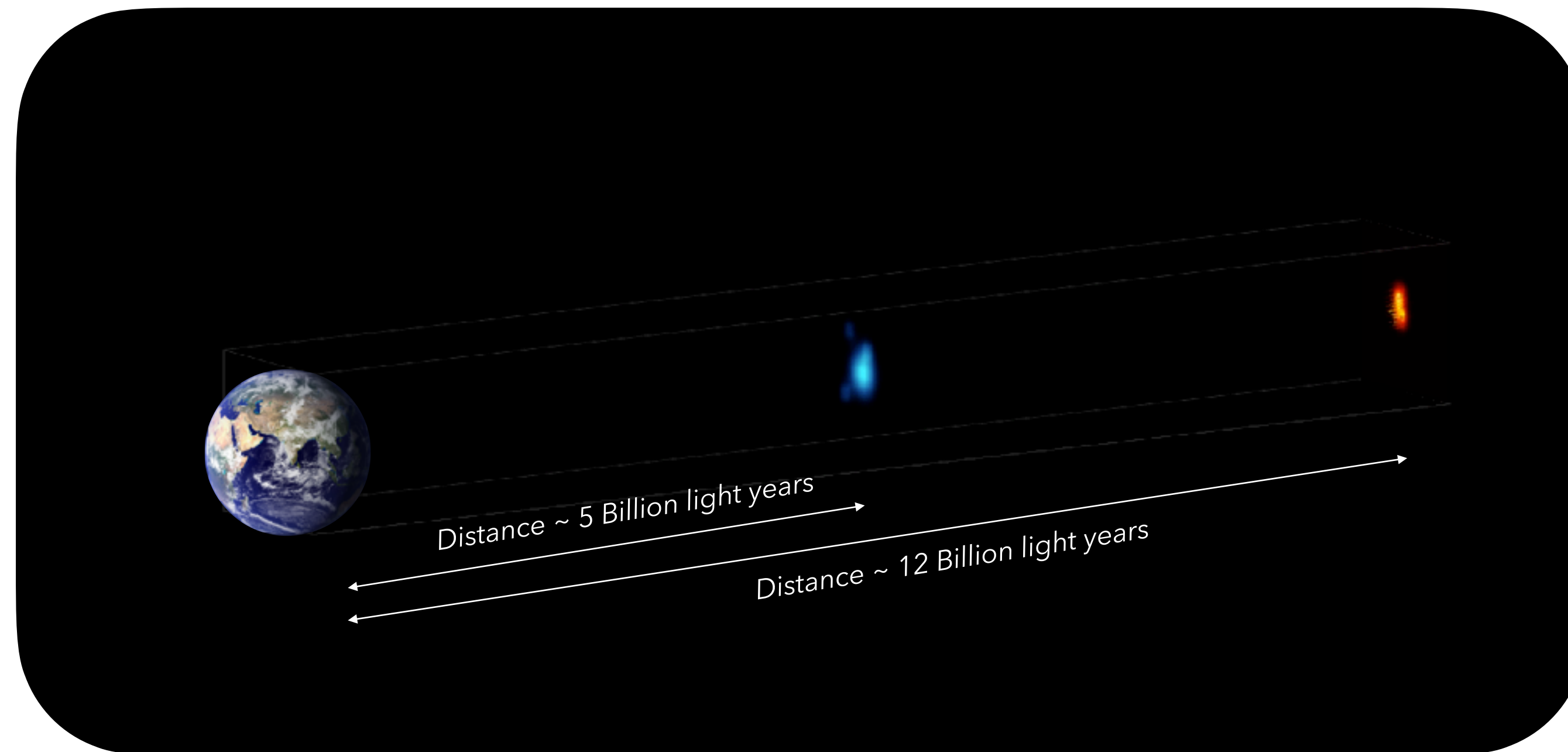
It's becoming more and more challenging to come up with analytical models complex enough to keep up with the increasing complexity of our data.

# GAME PLAN

1. Data-driven priors
2. Data-driven likelihoods
3. Accuracy metrics
4. Out-of-Distribution accuracy

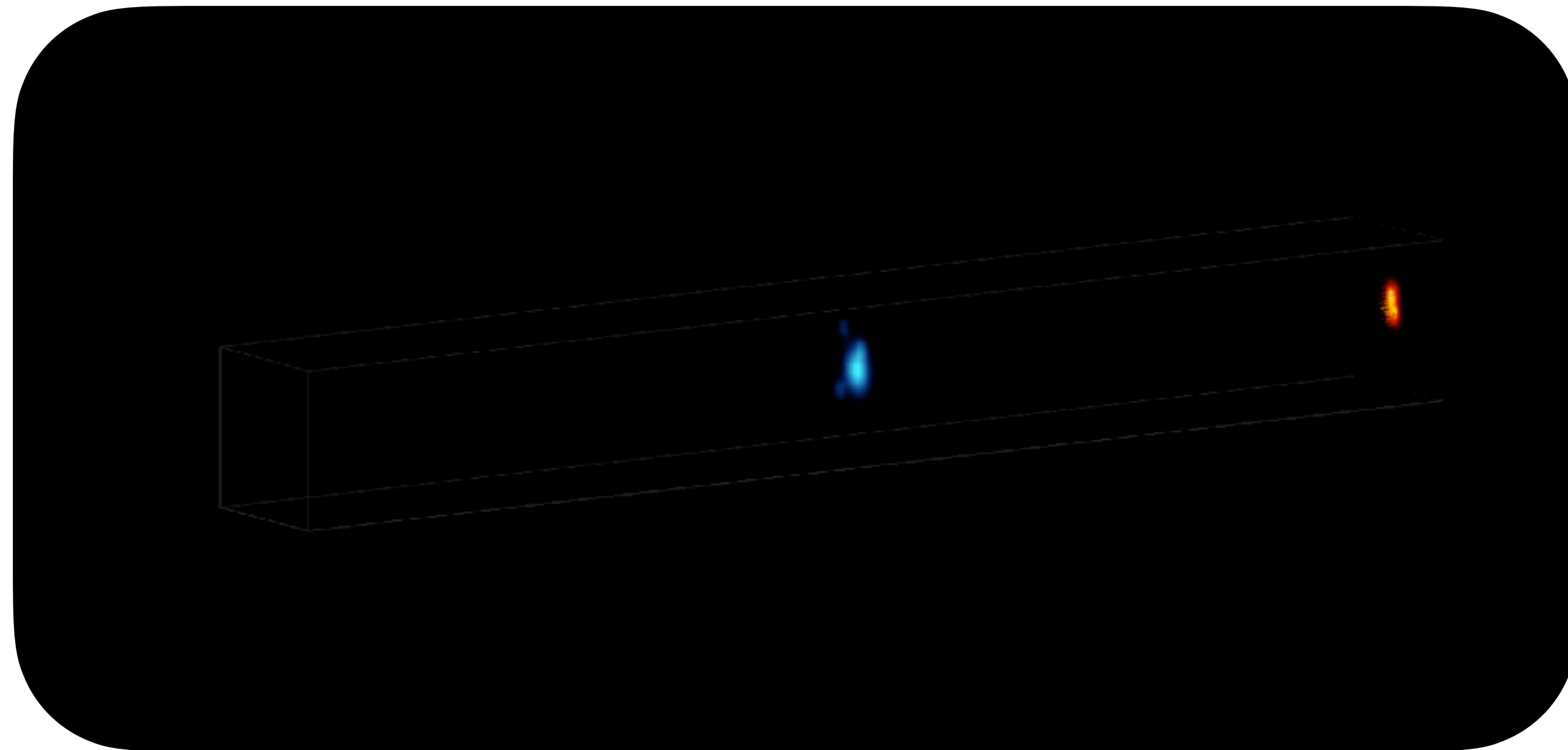
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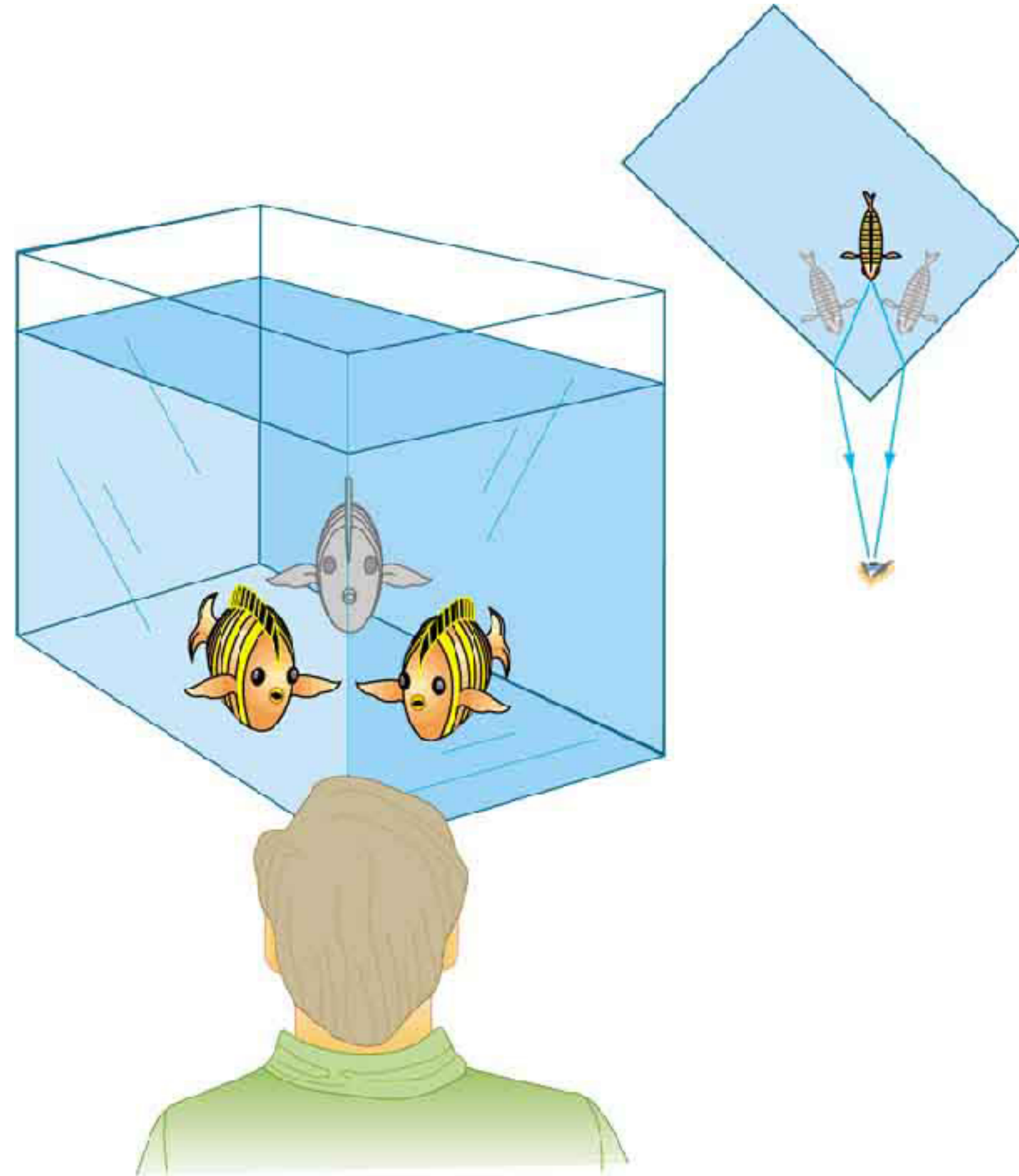
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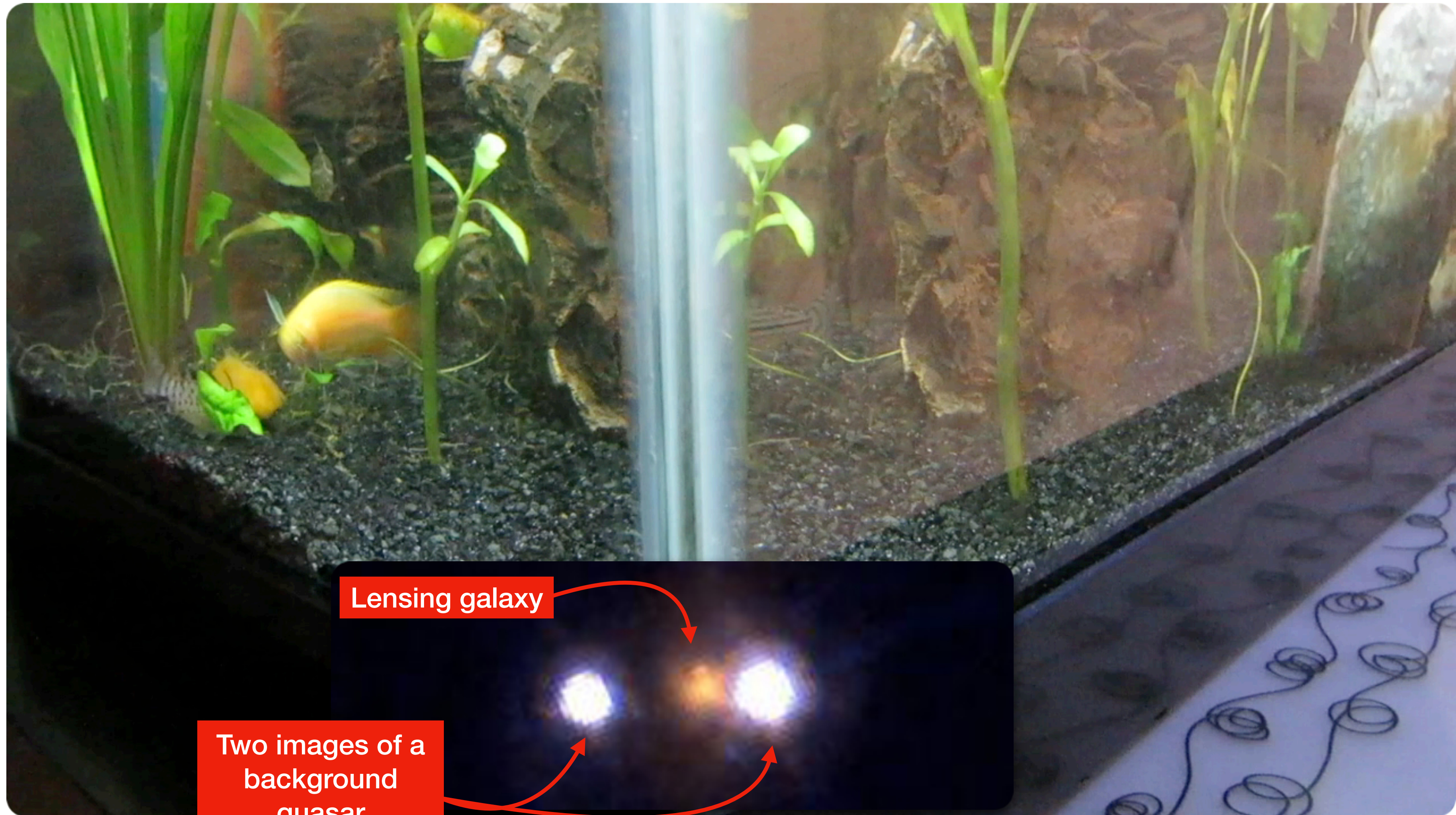
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Lensing galaxy

Two images of a background quasar



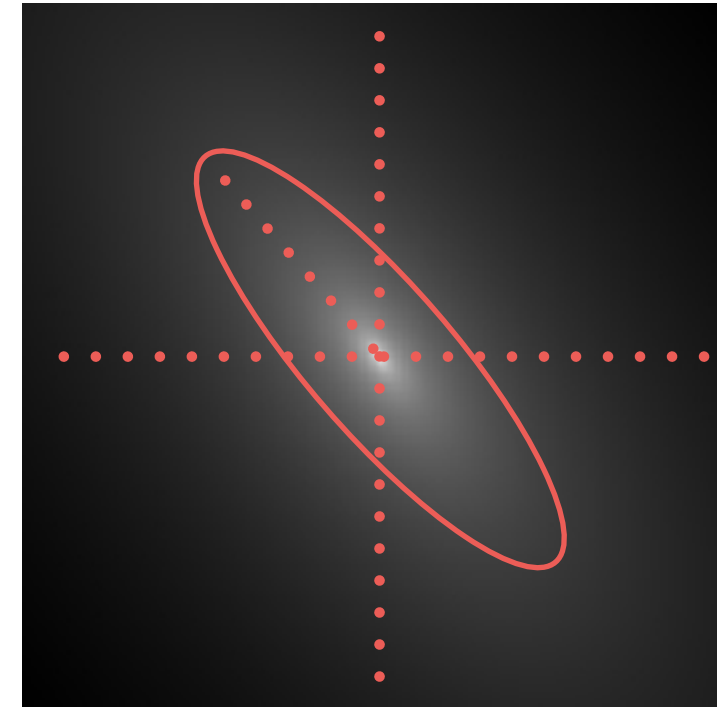
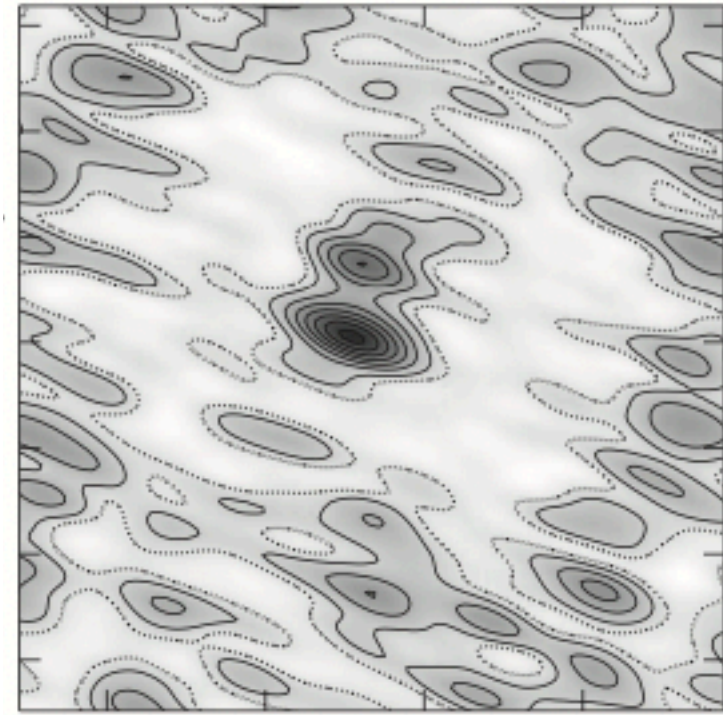




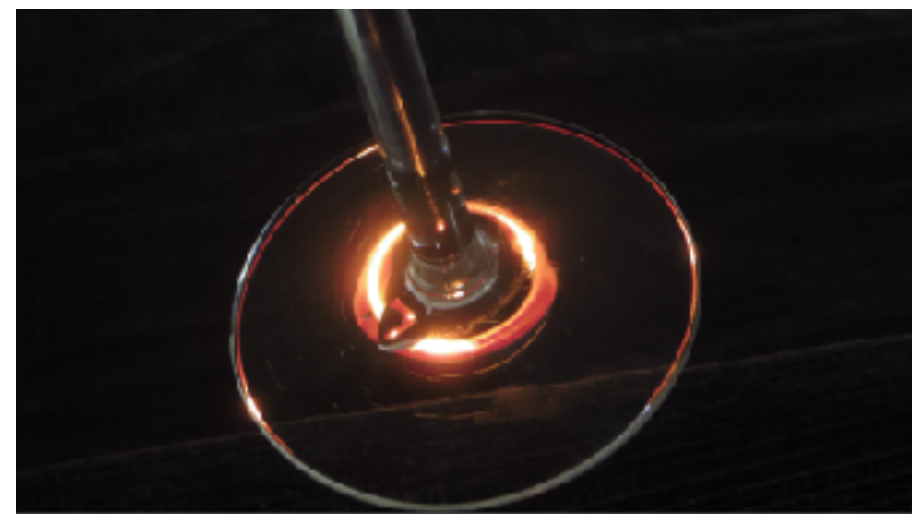


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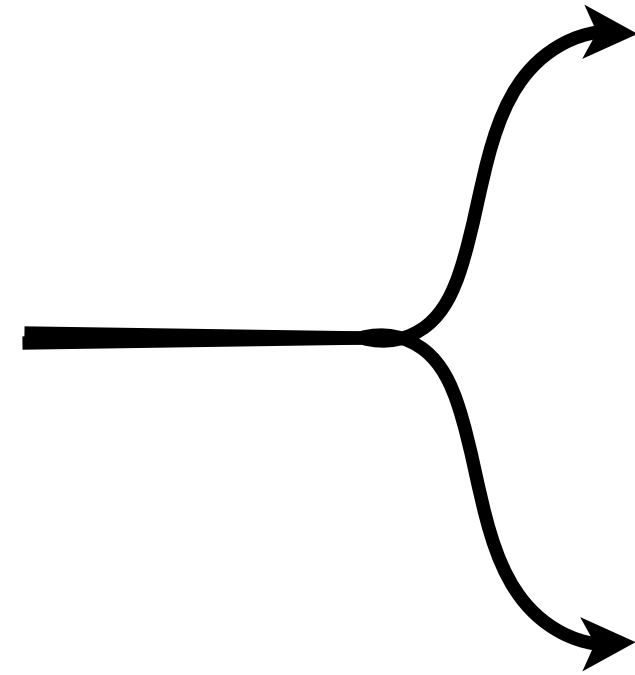
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# A SIMPLE EXAMPLE OF LENS MODELING



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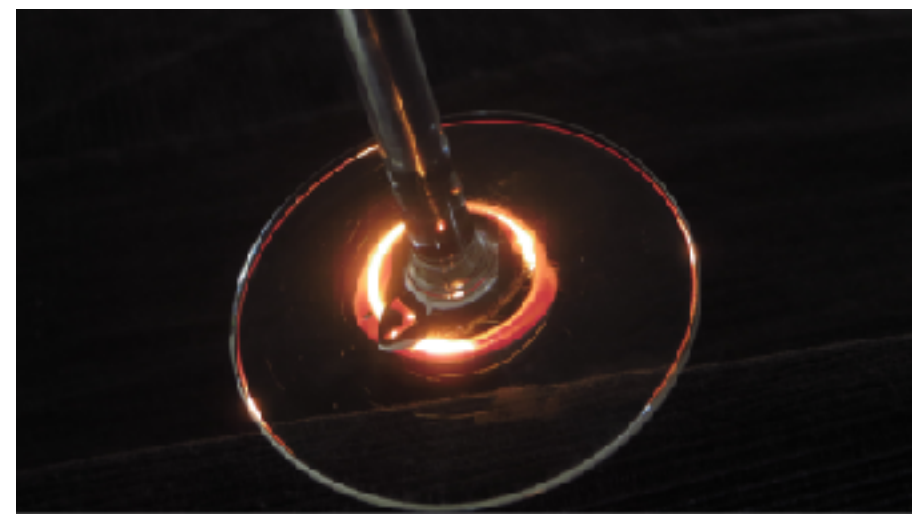


1: Morphology of the background source  
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2: Matter distribution in the lens  
(the shape of the wineglass)

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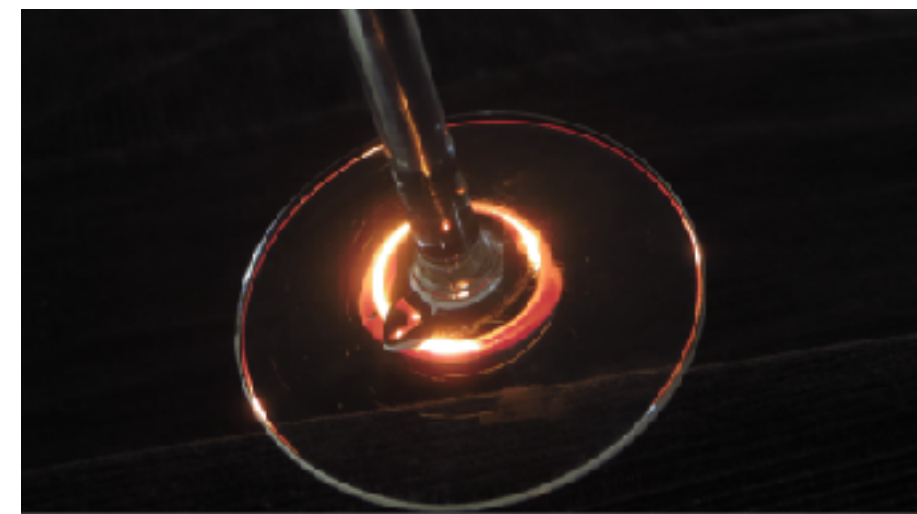
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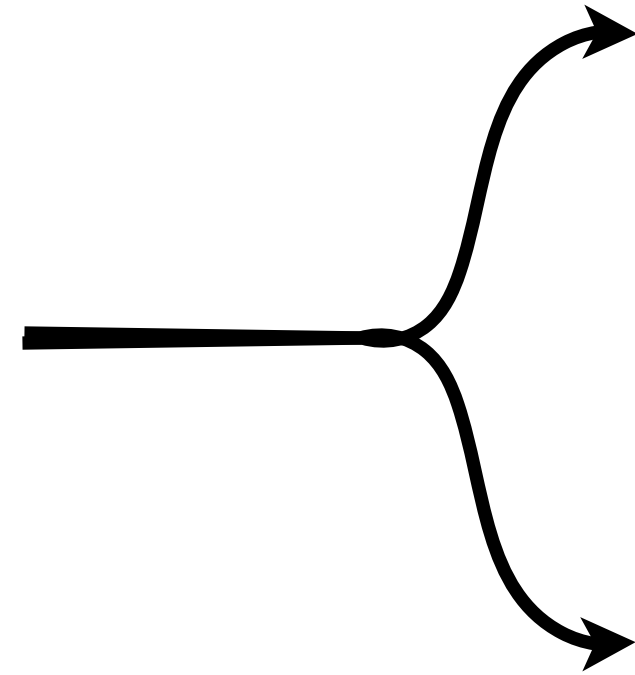
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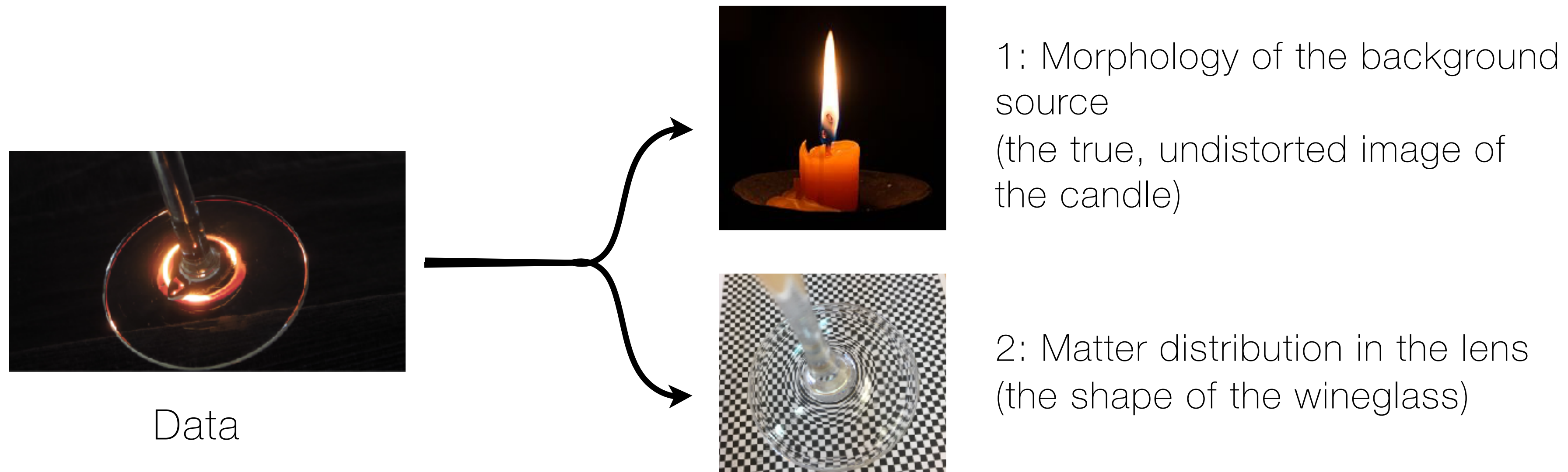
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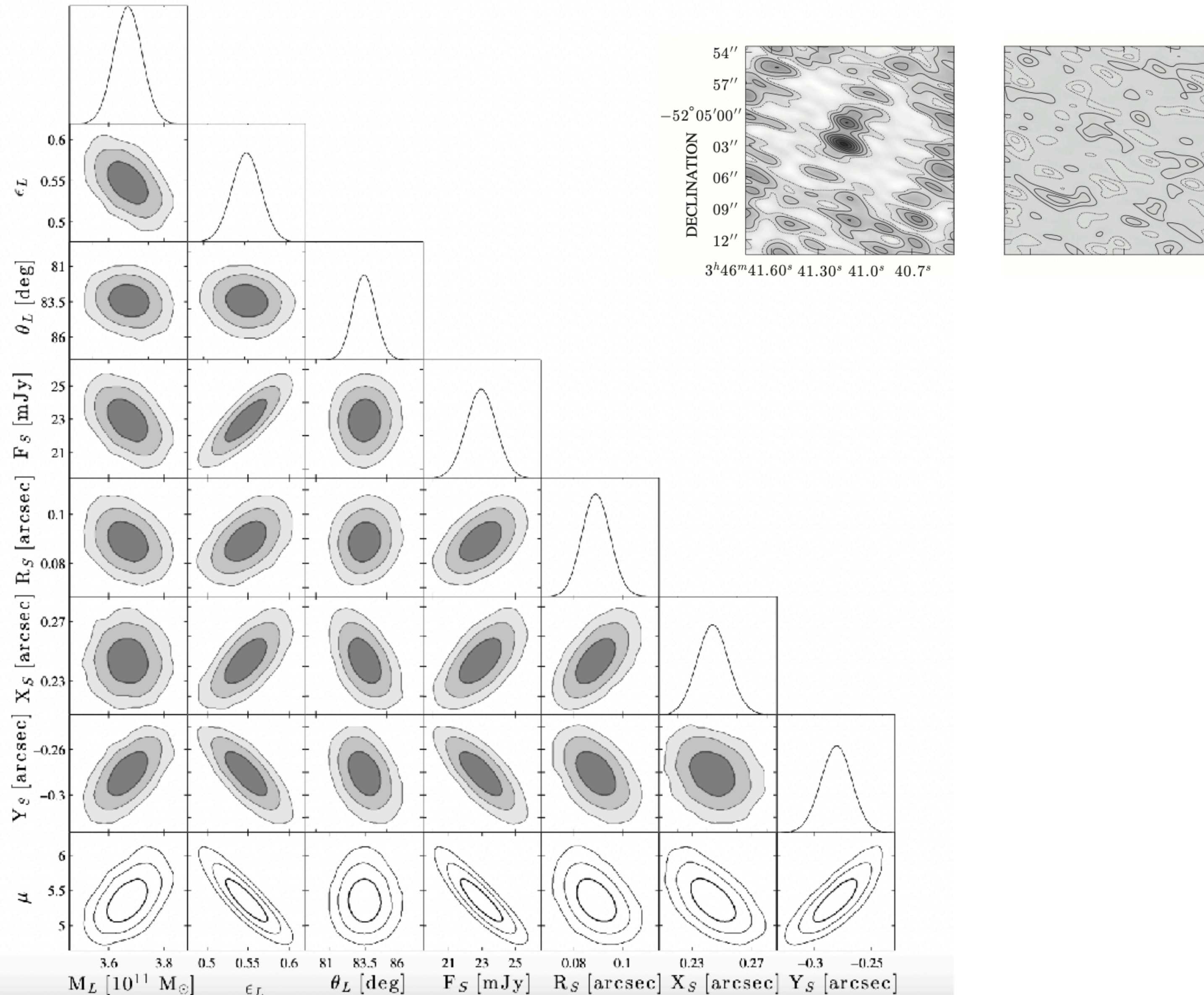


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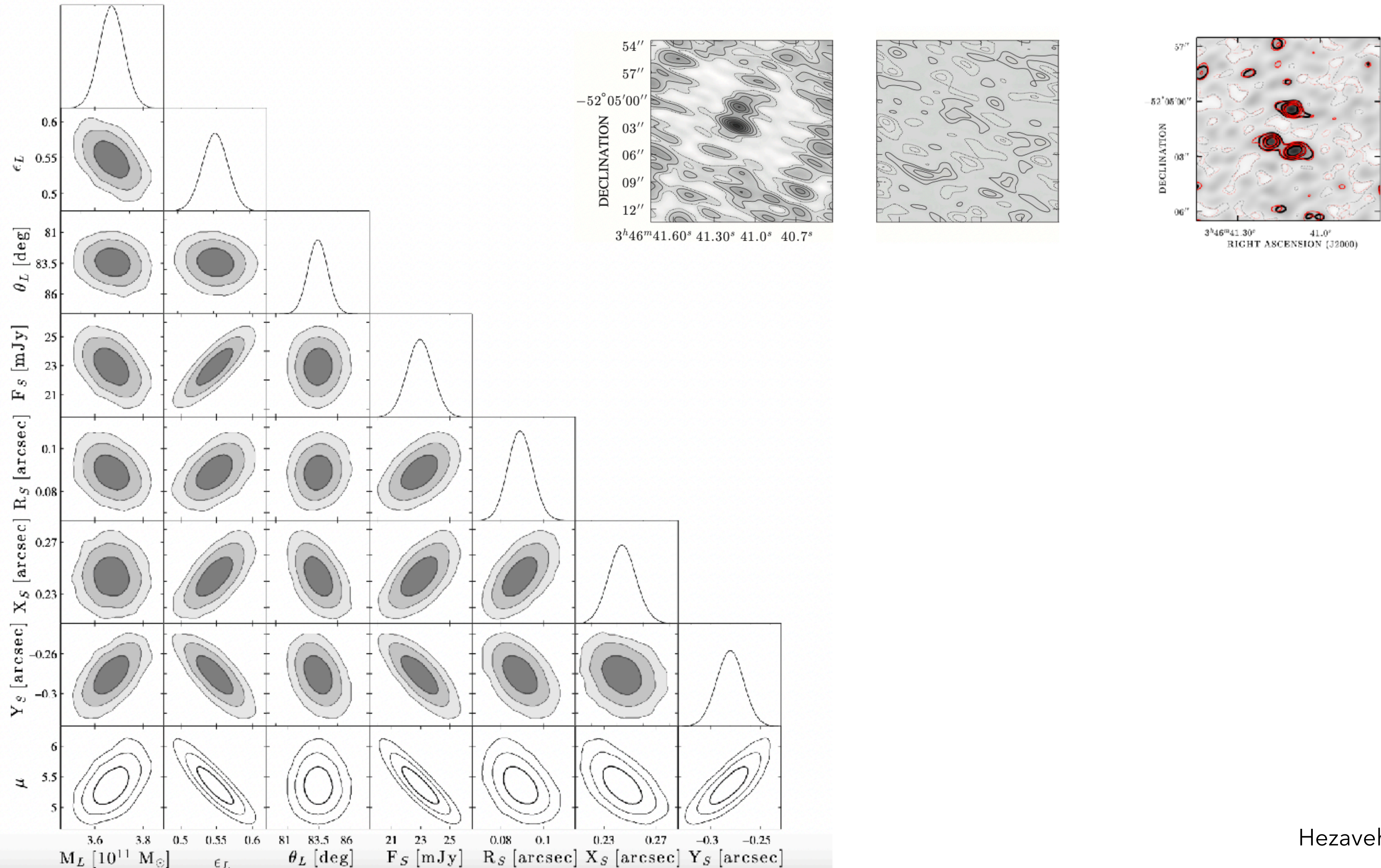
$\uparrow$   
 $\boxed{\text{Lens + Source Parameters (non-linear)}}$



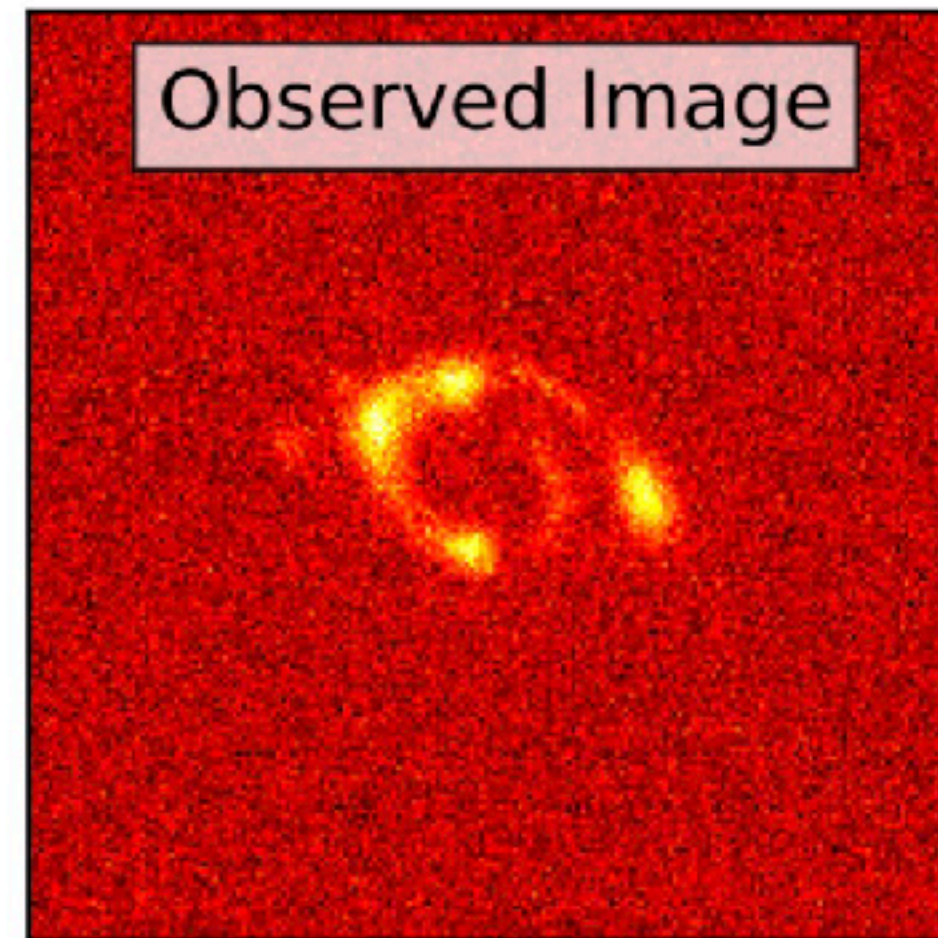
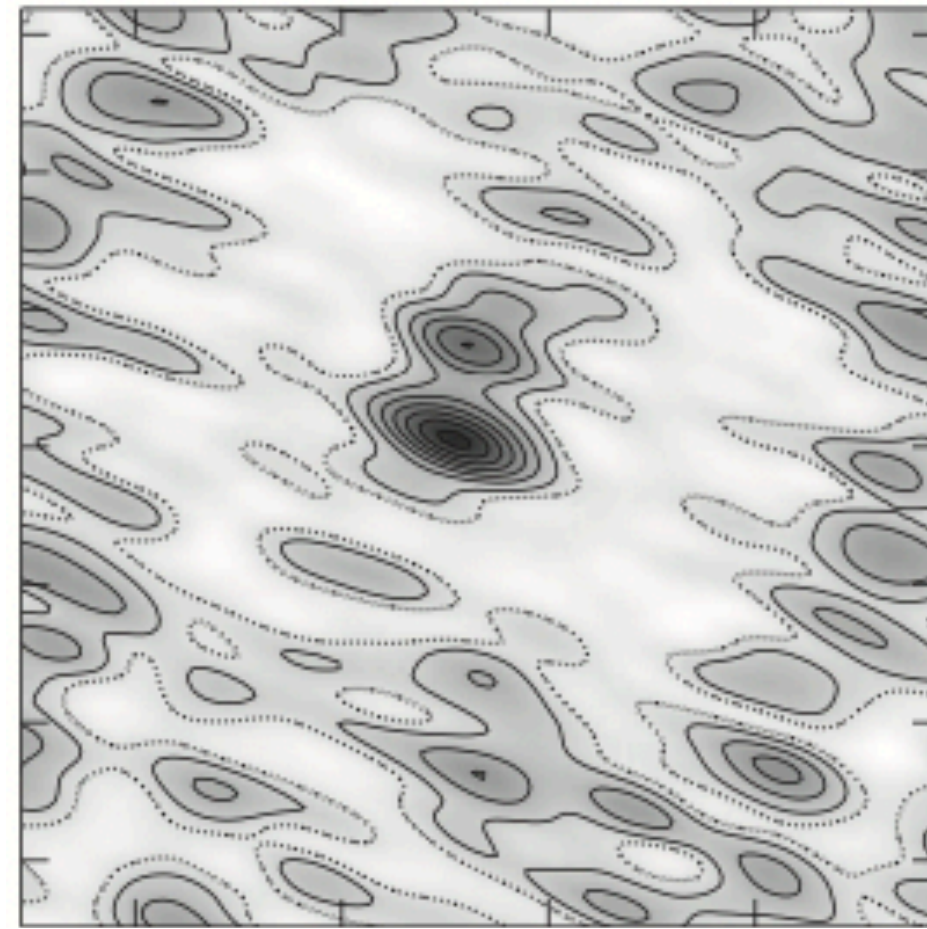
# PARAMETRIC BACKGROUND SOURCE RECONSTRUCTION



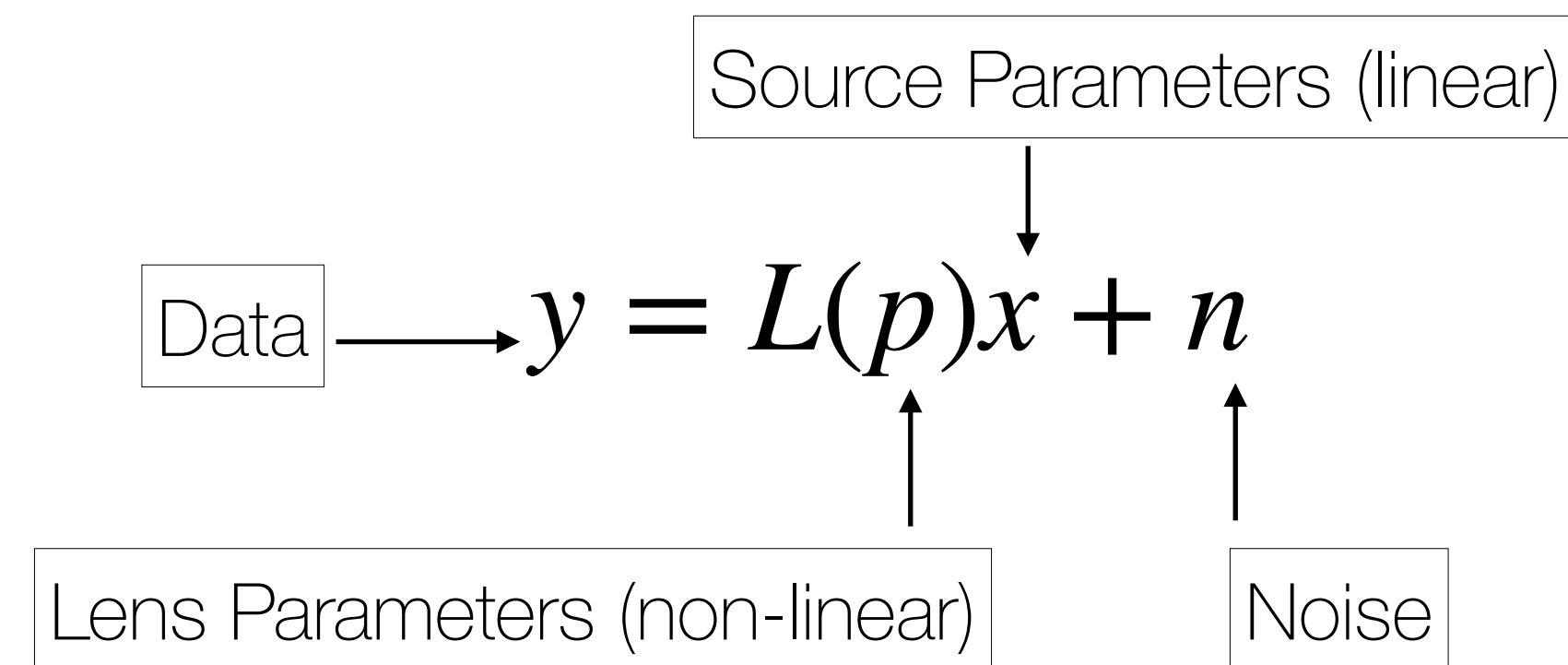
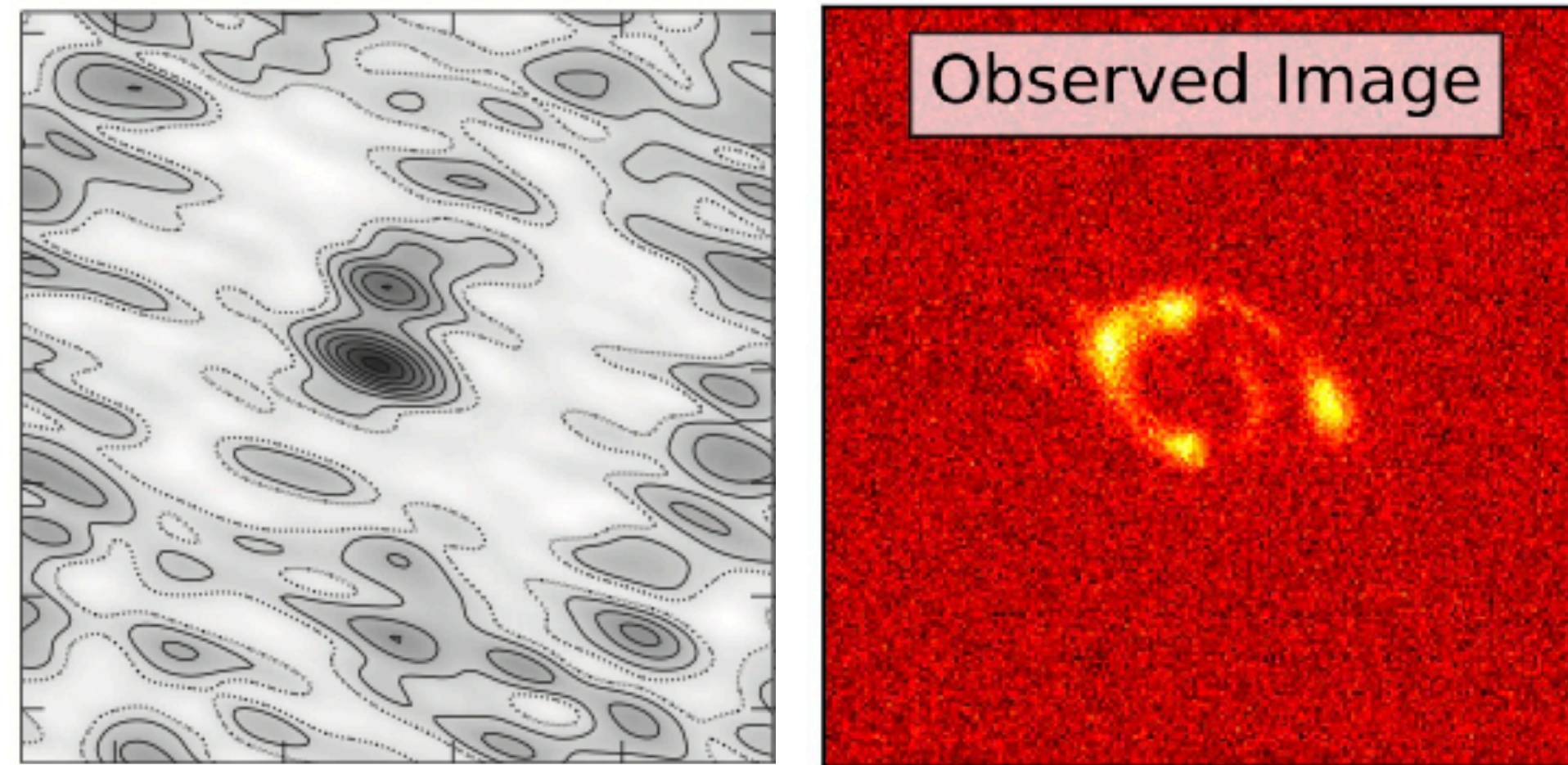
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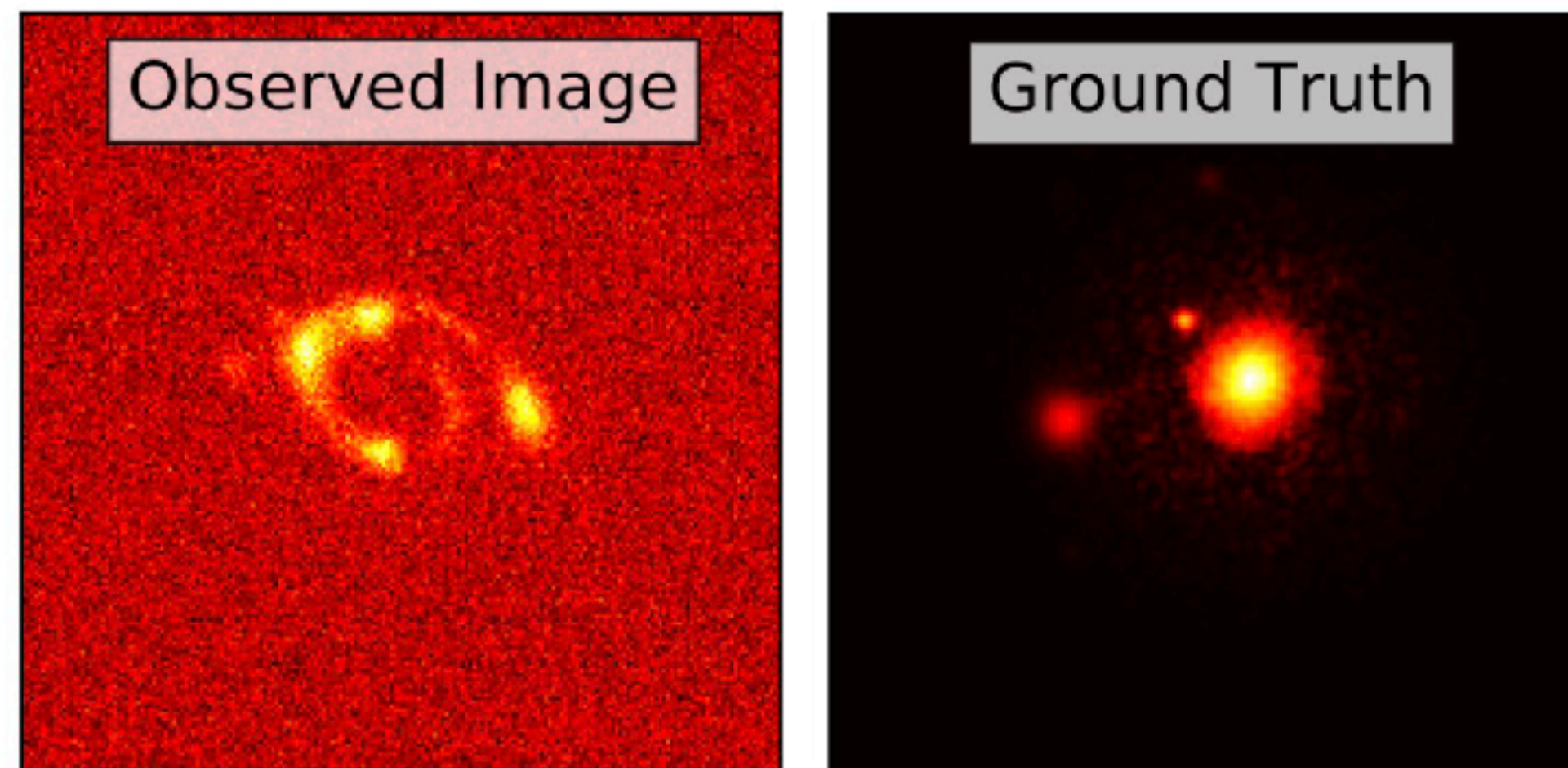
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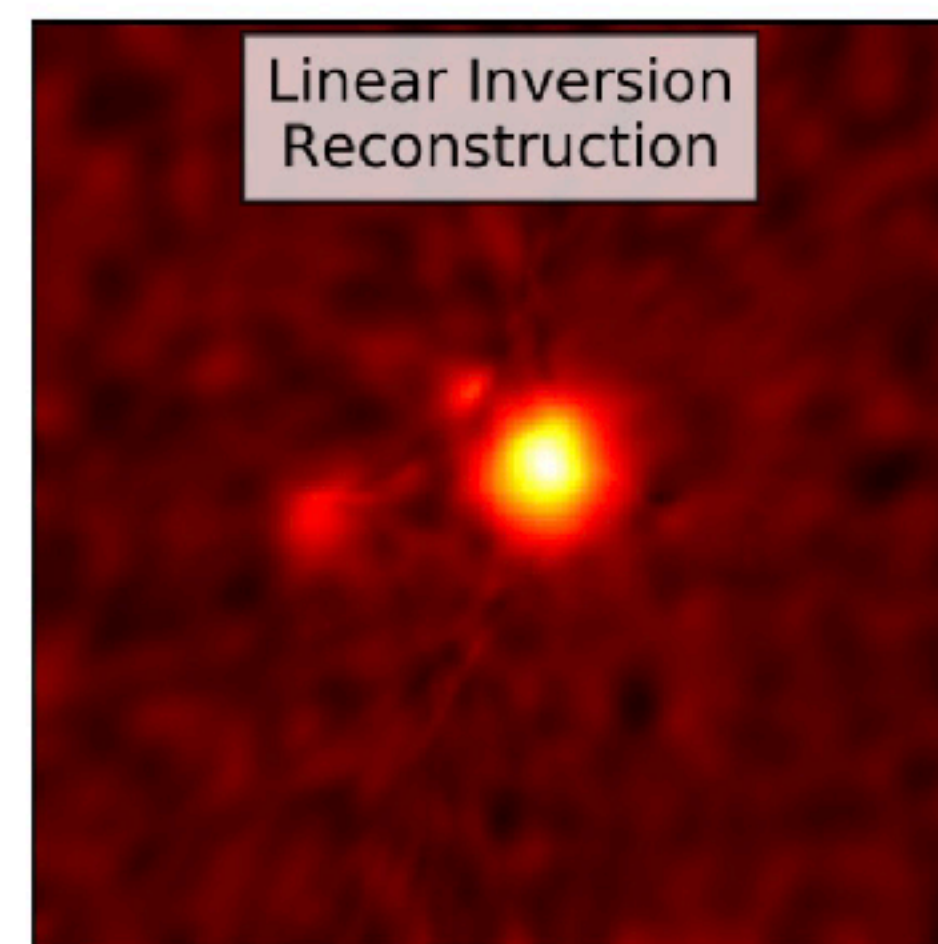
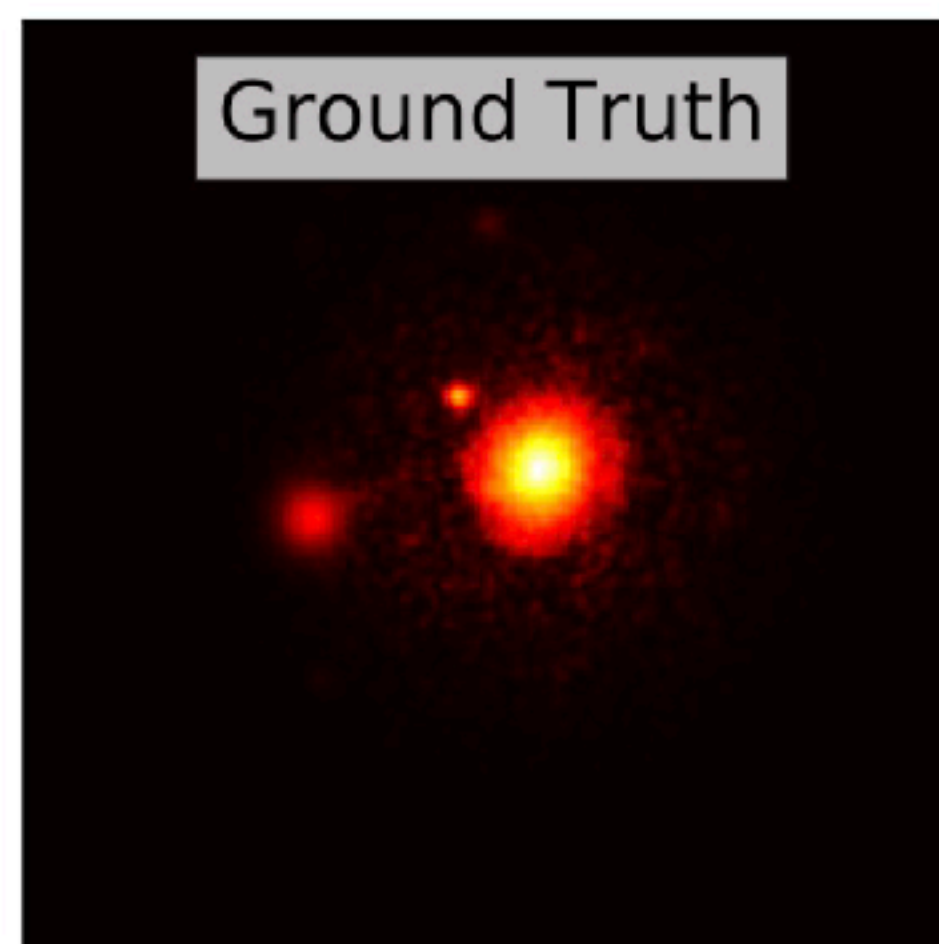
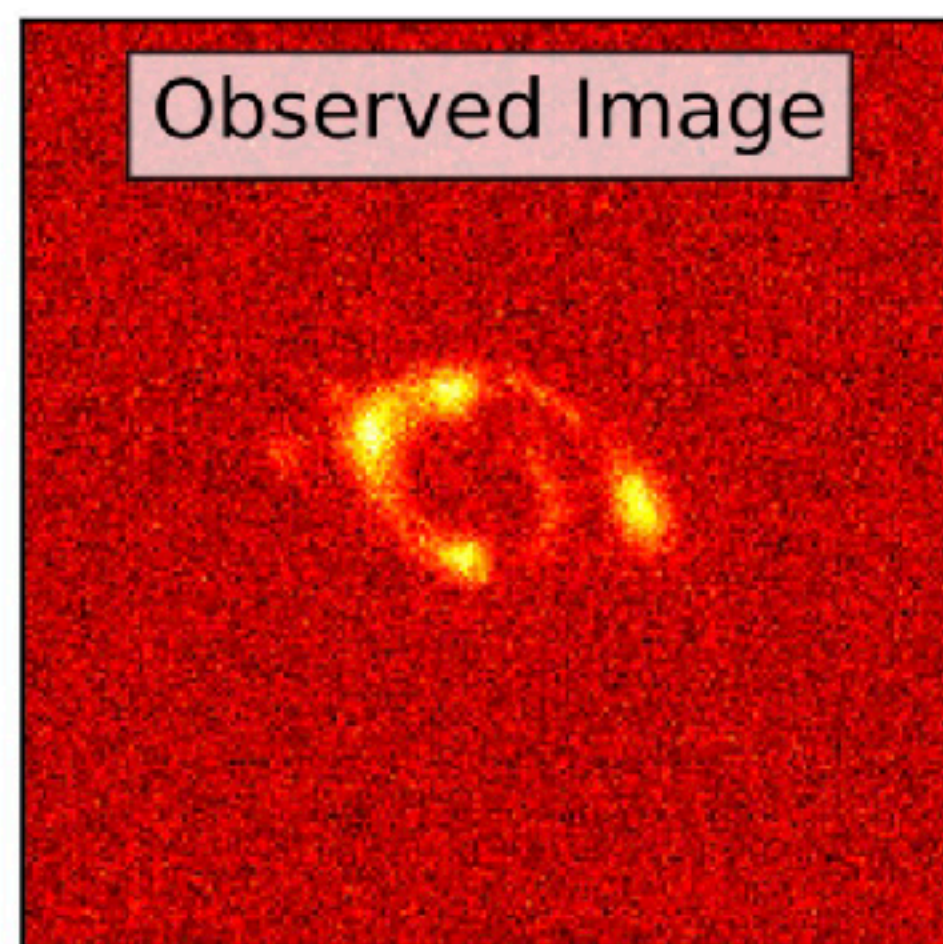
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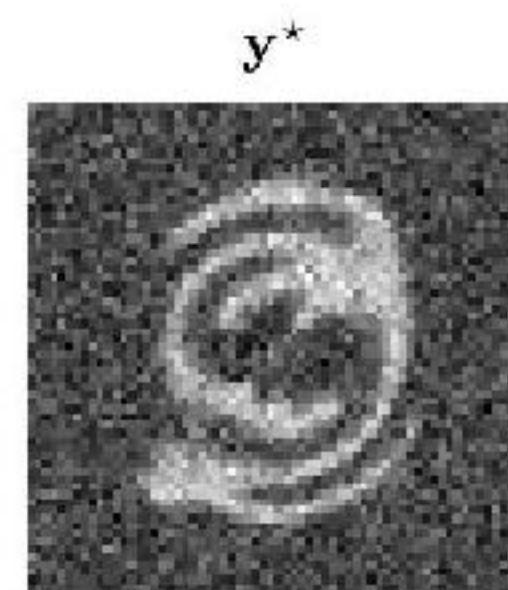
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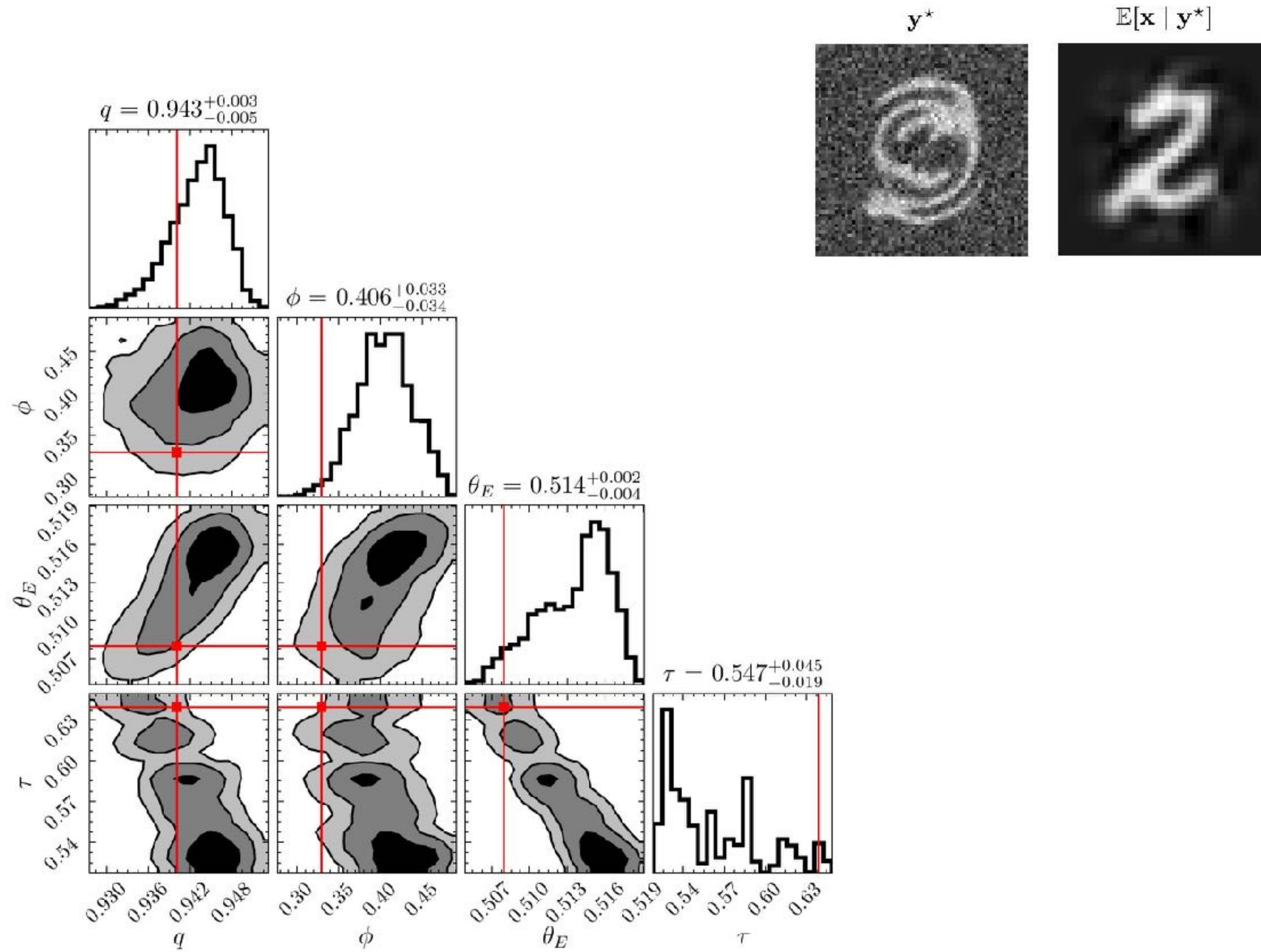
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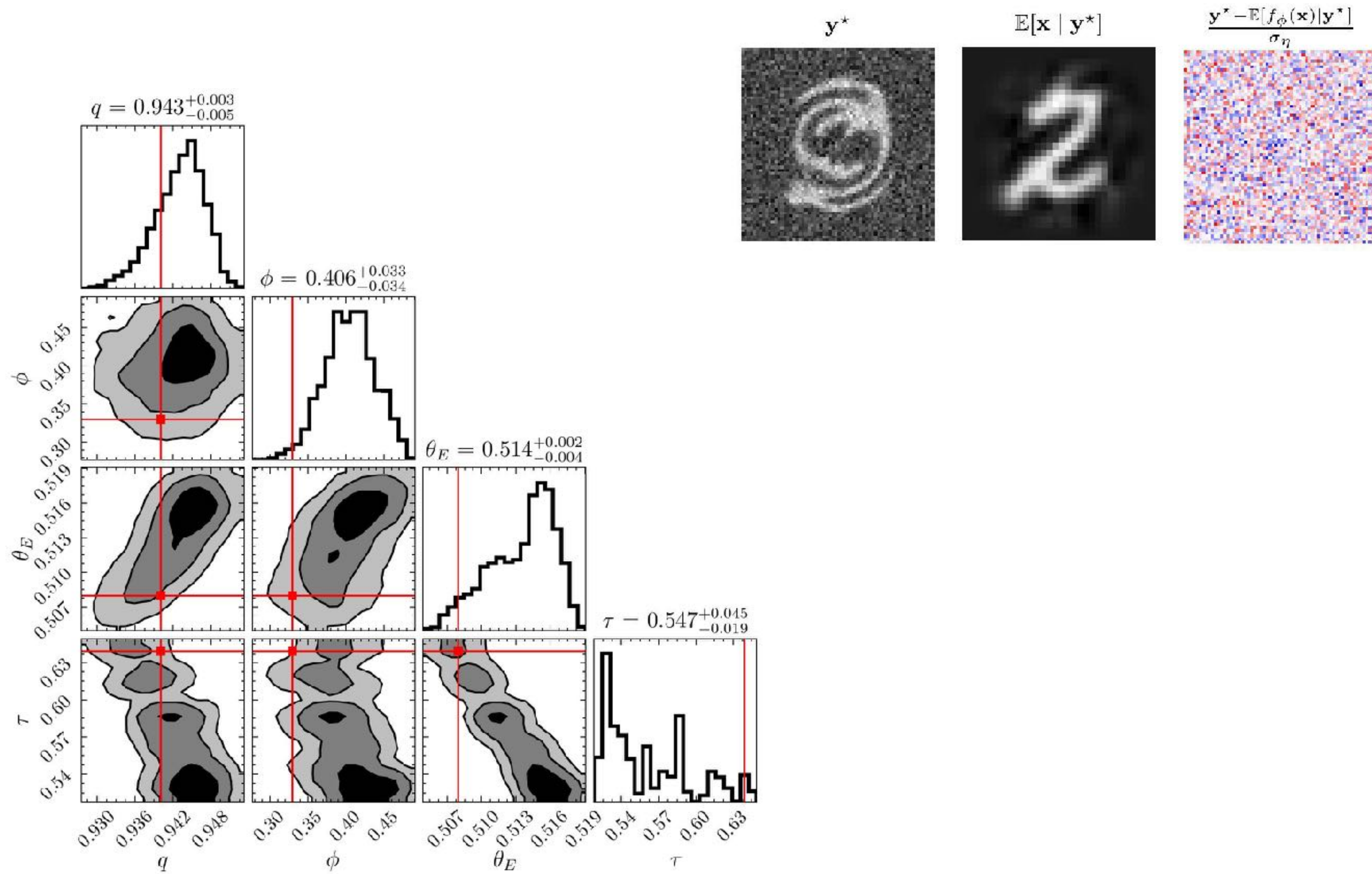
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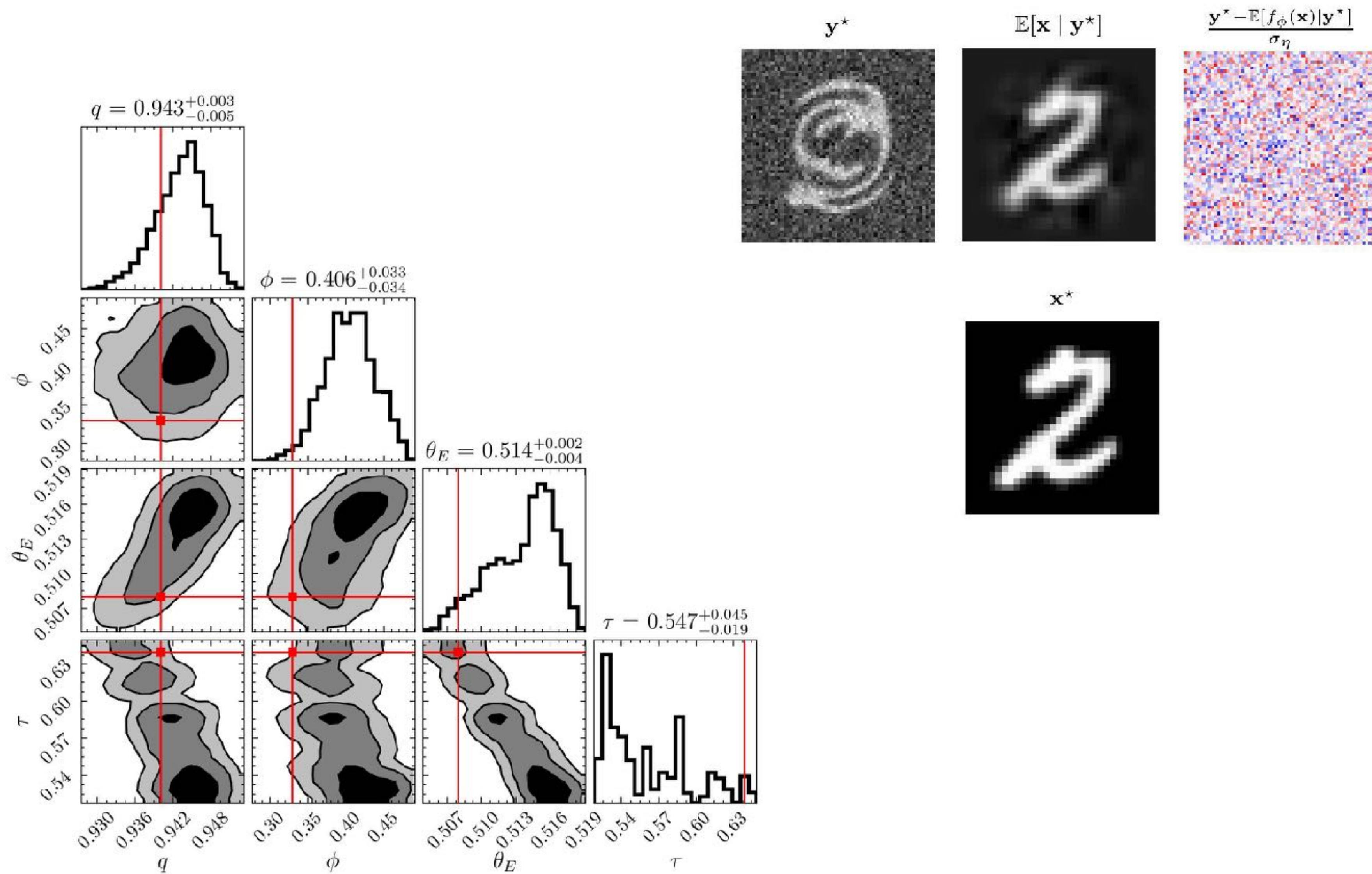
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# LEARNING HIGH-DIMENSIONAL PRIORS WITH SCORE MODELING

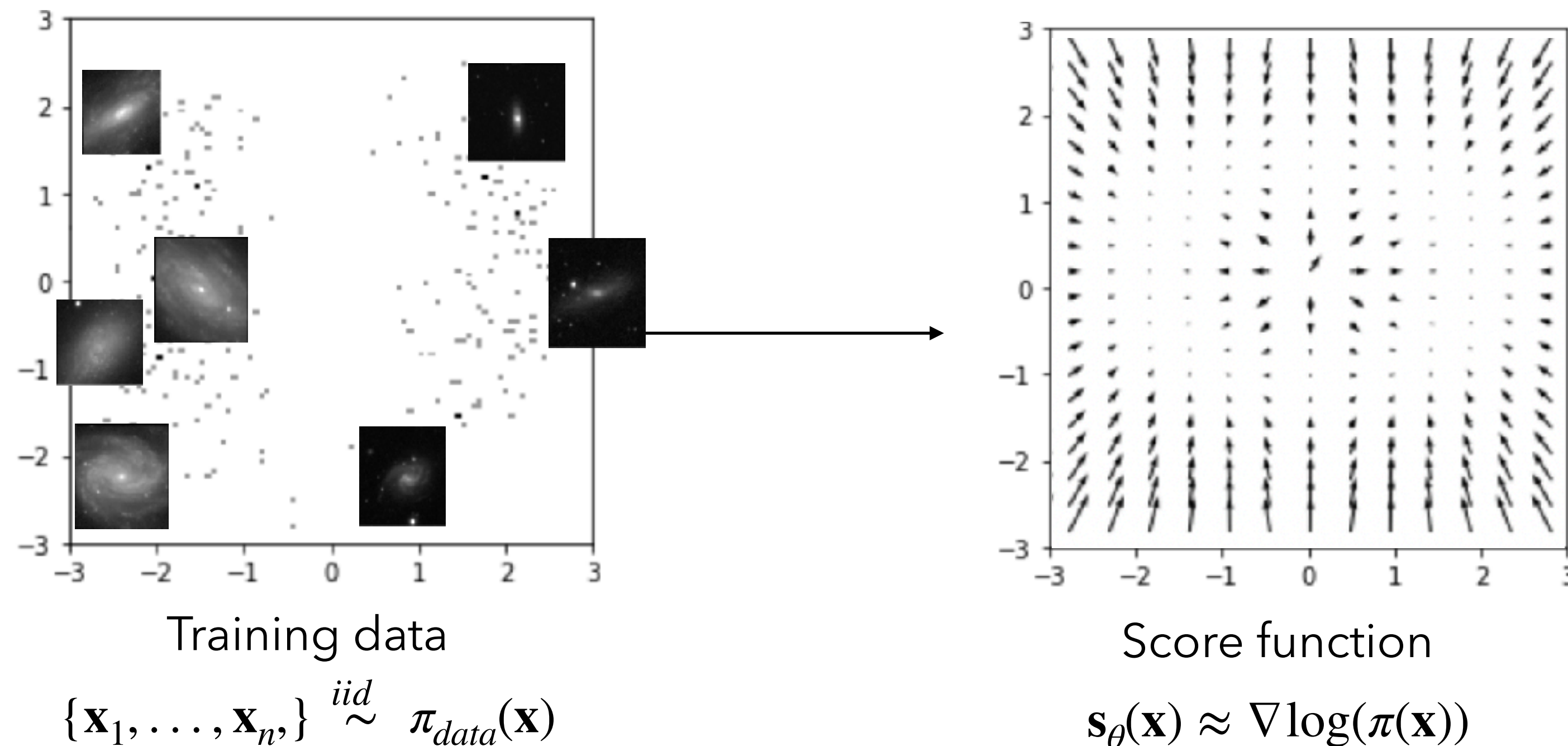
Can we learn our high-dimensional prior from data?

# LEARNING HIGH-DIMENSIONAL PRIORS WITH SCORE MODELING

Can we learn our high-dimensional prior from data?

Turns out that we can sample a distribution only knowing its **score**, which does not include the normalization constant and only uses local information.

$$\mathbf{s}(\mathbf{x}) = \nabla_{\mathbf{x}} \log(\pi(\mathbf{x}))$$



# SCORE-BASED MODELING



Alexandre Adam

We model the score of the prior

$$s_{\theta}(x) \equiv \nabla_x \log p_{\theta}(x)$$

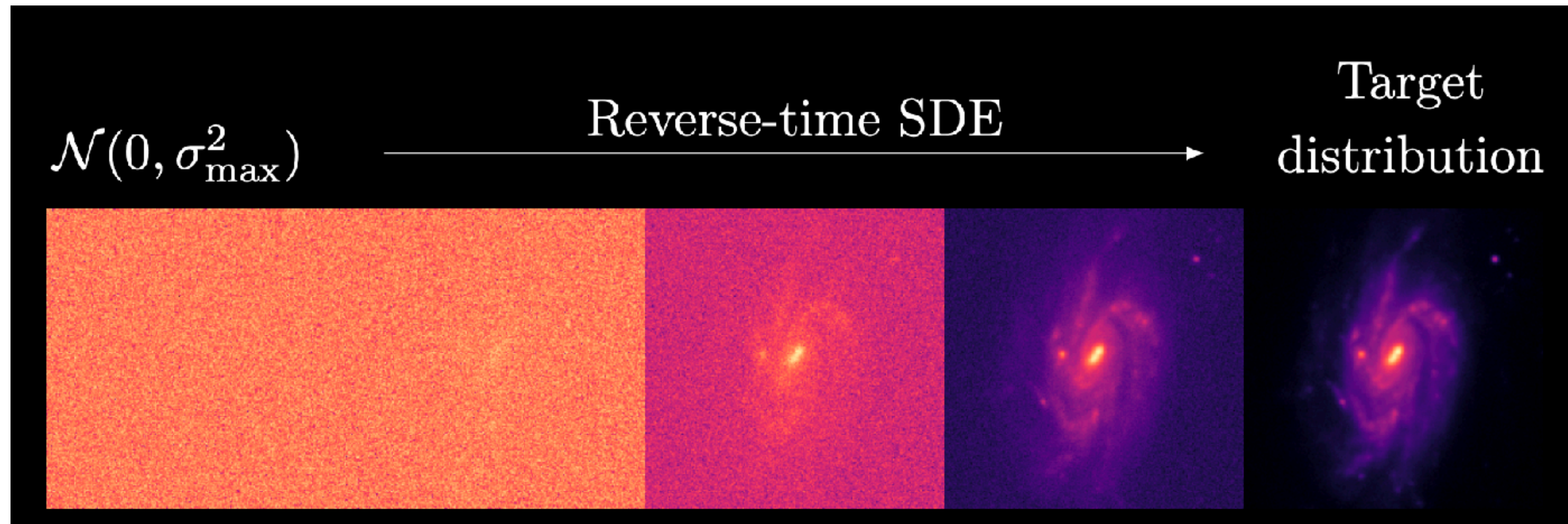
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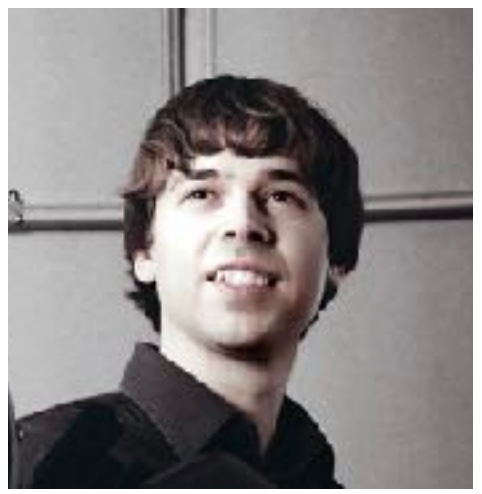
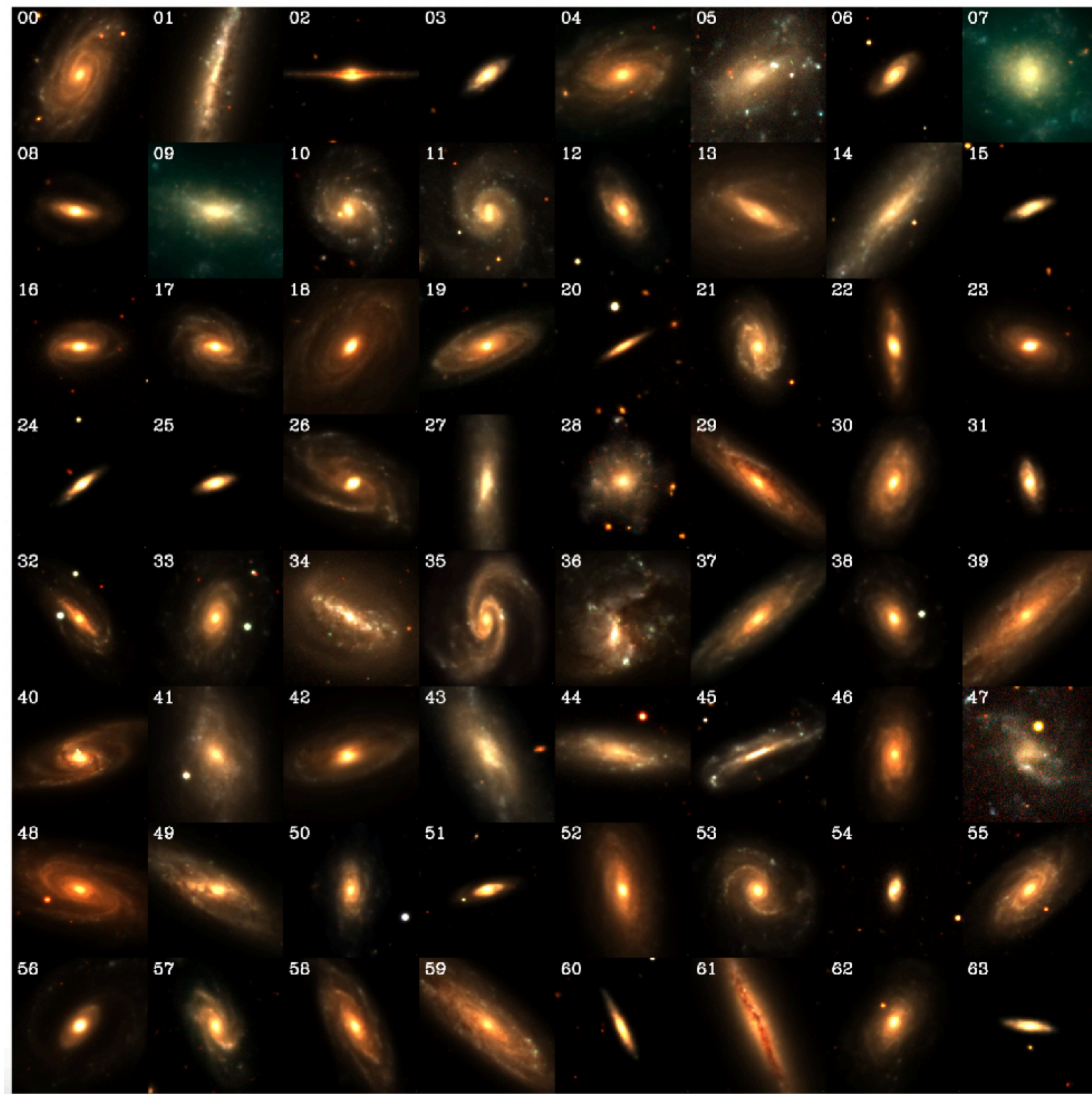


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Connor Stone

<http://www.mjjsmith.com/thisisnotagalaxy/>

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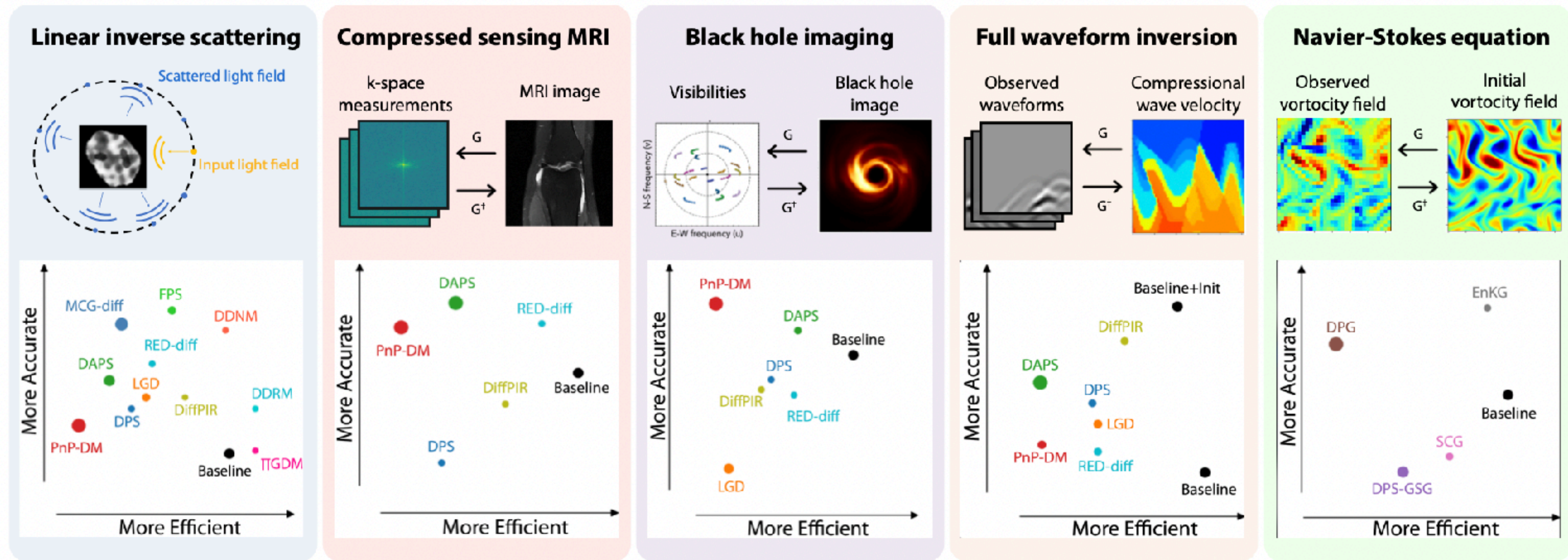
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## Sequential MC

Get approx samples, and reweigh them to get a new proposal. Iteratively converges

e.g : Trippe et al., 2023; Dou & Song, 2024, Cardoso et al., 2024

# THESE DIFFERENT METHODS PERFORM VERY DIFFERENTLY IN DIFFERENT PHYSICAL SCIENCE PROBLEMS



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$$\nabla_x \log p(x|y) = \nabla_x \log p(y|x) + \nabla_x \log p_\theta(x)$$

To a good approximation, we can calculate the likelihood score analytically if we assume it's Gaussian and we know the lensing matrix.

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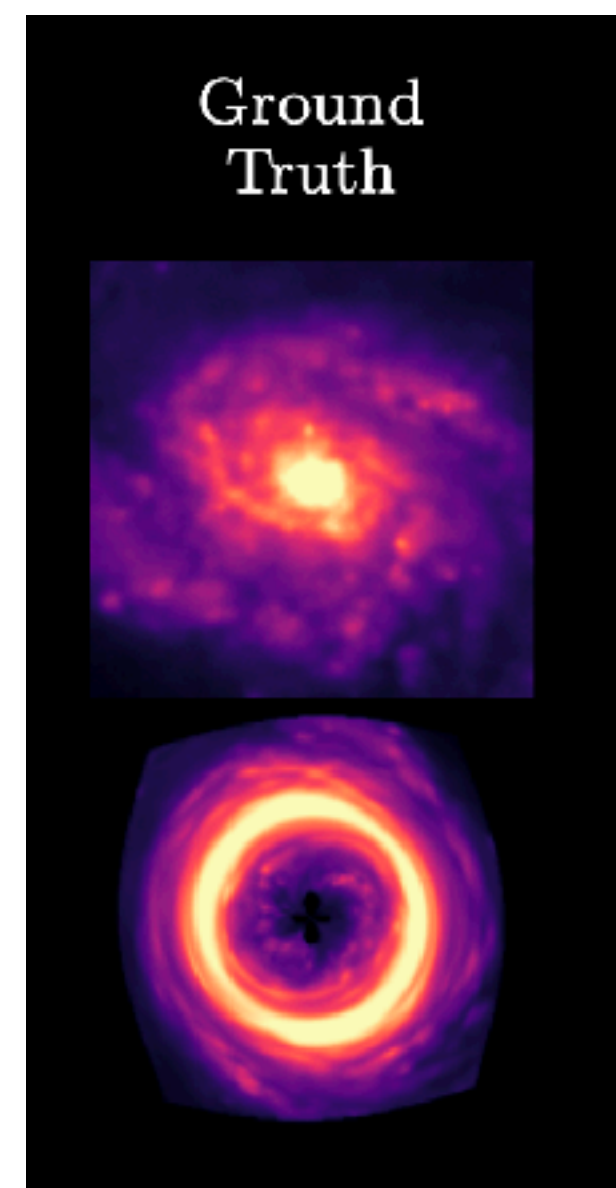
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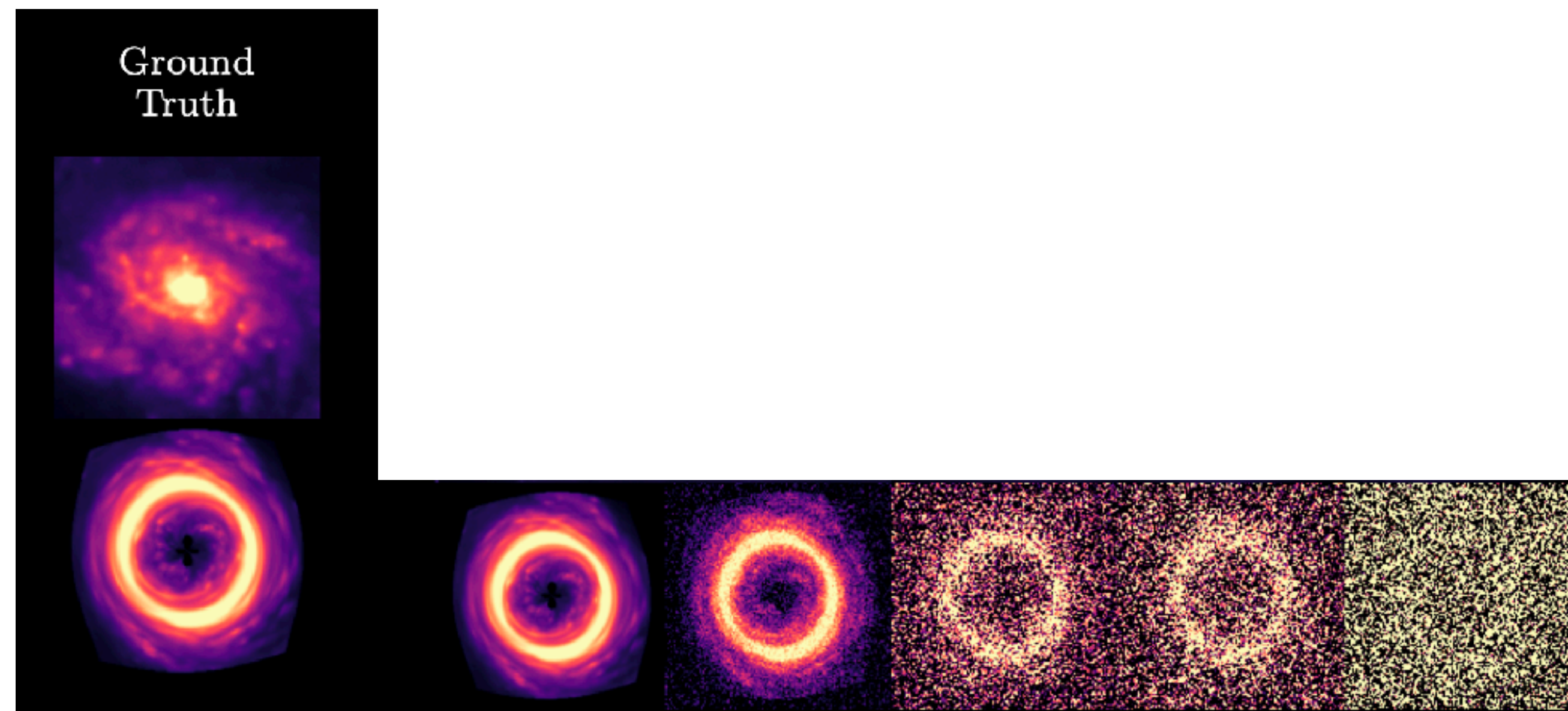
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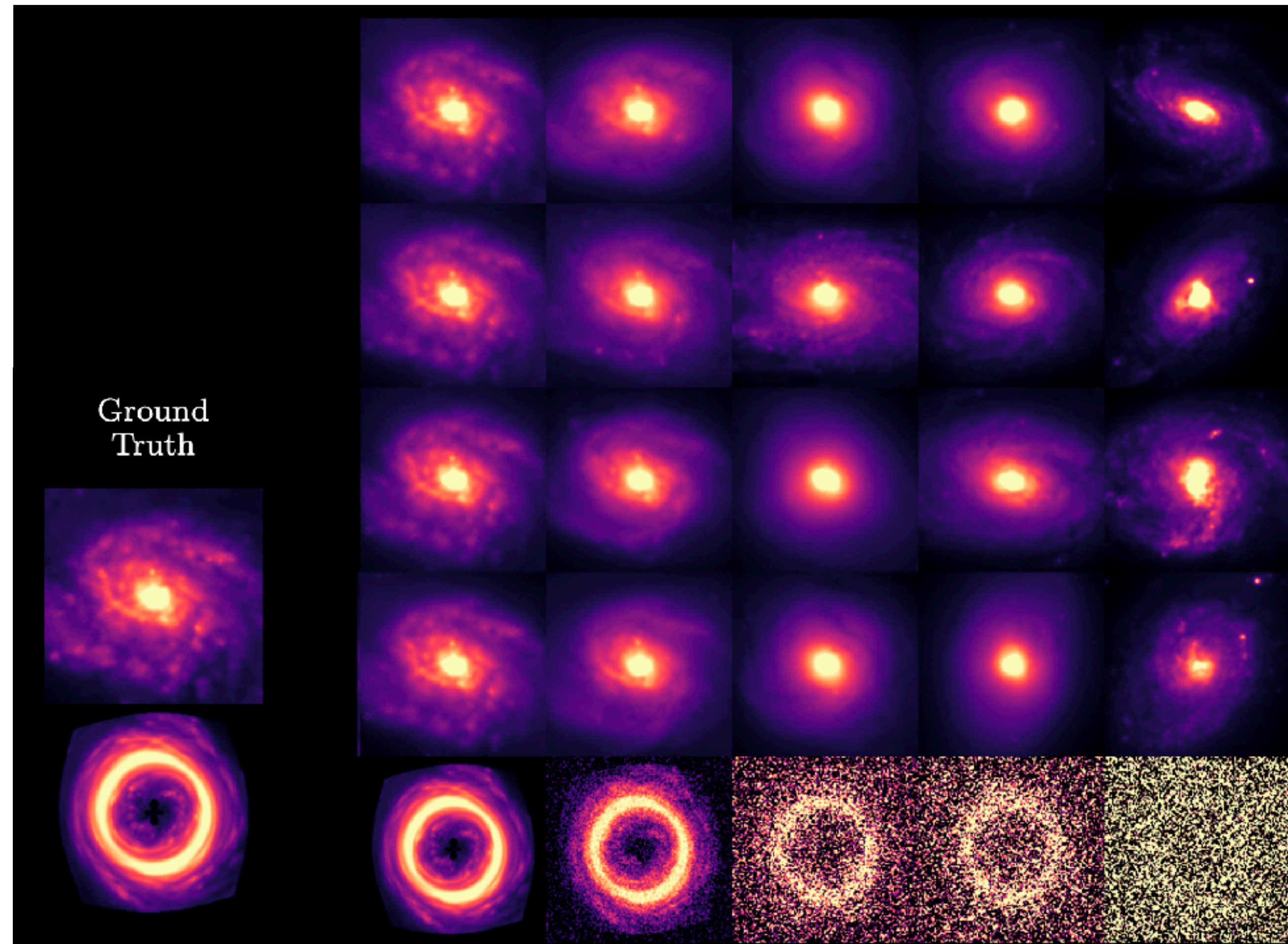
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# OUT OF DISTRIBUTION TESTS

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Alexandre Adam

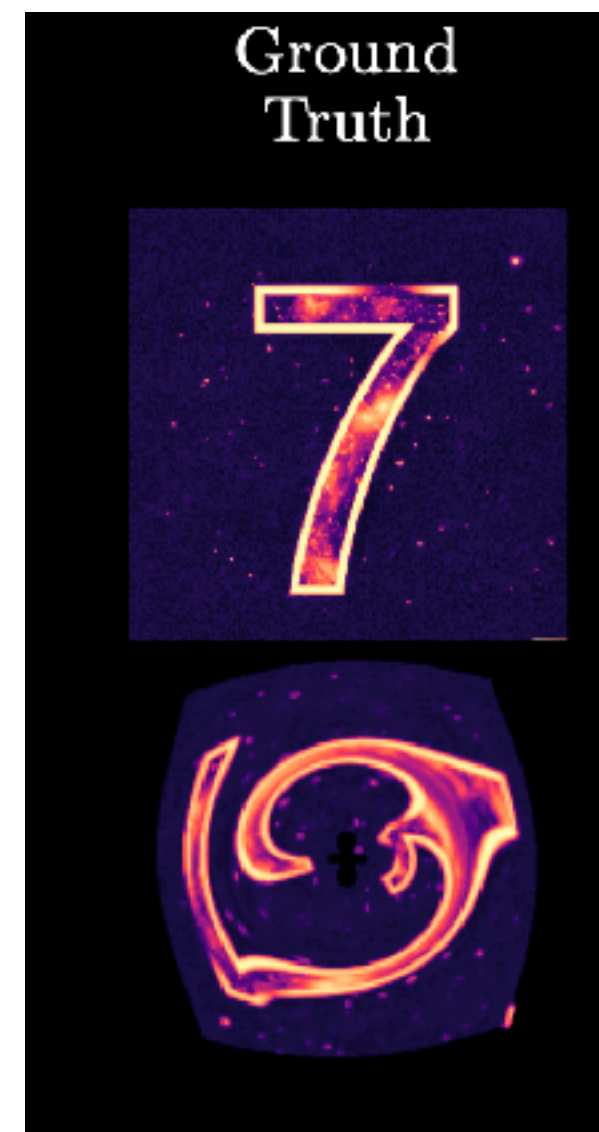
# OUT OF DISTRIBUTION TESTS



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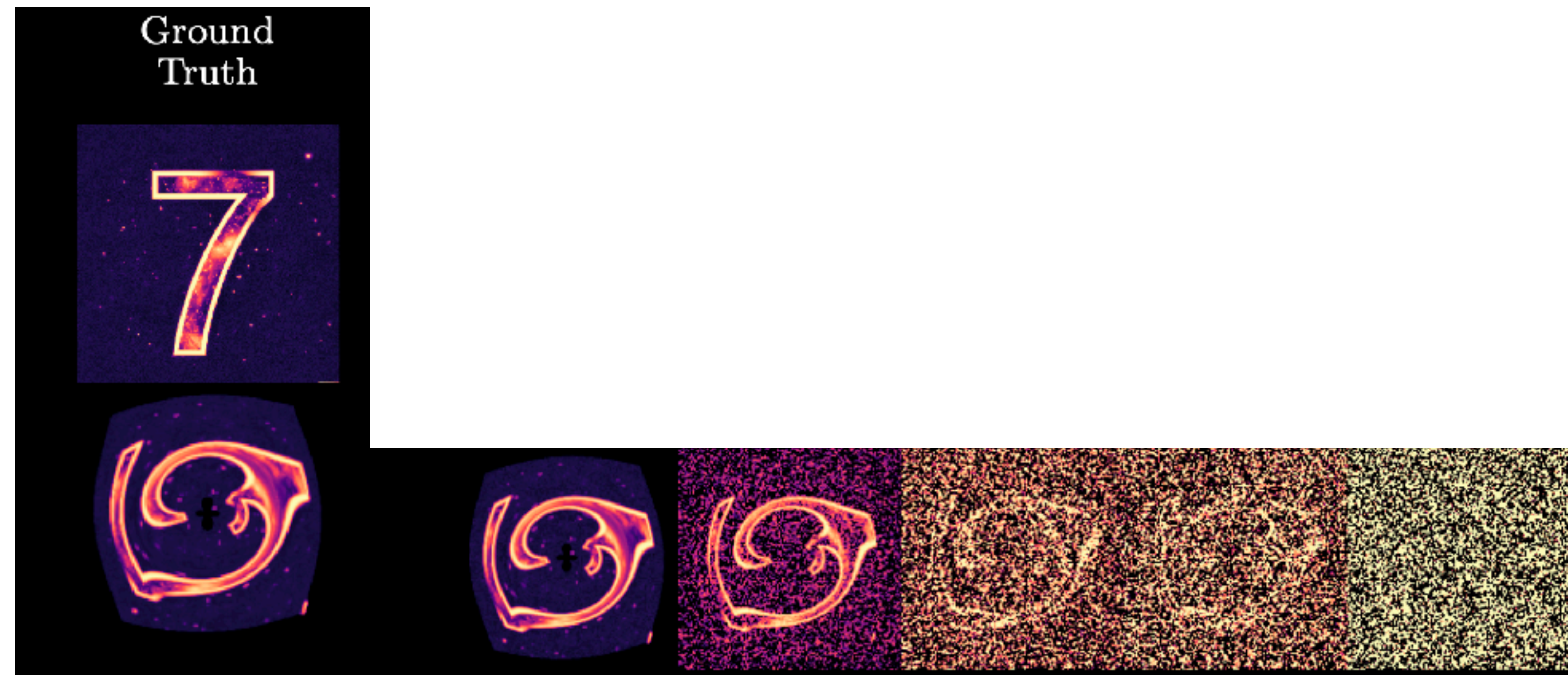
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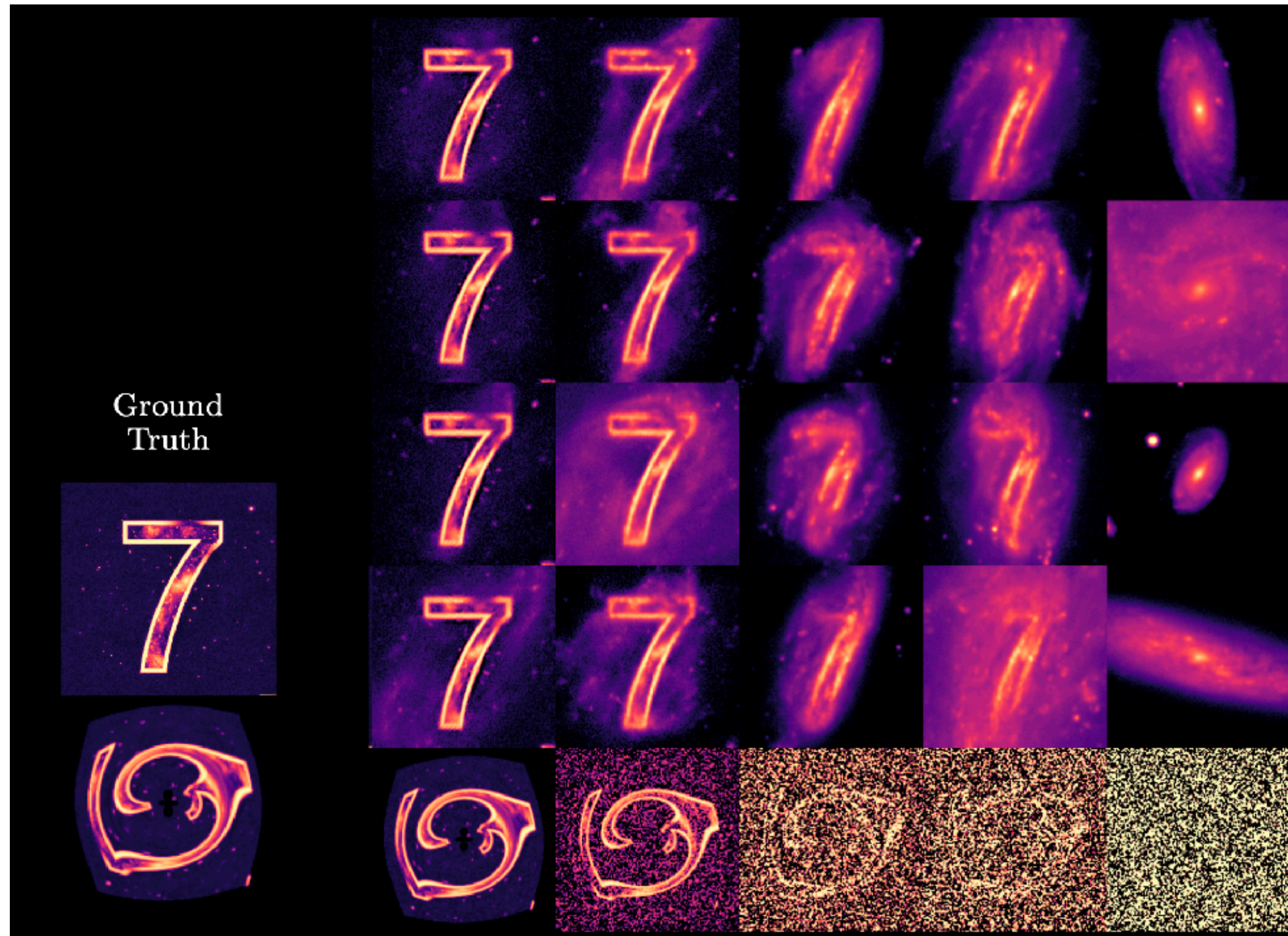
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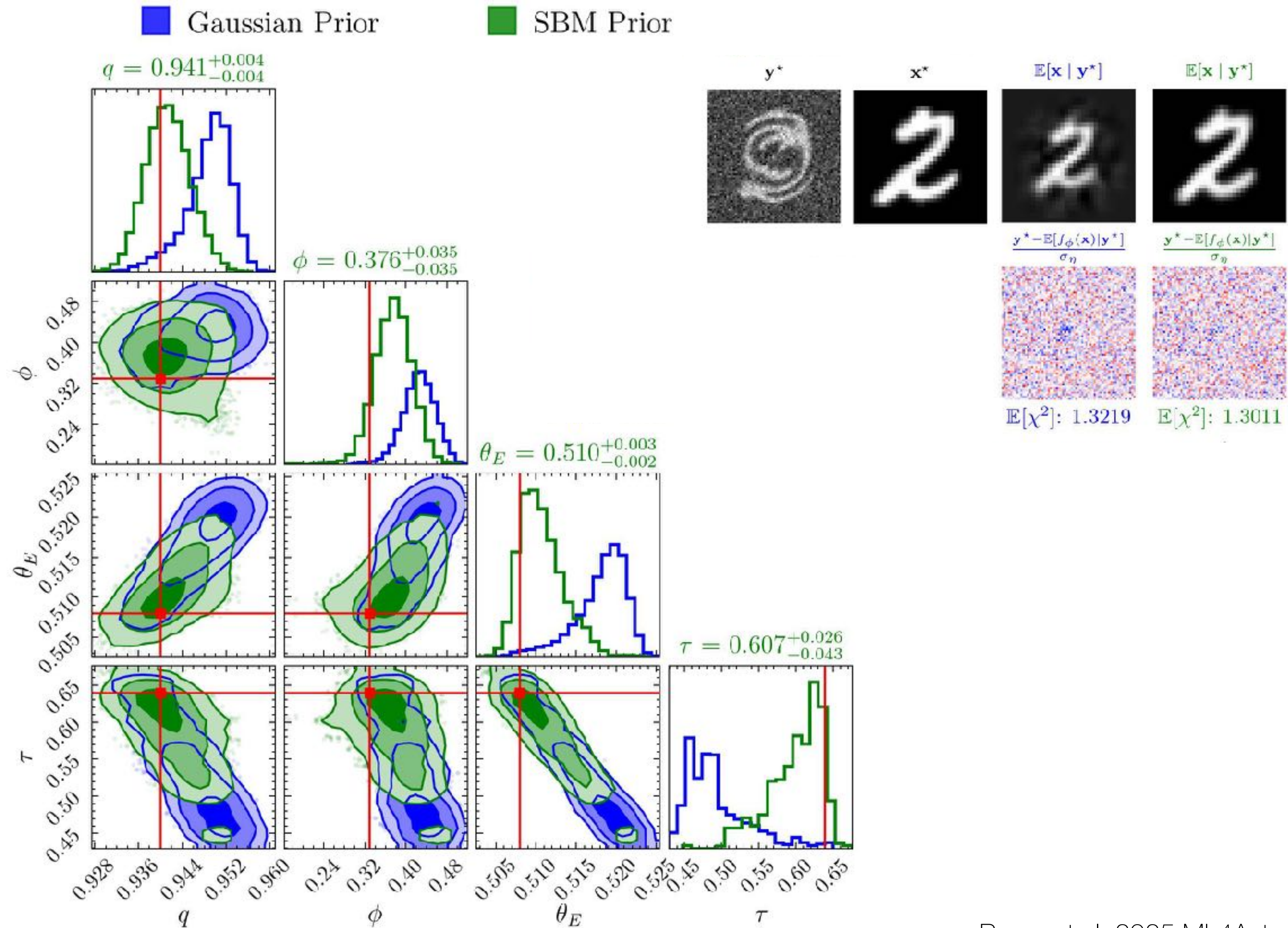
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# JOINT INFERENCE OF LENS AND SOURCE PARAMETERS



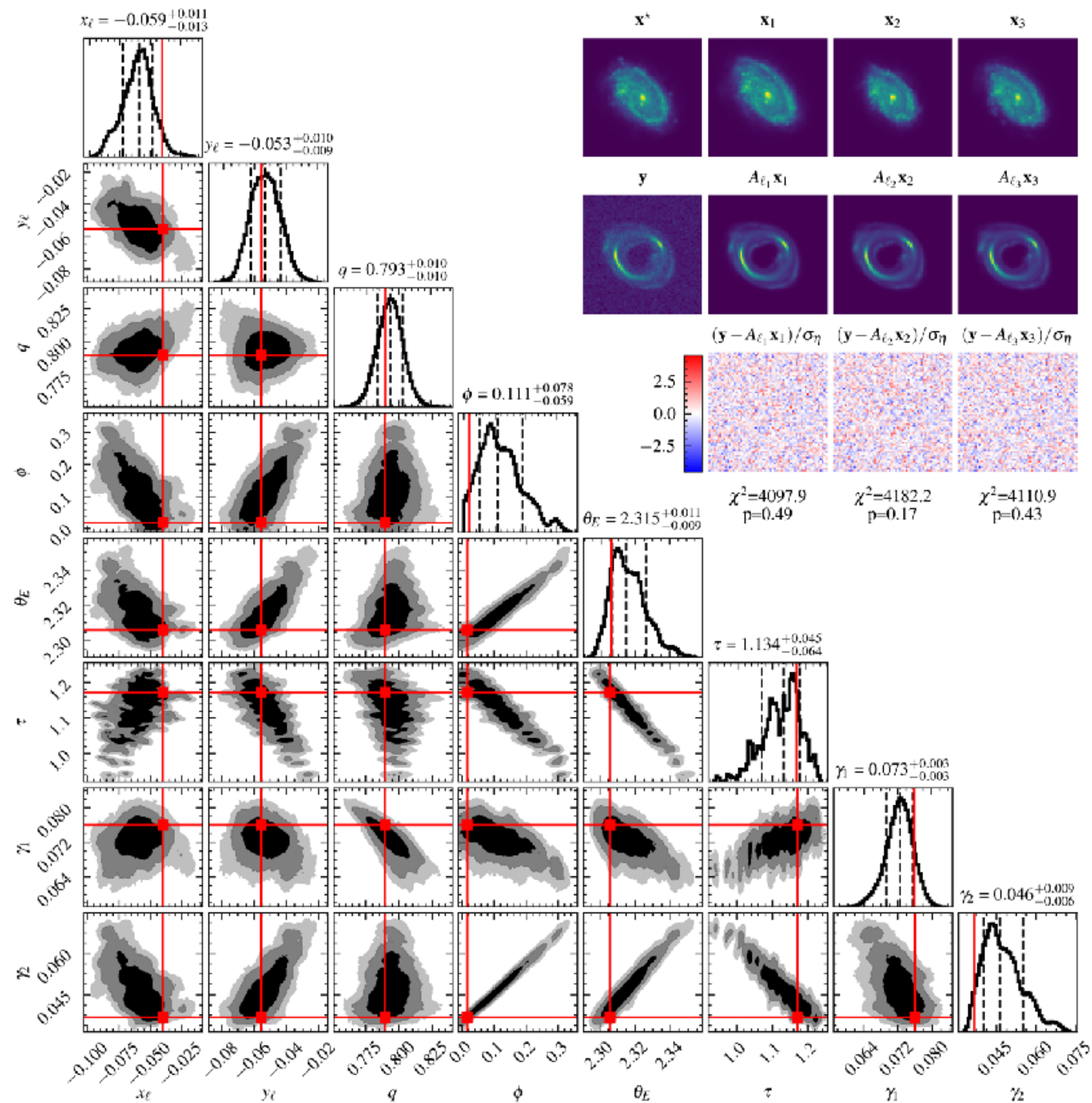
Missa Barco



# JOINT INFERENCE OF LENS AND SOURCE PARAMETERS



Missa Barco



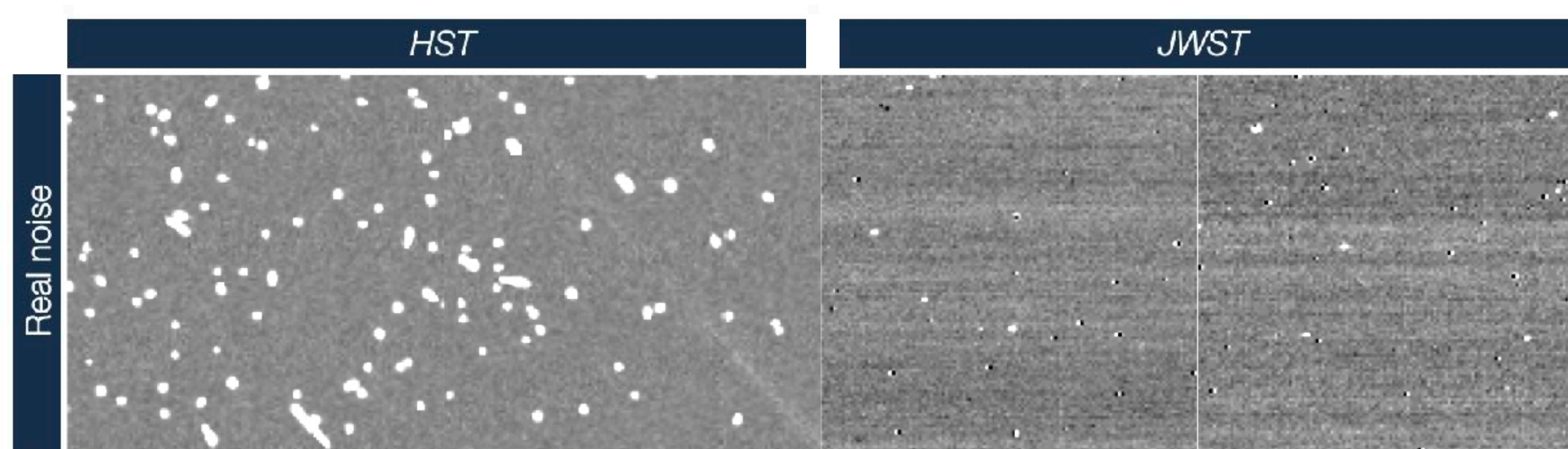
# GAME PLAN

1. Data-driven priors
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3. Accuracy metrics
4. Out-of-Distribution accuracy

# DEALING WITH REALISTIC NOISE: BEYOND GAUSSIANITY



Alexandre Adam Ronan Legin



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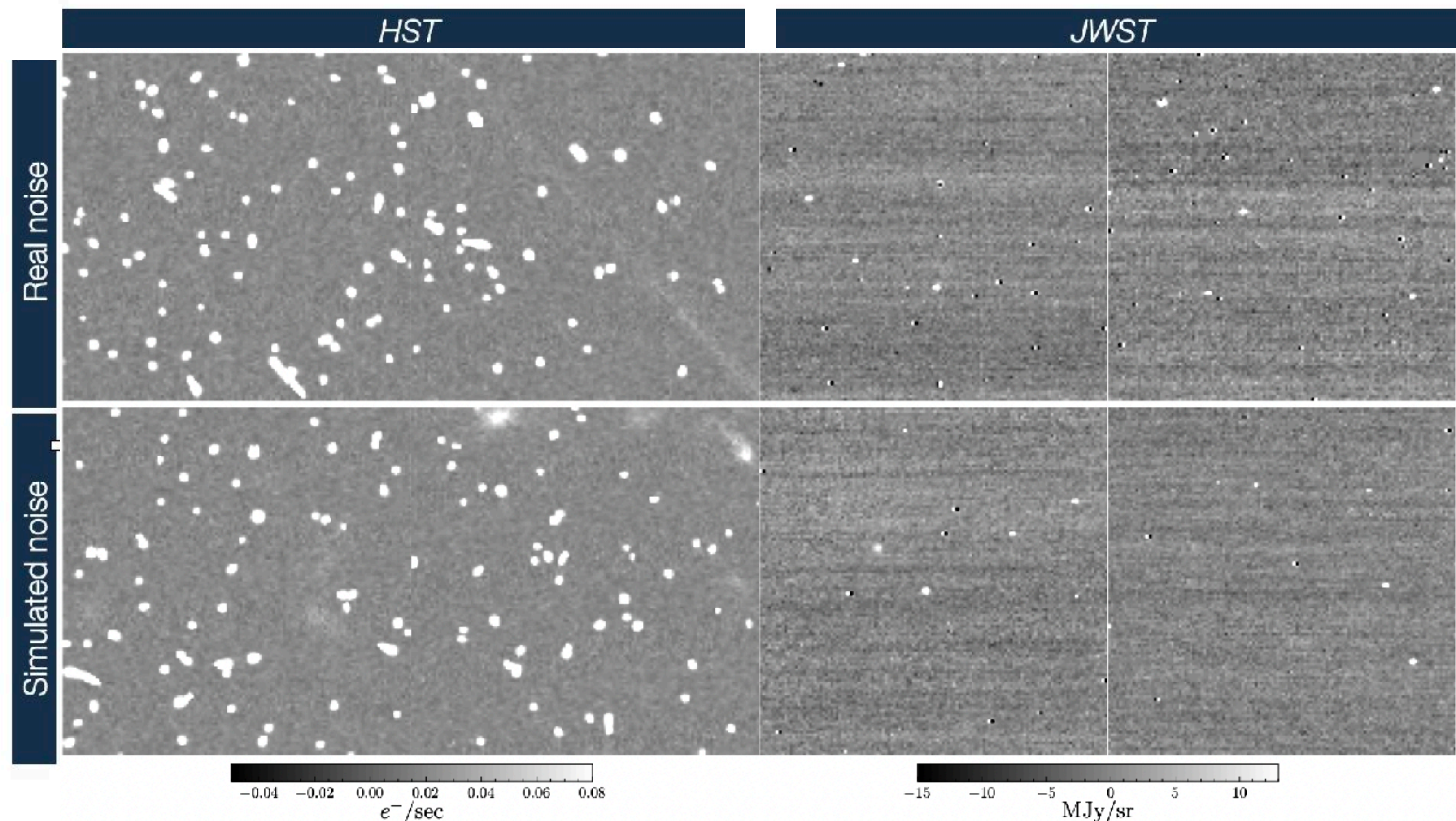
## SLIC: SCORE-BASED LIKELIHOOD CHARACTERIZATION



Alexandre Adam Ronan Legin

Since we have learnt a generative model of the additive noise, it can now be used in a simulation pipeline to get new, independent realizations of noise:

$$\mathbf{s}(\mathbf{x}) = \nabla_{\mathbf{x}} \log Q(\mathbf{x})$$



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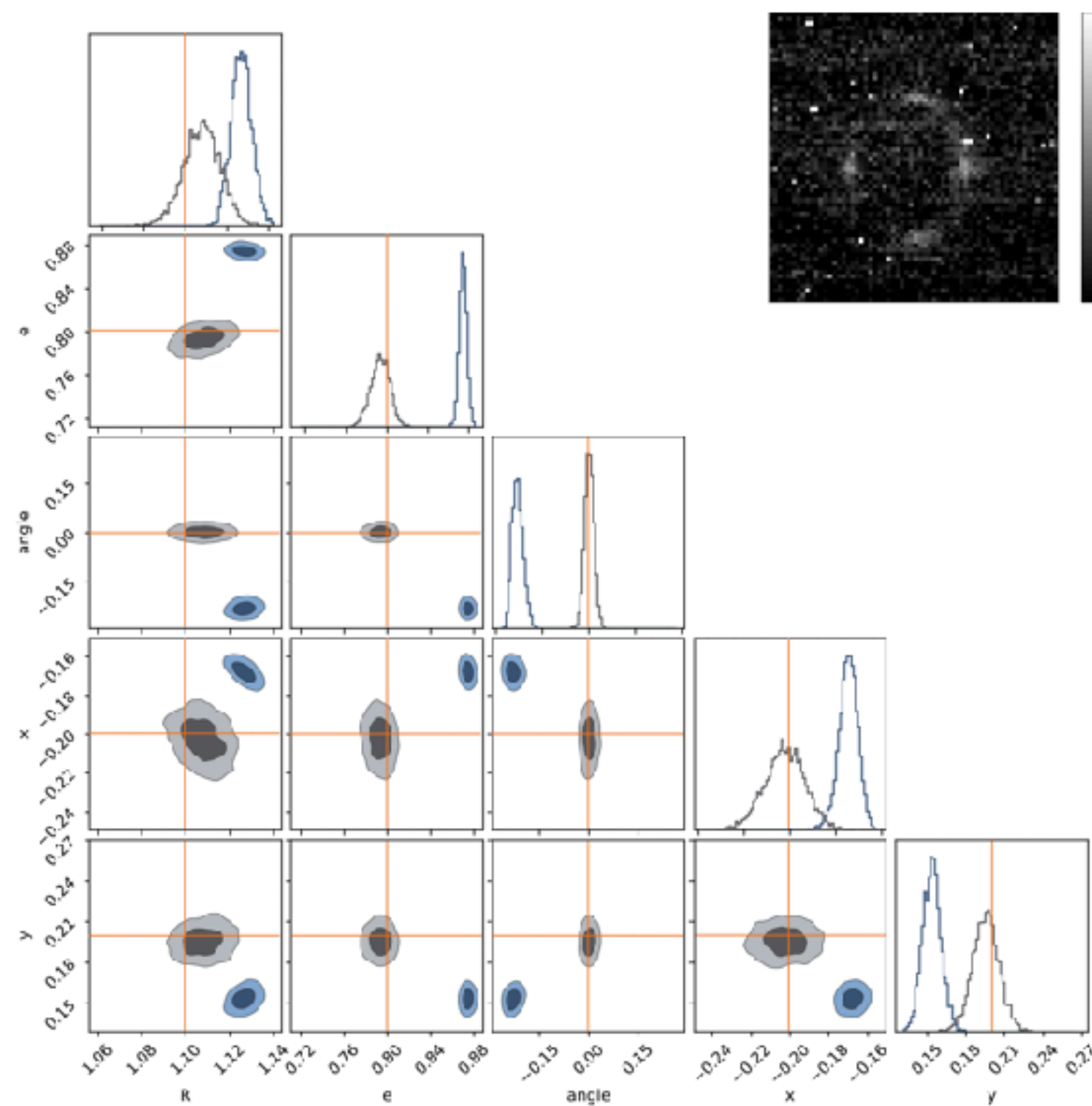


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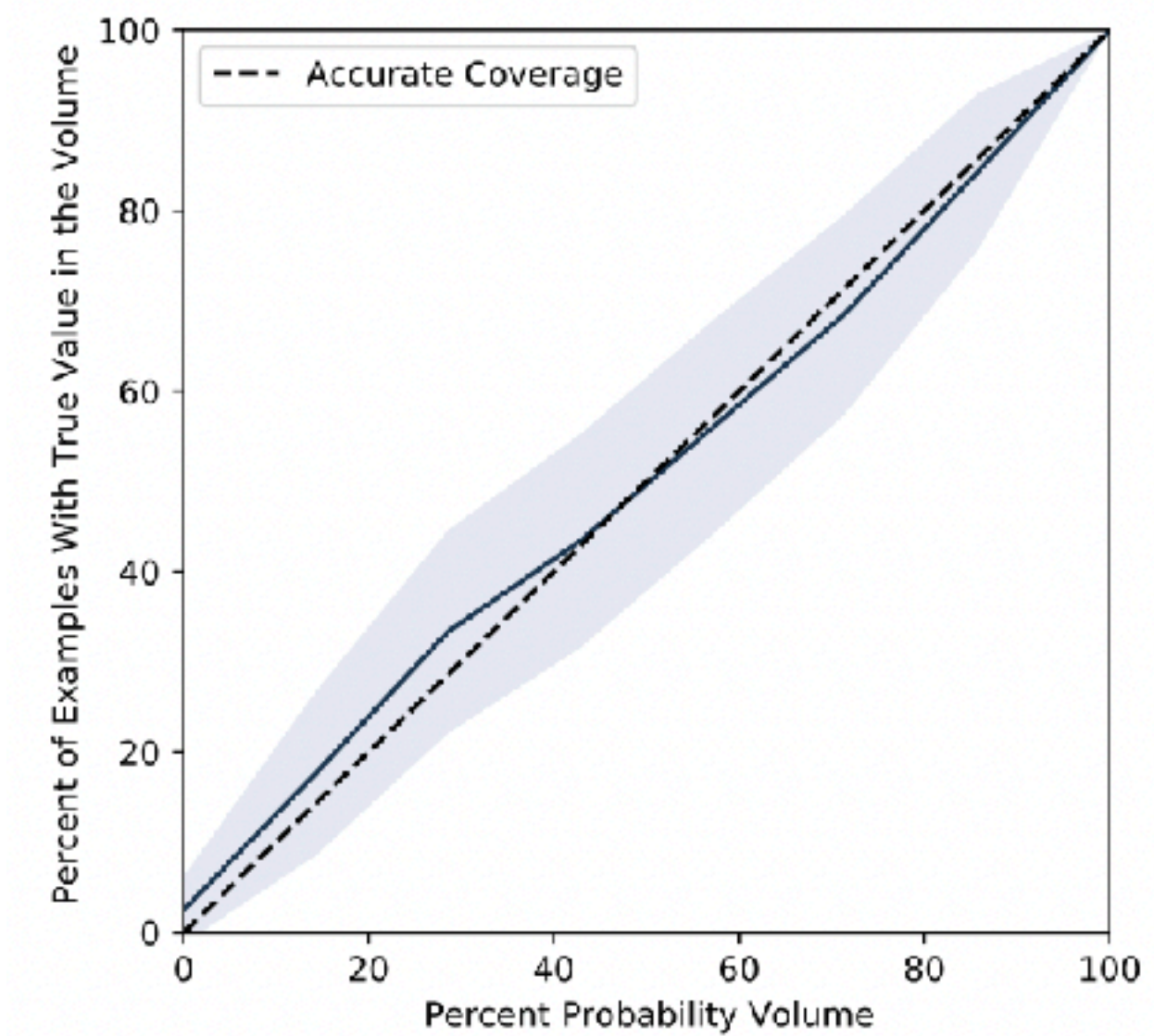
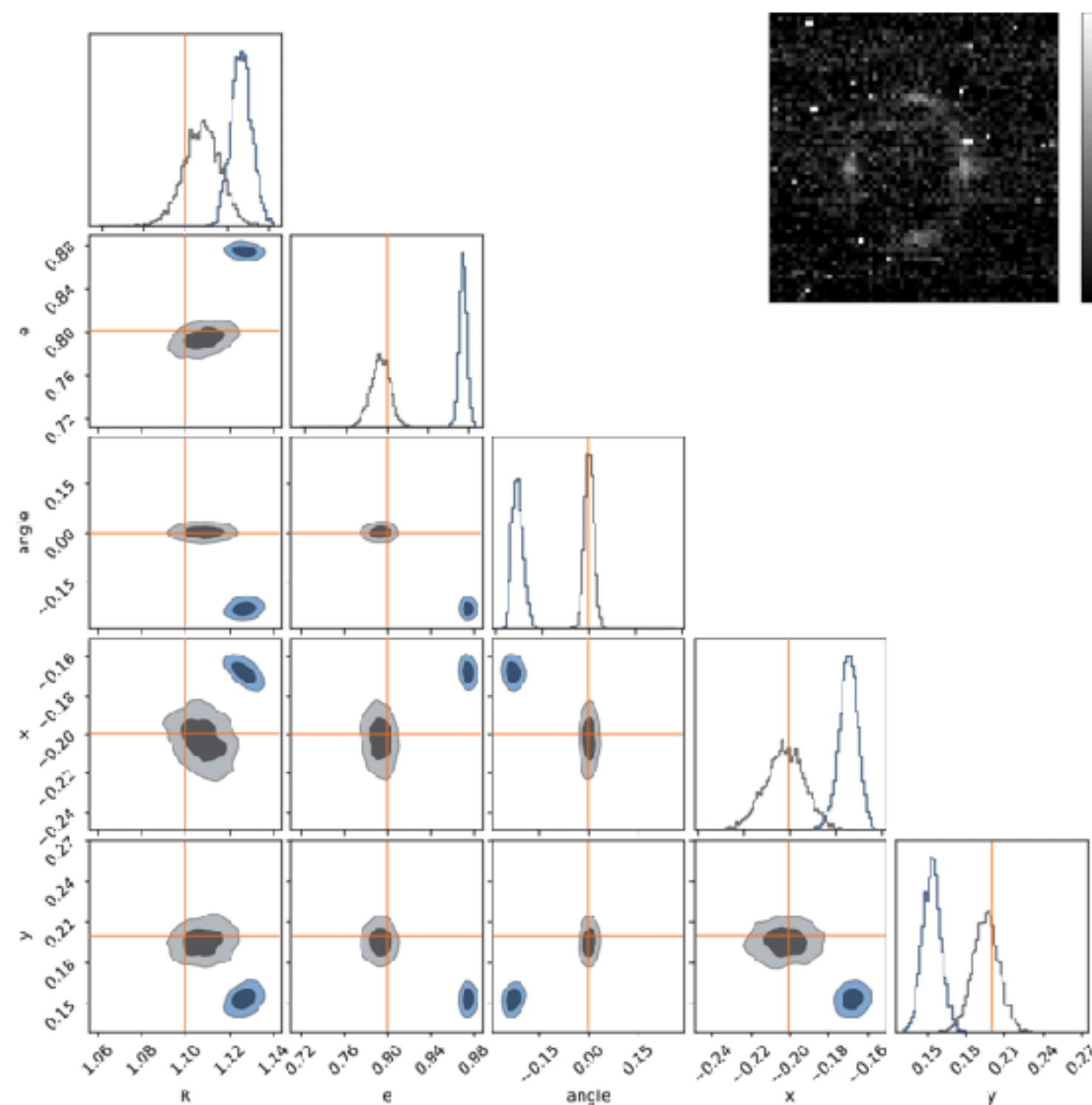


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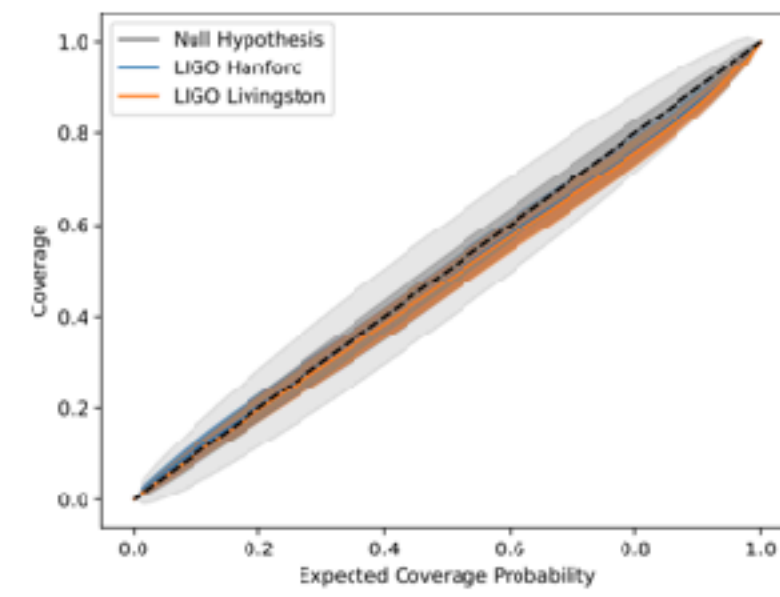
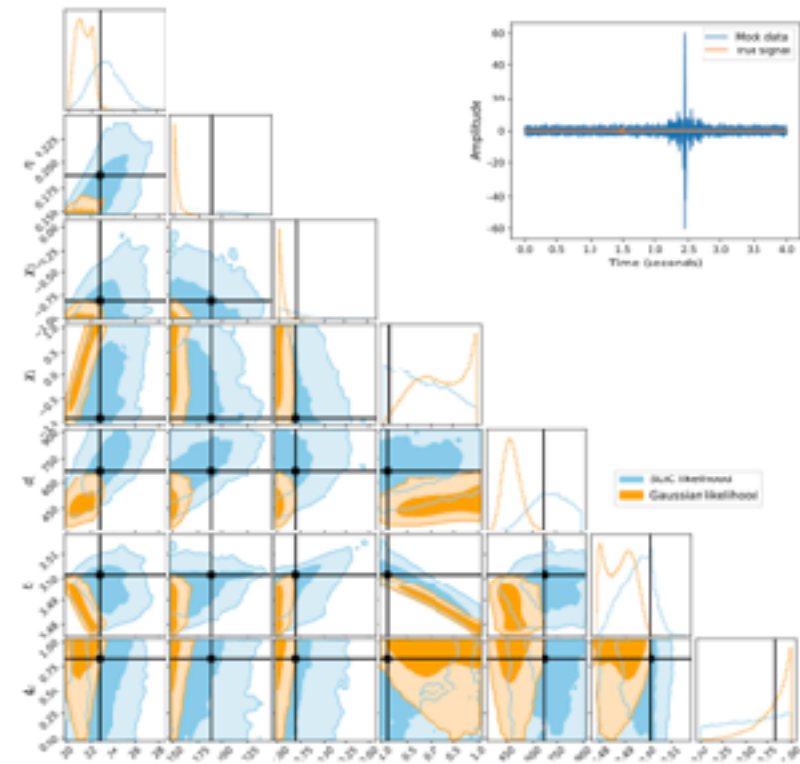
SLIC CAN BE APPLIED TO MULTIPLE DATASETS WHERE NON-GAUSSIAN NOISE IS AN ISSUE:

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## GRAVITATIONAL WAVE PARAMETER INFERENCE IN THE PRESENCE OF GLITCHES



Ronan Legin



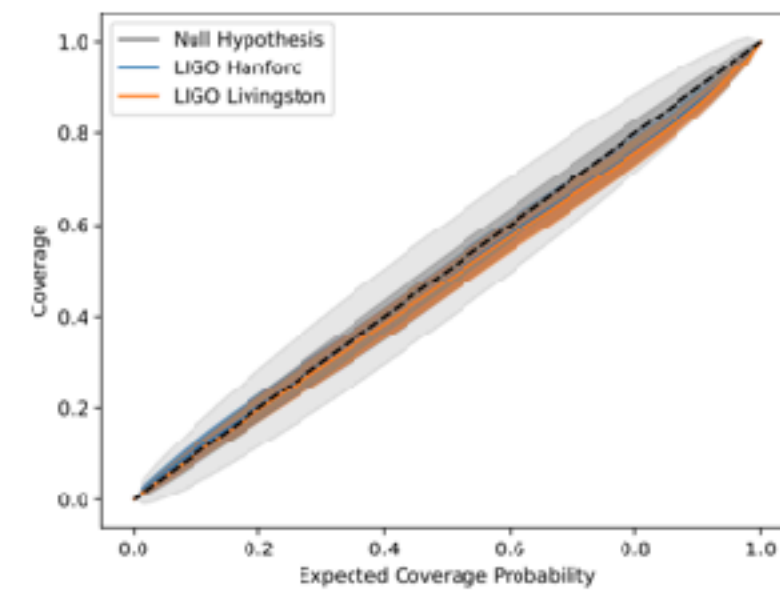
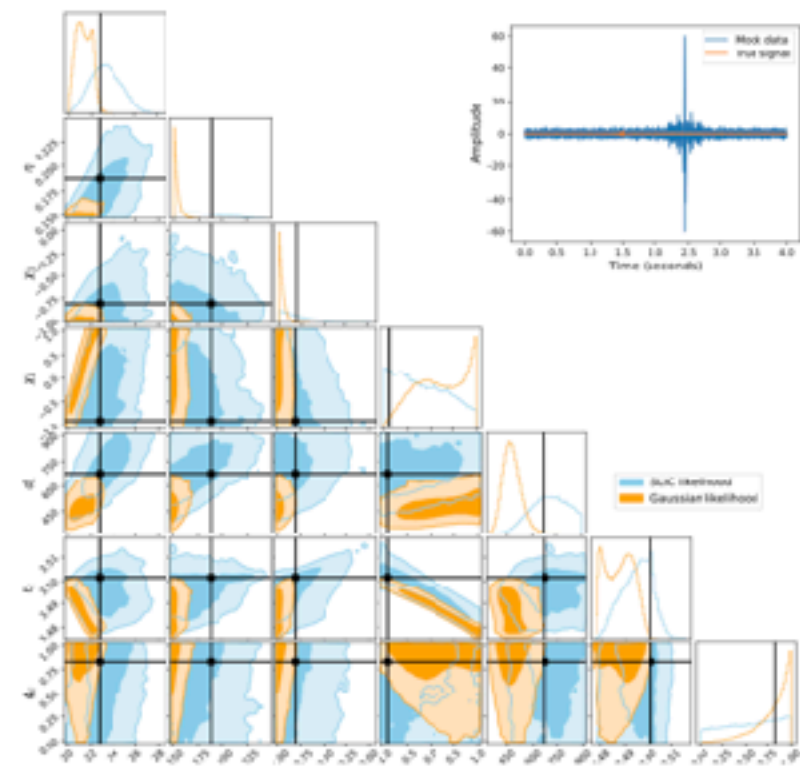
Legin et al, ApJ Letters 2026, 2410.19956

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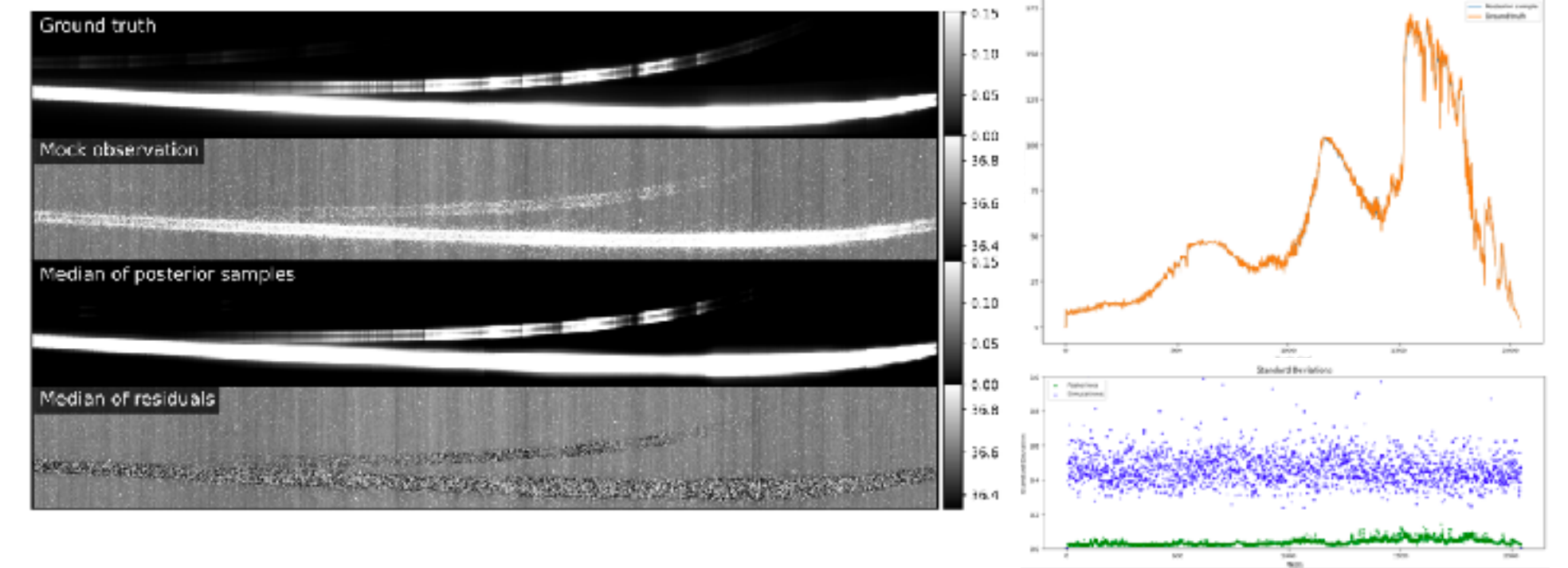


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## NOISE MODELING FOR JWST SPECTROSCOPY



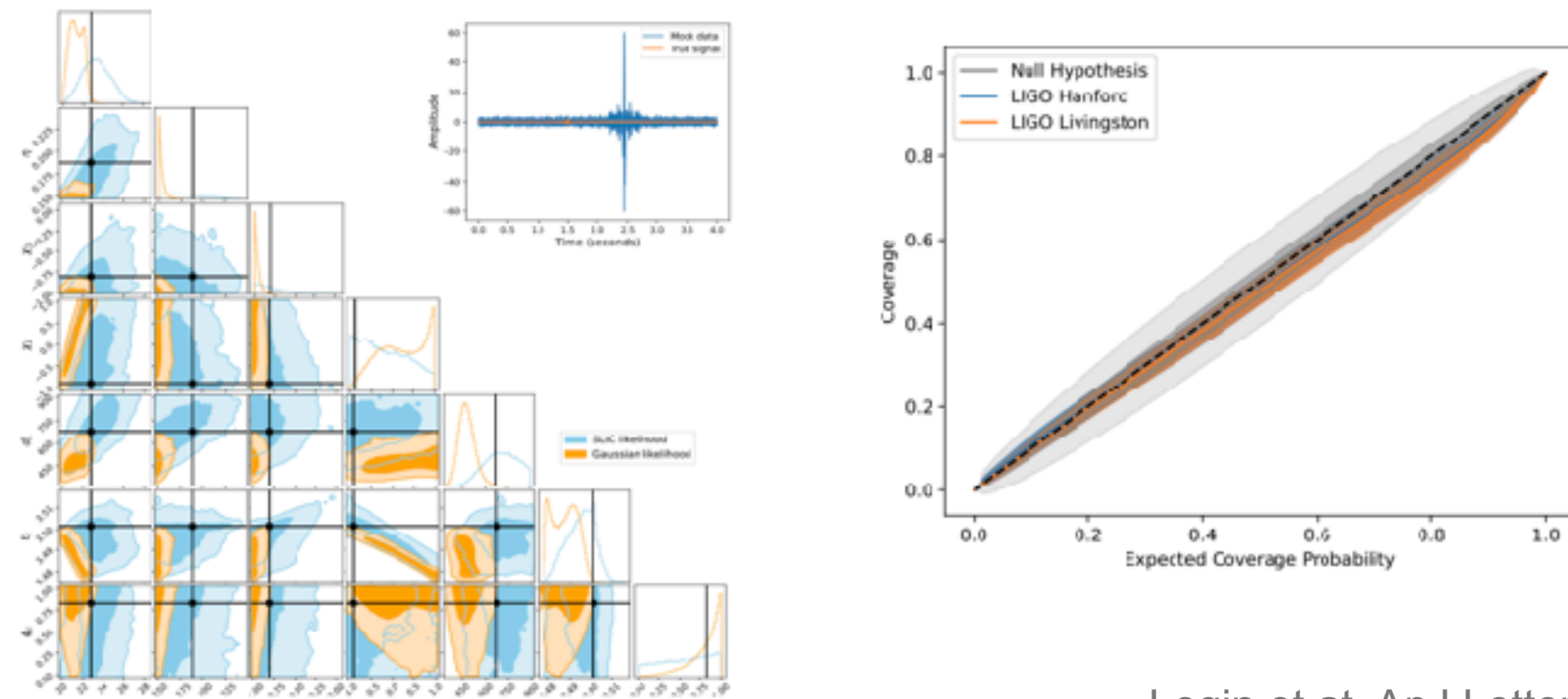
Salma Salhi



Salhi et al, NeurIPS ML4PS workshop 2024

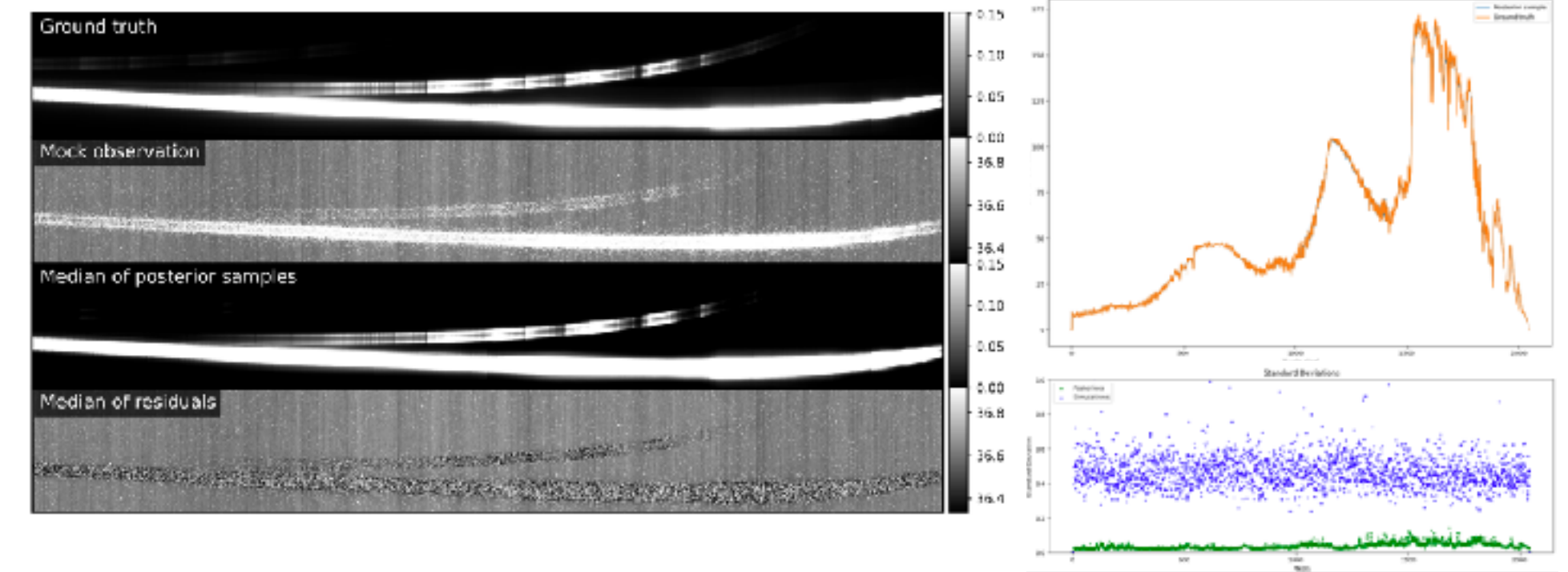
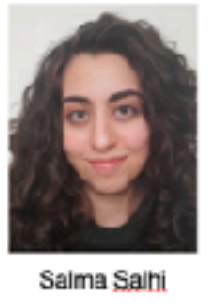
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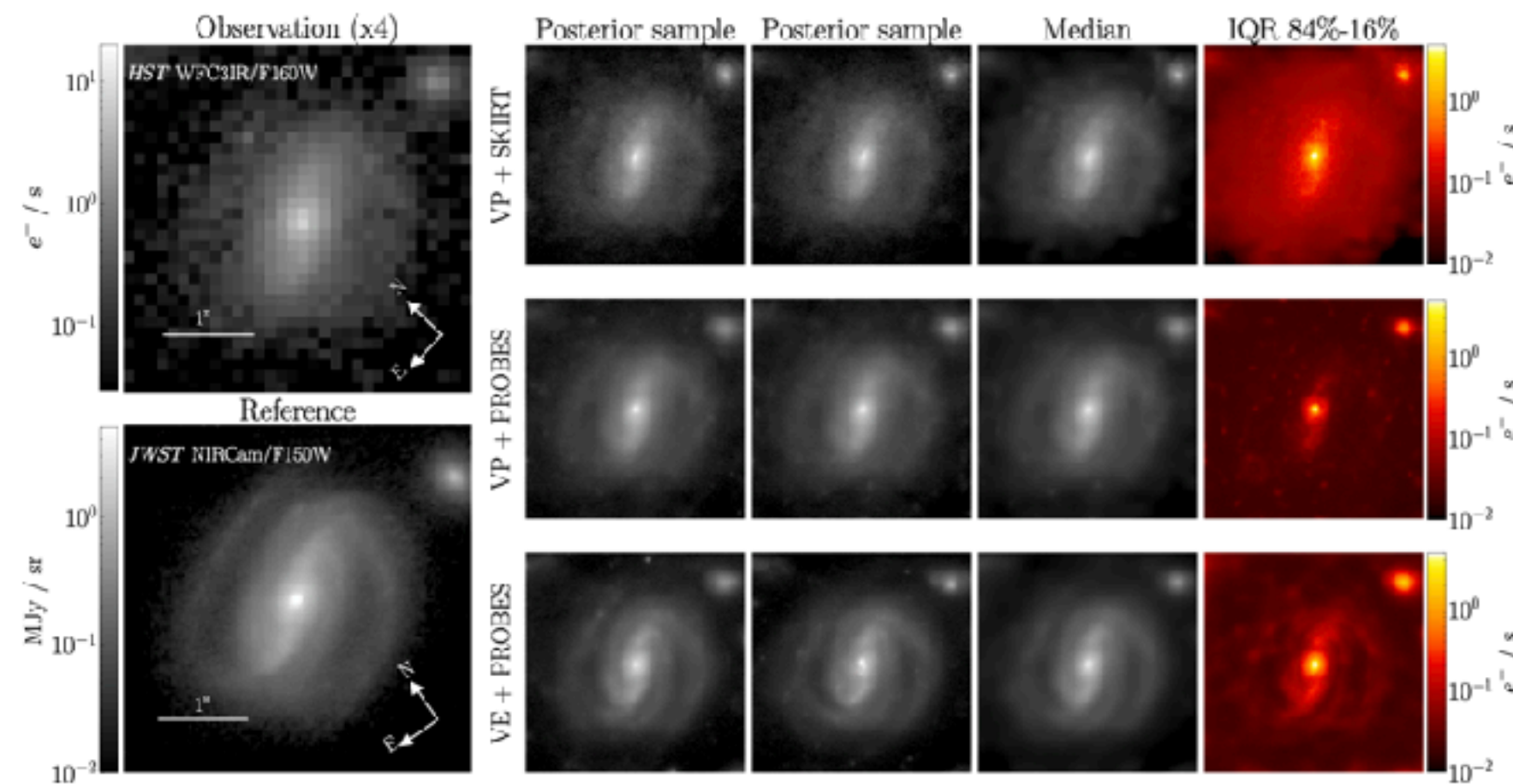
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Salhi et al, NeurIPS ML4PS workshop 2024

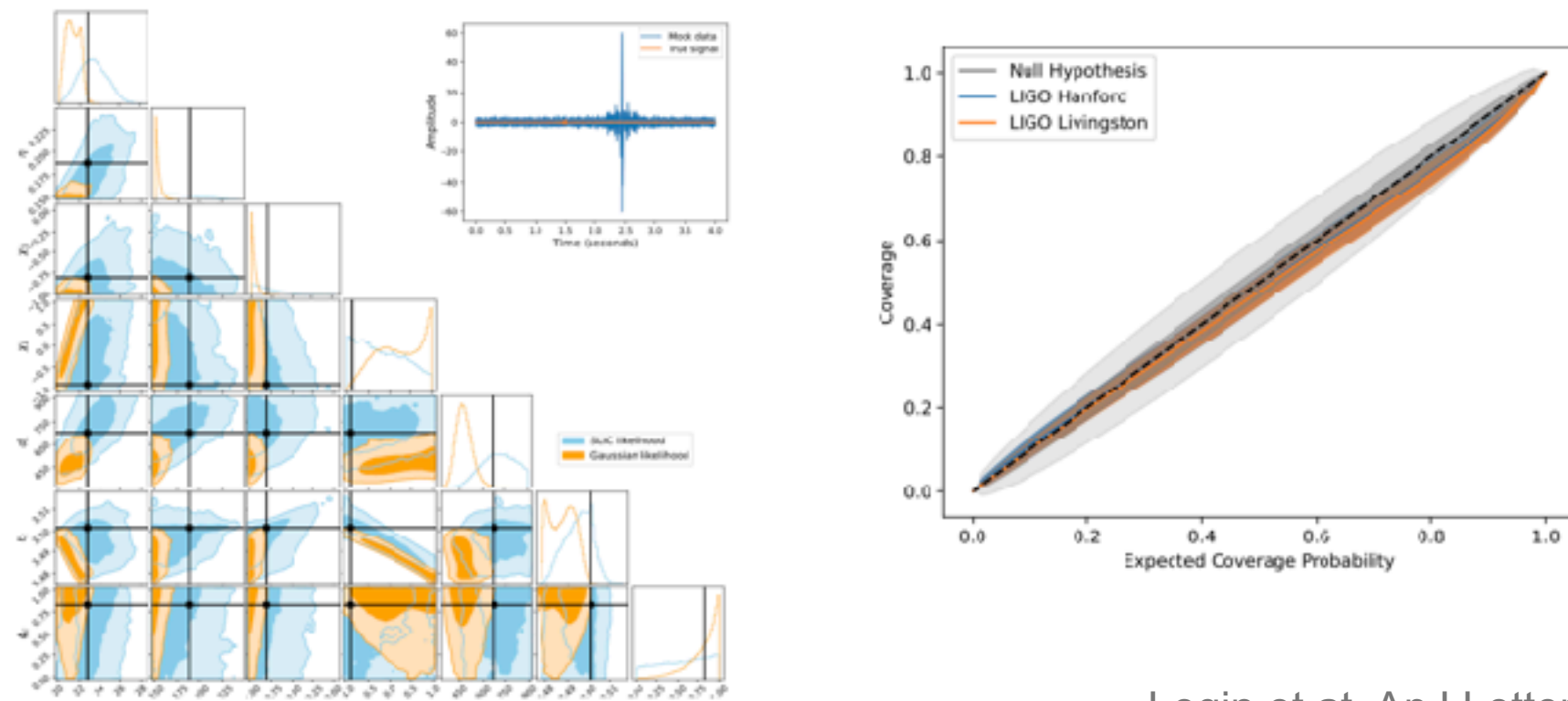
## PSF-DECONVOLUTION (FOR HST)



Adam et al. NeurIPS 2023 ML4PS workshop

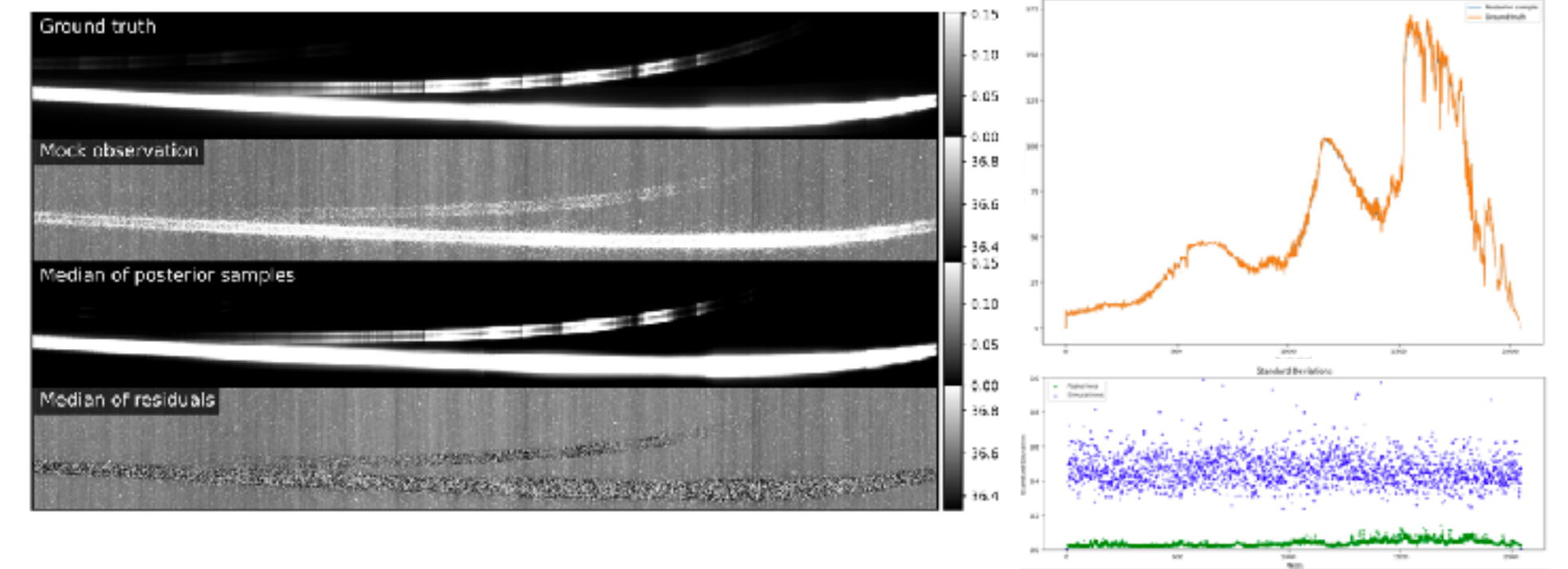
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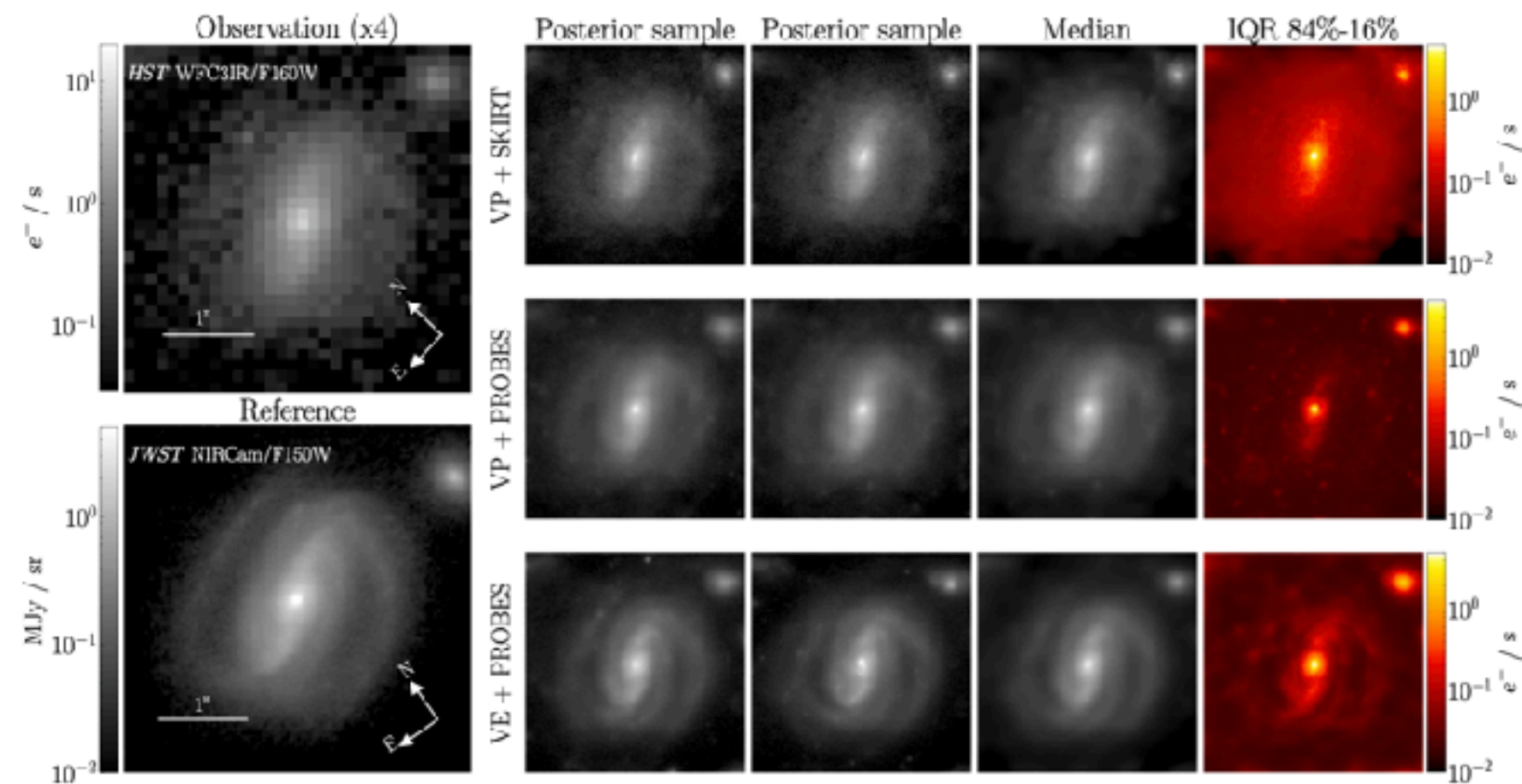
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Salhi et al., NeurIPS ML4PS workshop 2024

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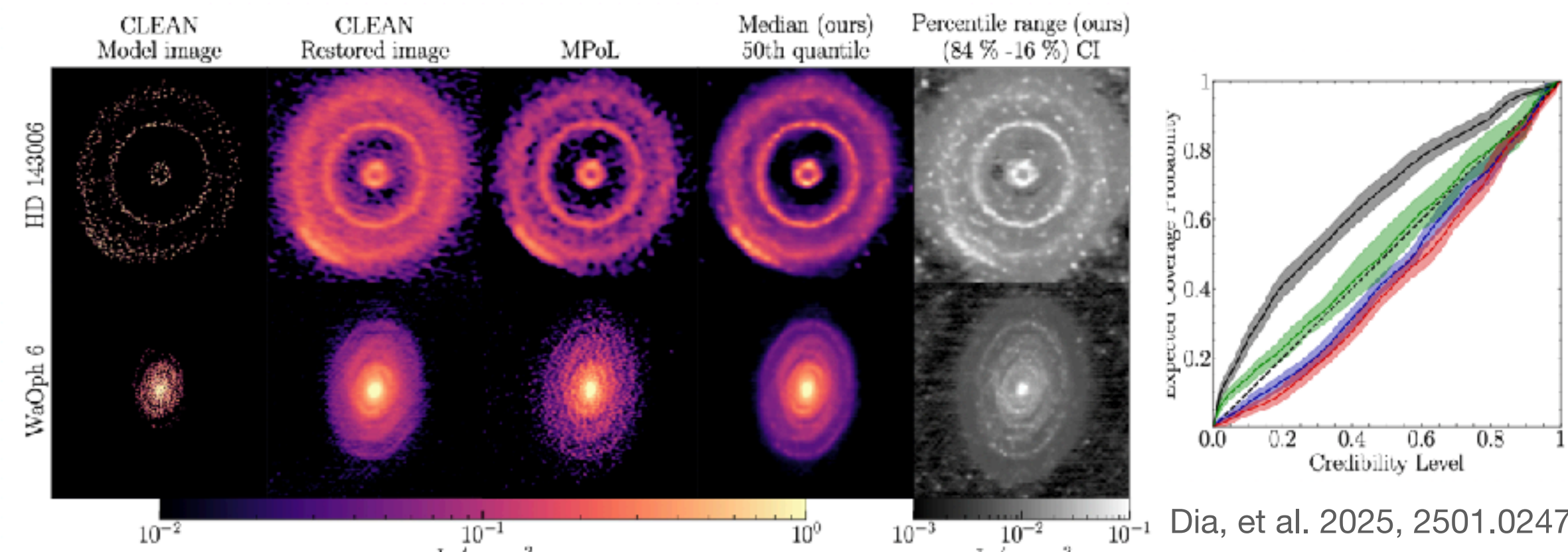


Adam et al. NeurIPS 2023 ML4PS workshop

## PSF-DECONVOLUTION FOR INTERFEROMETRIC DATA

With Score-Based Diffusion Models

$$\mathcal{V} = SFP_{\text{beam}}\mathbf{x} + \eta$$



Dia, et al. 2025, 2501.02473

# GAME PLAN

1. Data-driven priors
2. Data-driven likelihoods
3. Accuracy metrics
4. Out-of-Distribution accuracy

MACHINE LEARNING IS ENABLING US TO DO INFERENCE IN  
HIGH-DIMENSIONAL SPACES, AND TACKLE PREVIOUSLY  
INTRACTABLE QUESTIONS

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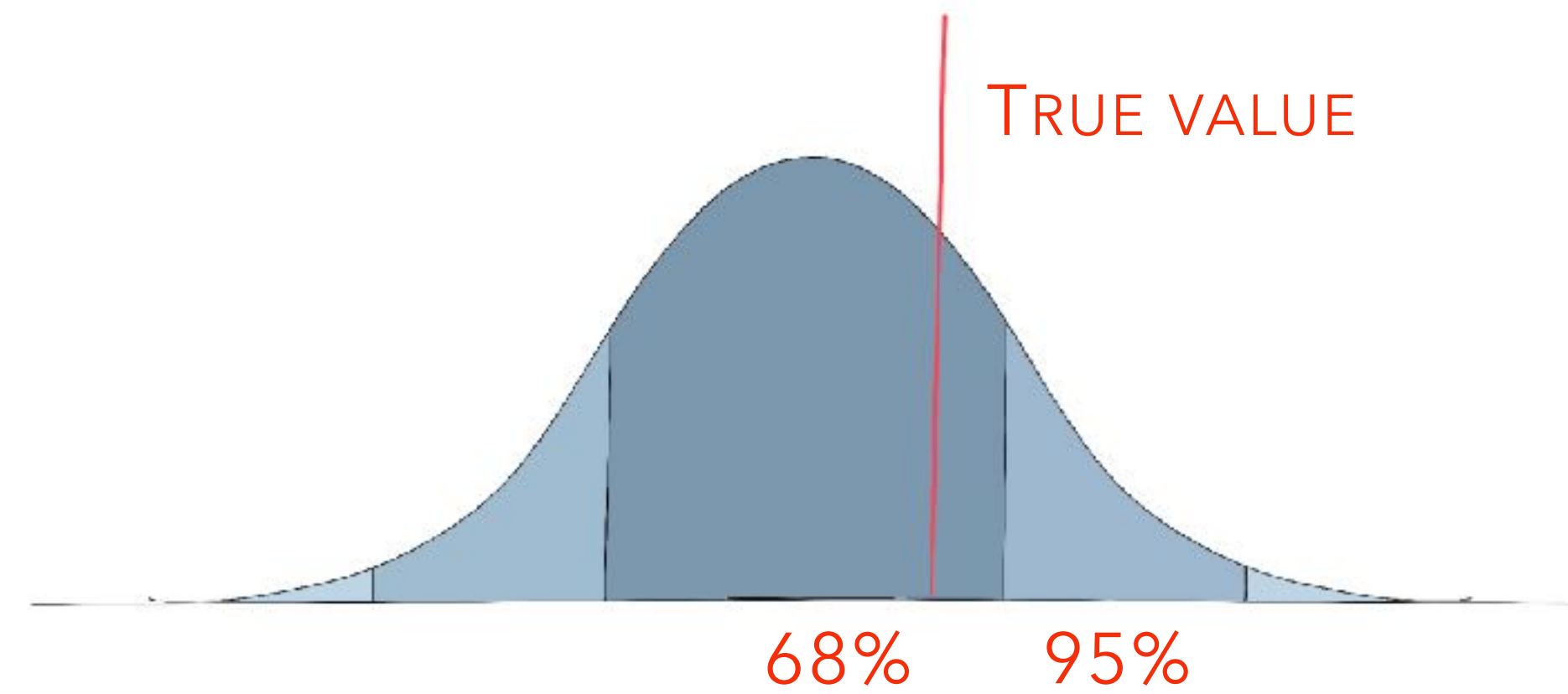
We need to answer 2 questions:

- If we have a pipeline that infers high-dimensional variables, like images of galaxies, how do we assess the accuracy of this pipeline?
- If we are using generative models as components or inputs to those inference pipelines, how do we quantify the accuracy with which they represent the underlying distribution?

# ARE THESE UNCERTAINTIES ACCURATE?

## COVERAGE TEST ASSESSMENT

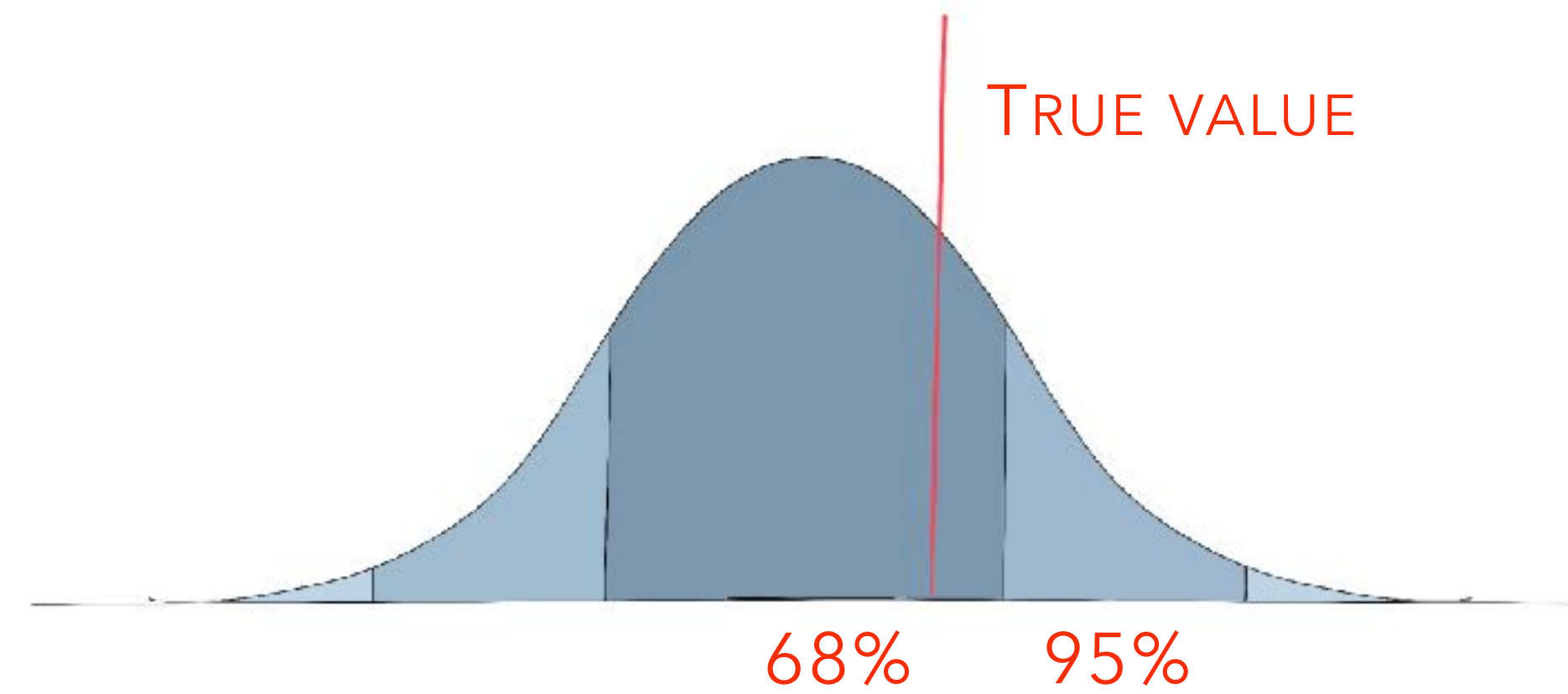
The expected coverage probability of a credible region is the proportion of the time that the region contains the true value of interest.



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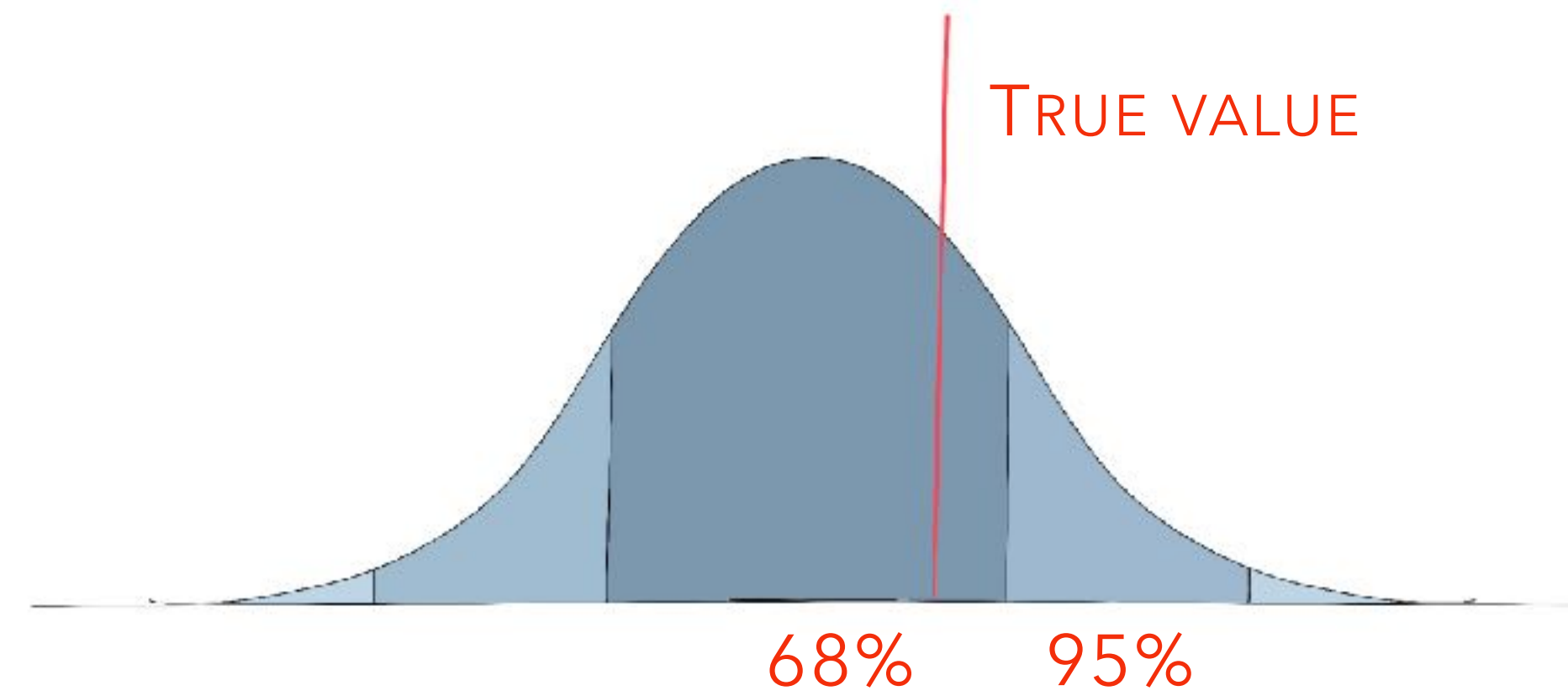


For an accurate posterior estimator, the expected coverage probability is equal to the probability mass of the credible region.

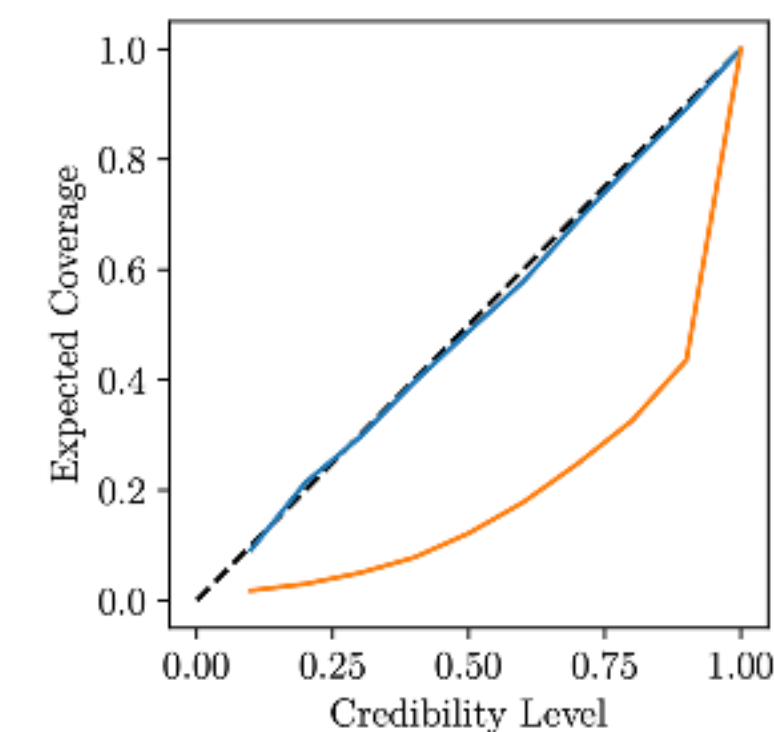
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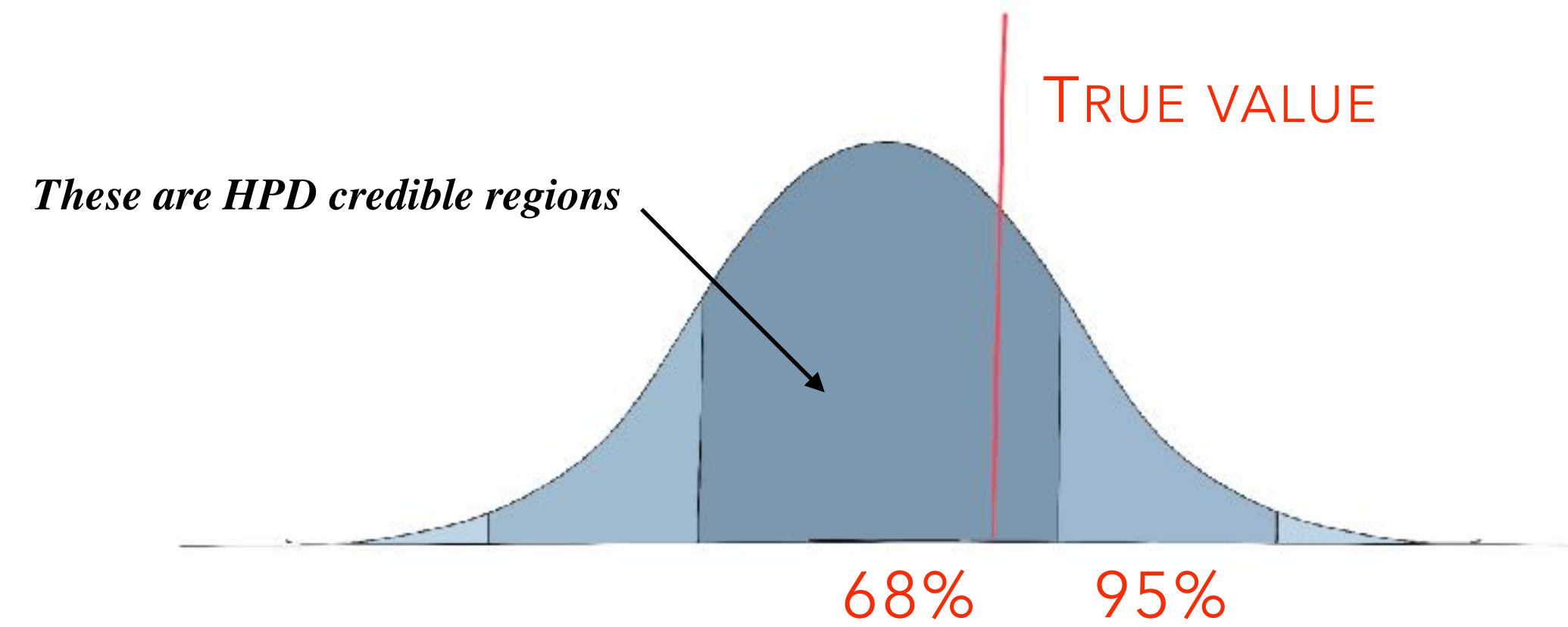
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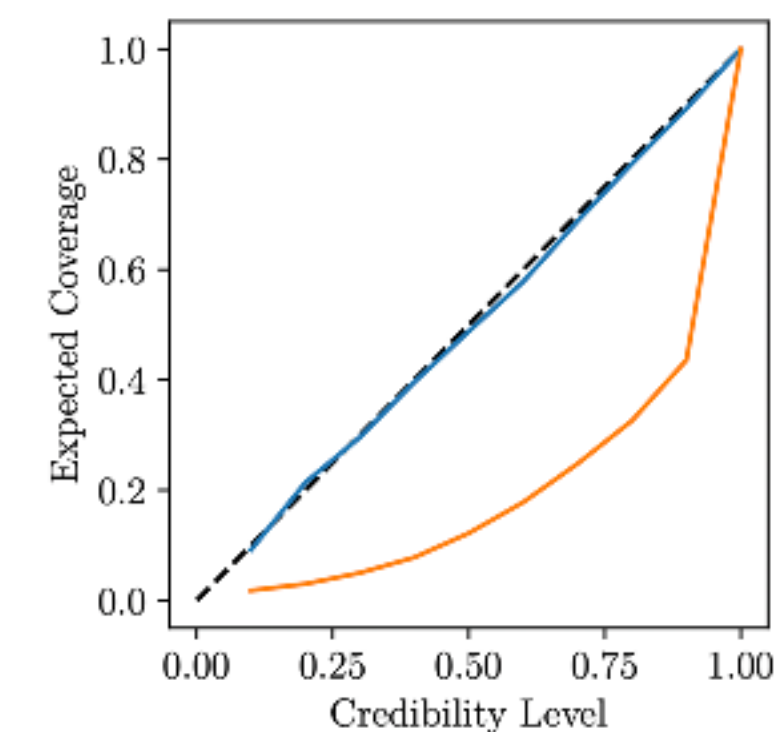
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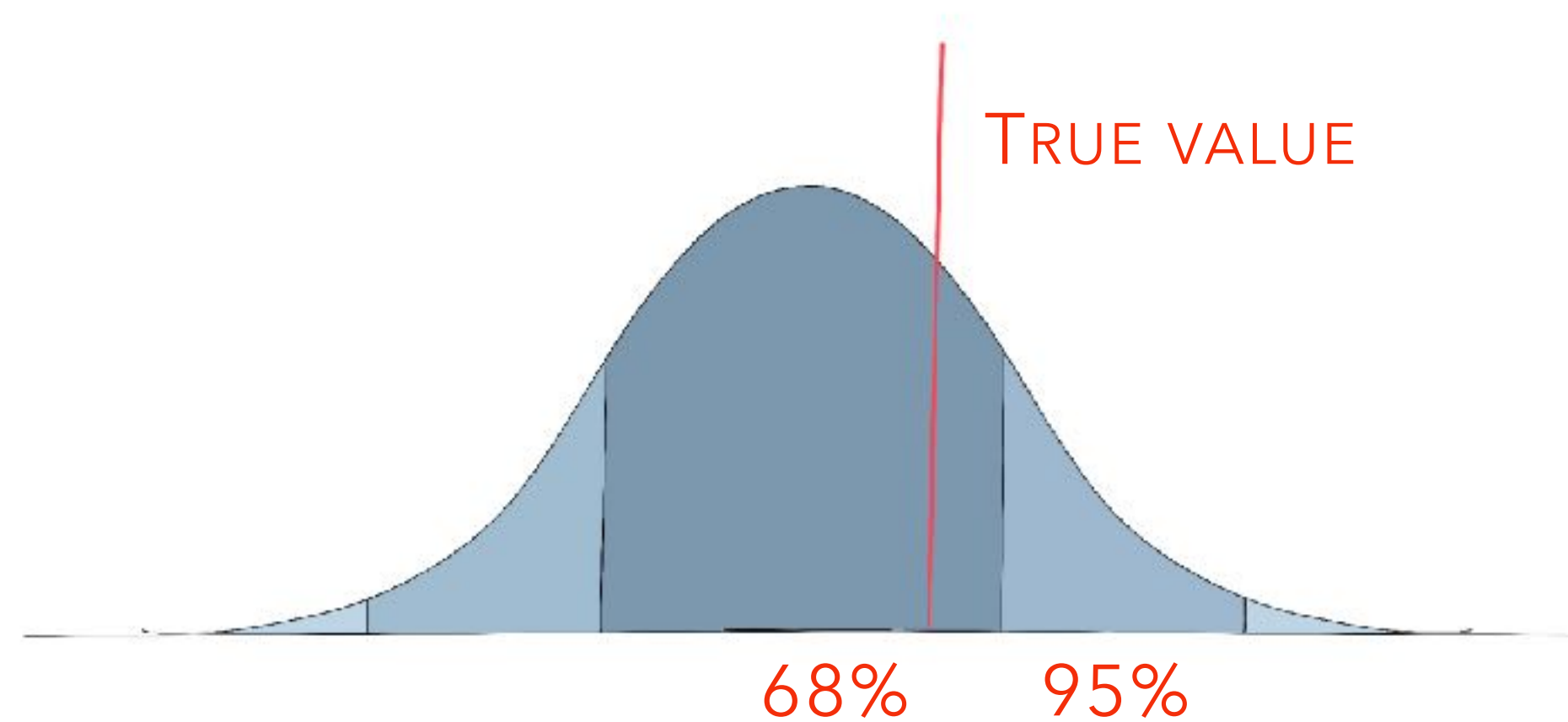
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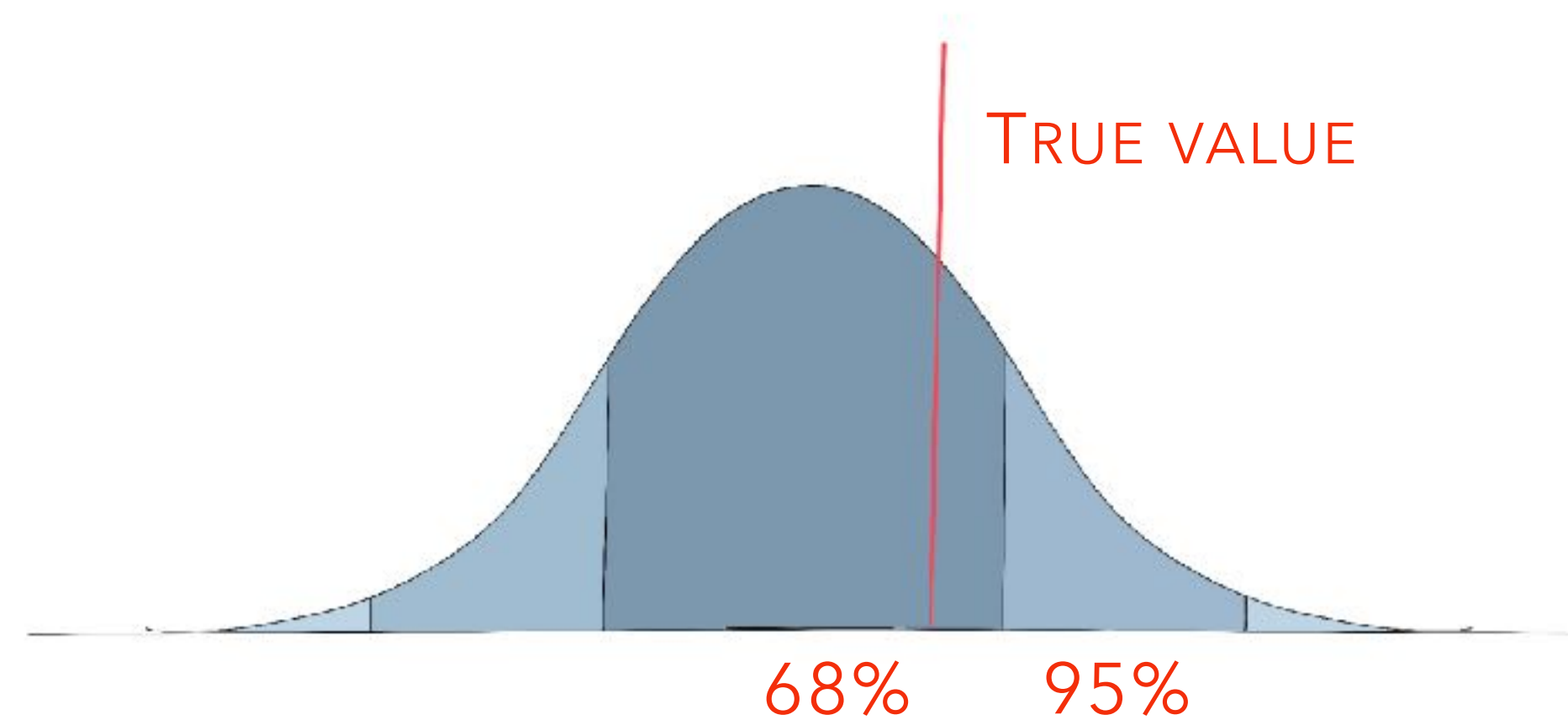
# PROBLEMS WITH TRADITIONAL HPD COVERAGE TESTS:



1. In high dimension, like in pixel space for an image, how do we find a the HDP region for a given volume?

We could estimate the density (e.g. with KDE) but very crude approximation with limited samples, especially over complex densities like images.

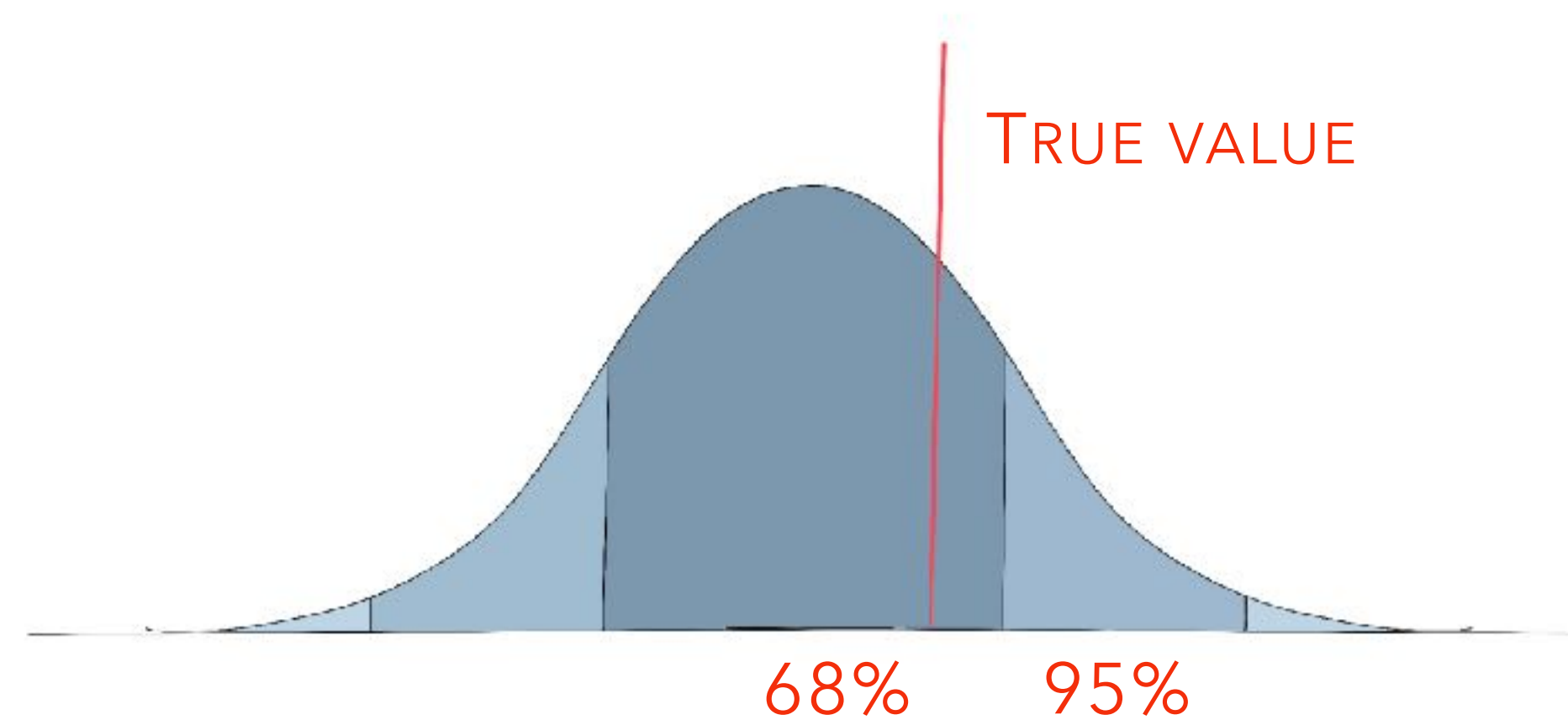
# PROBLEMS WITH TRADITIONAL HPD COVERAGE TESTS:



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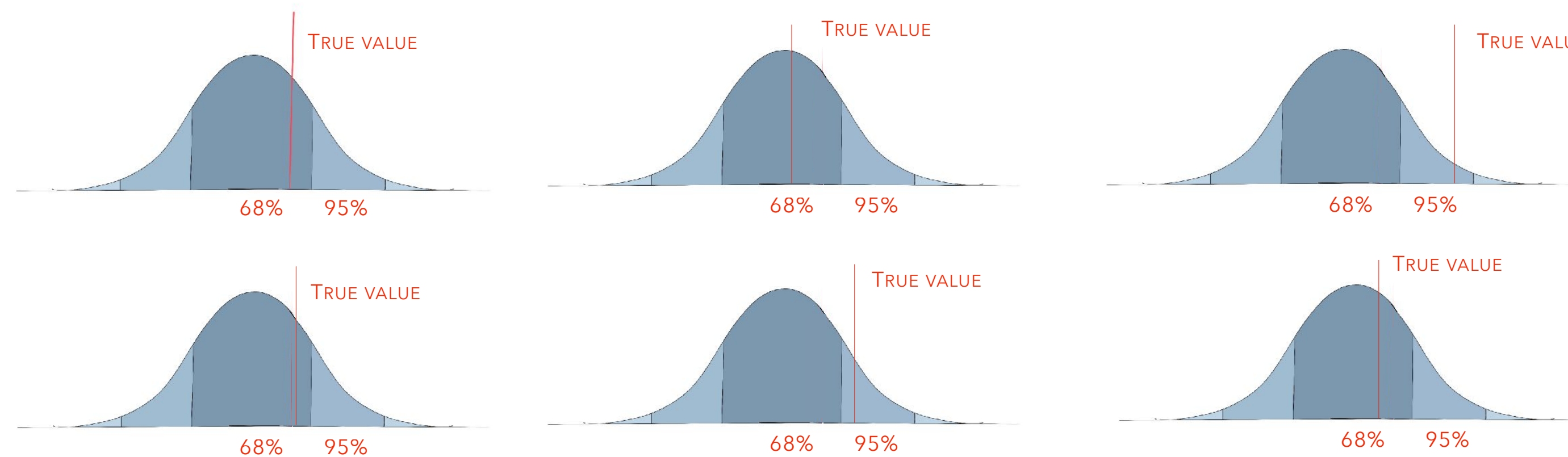
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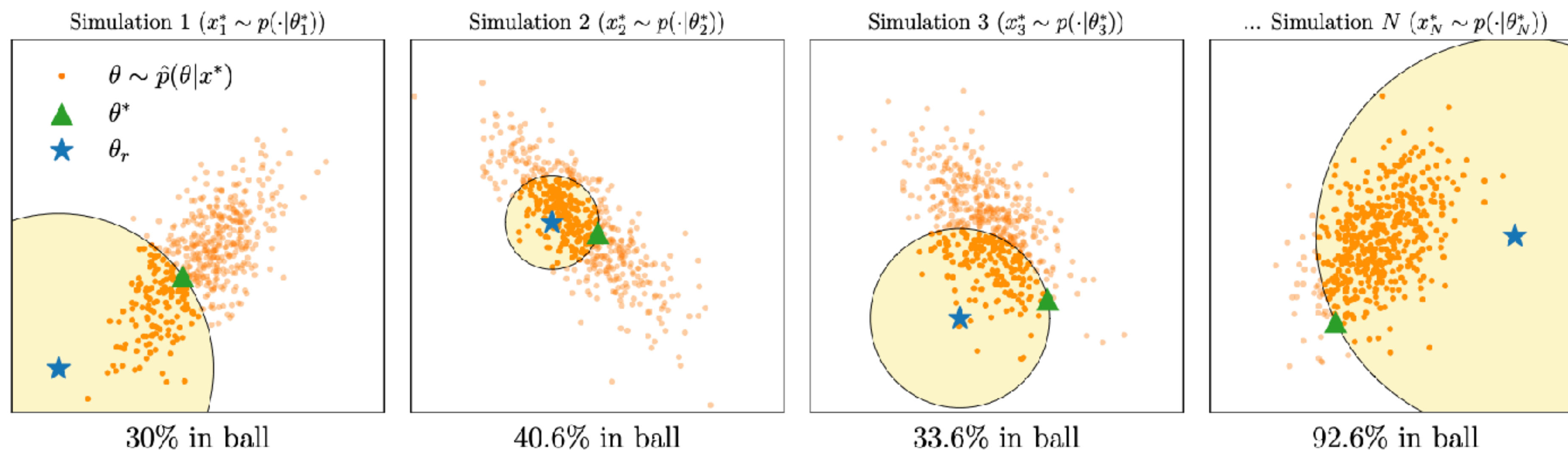
# COVERAGE TEST FOR ACCURACY WITH RANDOM POINTS -TARP-



Adam  
Coogan



Pablo  
Lemos

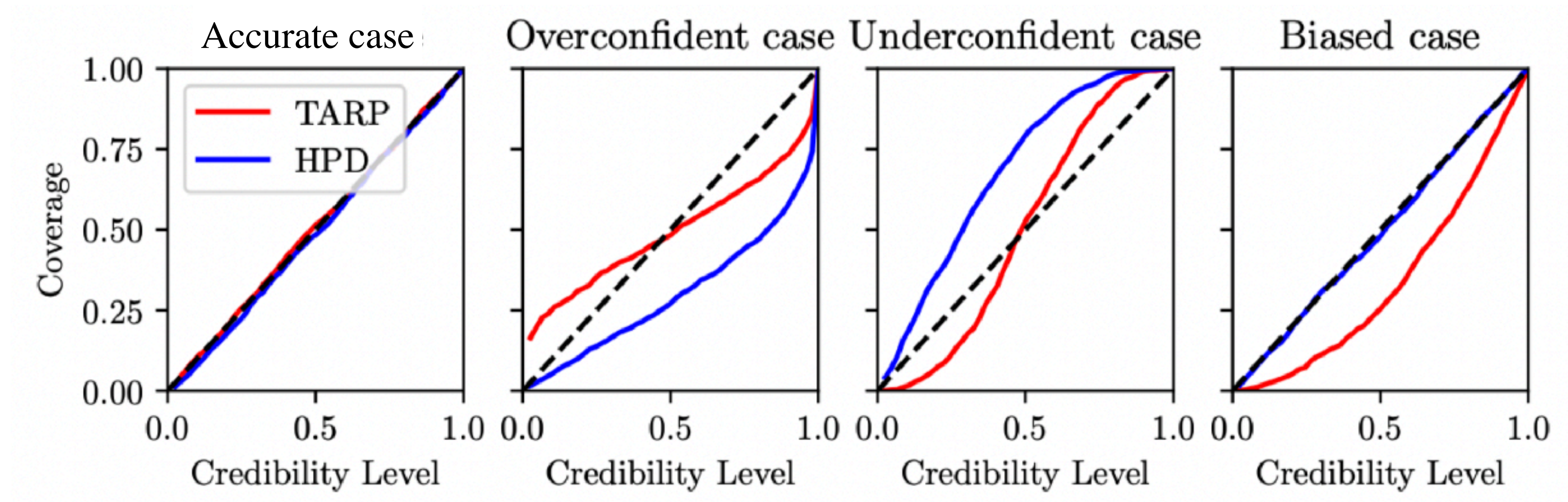


**pip install tarp**

# COVERAGE TEST FOR ACCURACY



Pablo Lemos



# PQMass: Probabilistic Assessment of Generative Models Using Probability Mass Estimation



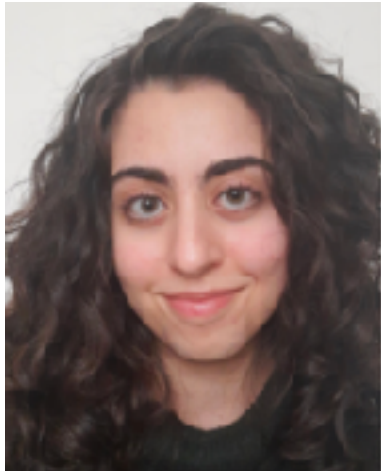
Sammy Sharief

Pablo Lemos

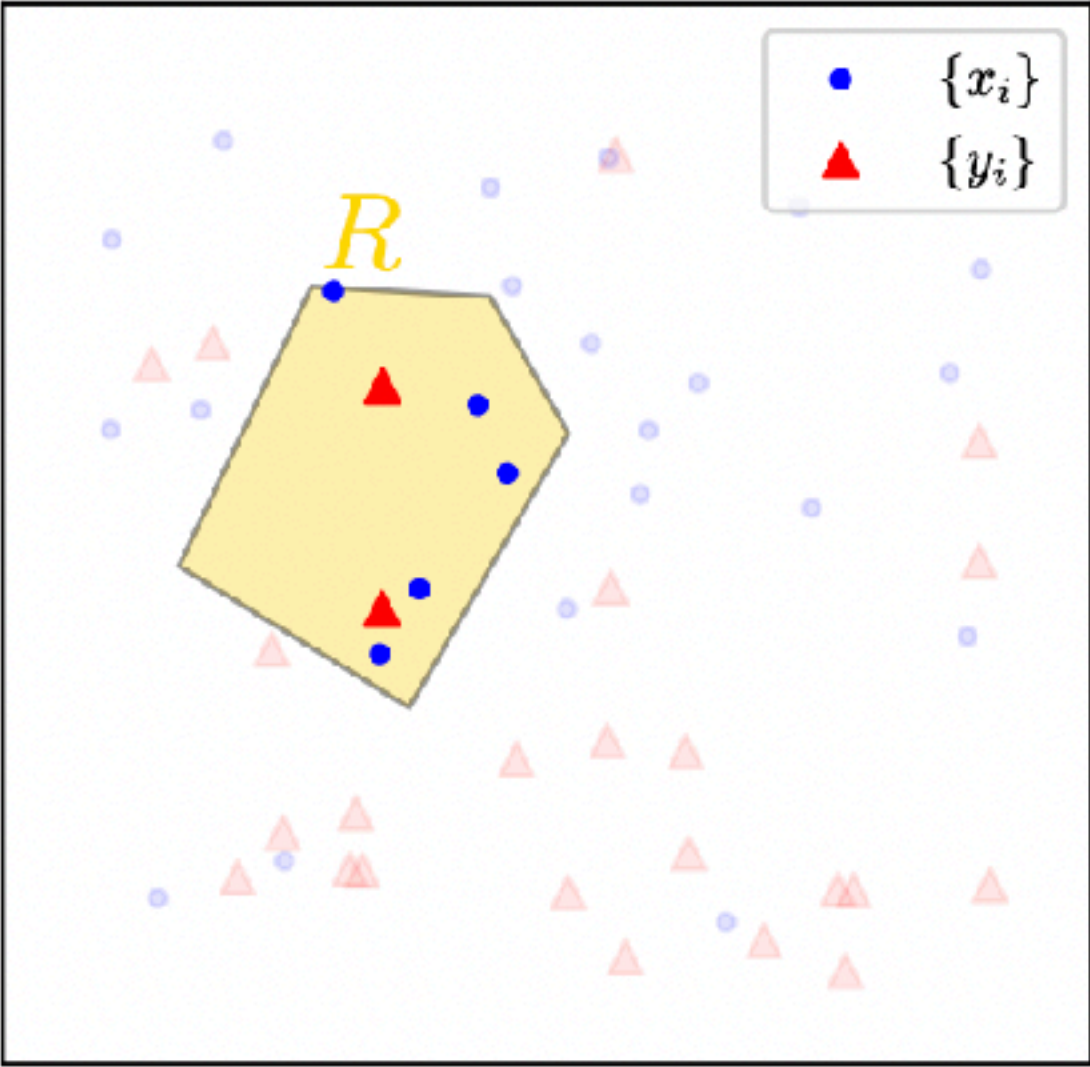
Kolya Malkin



Connor Stone



Salma Salhi



**pip install pqm**

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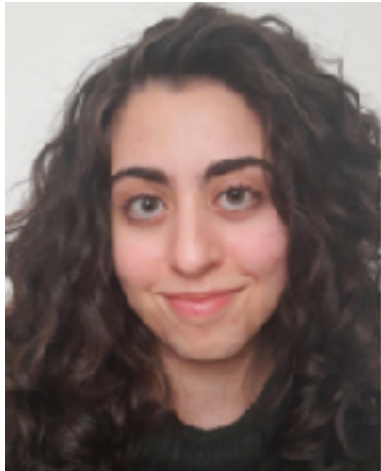
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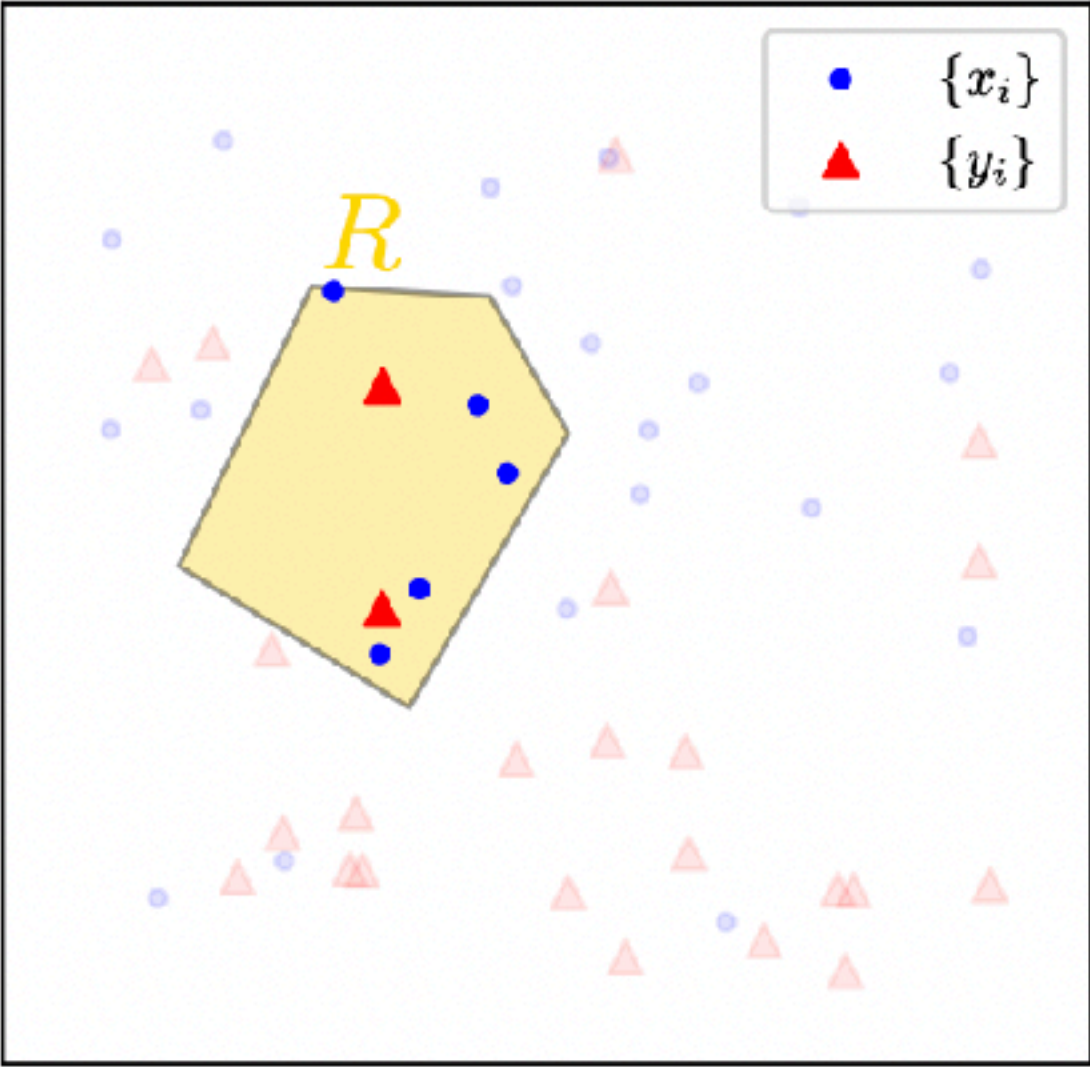
Kolya Malkin



Connor Stone



Salma Salhi



$$k(\mathbf{x}, R) \sim \mathcal{B}(n, \mathbb{P}_p(R))$$

**pip install pqm**

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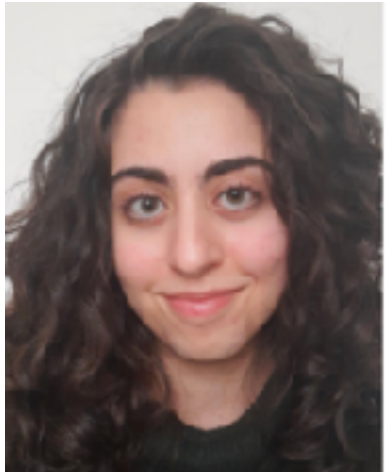
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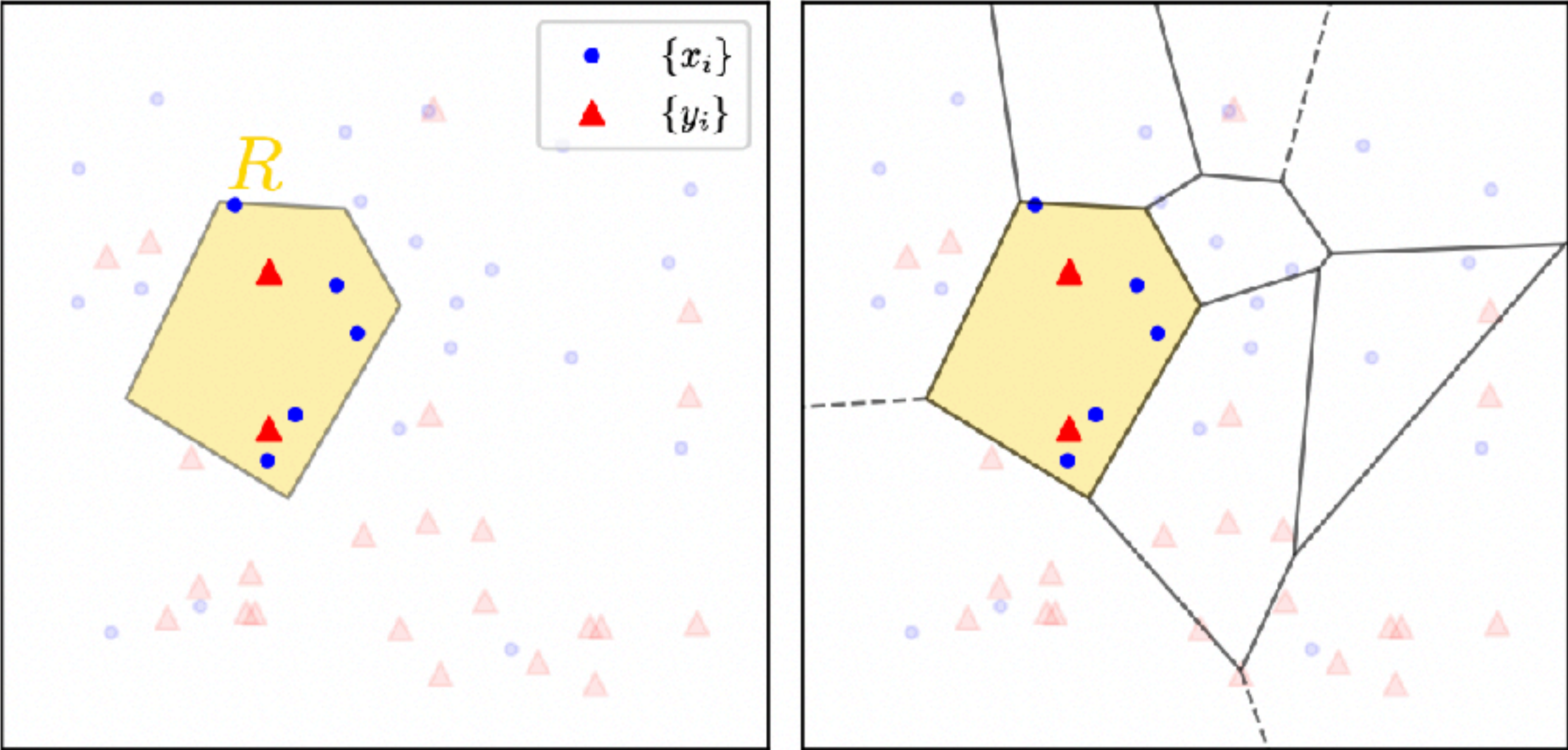
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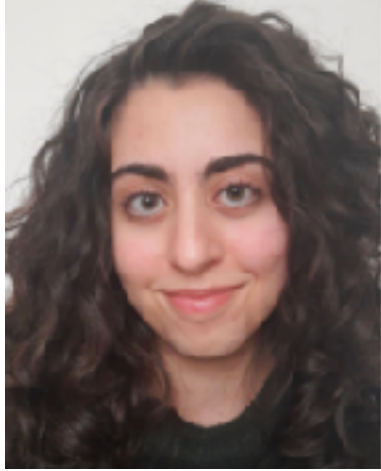
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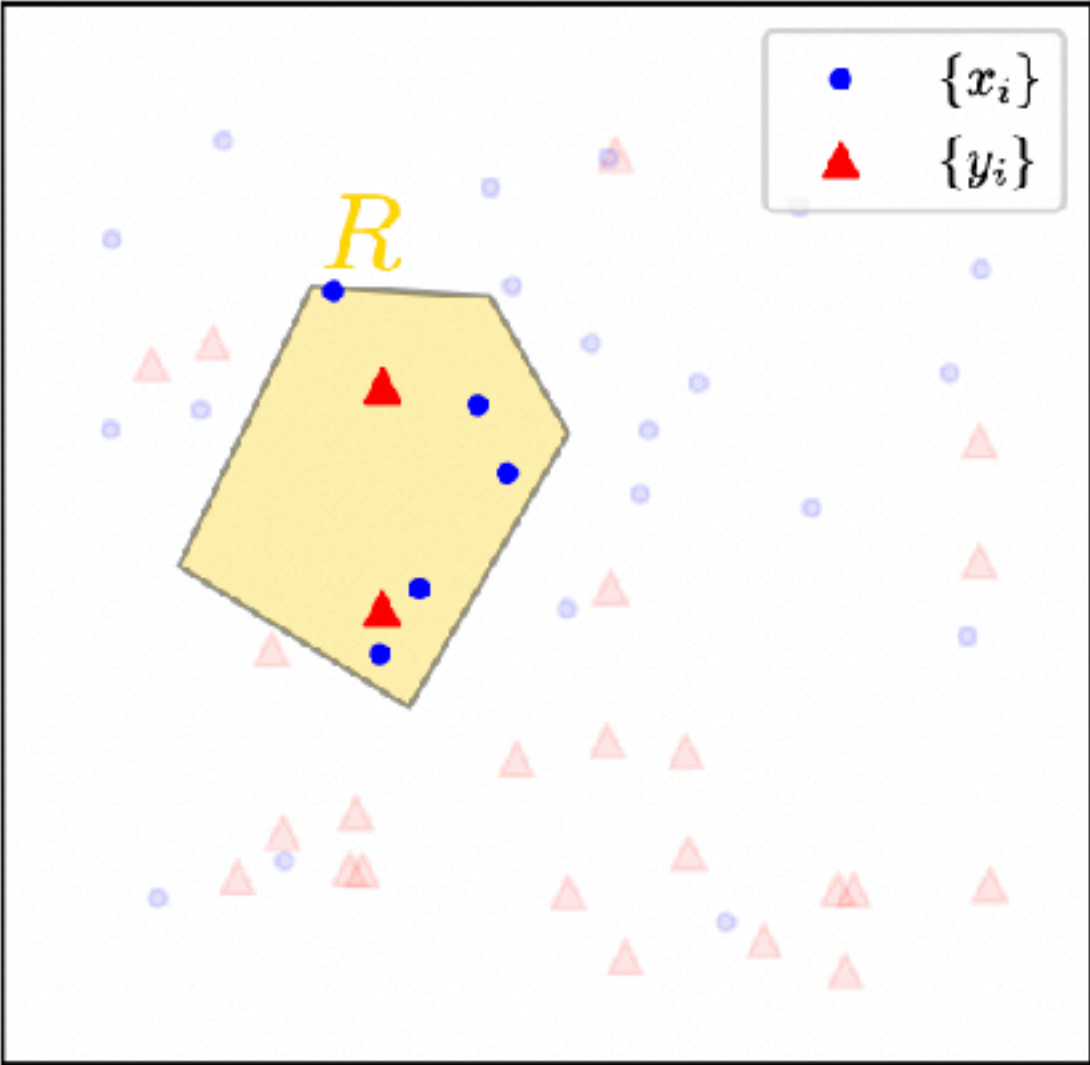
Kolya Malkin



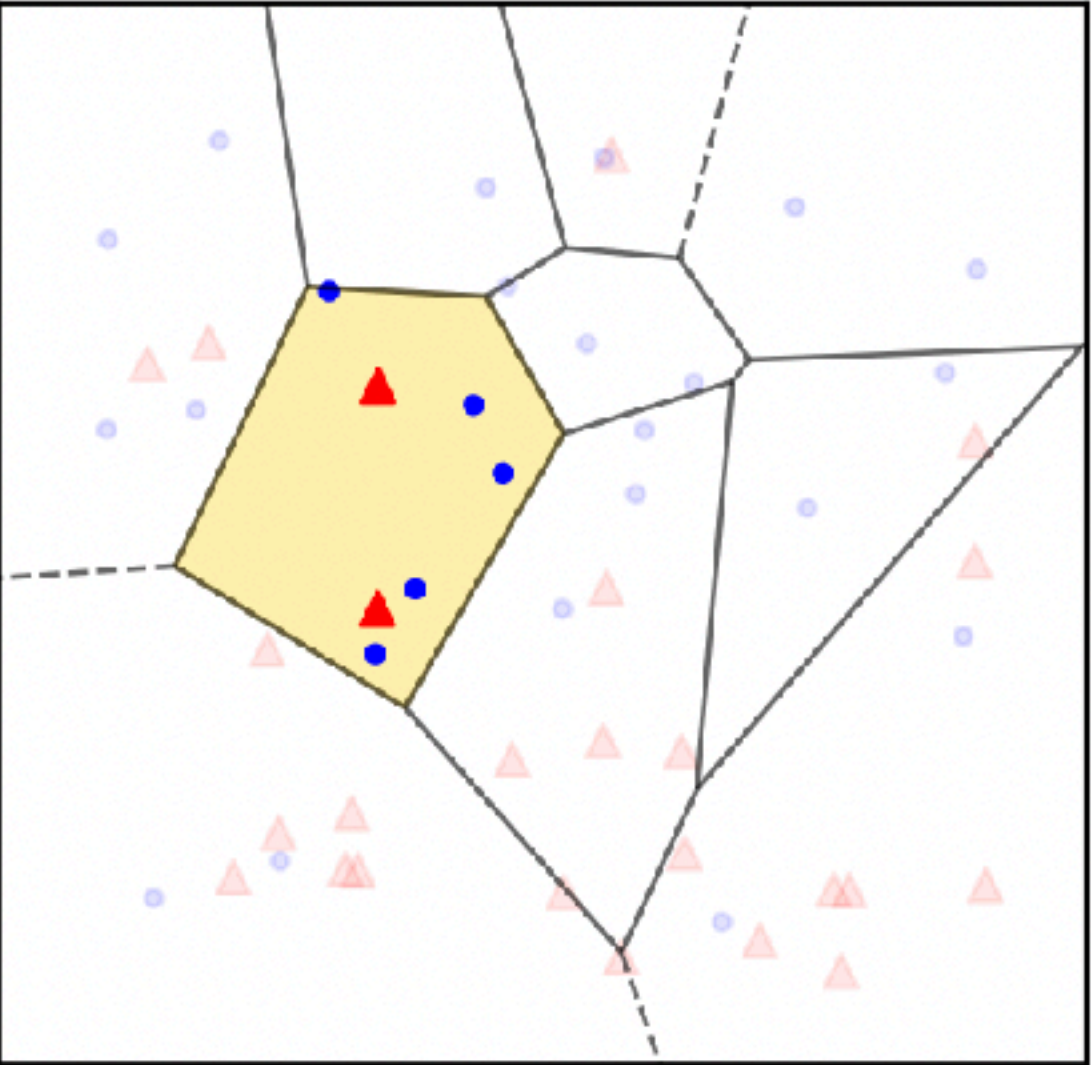
Connor Stone



Salma Salhi



$$k(\mathbf{x}, R) \sim \mathcal{B}(n, \mathbb{P}_p(R))$$



$$\{k(\mathbf{x}, R_i)\}_{i=1 \dots n_R} \sim \mathcal{M}\left(n, \{\mathbb{P}_p(R_i)\}_{i=1 \dots n_R}\right)$$

**pip install pqm**

# PQMASS: PROBABILISTIC ASSESSMENT OF GENERATIVE MODELS USING PROBABILITY MASS ESTIMATION



Sammy Sharief



Pablo Lemos



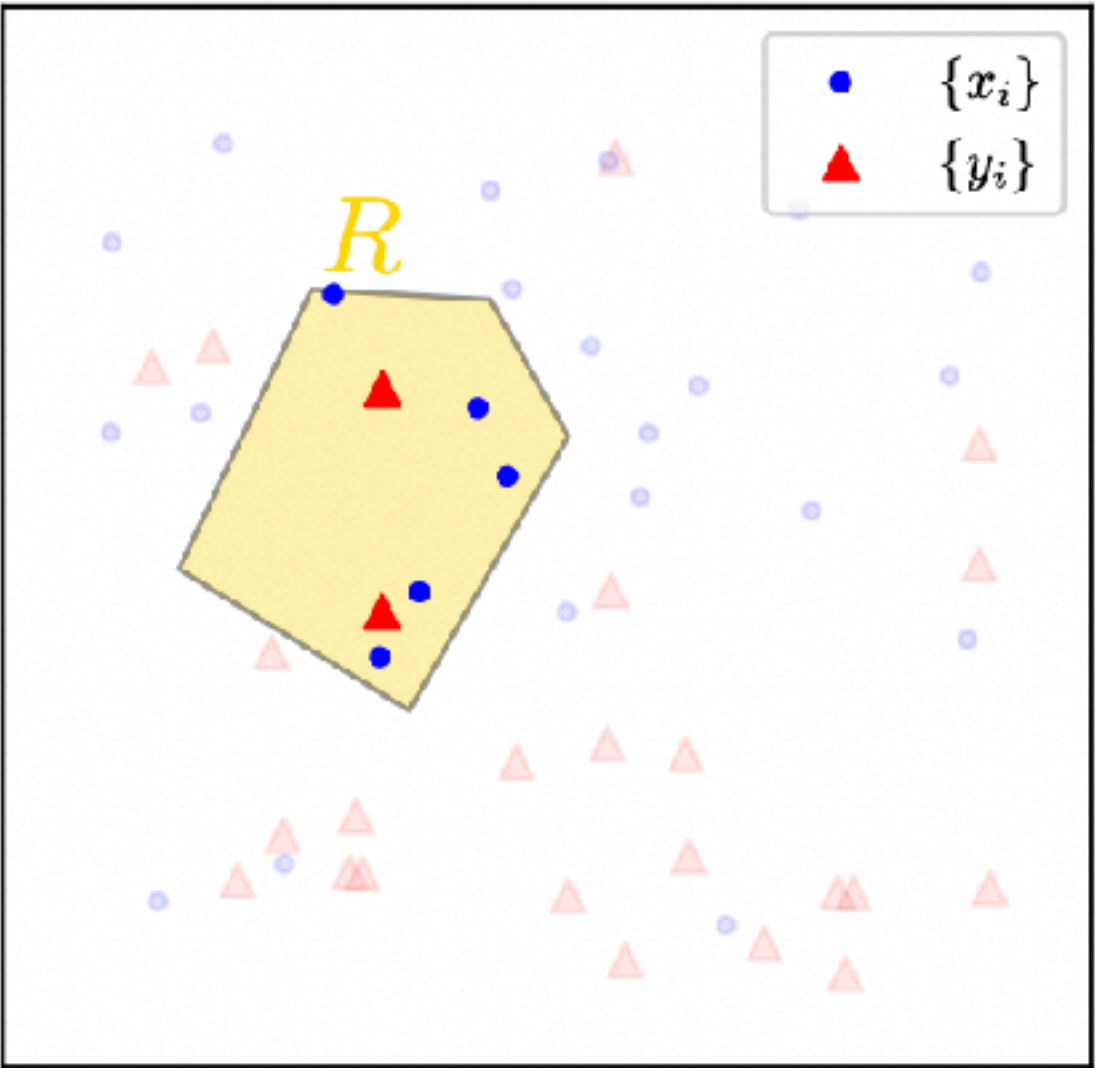
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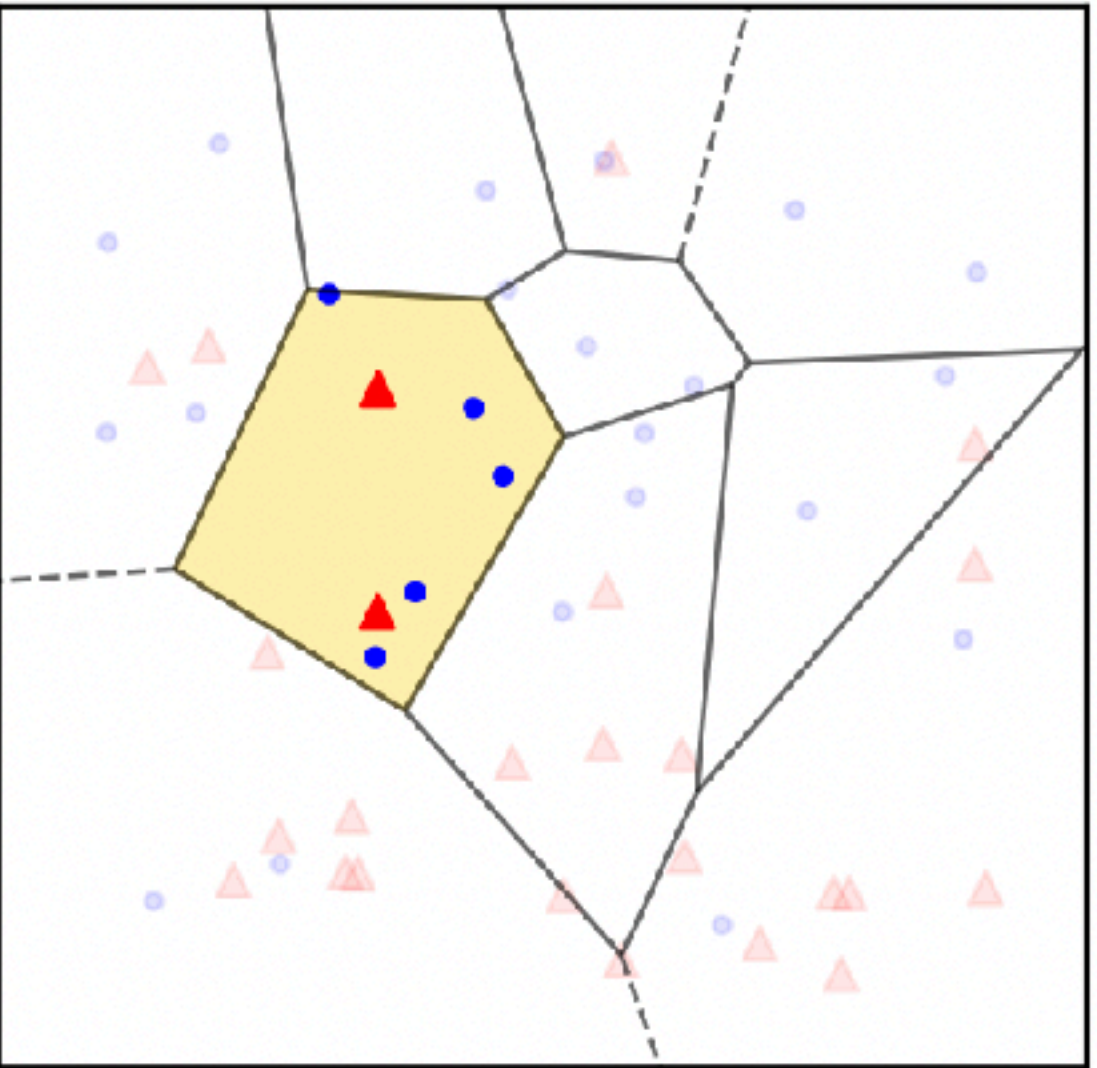
Connor Stone



Salma Salhi



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$$\{k(\mathbf{x}, R_i)\}_{i=1 \dots n_R} \sim \mathcal{M}(n, \{\mathbb{P}_p(R_i)\}_{i=1 \dots n_R})$$

$$\chi_{\text{PQM}}^2 \equiv \sum_{i=1}^{n_R} \left[ \frac{(k(\mathbf{x}, R_i) - \hat{N}_{x,i})^2}{\hat{N}_{x,i}} + \frac{(k(\mathbf{y}, R_i) - \hat{N}_{y,i})^2}{\hat{N}_{y,i}} \right]$$

**pip install pqm**

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Sammy Sharief

Pablo Lemos

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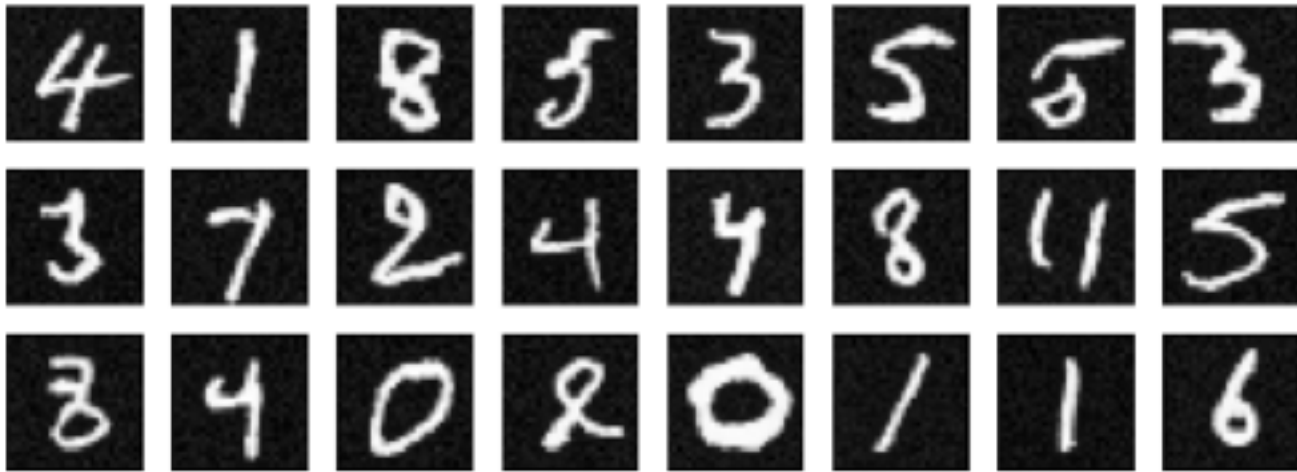
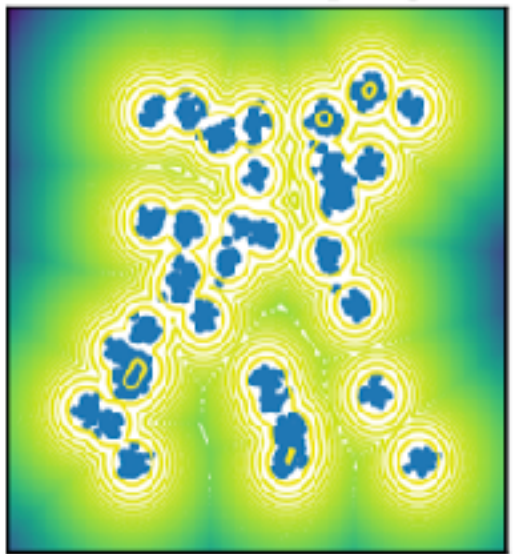
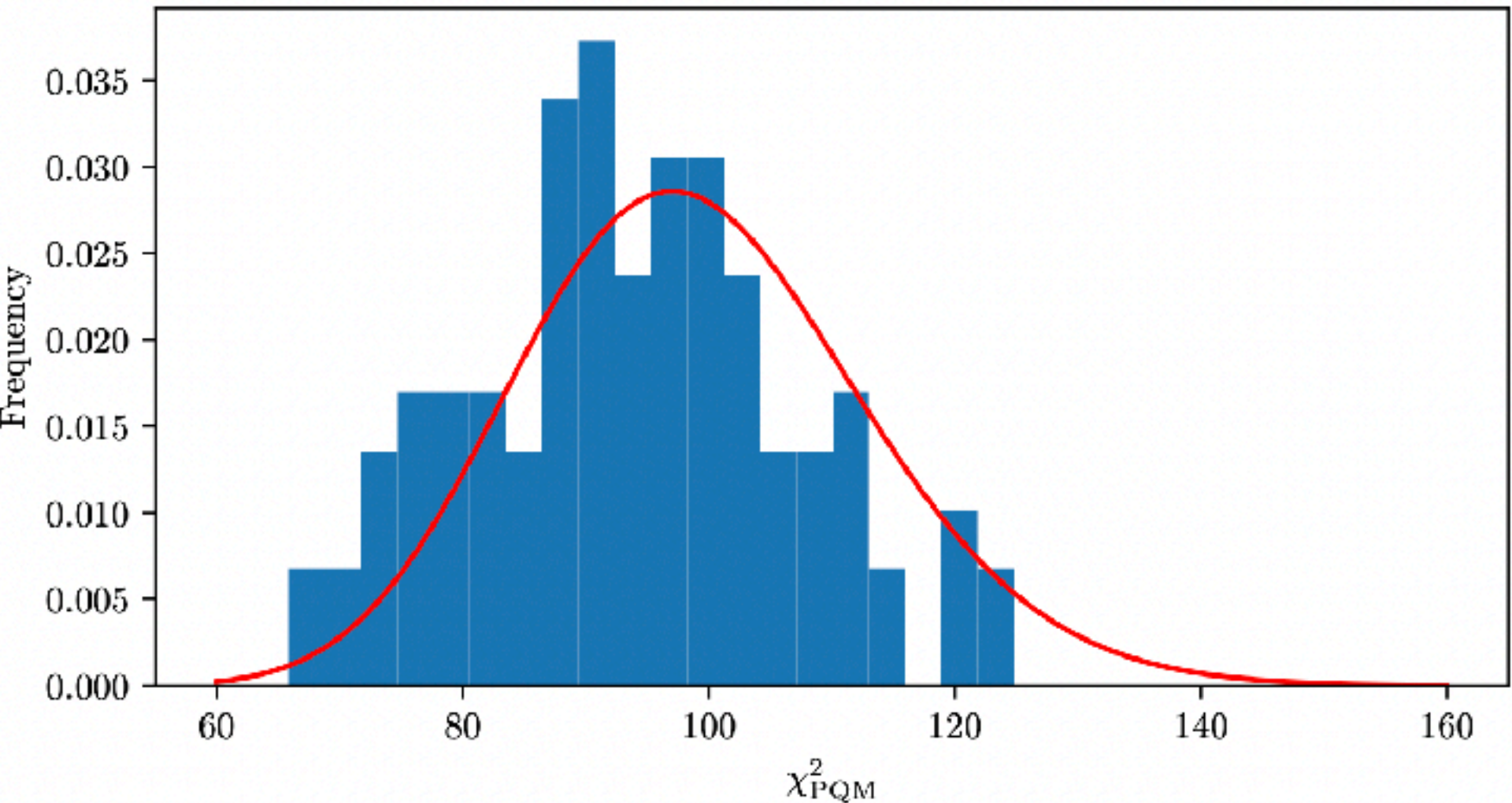
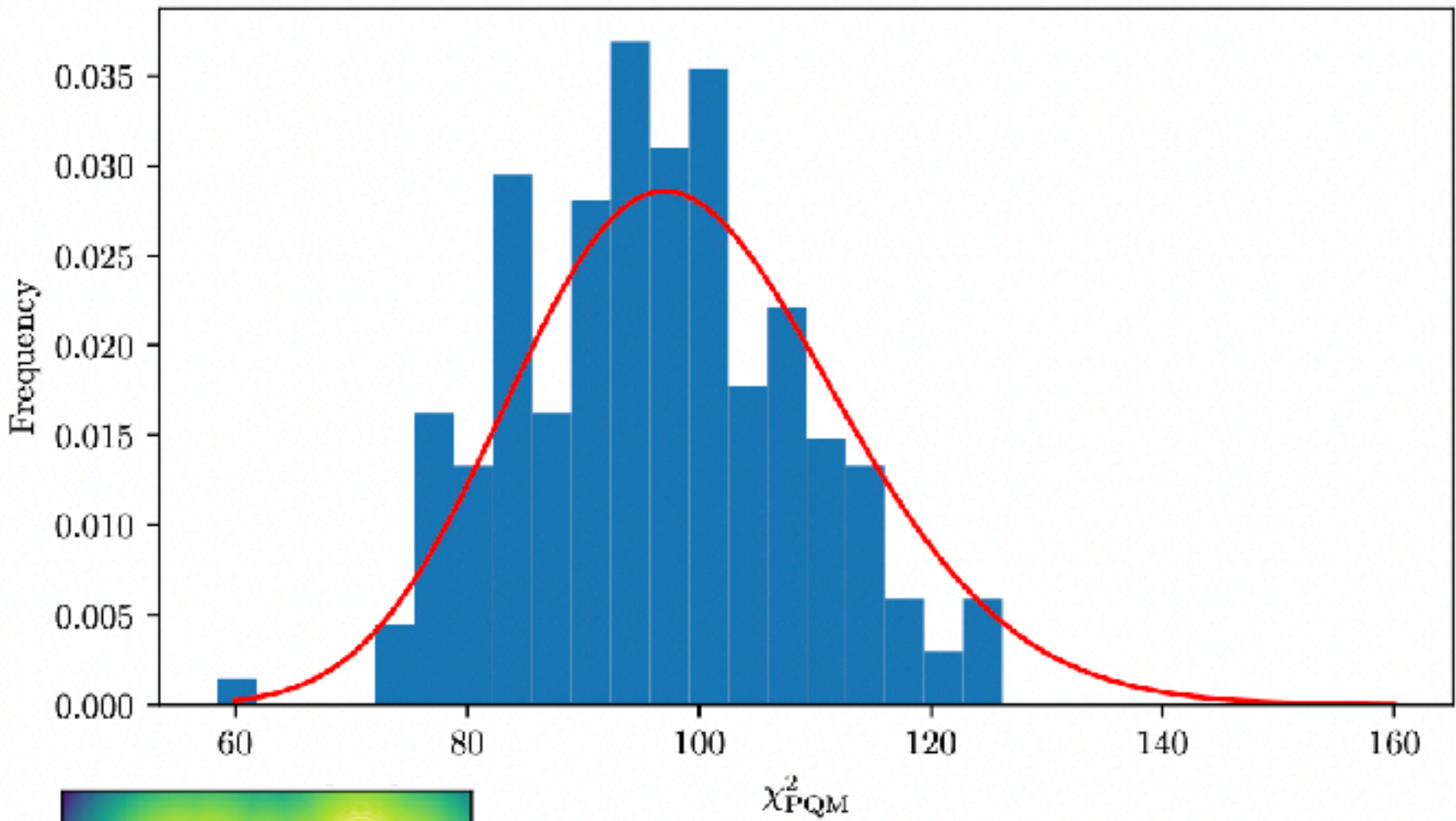


Connor Stone



Salma Salhi

*Given any sampling distribution, or generative model, if two sets of samples are generated from the same distribution, then the statistic  $\chi^2_{PQM}$  follows a chi-square distribution with  $n_R - 1$  degrees of freedom.*



# PQMass: Probabilistic Assessment of Generative Models Using Probability Mass Estimation



Sammy Sharief



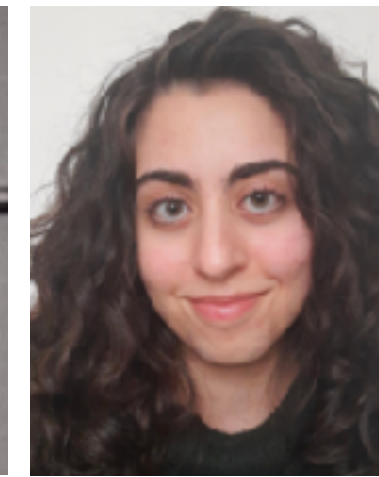
Pablo Lemos



Kolya Malkin



Connor Stone



Salma Salhi

## Power of PQMass:

- Works directly in pixel space (or other data space) without relying on dimensionality reduction
- Only relies on defining a distance between points, so scales extremely well to high-dimensional spaces (>500,000 dimensions)

# PQMASS: PROBABILISTIC ASSESSMENT OF GENERATIVE MODELS USING PROBABILITY MASS ESTIMATION



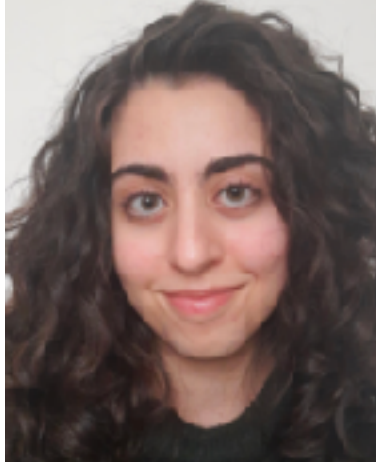
Sammy Sharief

Pablo Lemos

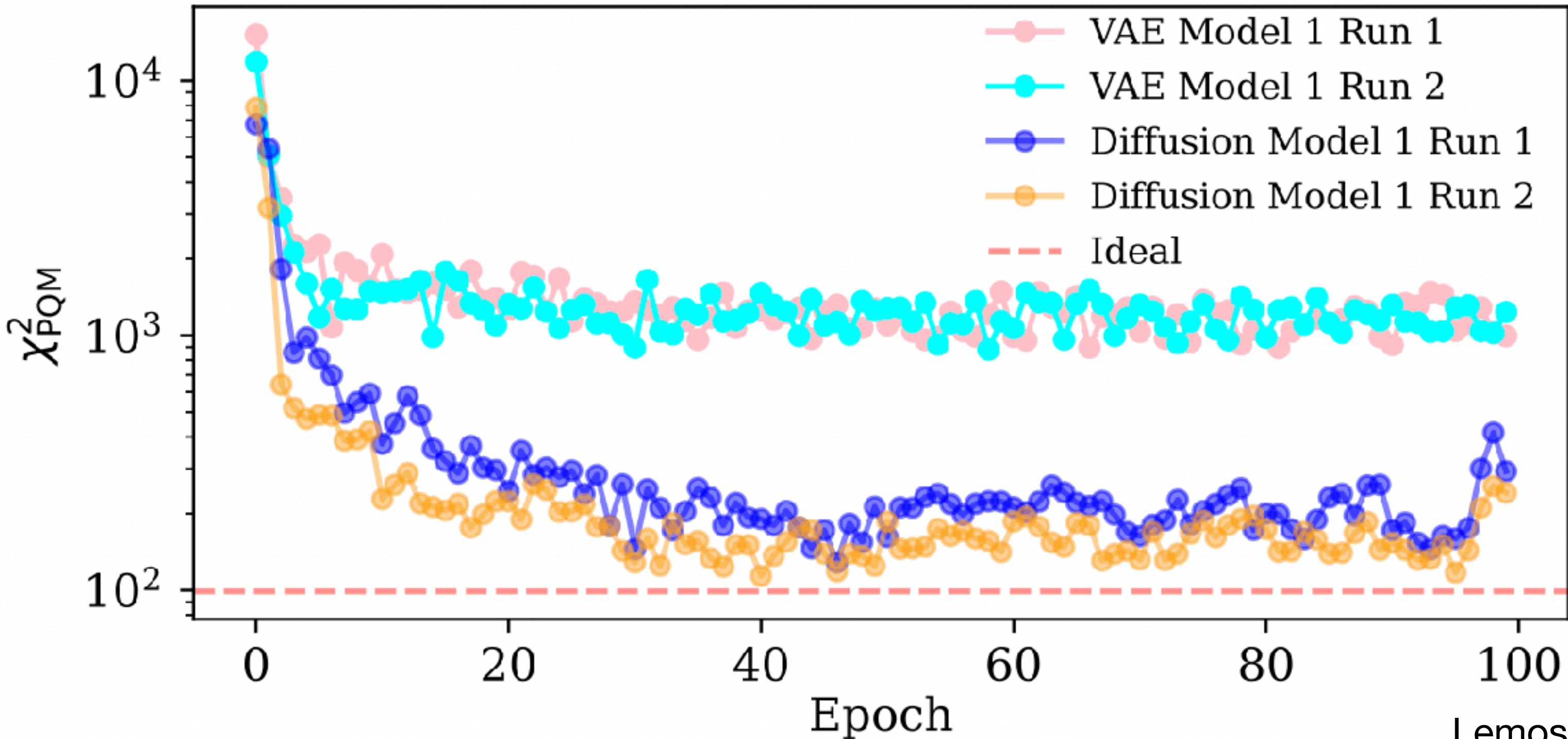
Kolya Malkin



Connor Stone



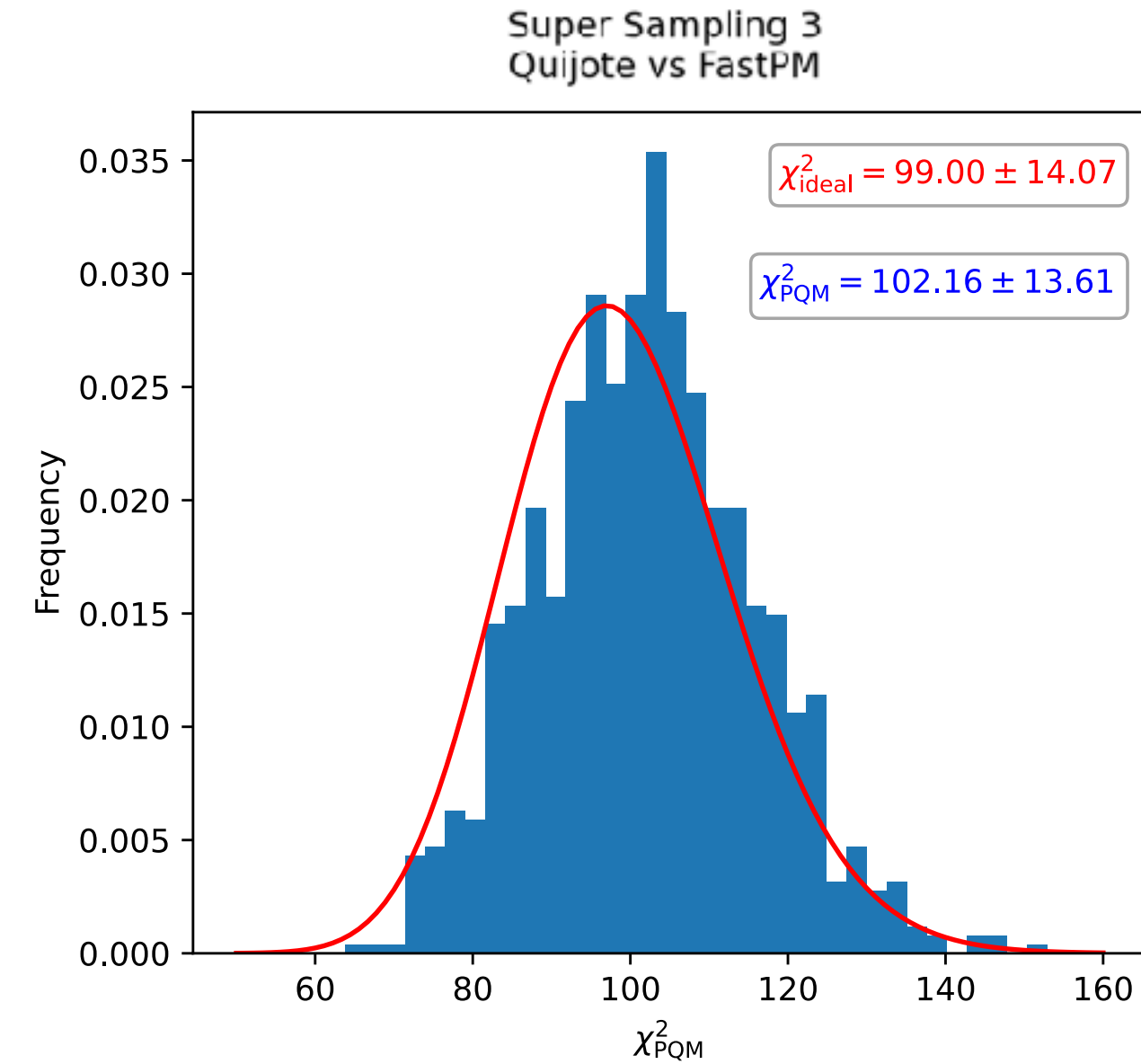
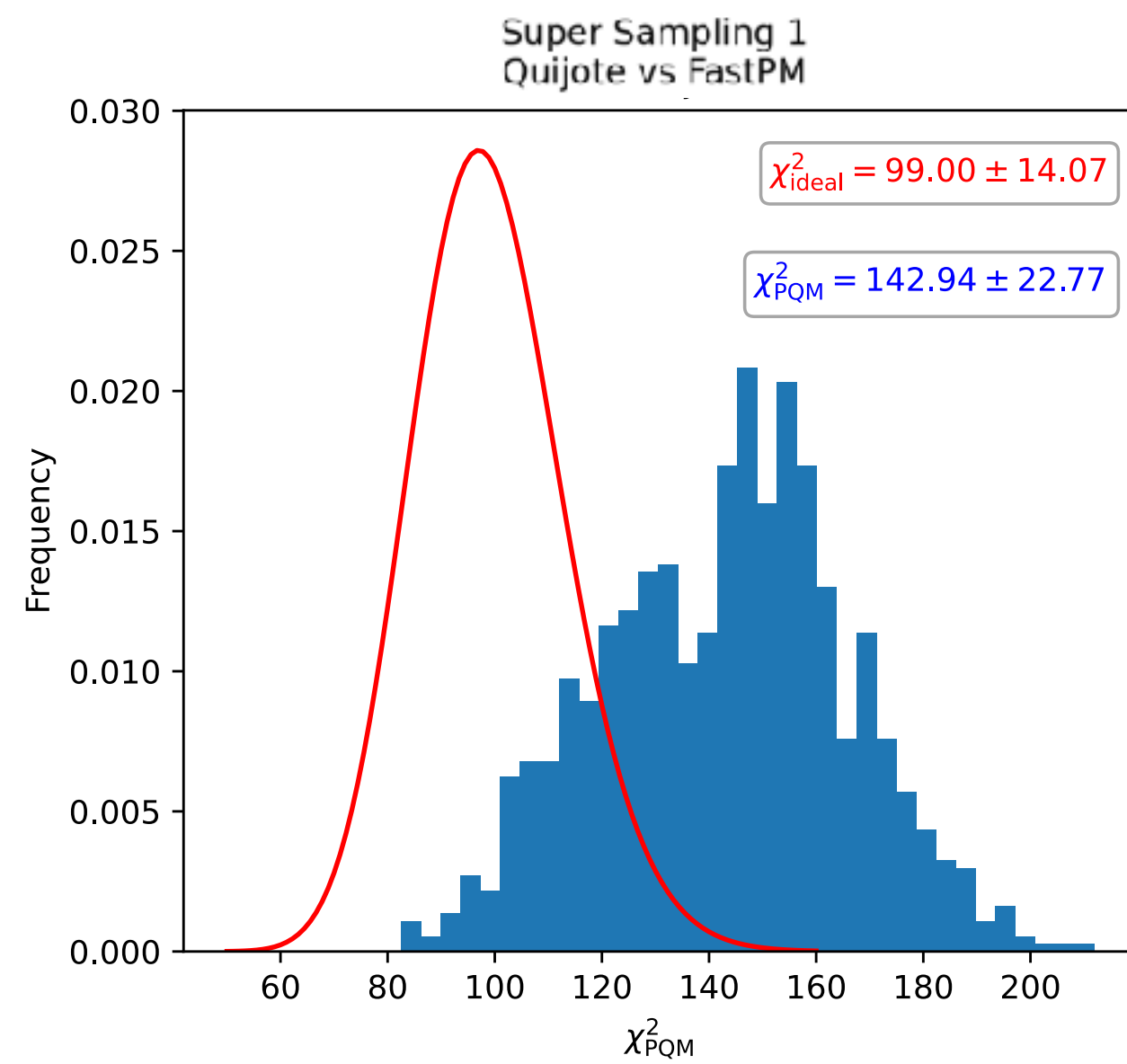
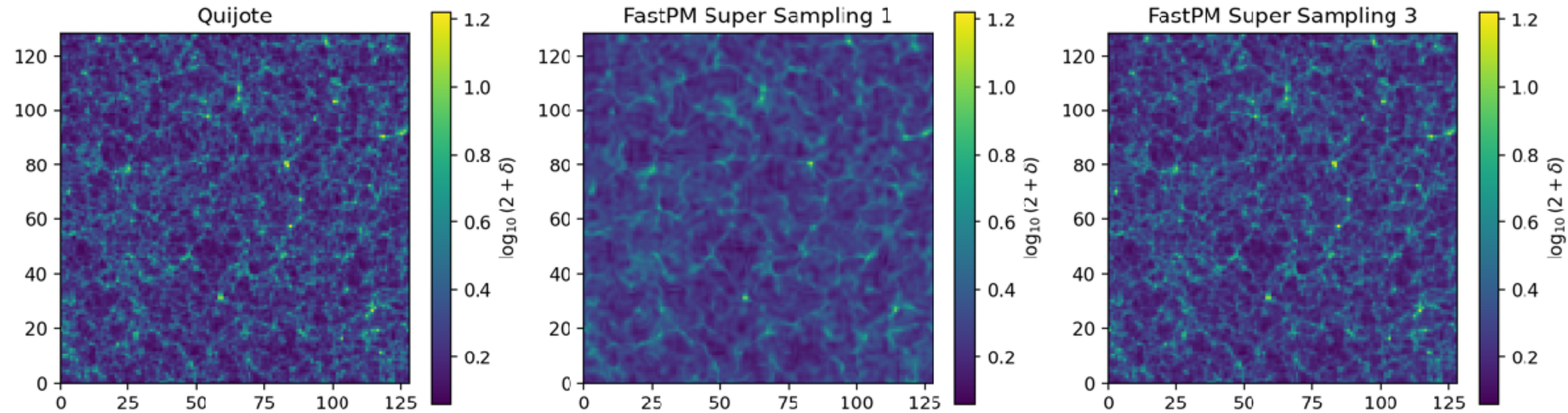
Salma Salhi



# PQMASS: EXAMPLE OF APPLICATION TO TEST ACCURACY OF SIMULATORS AND EMULATORS AT THE FIELD LEVEL



Sammy Sharief



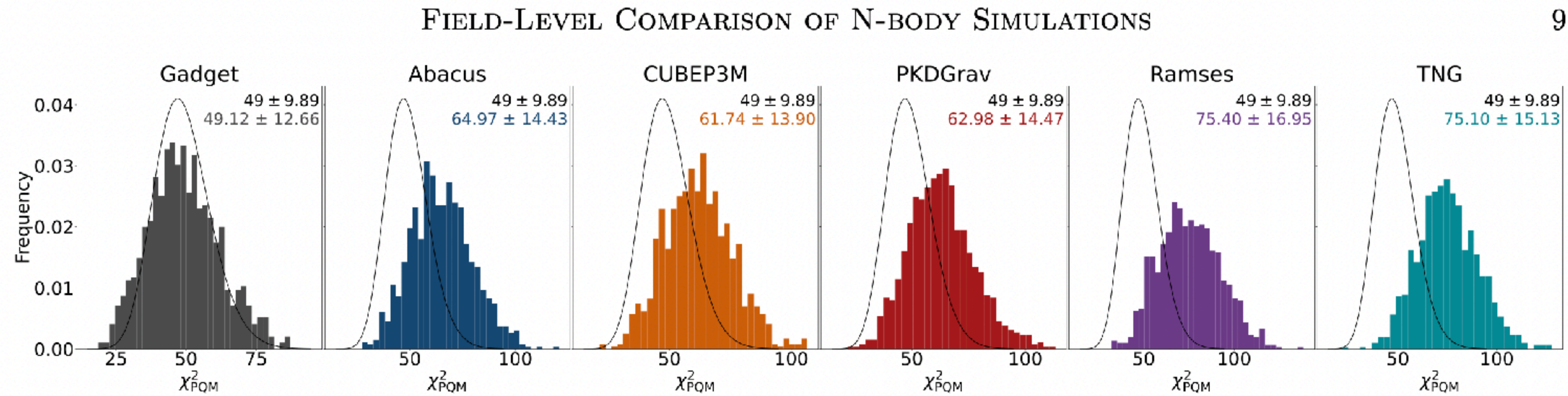
# PQMASS: EXAMPLE OF APPLICATION TO TEST ACCURACY OF SIMULATORS AND EMULATORS AT THE FIELD LEVEL



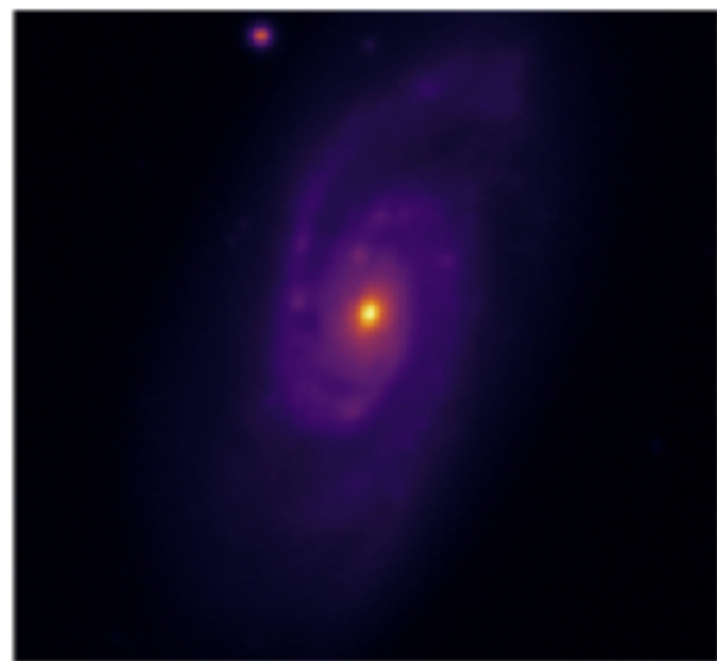
Adrian  
Bayer



Sammy  
Sharief



# PQMASS: CAN BE APPLIED TO DATA OF VARIED MODALITY

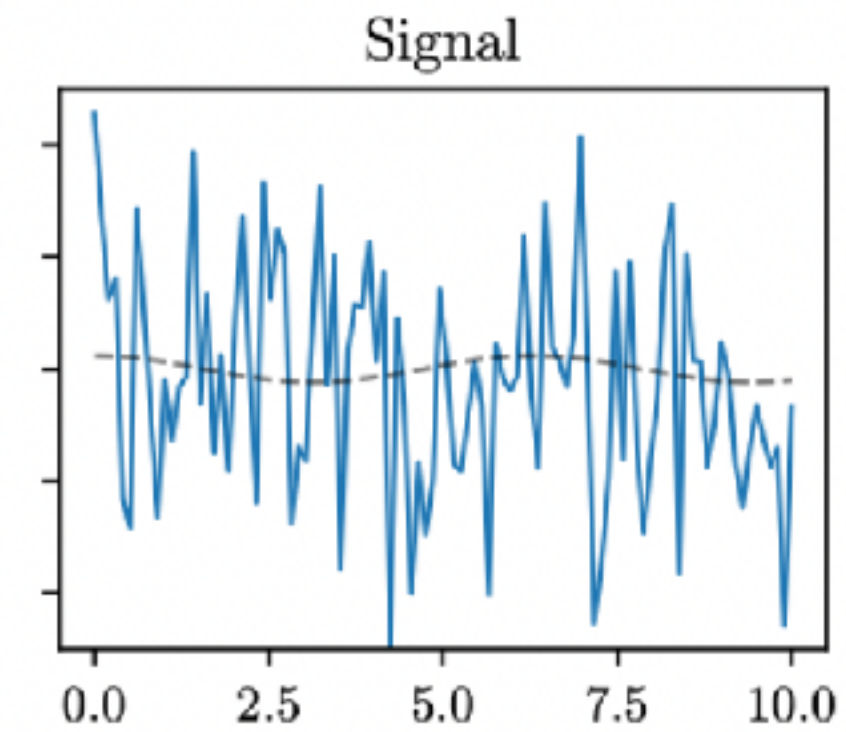


**Images**

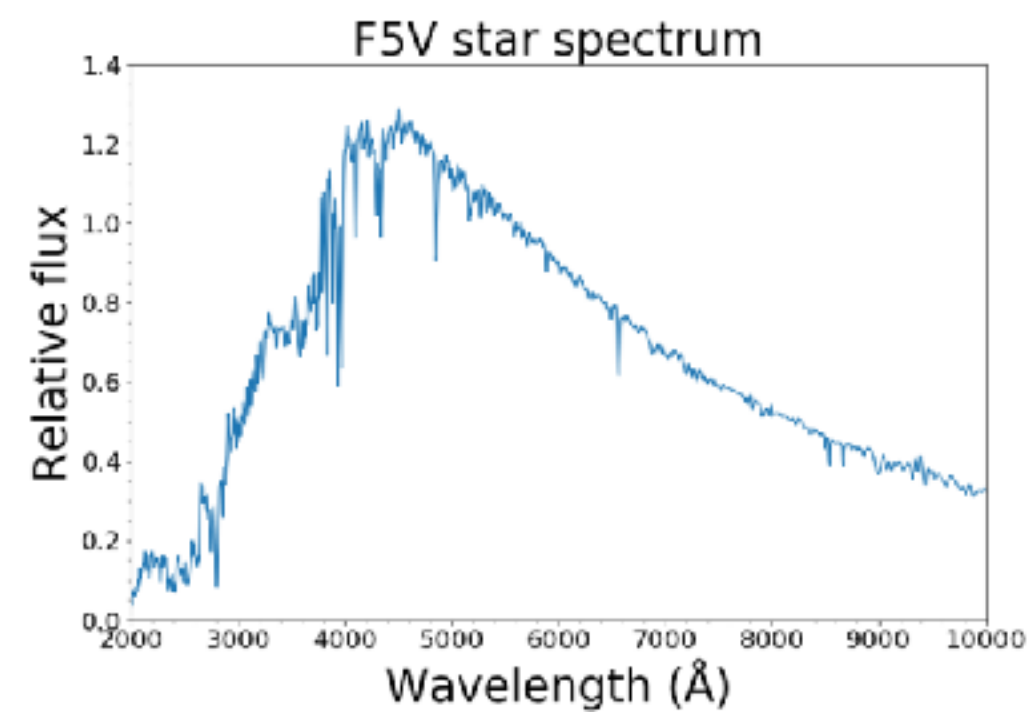
Variables Table

Variable Name	Role	Type	Demographic	Description
age	Feature	Integer	Age	N/A
workclass	Feature	Categorical	Income	Private, Self-emp-not-inc, Self-emp-inc, Federal-gov, Local-gov, State-gov, Without-pay, Never-worked.
fnlwgt	Feature	Integer		
education	Feature	Categorical	Education Level	Bachelors, Some-college, 11th, HS-grad, Prof-school, Assoc-acdm, Assoc-voc, 9th, 7th-8th, 12th, Masters, 1st-4th, 10th, Doctorate, 5th-6th, Preschool.
education-num	Feature	Integer	Education Level	

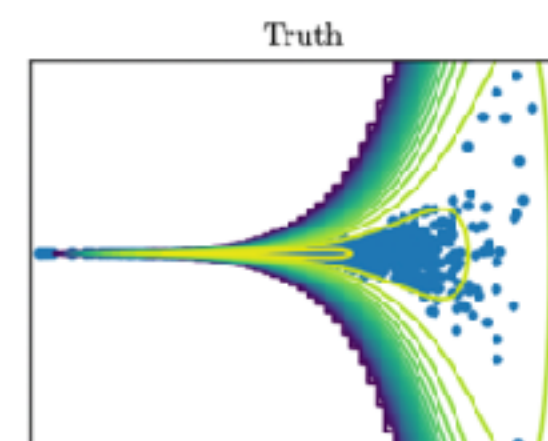
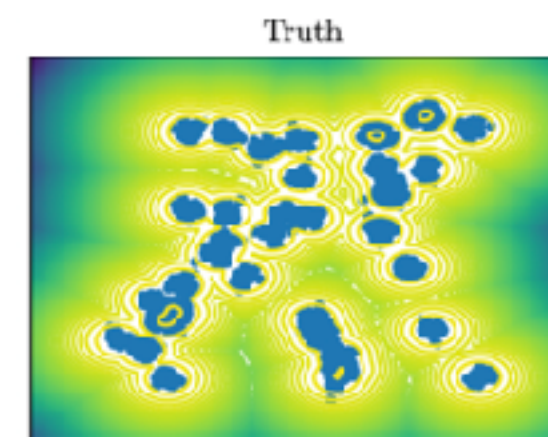
**Tabular data**



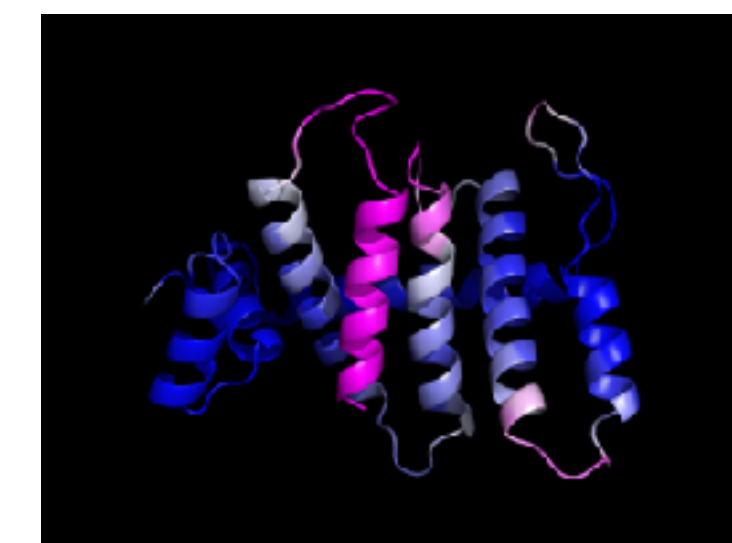
**Time series data**



**Spectra**

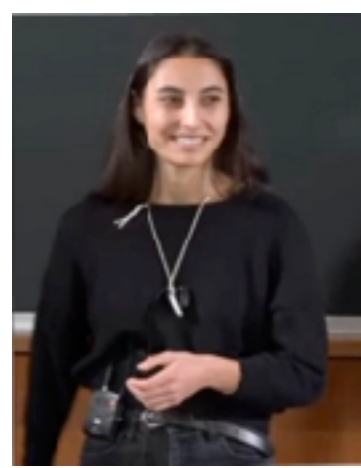


**Low-dimensional parametric distributions**



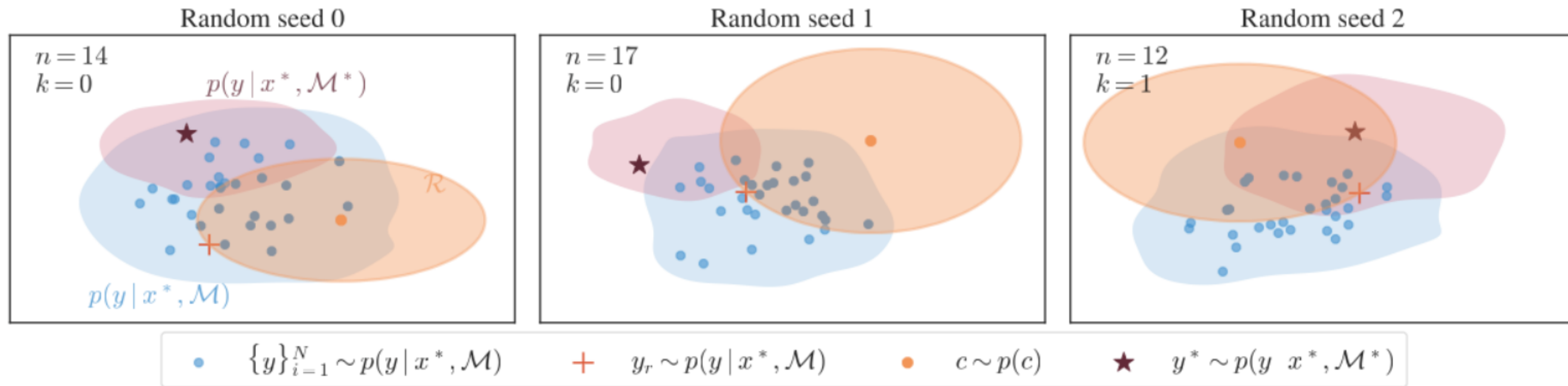
**Protein sequences**

# MIRA: PQMASS FOR POSTERIOR INFERENCE AND CONDITIONAL MODELS TRANSFORMING TARP INTO AN ACCURACY METRIC



Sammy Sharief

Justine Zeghal



$$p(k | n) = \frac{n + 1}{N + 2} \mathbb{1}(k = 1) + \frac{N - n + 1}{N + 2} \mathbb{1}(k = 0)$$

$$\mu_{\text{Mira}}(\mathcal{M}) = \mathbb{E}_{p(Z)} \left[ \mathbb{E}_{p(k,n|Z)} [p(k | n)] \right]$$

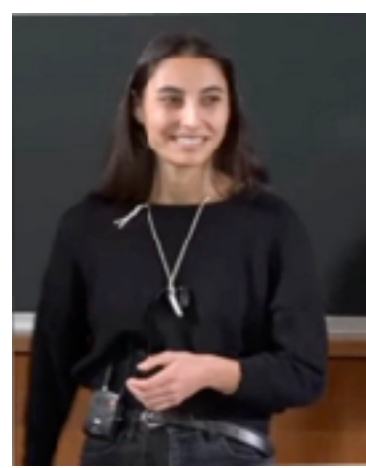
$$\mathbb{E}_{p(k,n)} [P_N] \rightarrow \frac{2}{3} \quad \text{as } N \rightarrow \infty$$

$$\text{Var}_{p(k,n)} [P_N] \rightarrow \frac{1}{18} \quad \text{as } N \rightarrow \infty$$

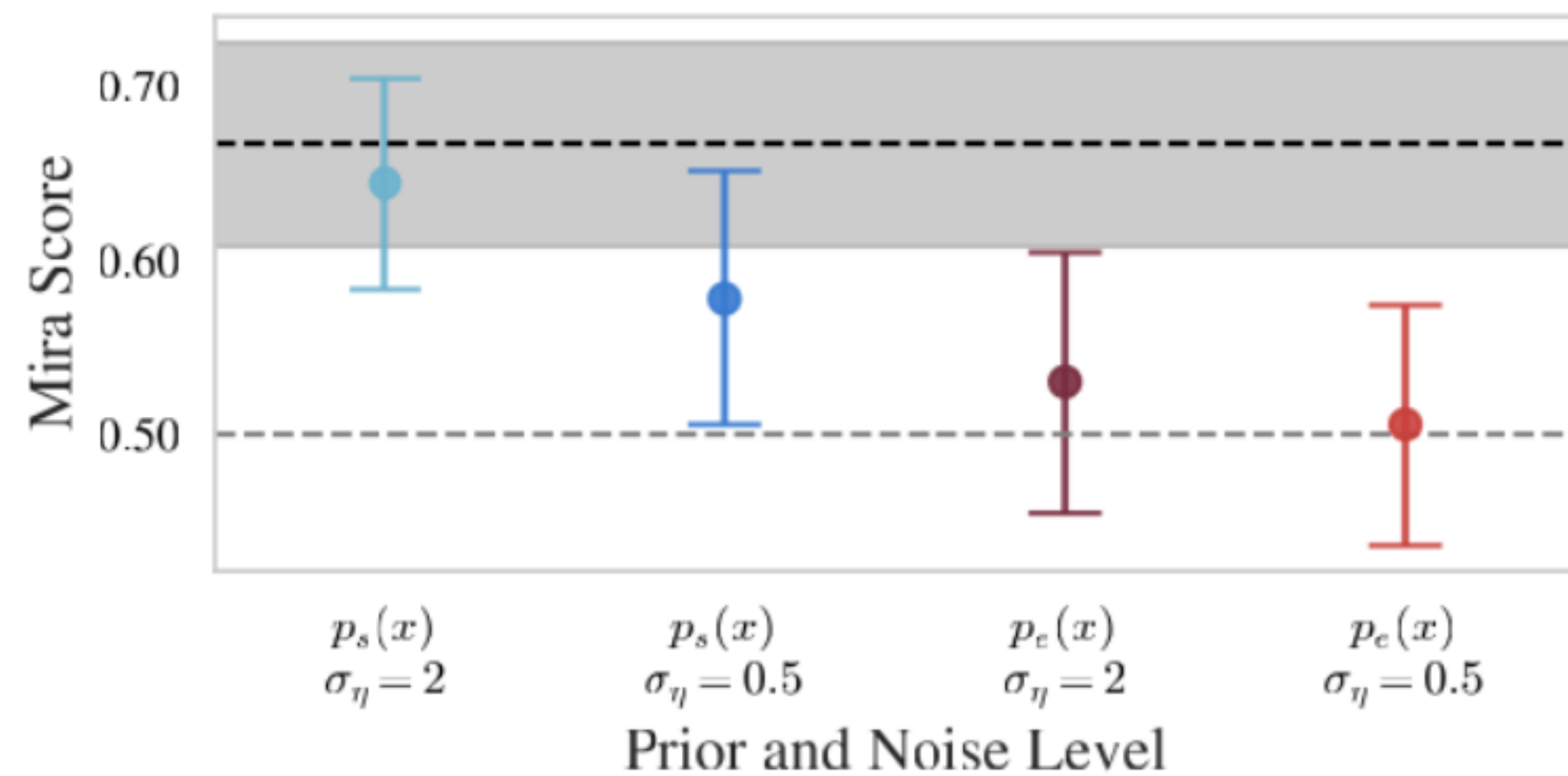
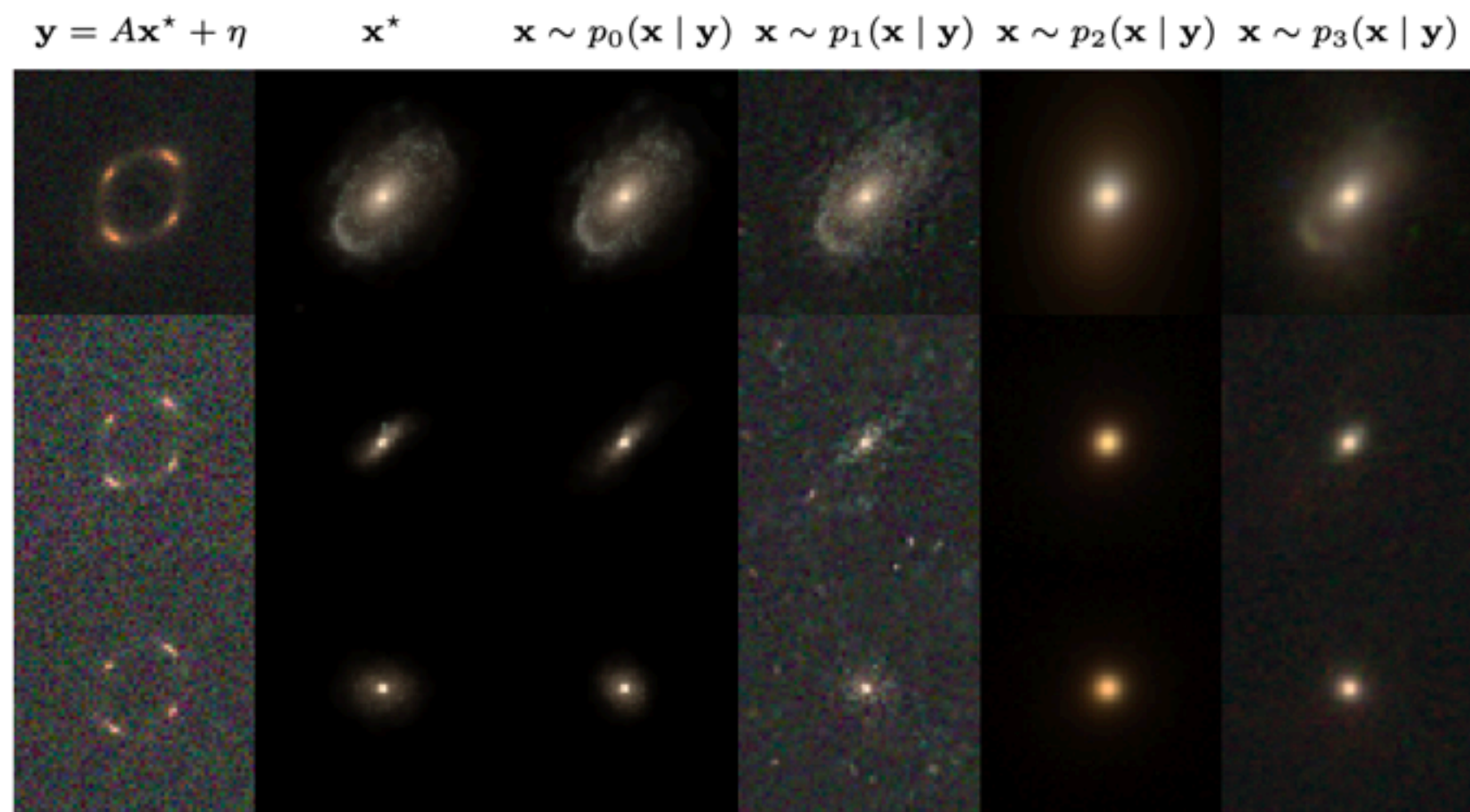
# MIRA (MASS IN RANDOM AREAS): A SCORE FOR CONDITIONAL DISTRIBUTION ACCURACY AND MODEL COMPARISON



Sammy Sharief



Justine Zeghal

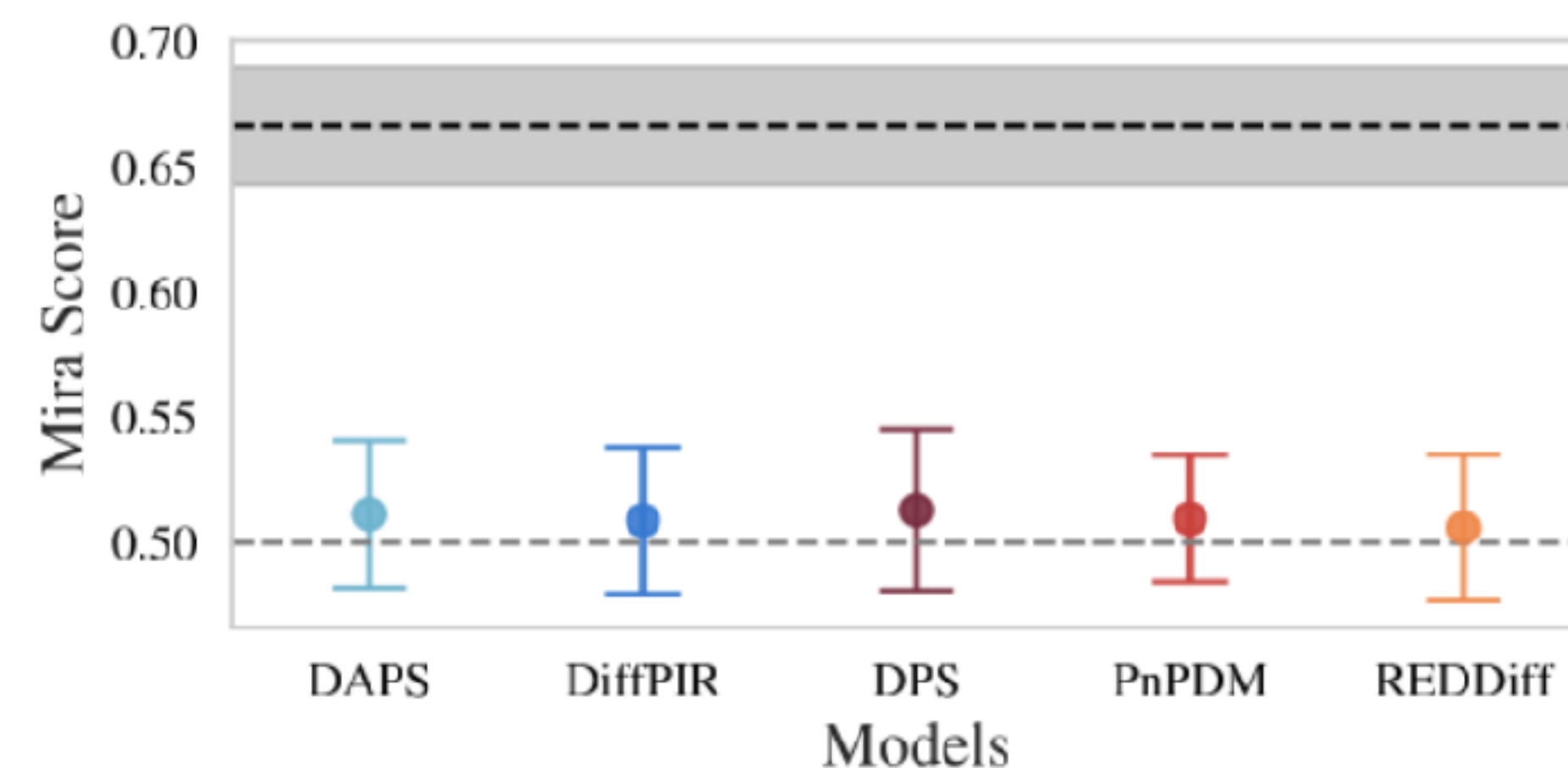
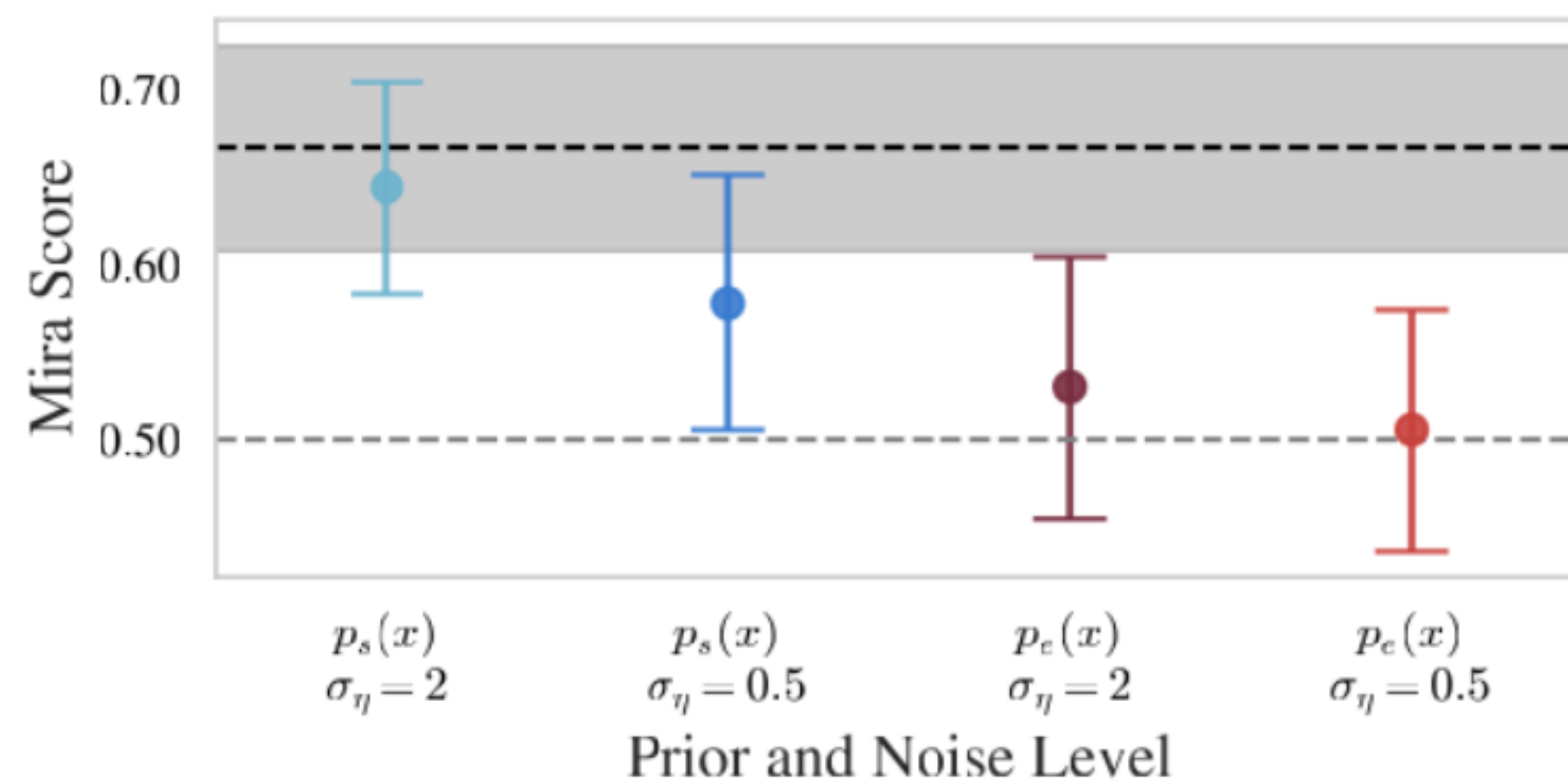
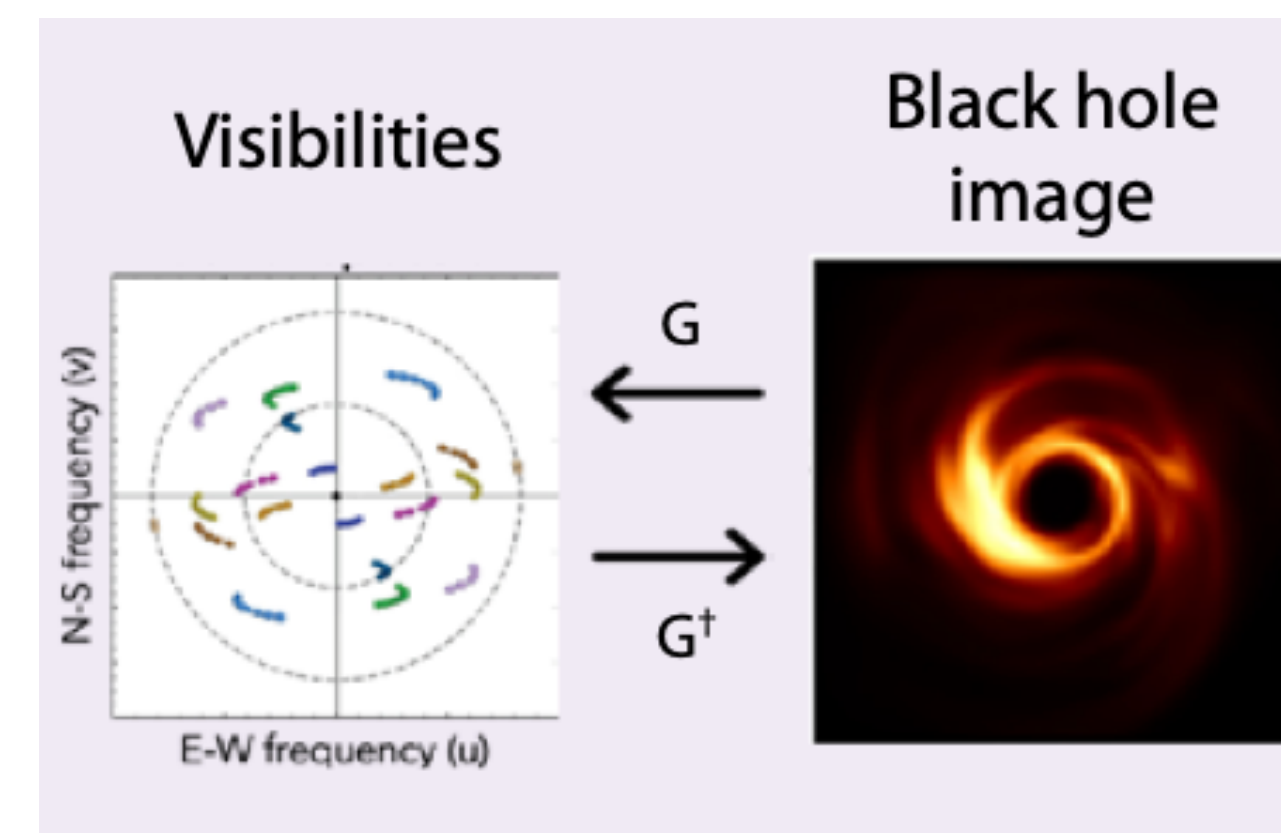
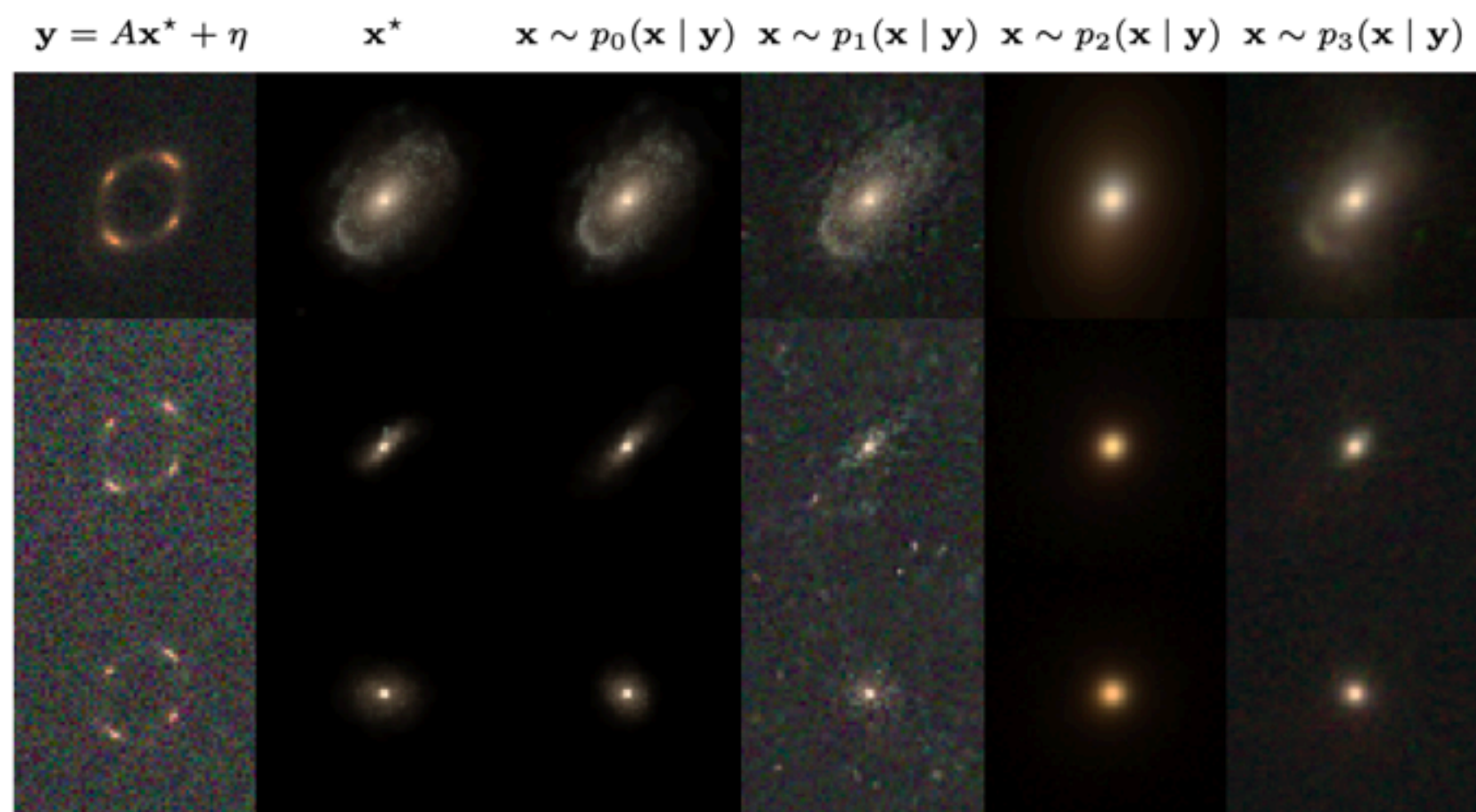


# MIRA (MASS IN RANDOM AREAS): A SCORE FOR CONDITIONAL DISTRIBUTION ACCURACY AND MODEL COMPARISON



Sammy Sharief

Justine Zeghal



# GAME PLAN

1. Data-driven priors
2. Data-driven likelihoods
3. Accuracy metrics
4. Out-of-Distribution accuracy

# ADDRESSING DISTRIBUTIONAL SHIFTS: PRIOR MISSPECIFICATION

So far, we've used **data** (corrupted by noise, psf, etc. ) or **simulations** (potentially out of distribution) to learn our prior distributions.

**Should we worry?**

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## **Should we worry?**

We will soon be in the regime where we have  $\mathcal{O}(10^5)$  strong lenses.

PQMass can allow us to detect if our posterior reconstructions are significantly shifted with respect to our prior  
=> we can detect if the prior is out of distribution even in very high dimensional spaces.

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Can we use some of our observations to learn the prior hierarchically?

# ADDRESSING DISTRIBUTIONAL SHIFTS: PRIOR MISSPECIFICATION



Missa  
Barco

What we would like to do:

$$P(\theta | d_{1,\dots,i}) \propto \prod_i \int P(d_i | x_i) p(x_i | \theta) p(\theta) dx$$

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However, in our case, evaluating  $p(x_i | \theta)$  is not straightforward.

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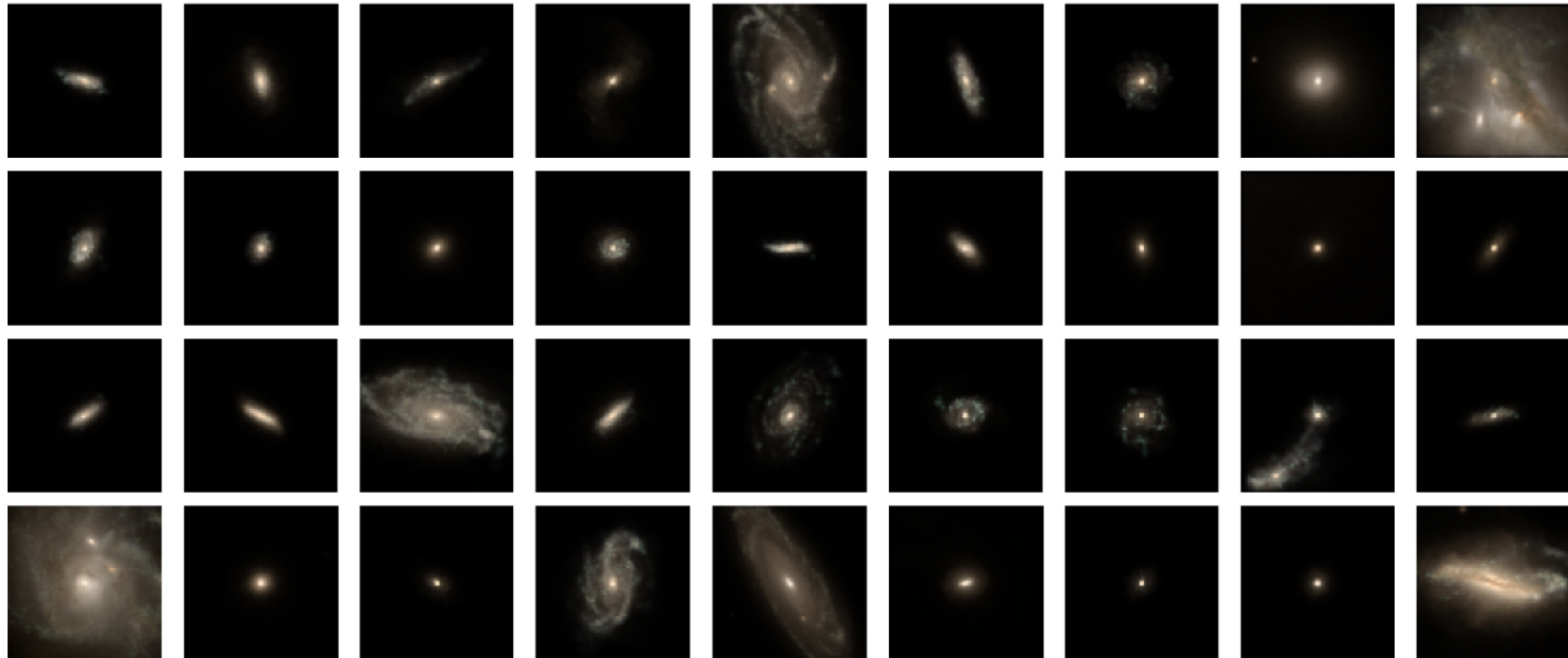
And we do this iteratively using posterior samples to retrain a new prior.

# ADDRESSING DISTRIBUTIONAL SHIFTS: PRIOR MISSPECIFICATION



Missa  
Barco

Applications to galaxies: can we learn that spiral galaxies exist starting from a prior with just (real, noisy) ellipticals?

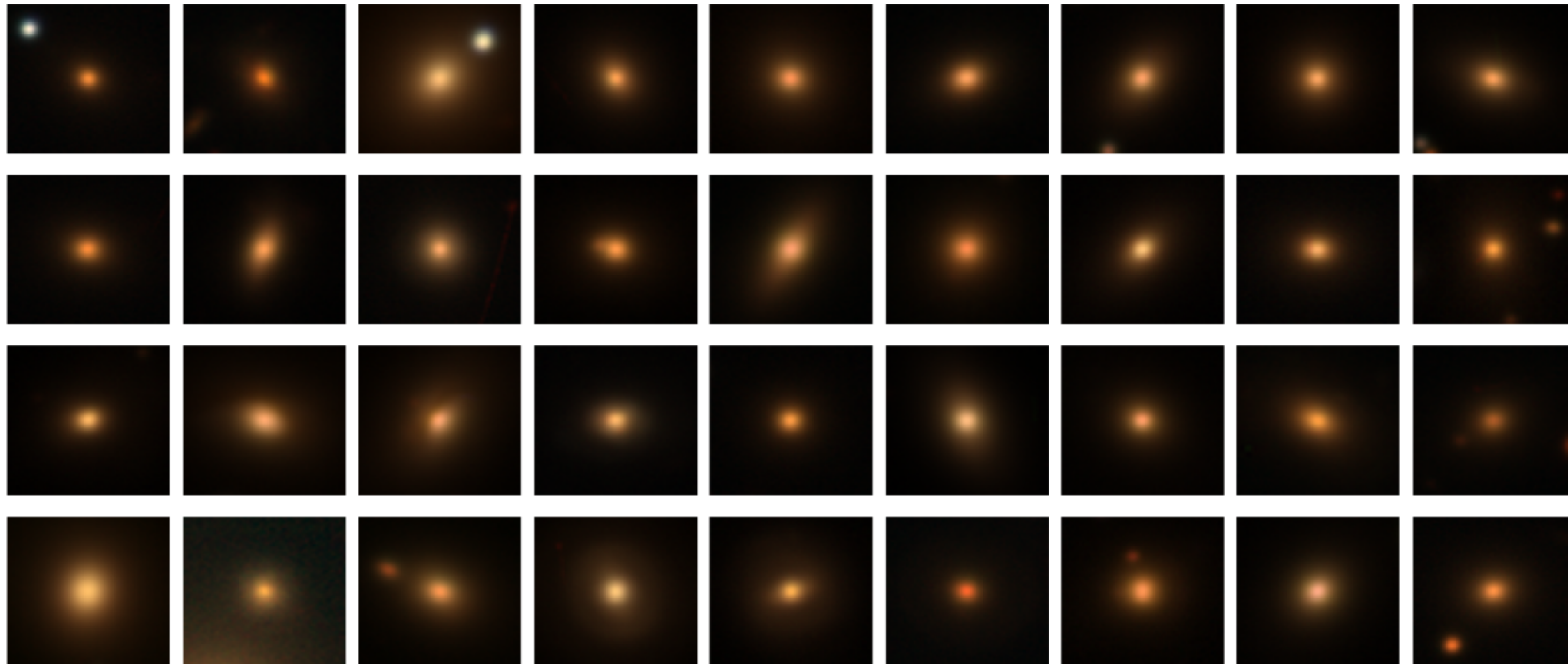


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Barco

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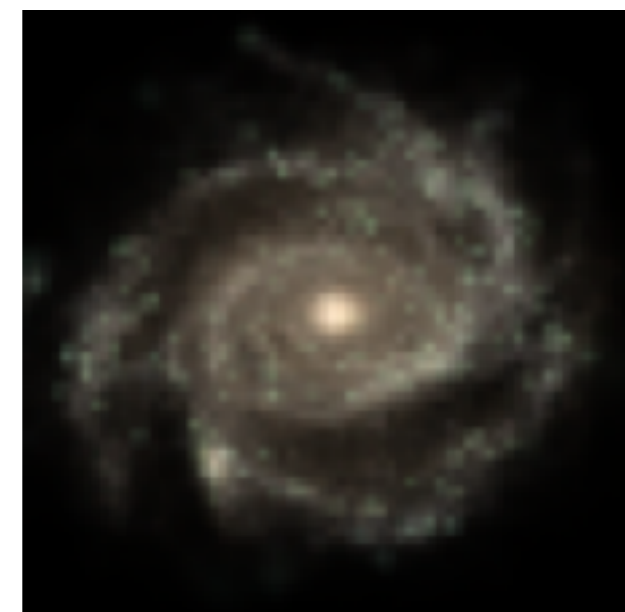
## Observation



Observation



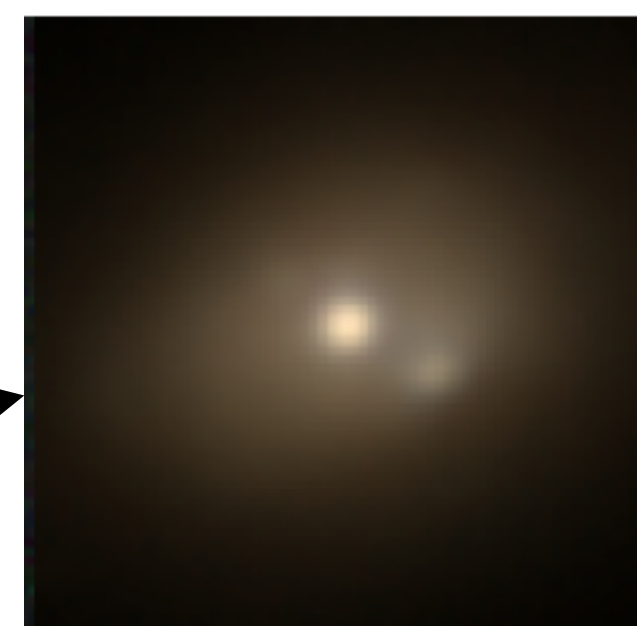
True  
Galaxy



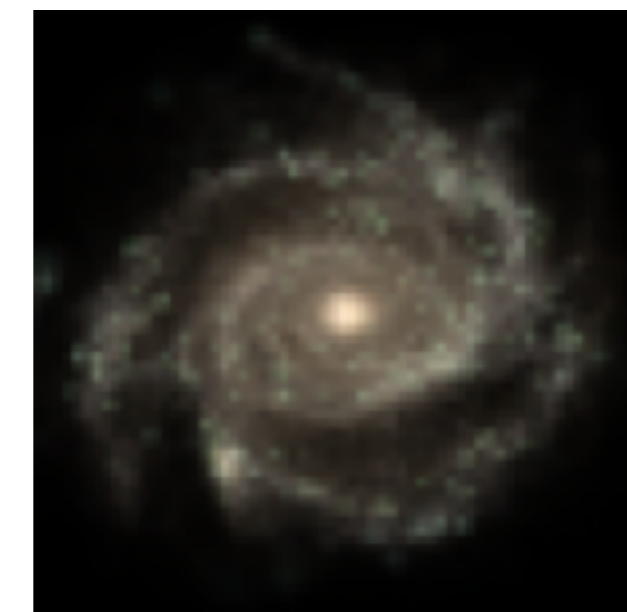
Observation

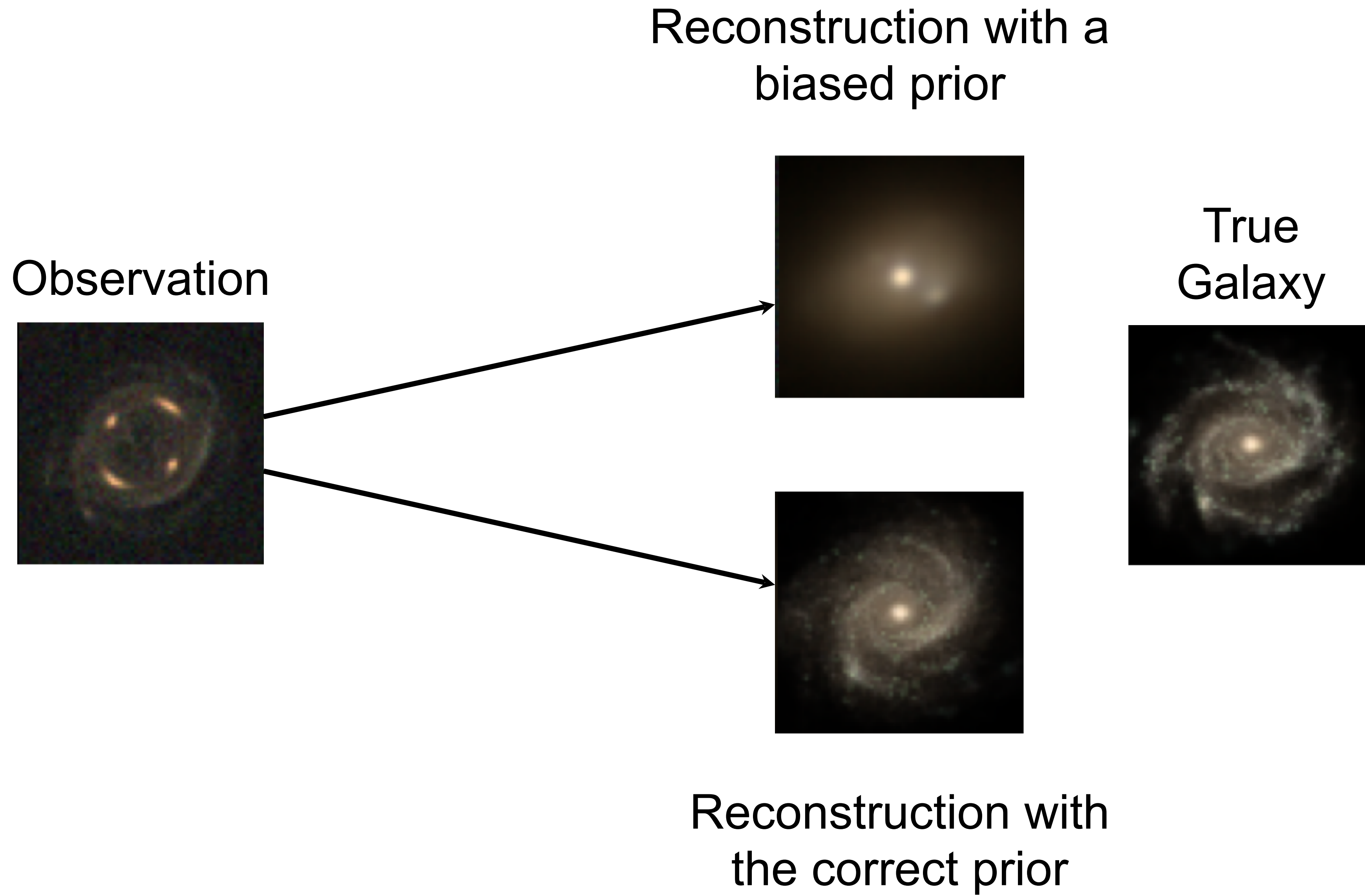


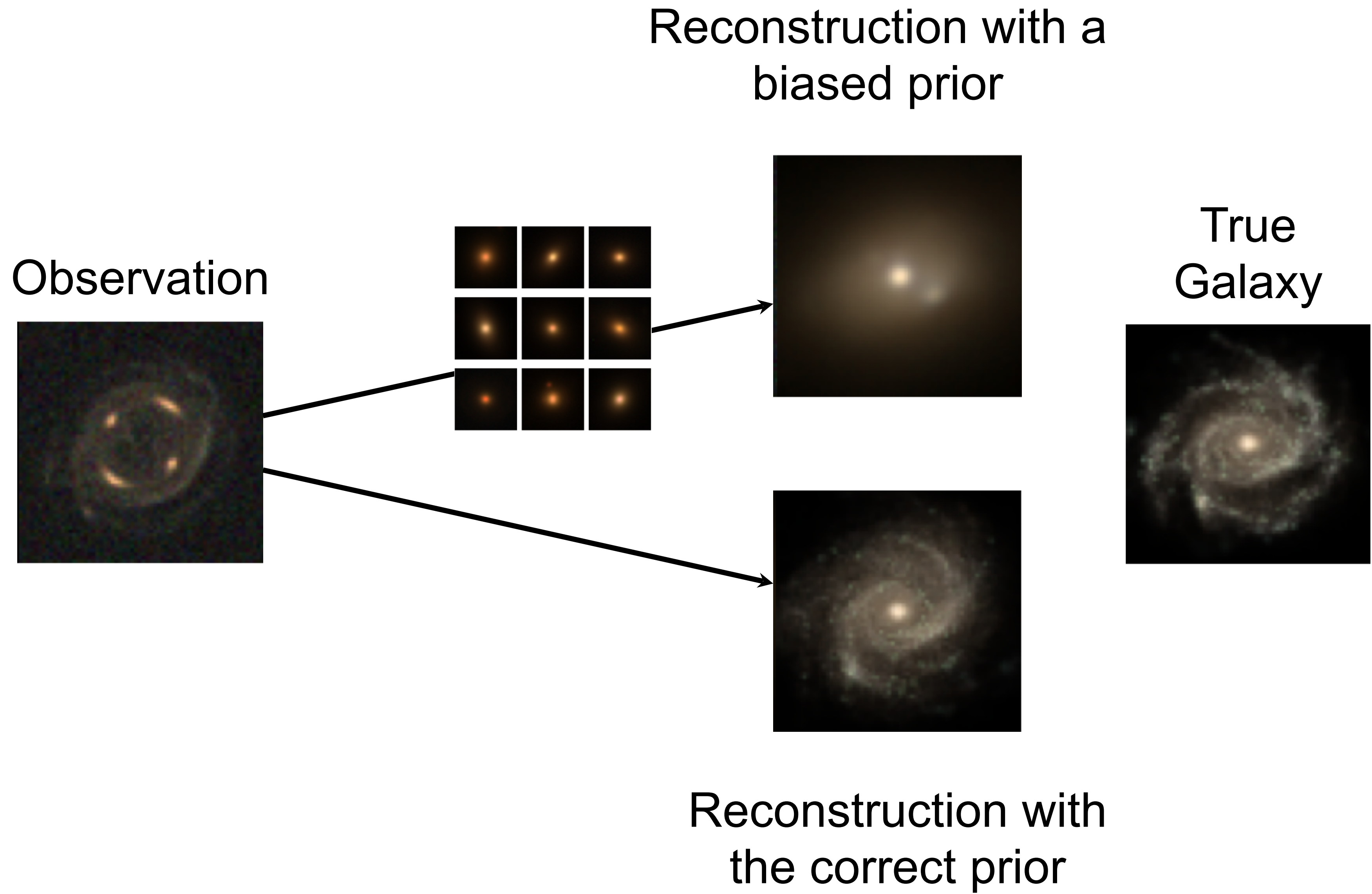
Reconstruction with a  
biased prior

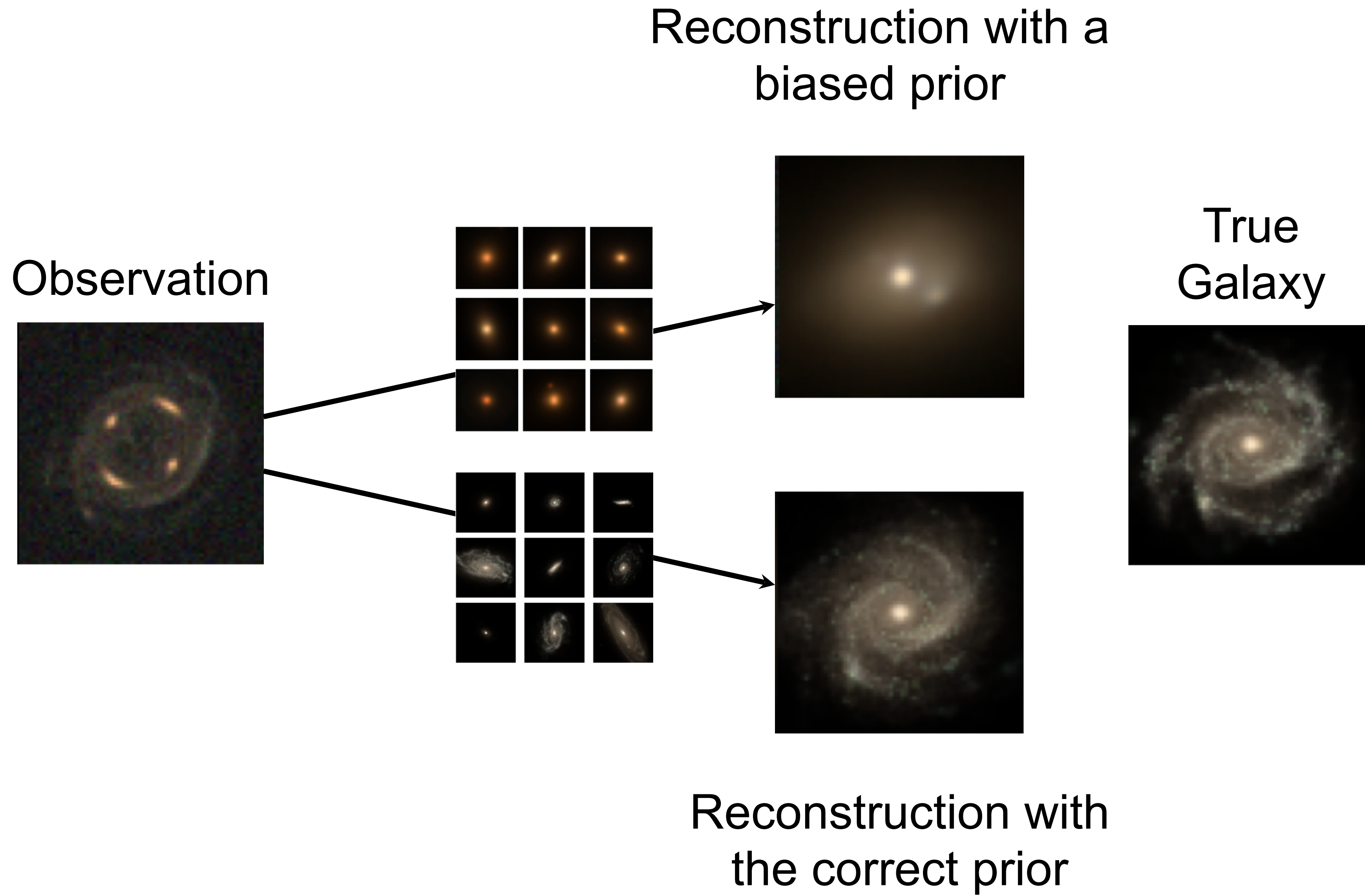


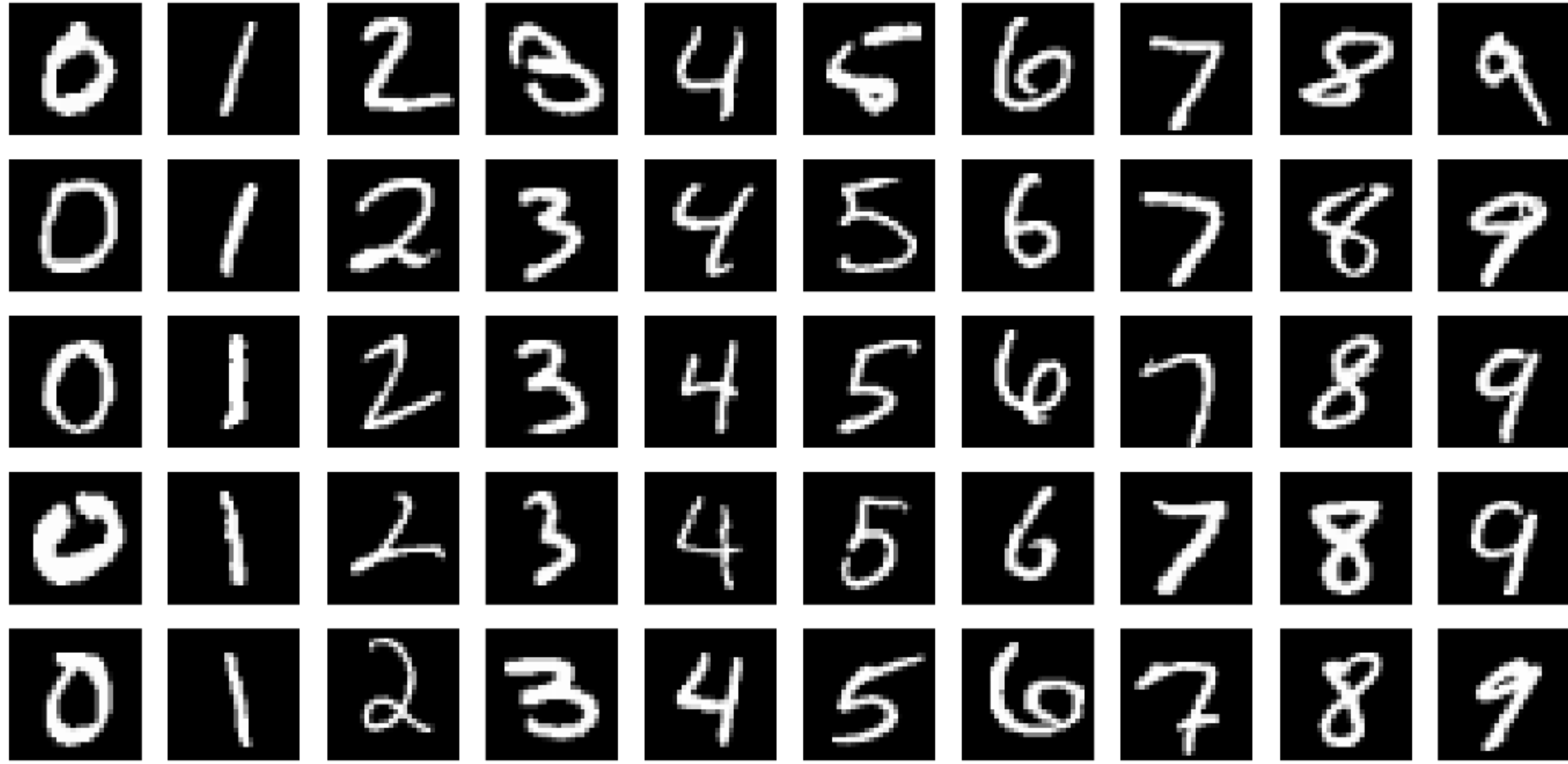
True  
Galaxy











0 1 2 3

0 1 2 3

0 1 2 3

0 1 2 3

0 1 2 3

?

5 6 7 8 9

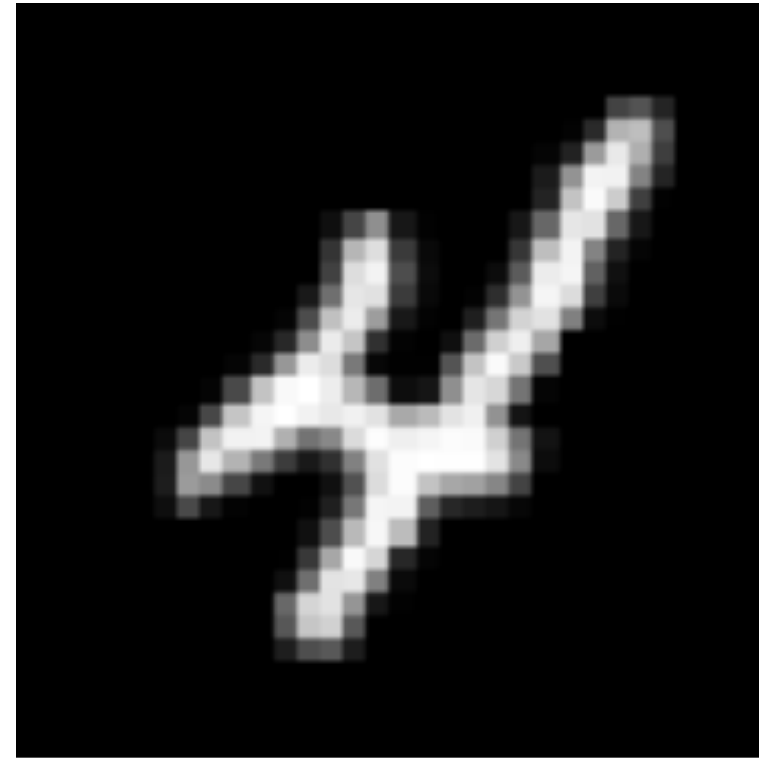
5 6 7 8 9

5 6 7 8 9

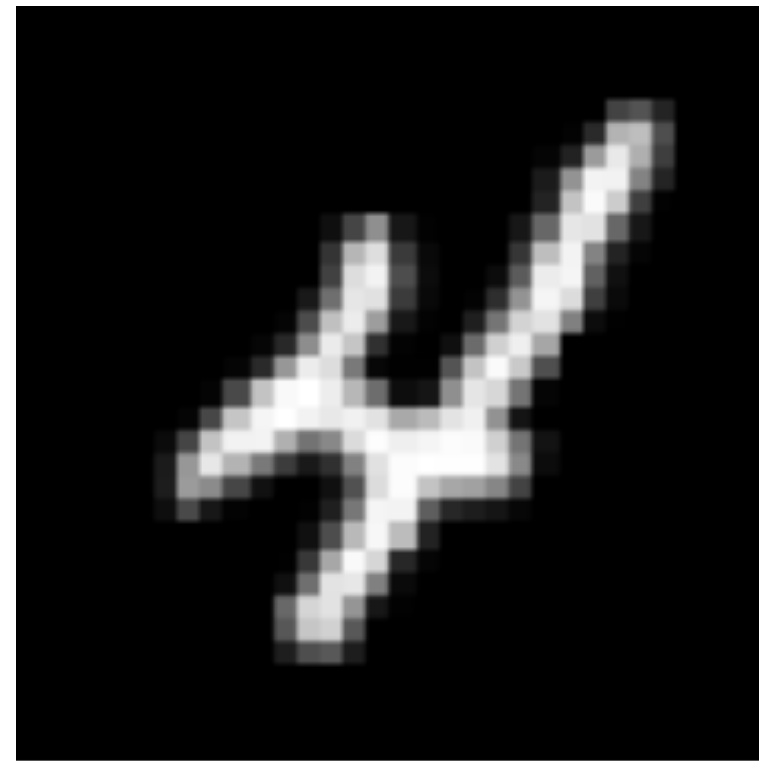
5 6 7 8 9

5 6 7 8 9

True digit



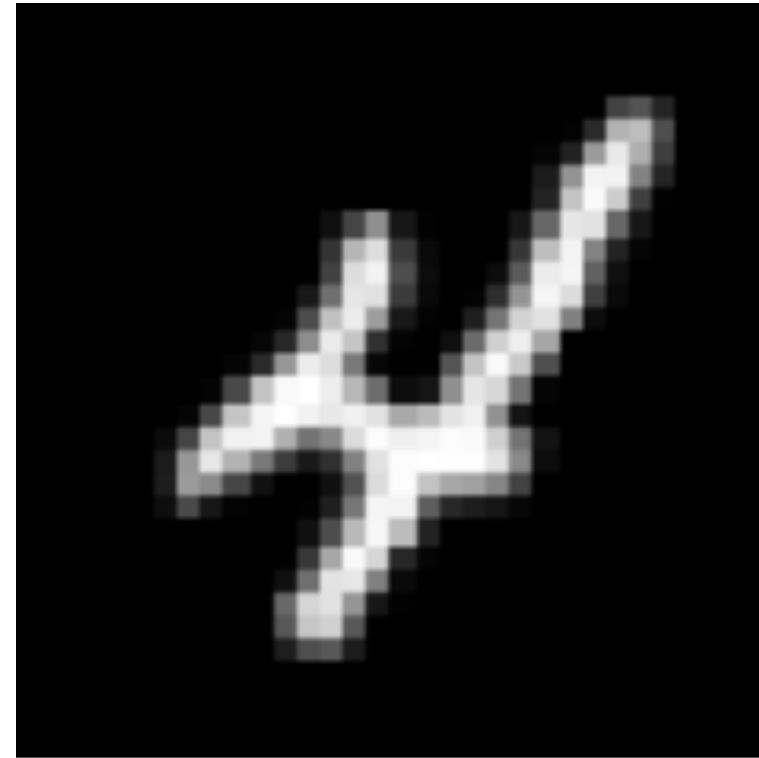
True digit



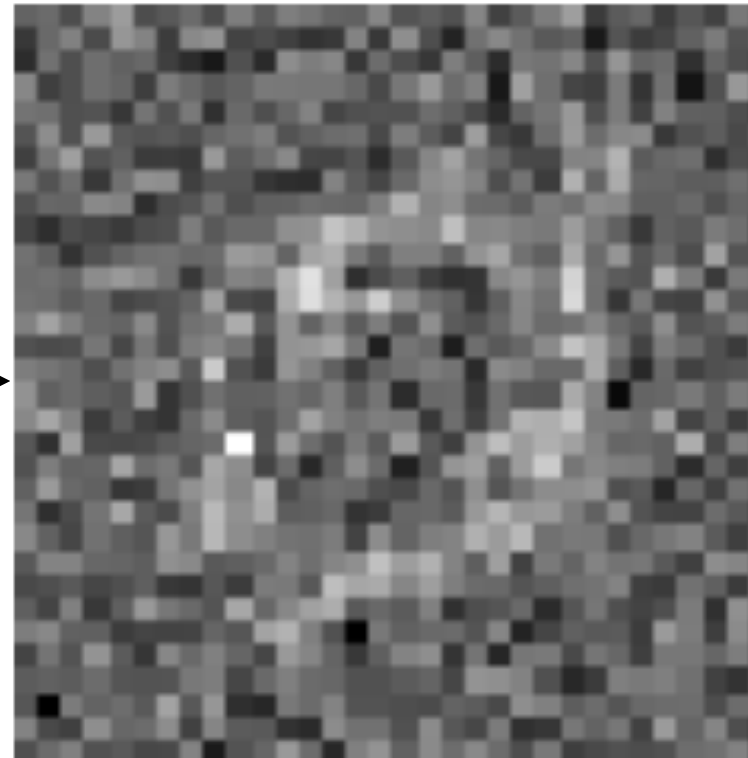
Observation



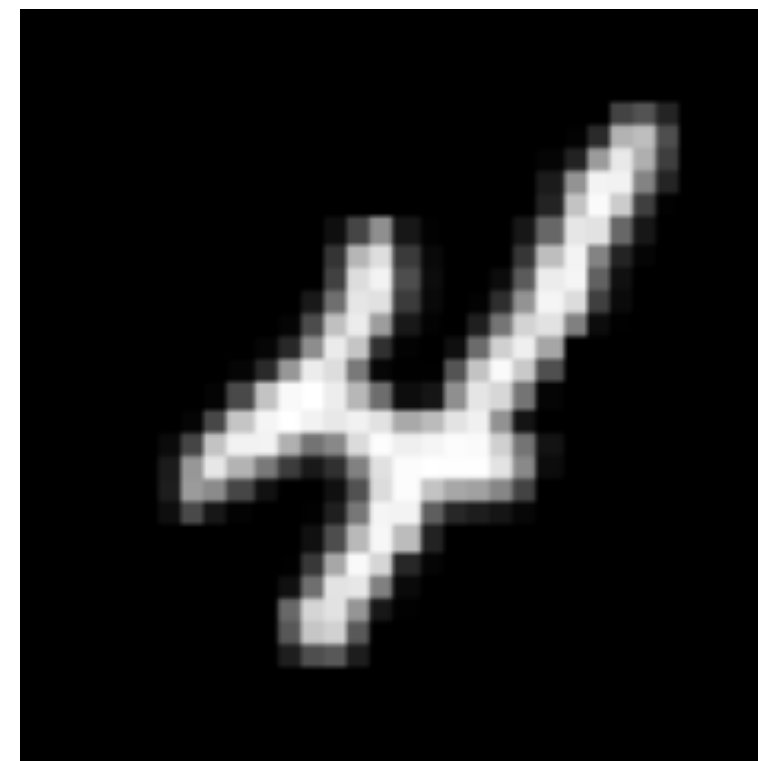
True digit



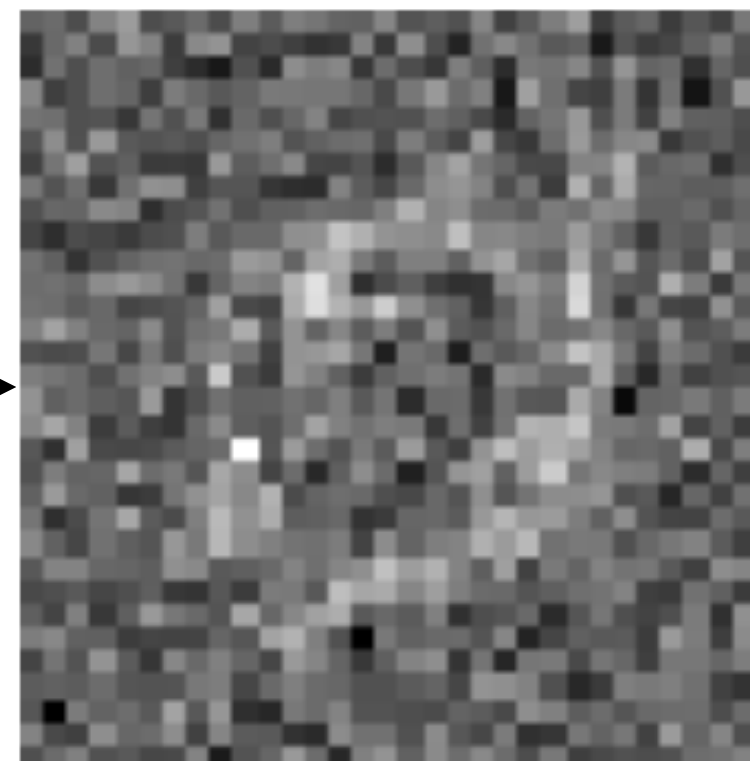
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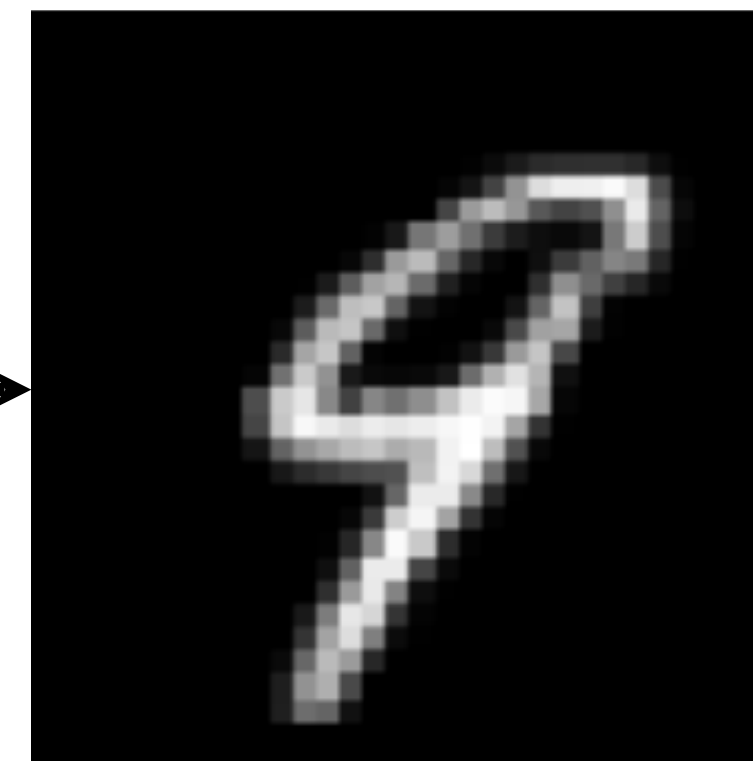
True digit



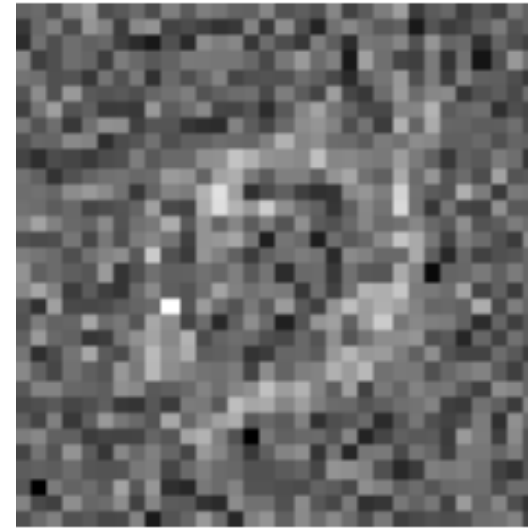
Observation



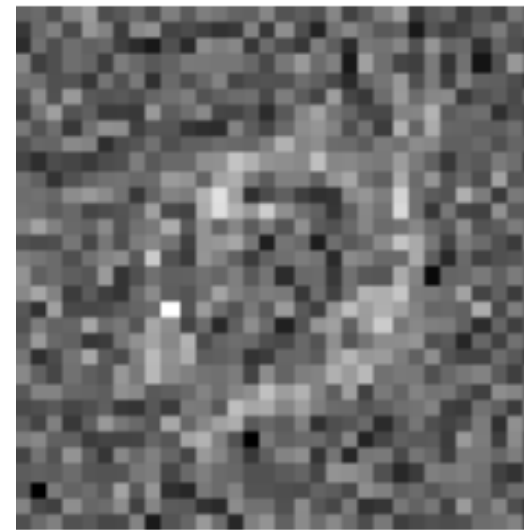
Reconstruction



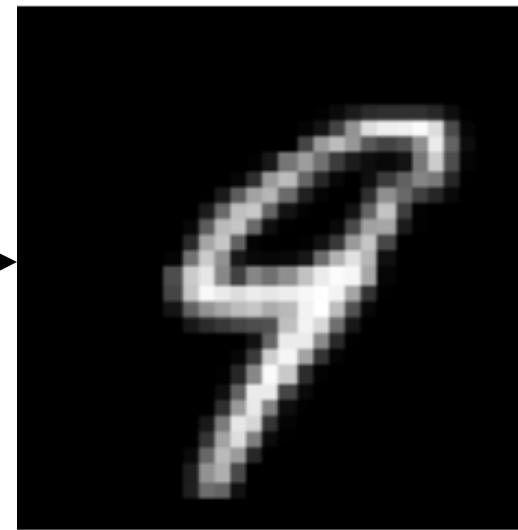
Observation



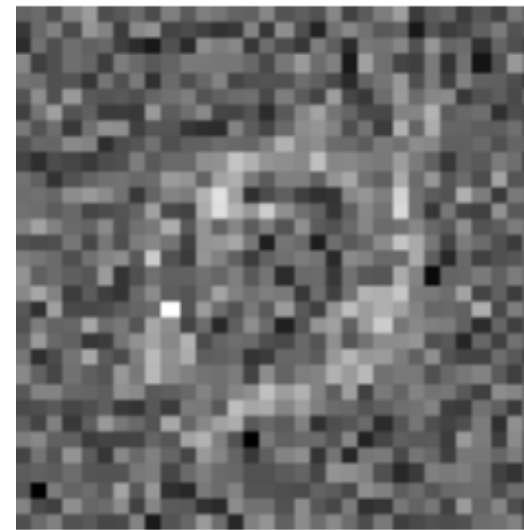
Observation



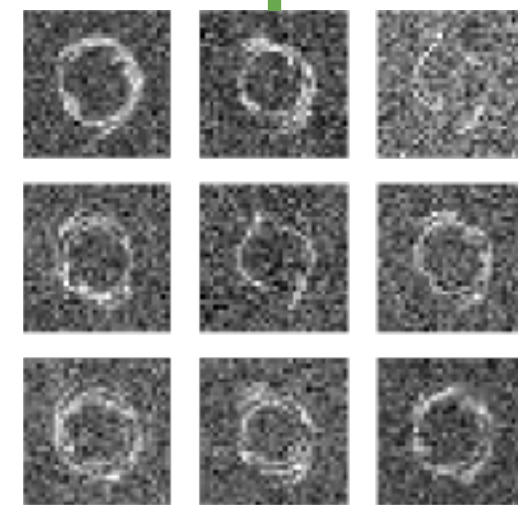
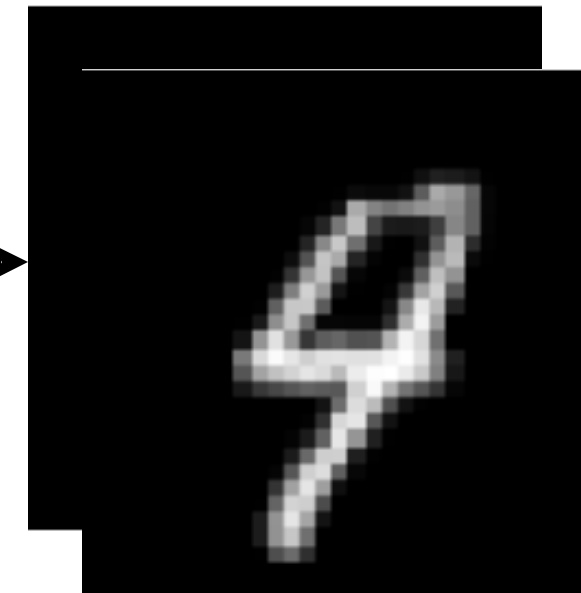
Reconstruction



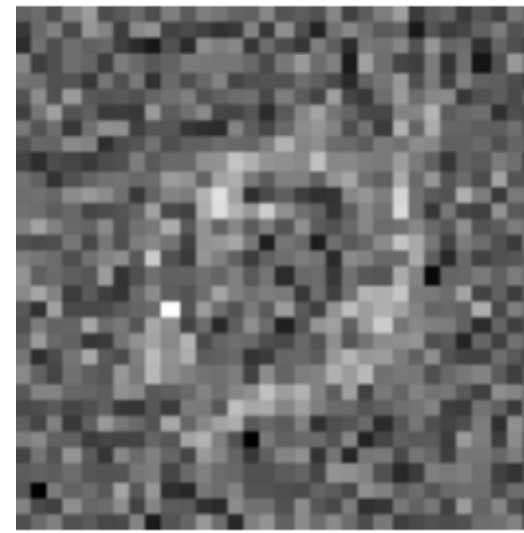
Observation



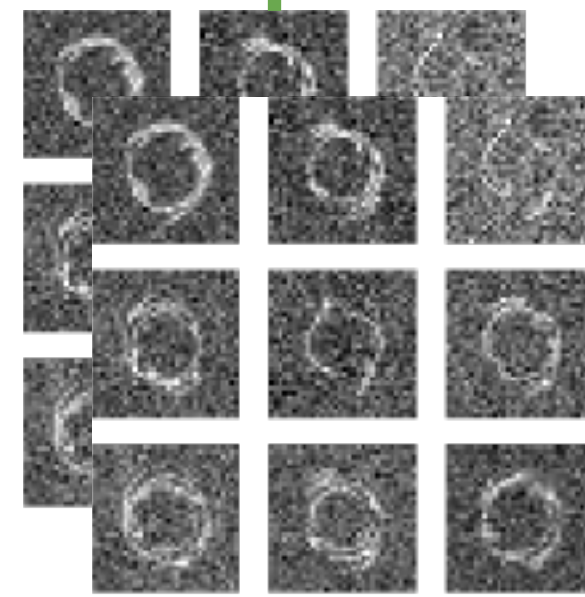
Reconstruction



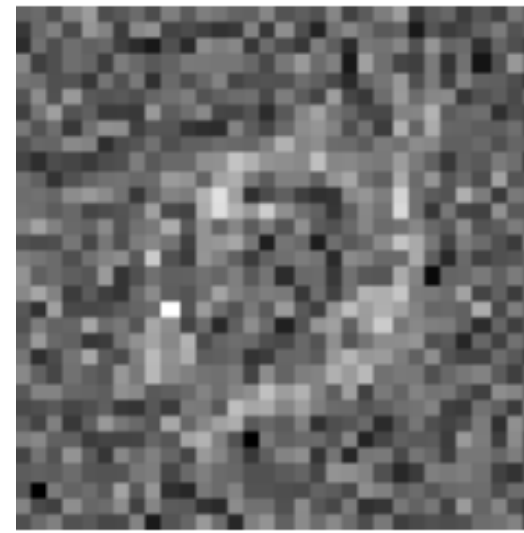
Observation



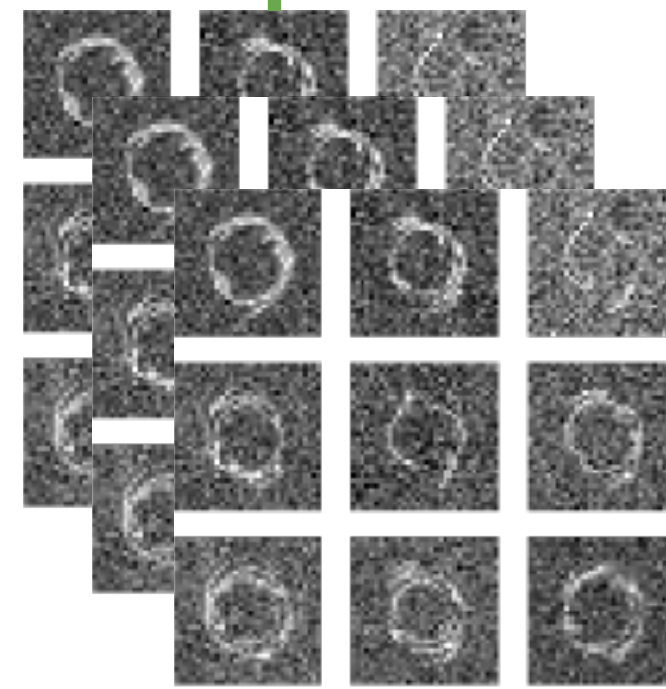
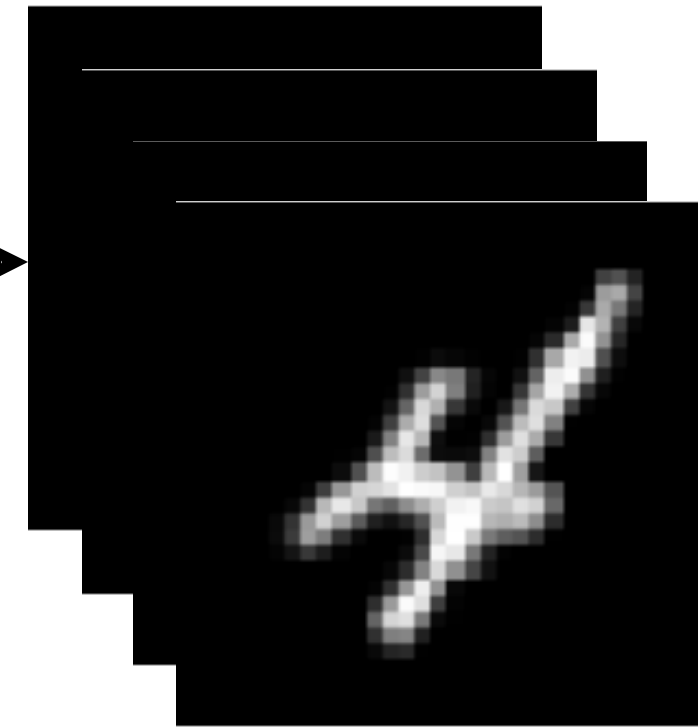
Reconstruction



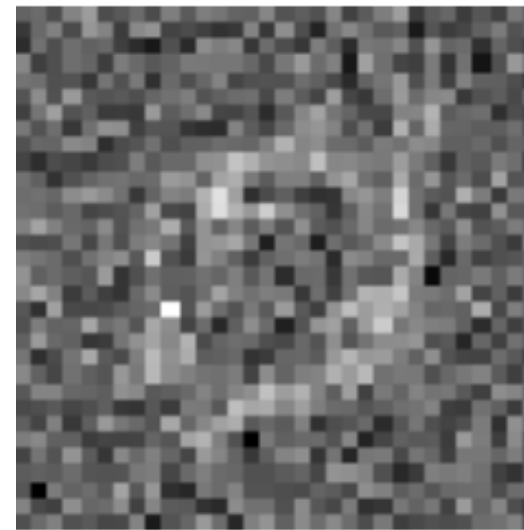
Observation



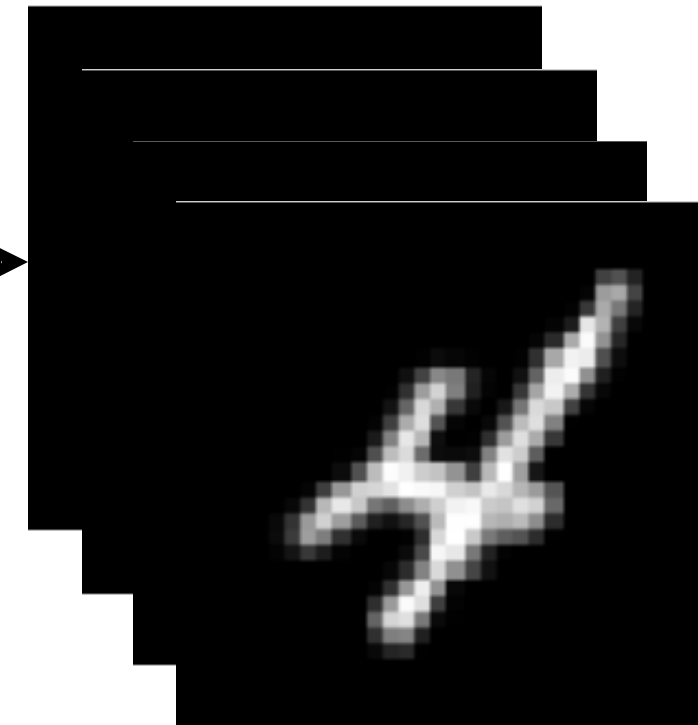
Reconstruction



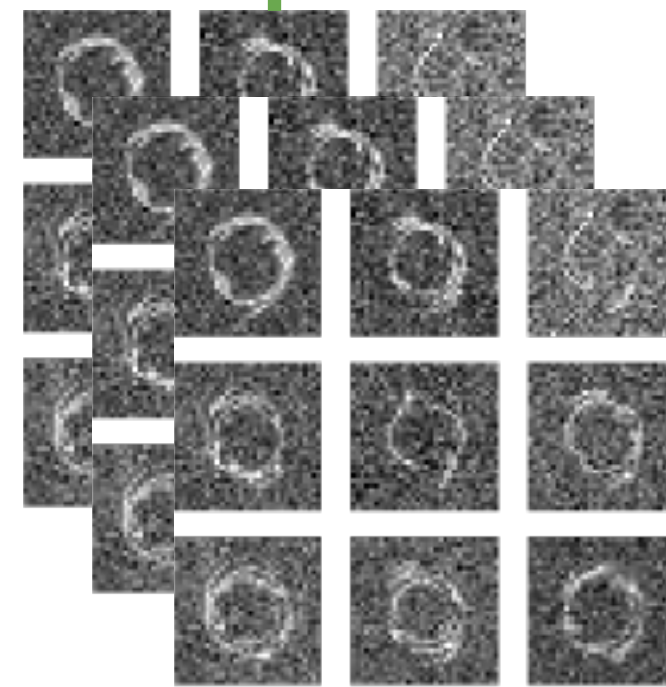
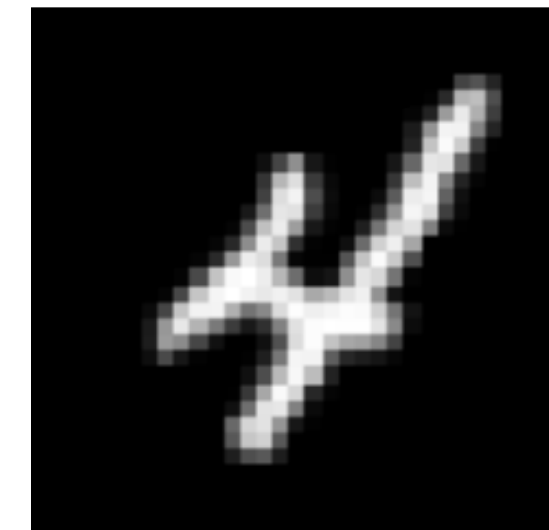
Observation



Reconstruction



True



# ADDRESSING DISTRIBUTIONAL SHIFTS: PRIOR MISSPECIFICATION



Missa  
Barco

Applications to galaxies: can we learn that spiral galaxies exist starting from a prior with just (real, noisy) ellipticals?

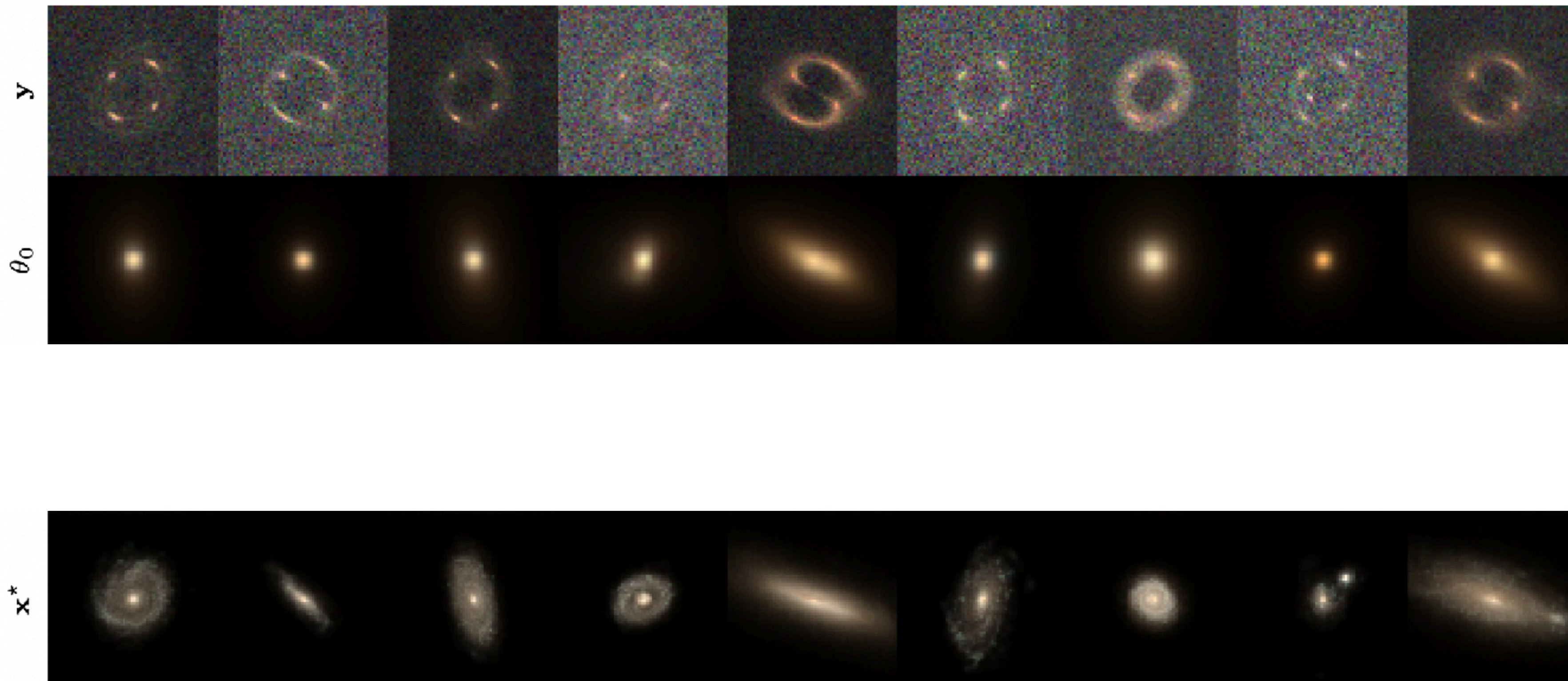


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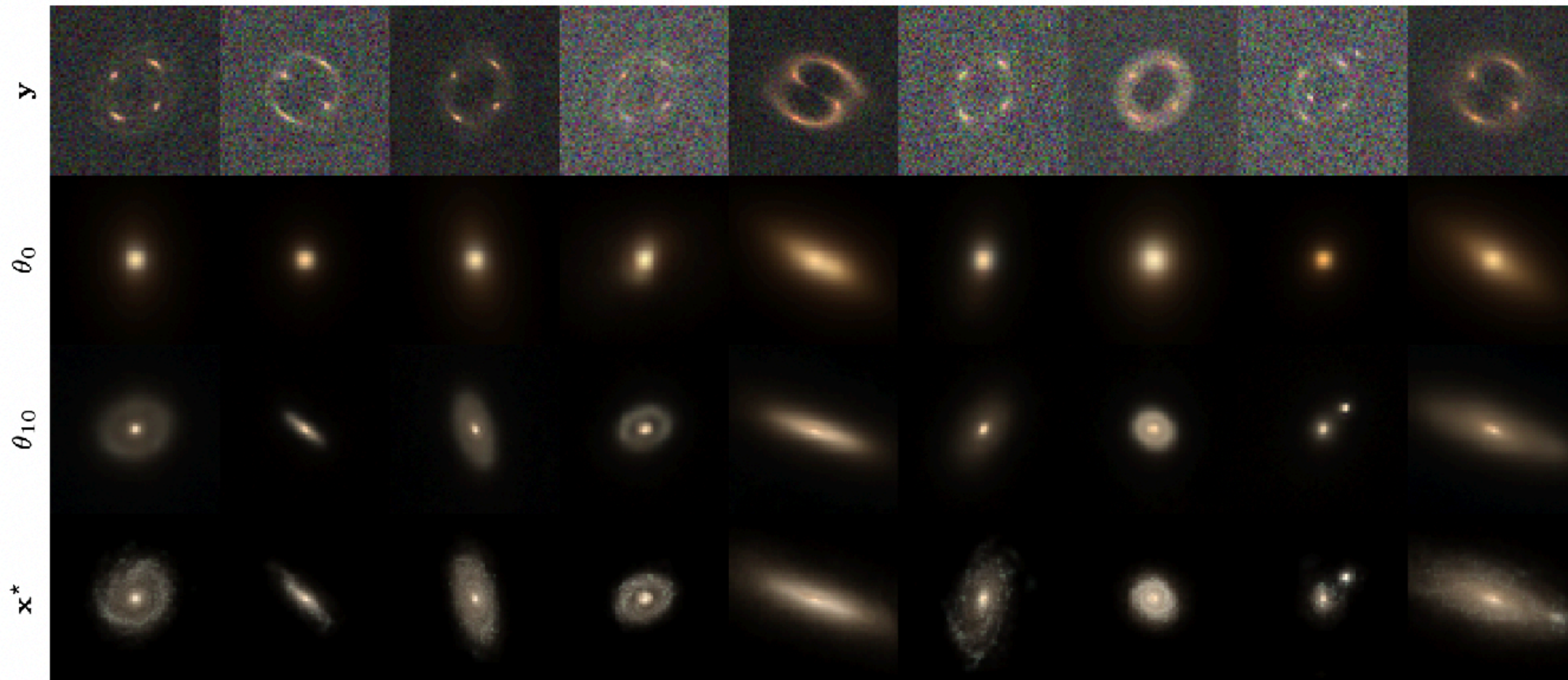


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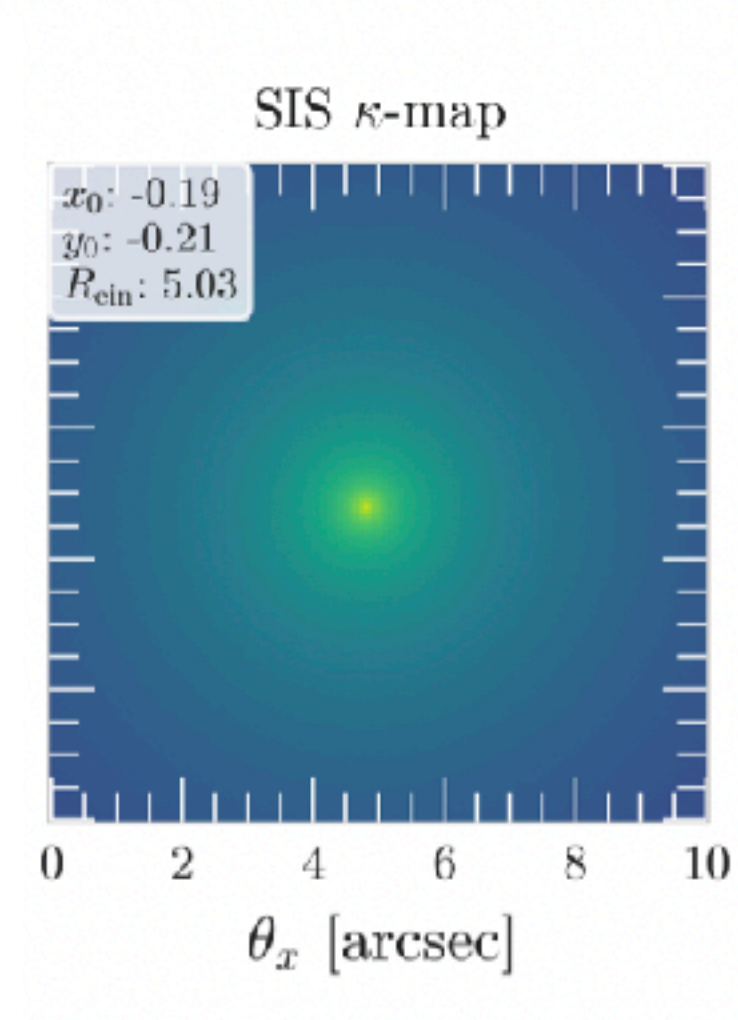
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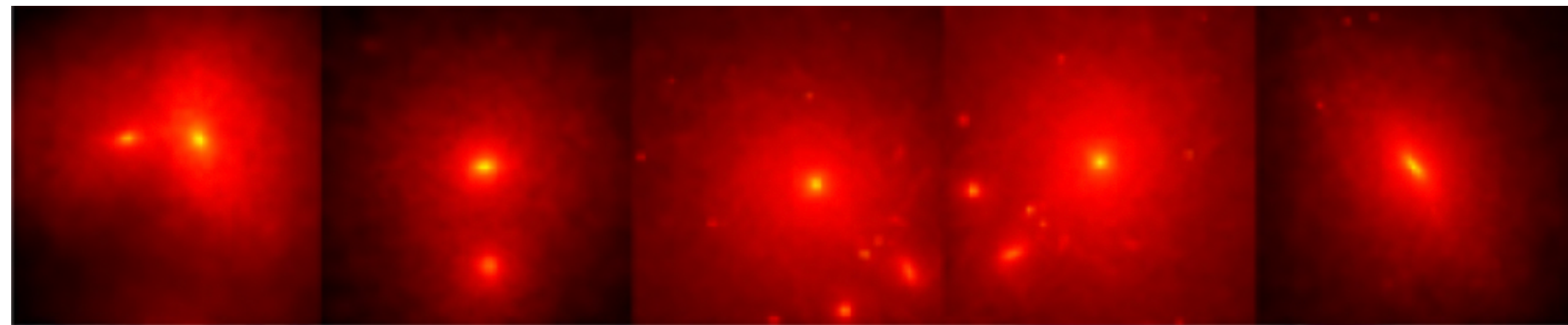
Nicolas  
Payot

Could we use a similar framework to learn misspecified ***physical models***?

Parametric model for the lens:



Realistic mass map from simulations:



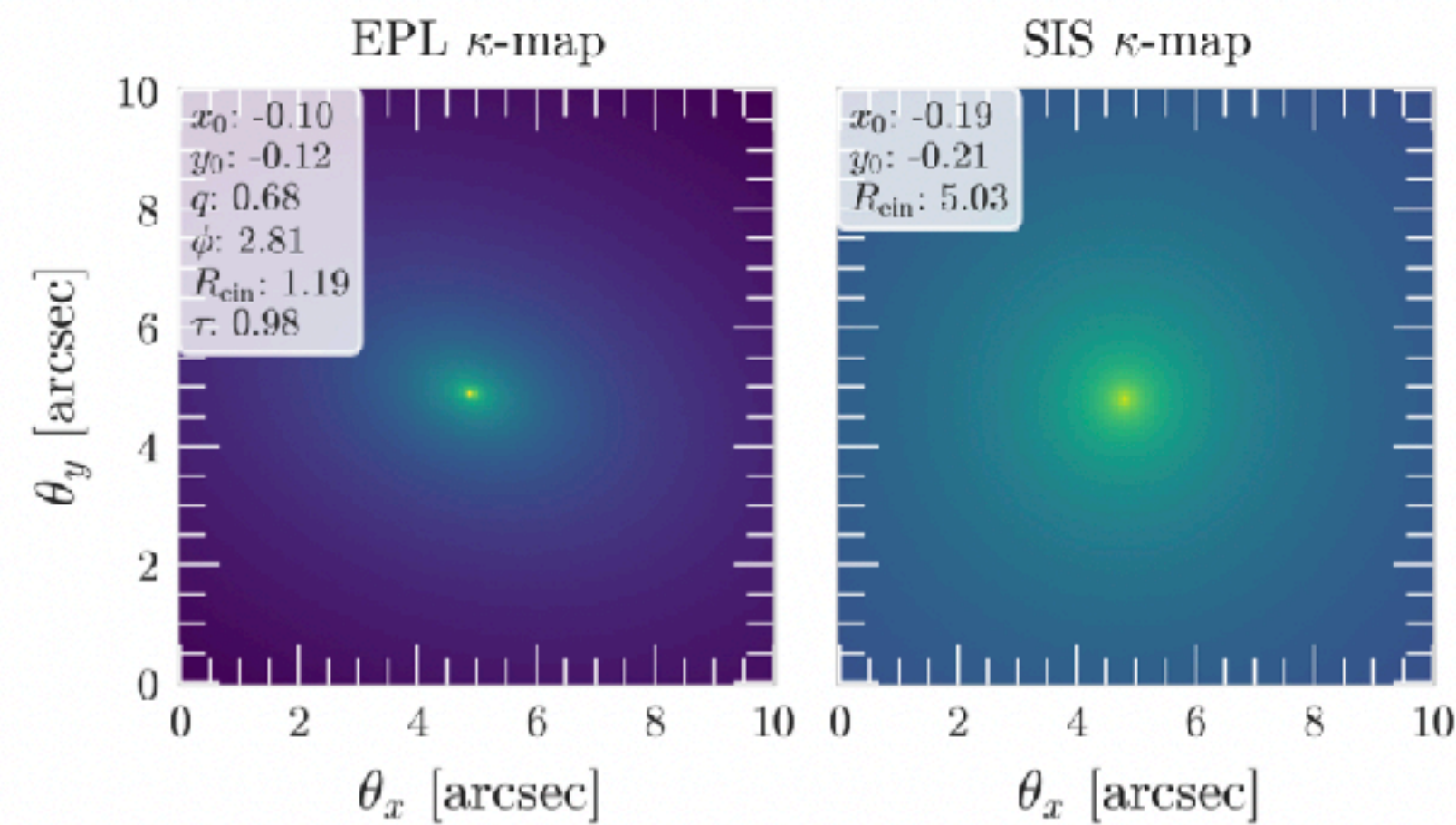
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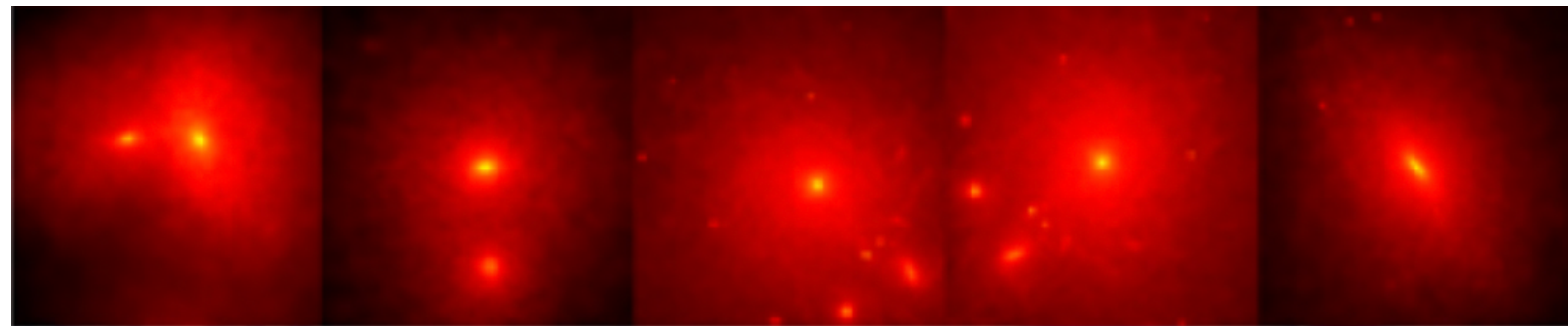
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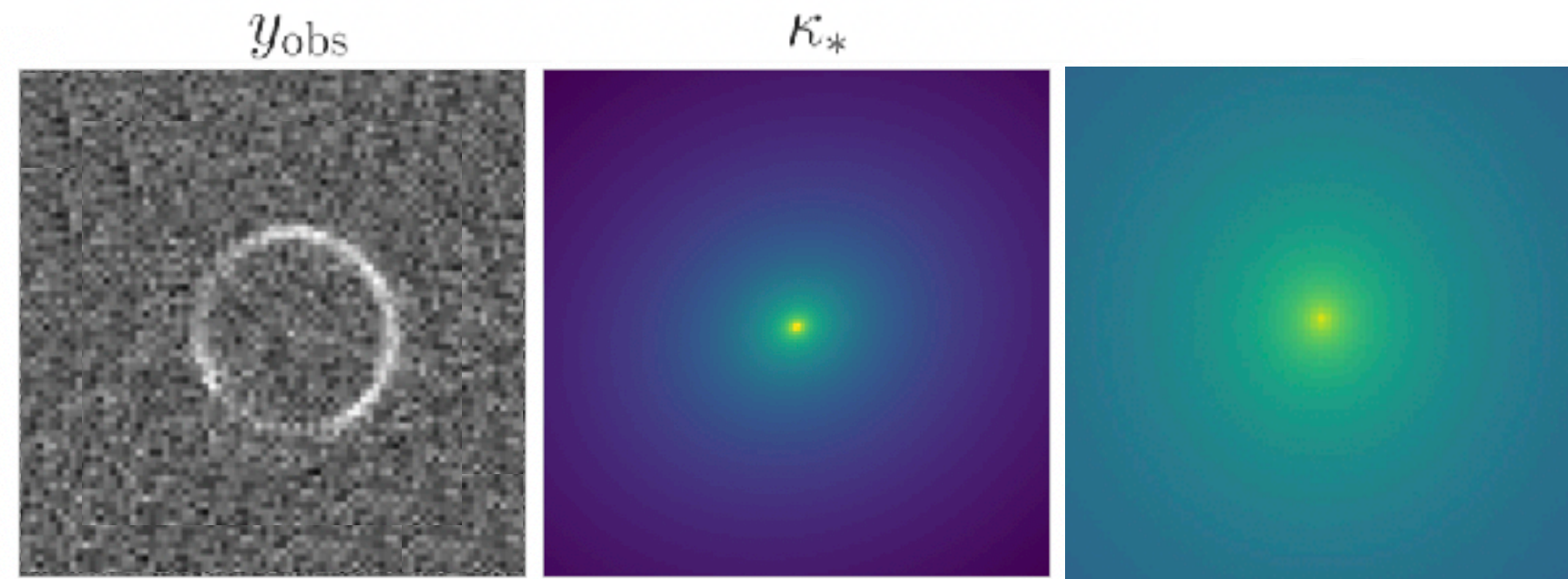


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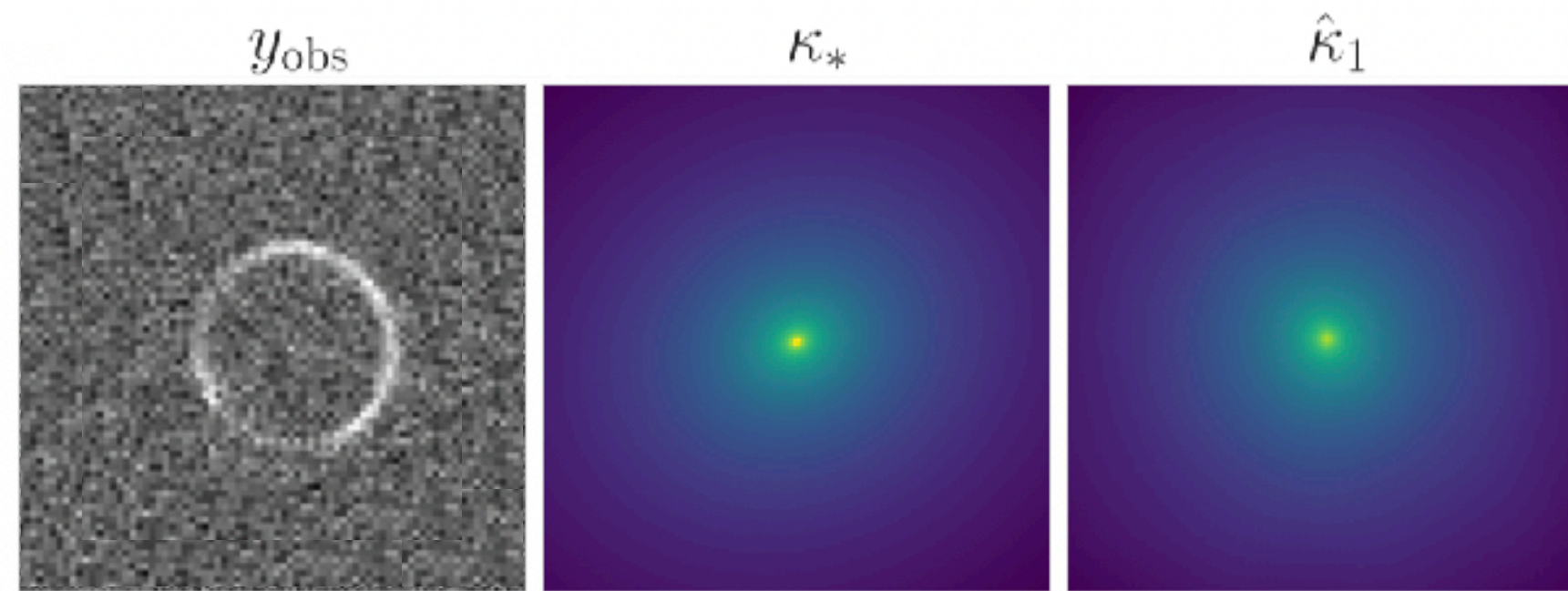
We can map our over-constrained physical model to an over-parametrized latent space, and apply the same technique!



Nicolas  
Payot

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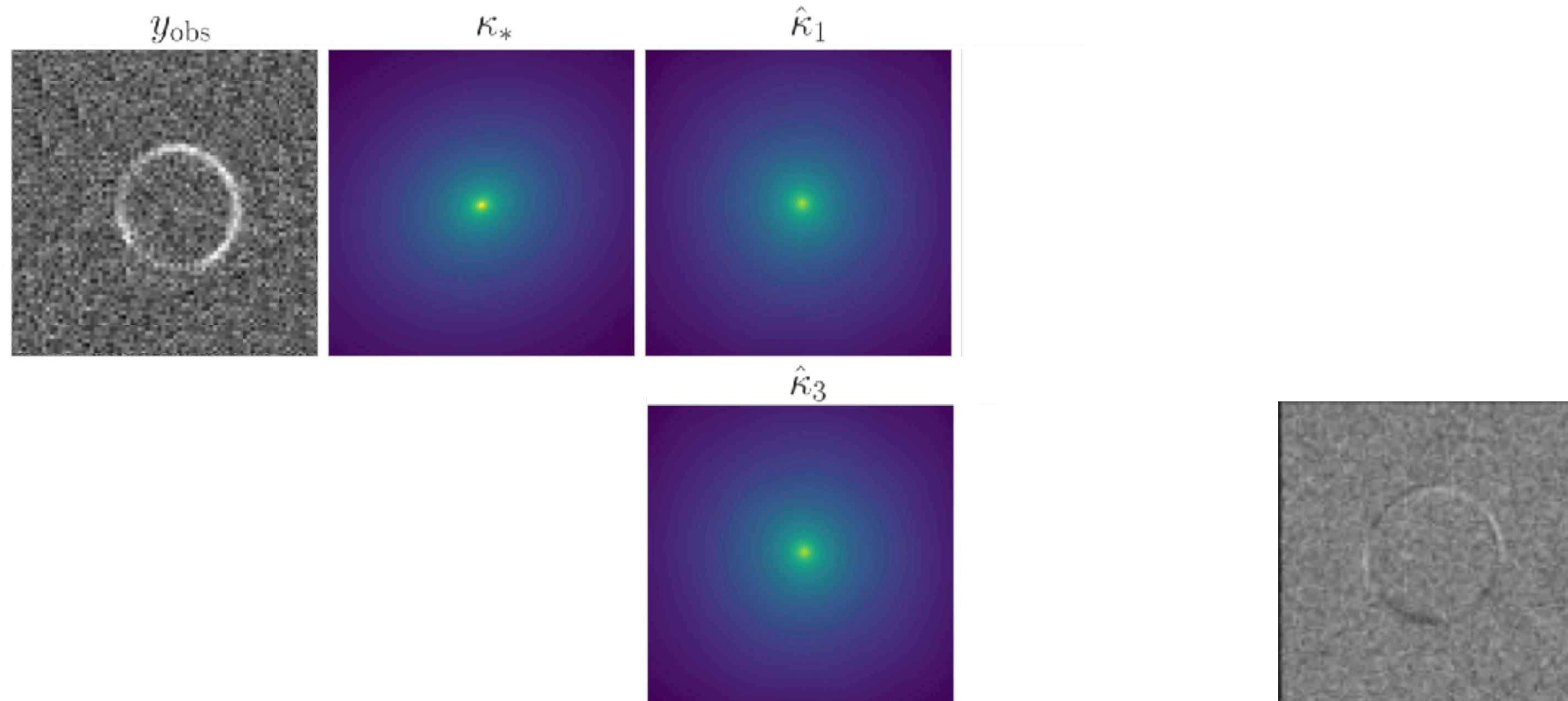
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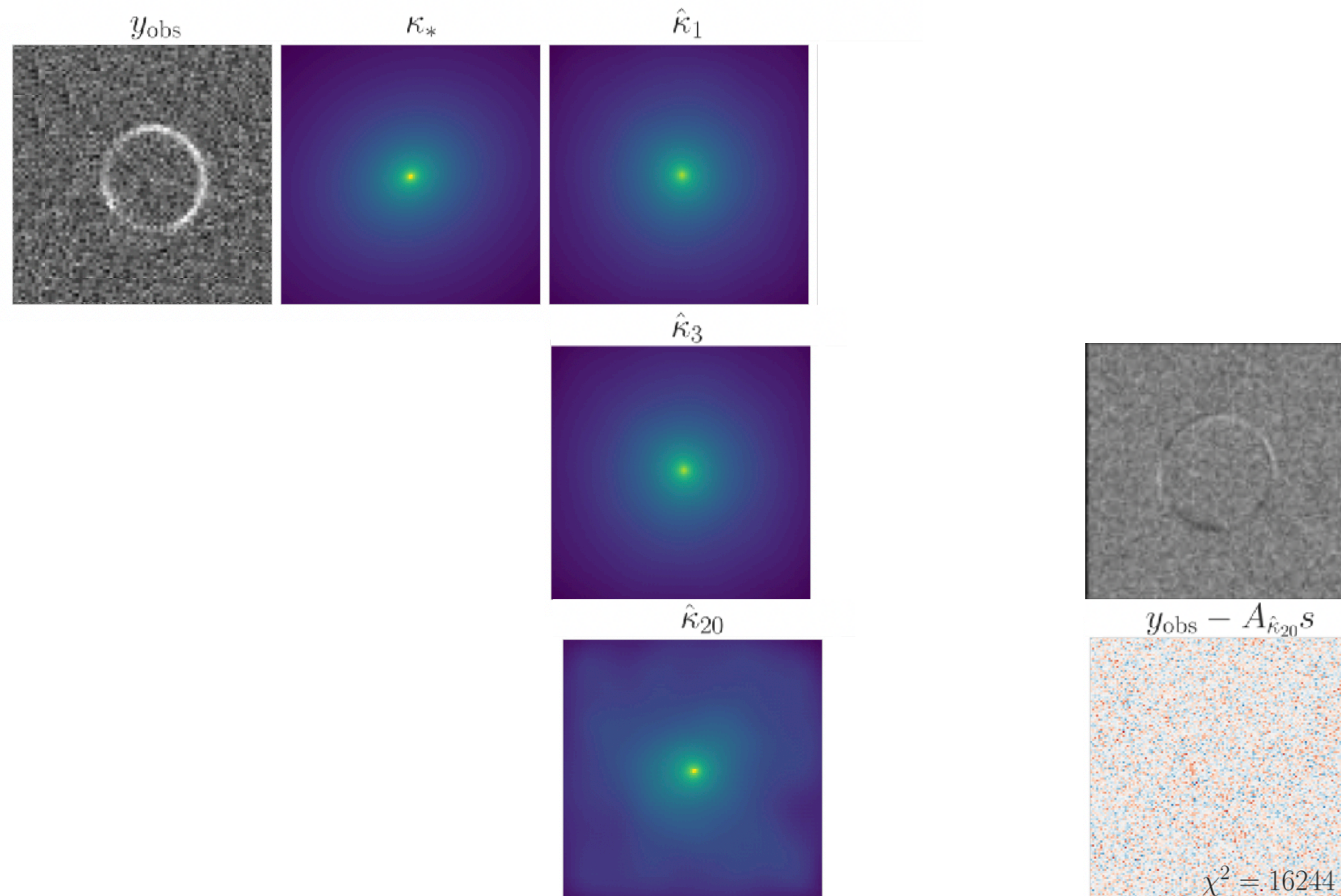
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Payot

# ADDRESSING DISTRIBUTIONAL SHIFTS: MODEL MISSPECIFICATION?

We can map our over-constrained physical model to an over-parametrized latent space, and apply the same technique!



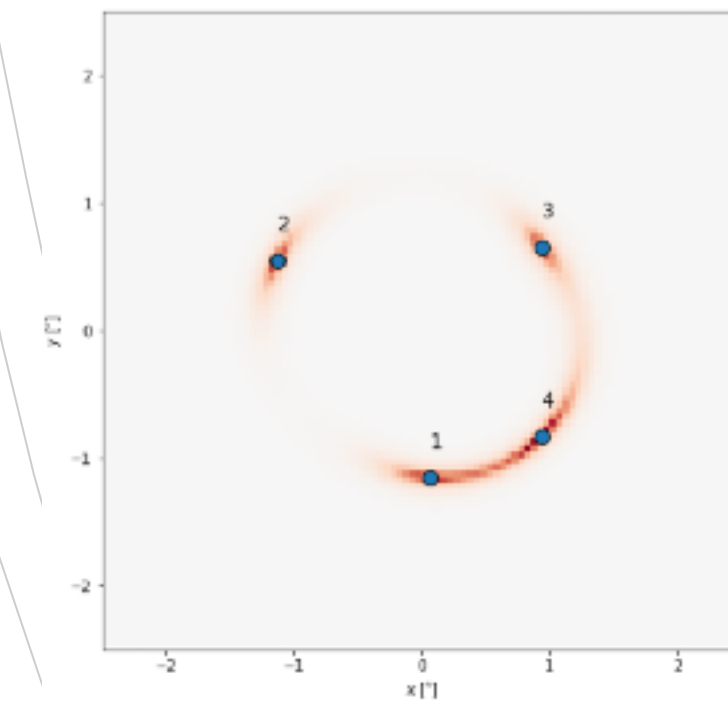
Nicolas  
Payot



# STRONG LENSING SIMULATION PIPELINE: CAUSTIC

**A fast, GPU-compatible, auto-differentiable, extremely modular simulation pipeline for all your strong lensing needs.**

- 1) Lens and source from analytic profiles or pixelated images/densities
- 2) Multiplane lensing
- 3) Line of sight mass distributions
- 4) Fast microlensing simulations
- 5) Time-delays



Connor  
Stone



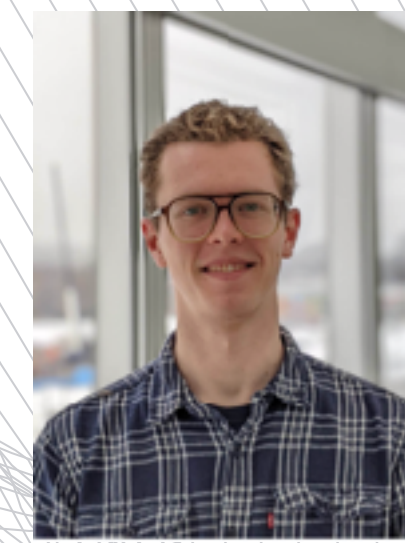
Adam  
Coogan



Andi  
Filipp



Alex  
Adam



Misha  
Barth



Charles  
Wilson

<https://github.com/Ciela-Institute/caustics>

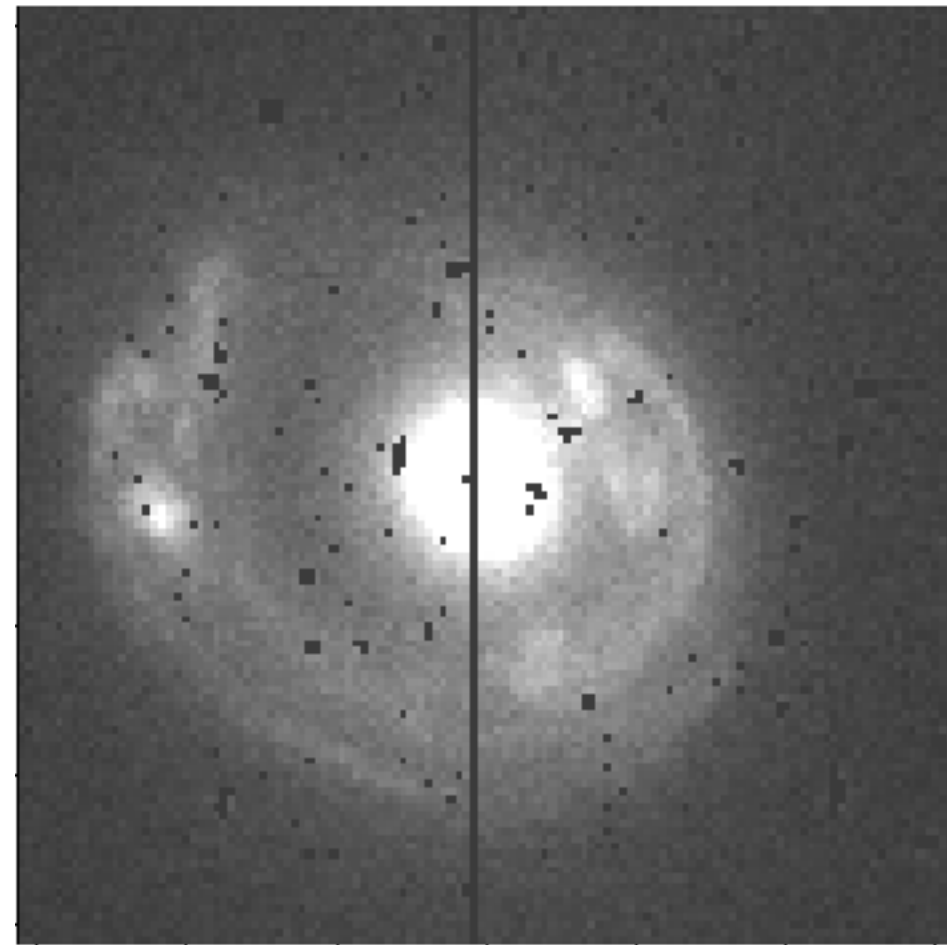
<https://caustics.readthedocs.io/en/latest/intro.html>

# BONUS!

1. Data-driven priors
2. Data-driven likelihoods
3. Accuracy metrics
4. Out-of-Distribution accuracy
5. Data!

# TEST CASE: SDSSJ1430+4105

Lens Observation



**Discovered in 2006 from SDSS;**

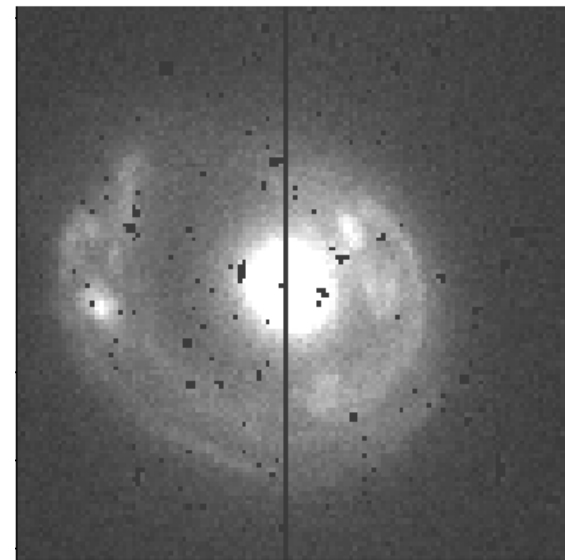
**Hubble Space Telescope Imaging;**

**Distance:  $z_l \sim 0.285$ ,  $z_s \sim 0.575$ .**

RECONSTRUCTING THE SOURCE GALAXY IS CHALLENGING,  
EVEN WITH NON-MNIST SOURCES.

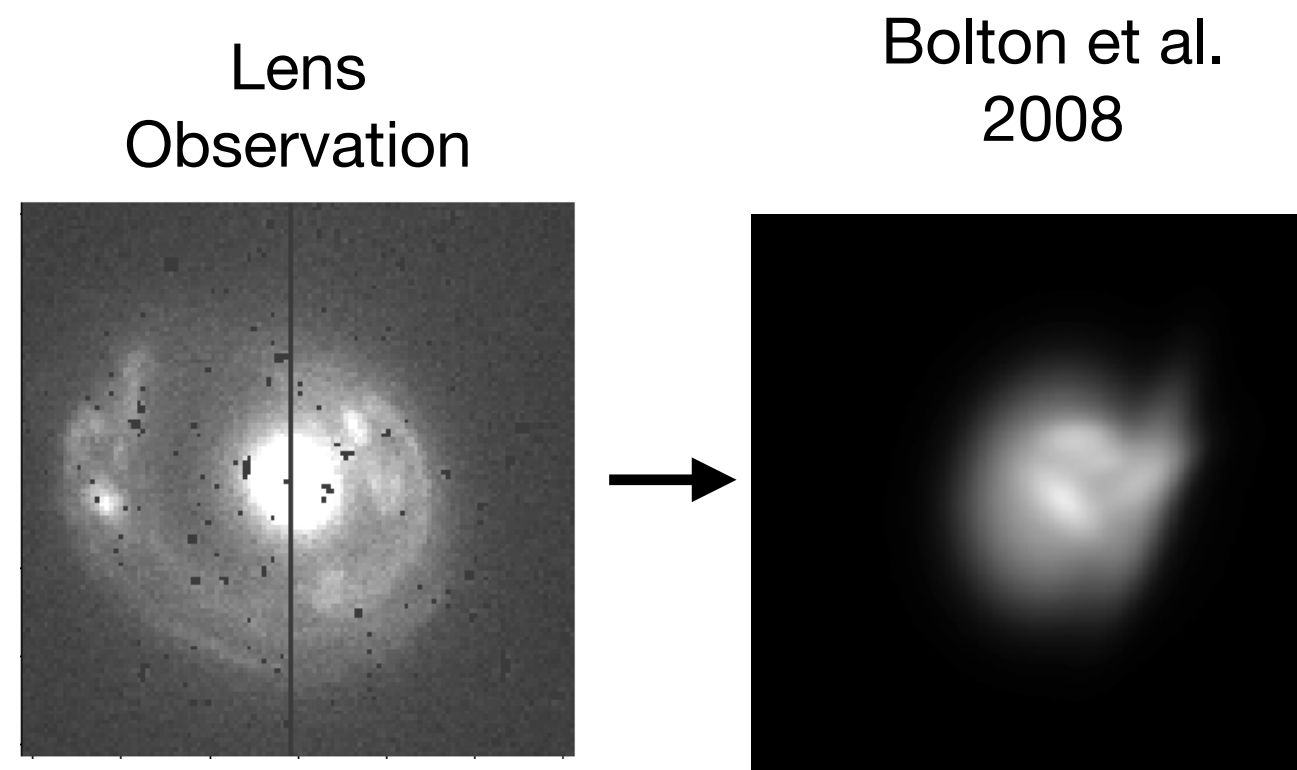
Joint inference of lens and source

Lens  
Observation



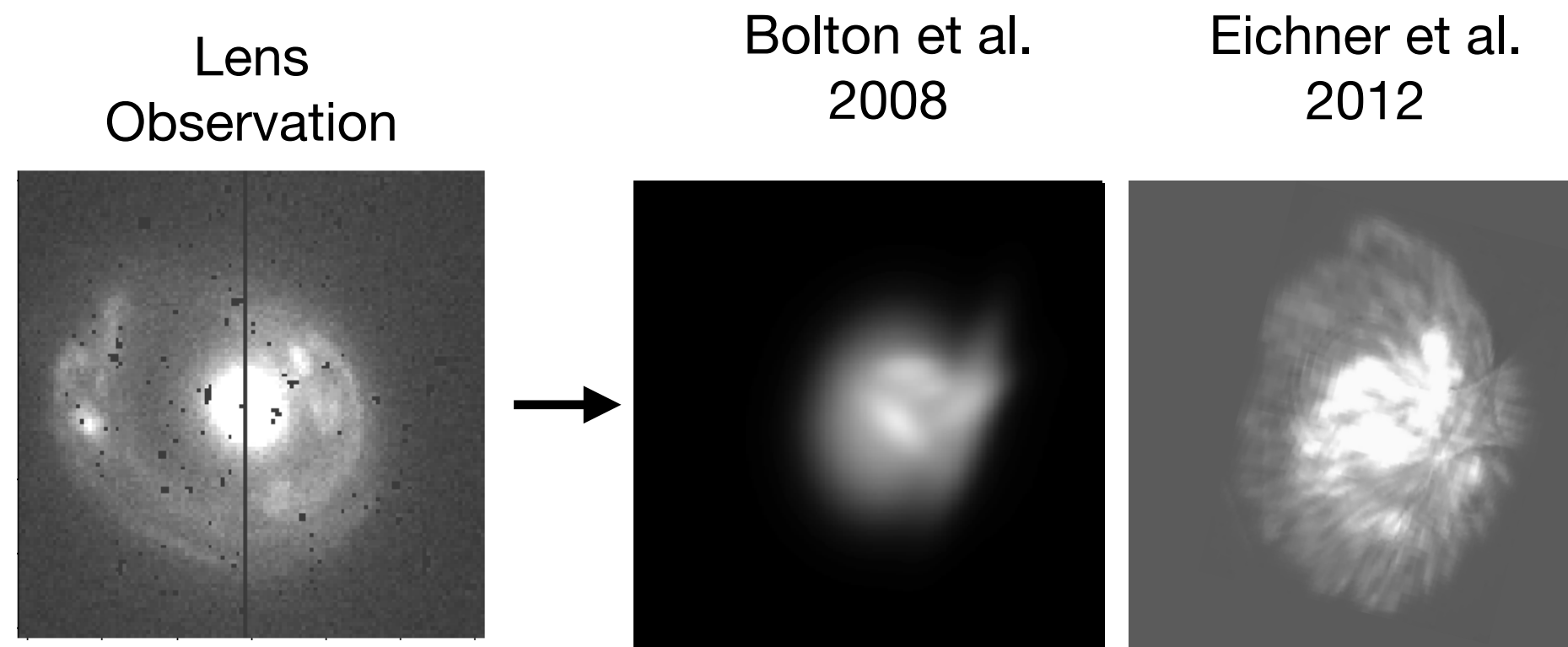
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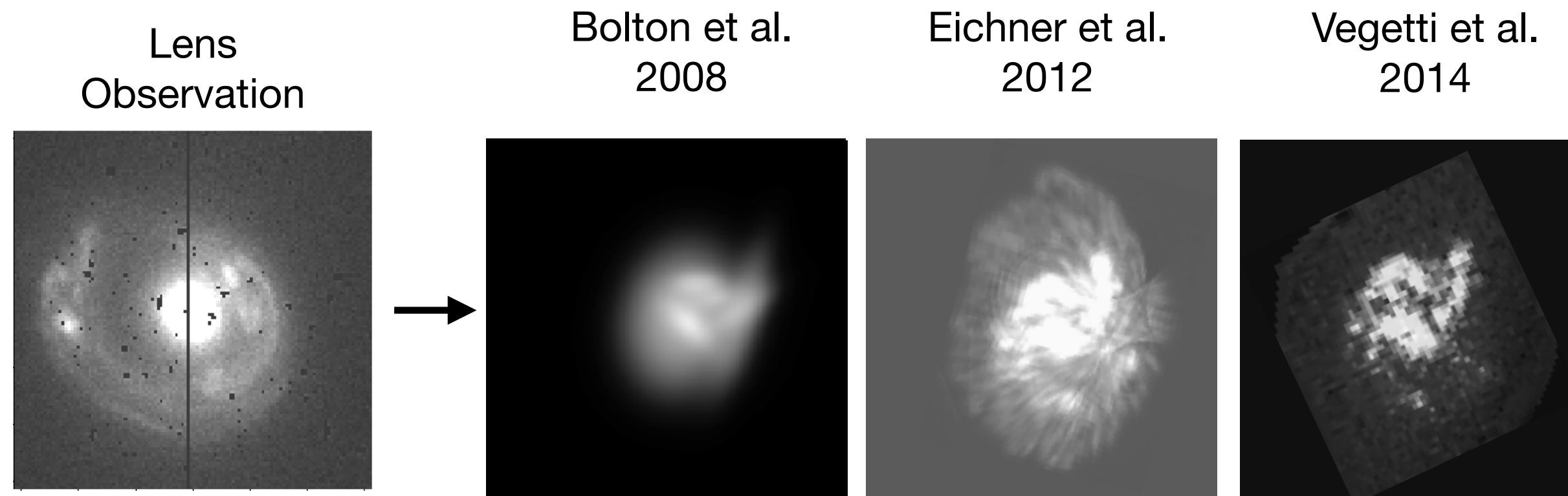
RECONSTRUCTING THE SOURCE GALAXY IS CHALLENGING,  
EVEN WITH NON-MNIST SOURCES.

Joint inference of lens and source



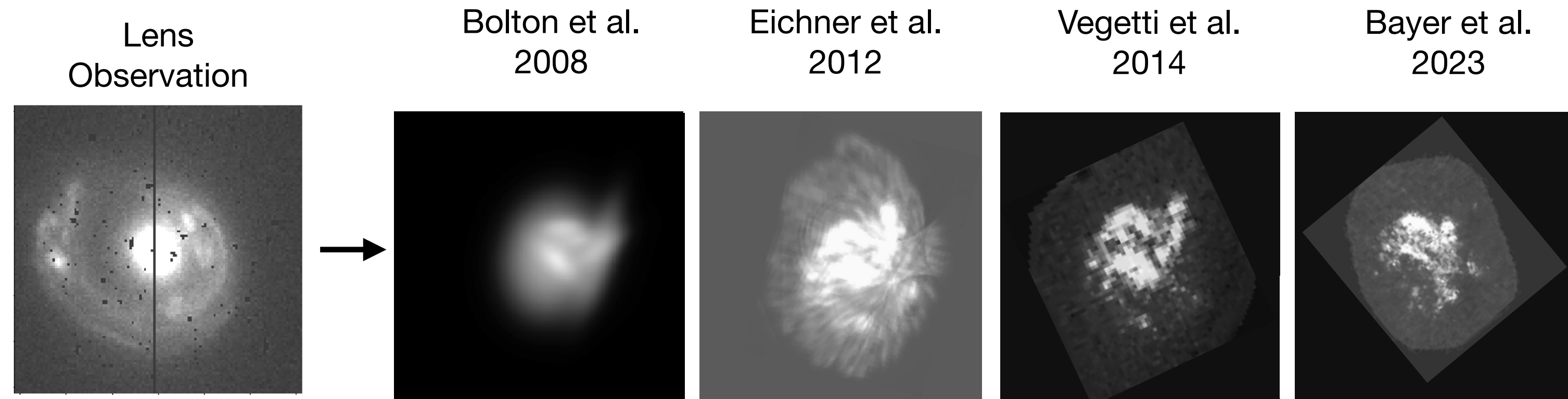
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## Joint inference of lens and source



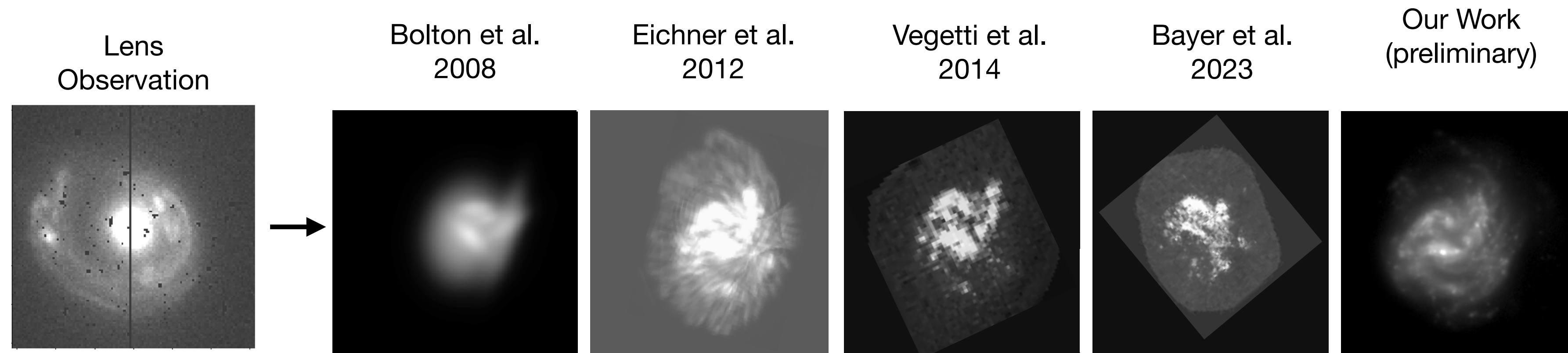
# RECONSTRUCTING THE SOURCE GALAXY IS CHALLENGING, EVEN WITH NON-MNIST SOURCES.

## Joint inference of lens and source



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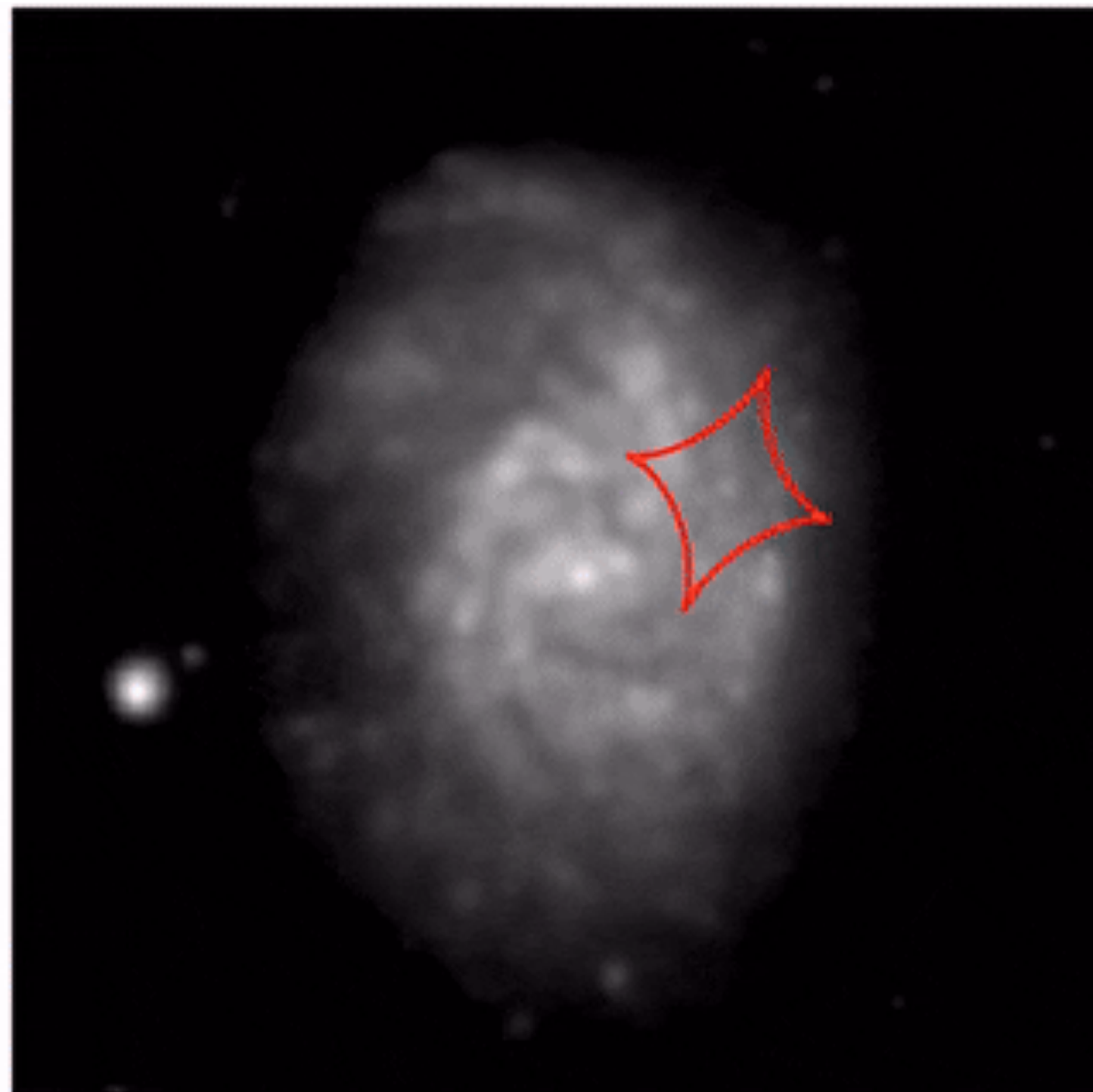
## Joint inference of lens and source



WE CAN SAMPLE THE POSTERIOR OVER POSSIBLE SOURCES, MARGINALIZING OVER POSSIBLE LENS MODELS

Uncertainties show that the complex morphology is reconstructed with high confidence.

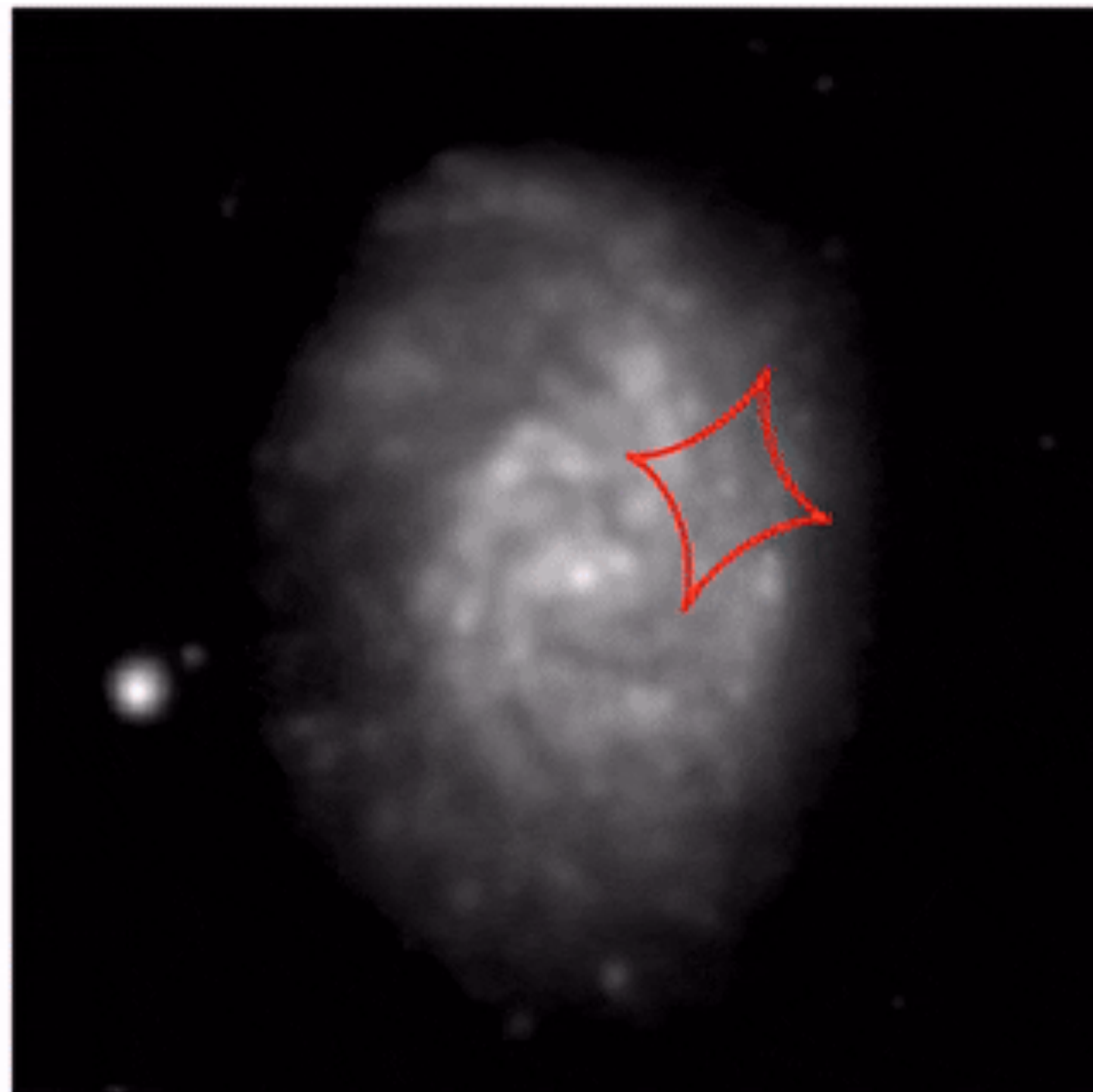
Galaxy Model



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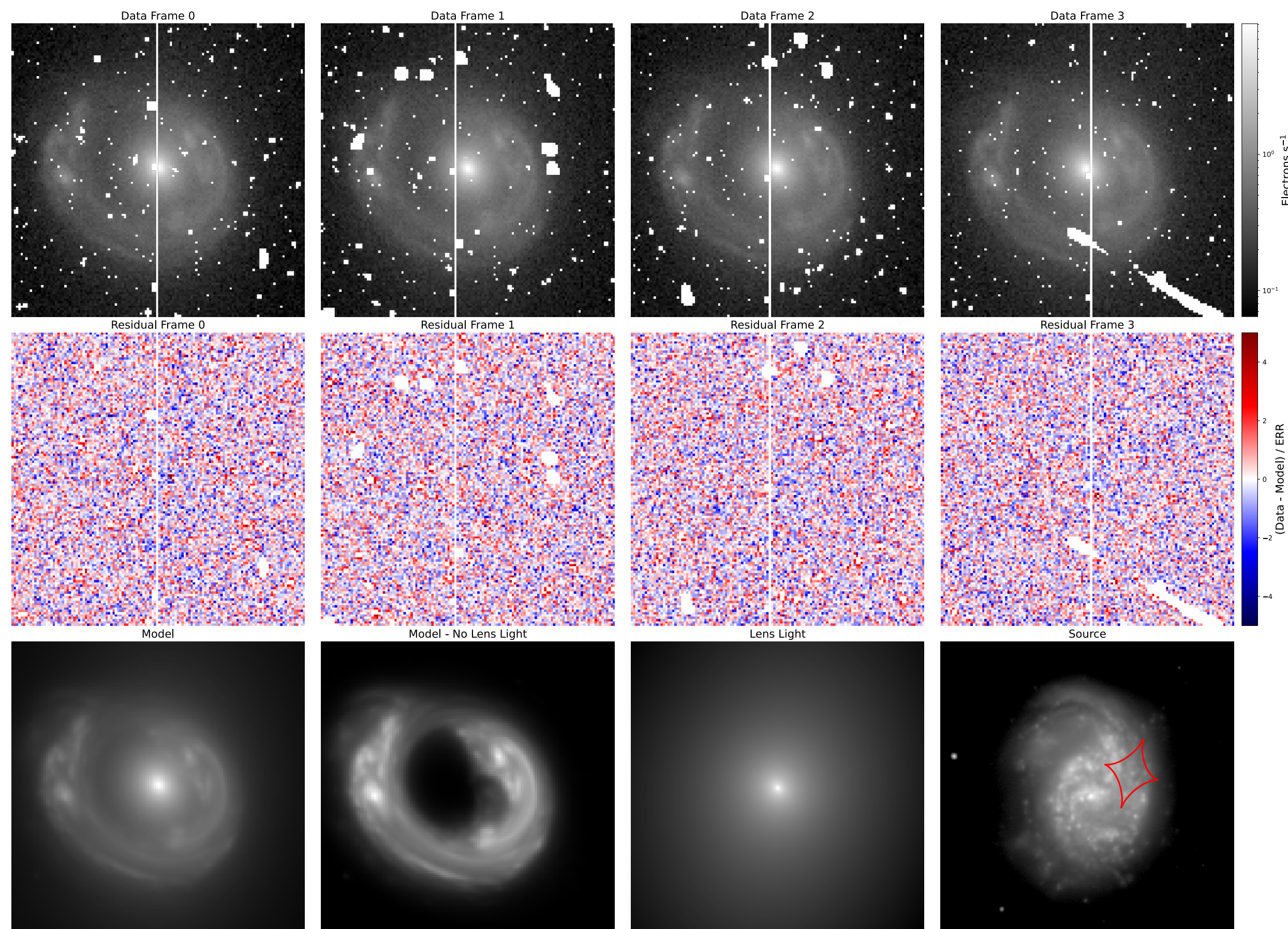
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Galaxy Model



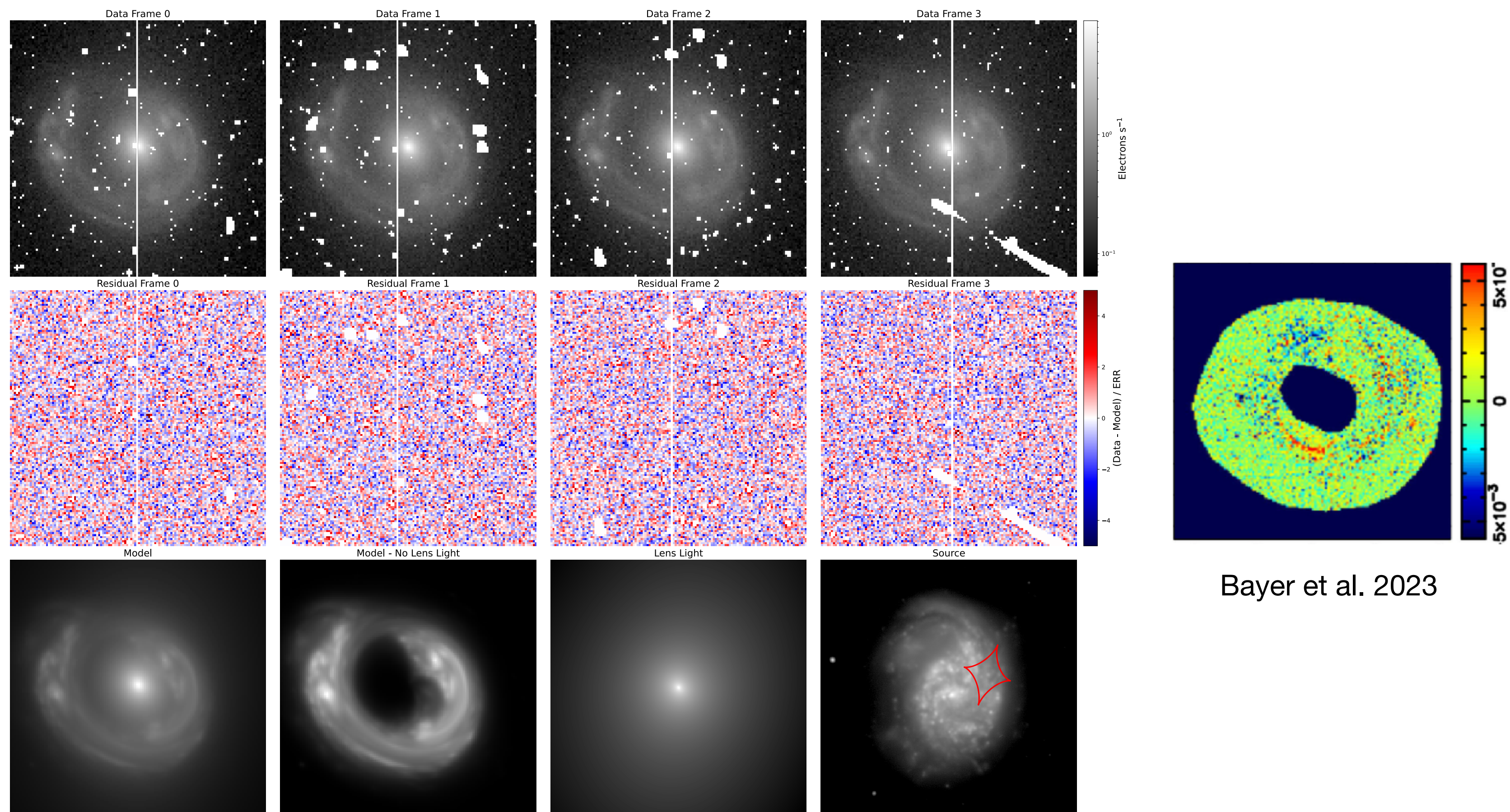
# HOW WELL DOES OUR GALAXY MODEL RECONSTRUCT THE LENSED OBSERVATION?

We can extract near-maximal information from the data.



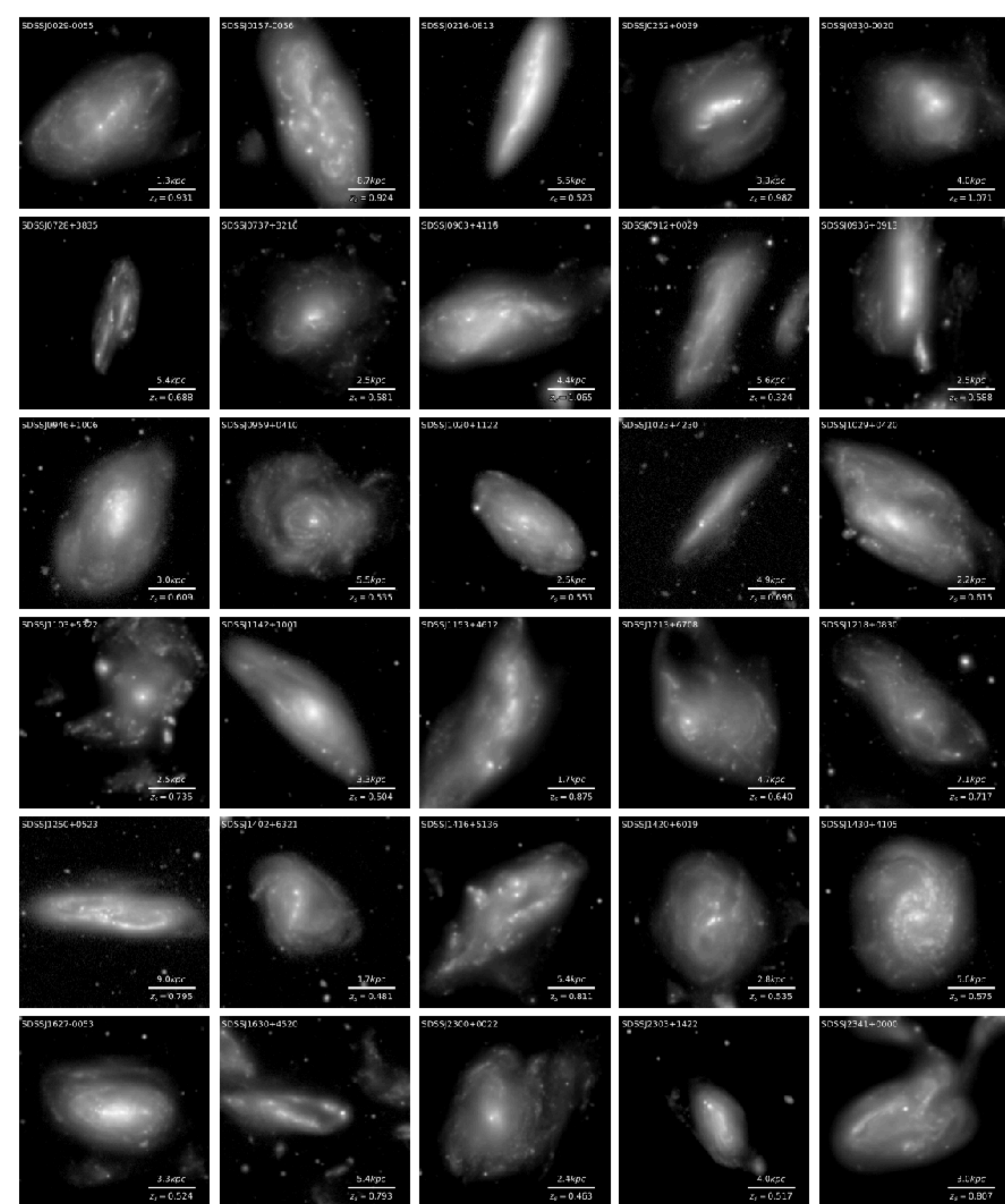
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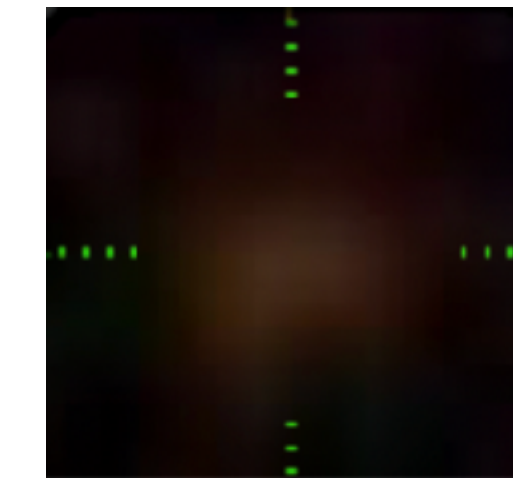


Bayer et al. 2023

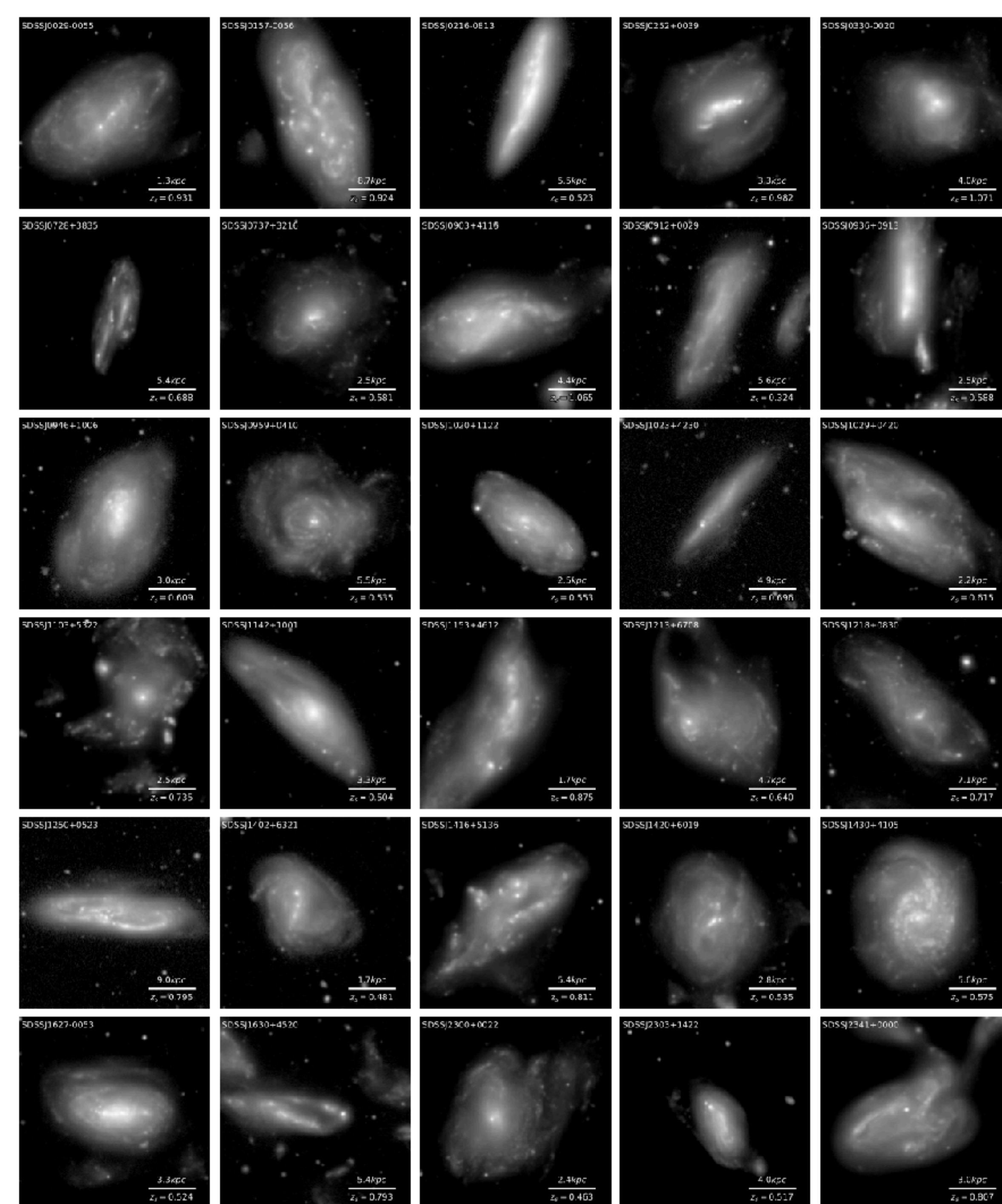
# 'FAMILY PHOTO' OF RECONSTRUCTED GALAXIES $z \sim 0.6$ TO 1 (5 TO 8 BILLION LIGHTYEARS AWAY)



# 'FAMILY PHOTO' OF RECONSTRUCTED GALAXIES $z \sim 0.6$ TO 1 (5 TO 8 BILLION LIGHTYEARS AWAY)



Source: SDSS DR10  
 $z=0.7$  About 6.3 billion light-years away



## FINAL THOUGHTS...

I presented a lot of examples related to strong lensing data analysis, but the structure of many inverse problems in astrophysics and cosmology is similar, and they face similar challenges.

By tackling to the problem of explicit, high-dimensional inference, we have access to our posterior samples, and that allows us to open the black box, and understand where the information in our inference is coming from. This is (one of) the way we are going to build trust in inference methods that incorporate ML.