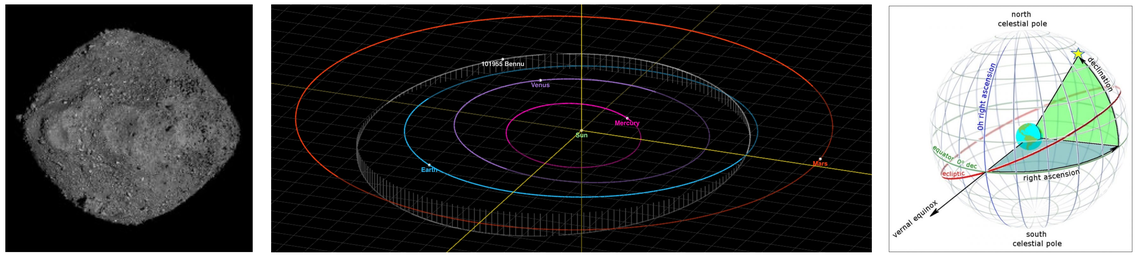
Statistical Methods in Orbit Determination and Planetary Defense

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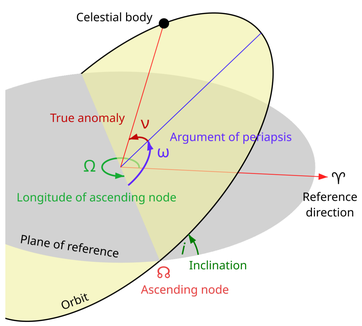
Asteroids and Their Observations



Left: Asteroid **101955 Bennu** is a rubble-pile near-earth asteroid. (NASA/Goddard/University of Arizona) Middle: The orbit of Bennu is depicted as a white ellipse along with the orbits of the inner planets (https://ssd.jpl.nasa.gov/ov). Right: The celestial sphere defining the on-sky coordinate system (University of Iowa).

Asteroids are collections of rocks and ice that are left over remnants from the formation of our solar system.
Asteroids move according to the laws of gravity, orbiting the center of mass of the solar system.
We observe asteroids from the surface of Earth in an on-sky coordinate system at different moments in time. Asteroids have variable positions on the sky over time since they move relative to the Earth. They are observed by telescopes at positions (α, δ) at time t with associated uncertainties $(\sigma_\alpha, \sigma_\delta)$.

Motion Modeling: Orbits



A visualization of orbital parameters (Lasunncty 2023).

We model asteroid motion as two-body Keplerian dynamics about the Sun:

$$\dot{\mathbf{r}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \mathbf{a}(\mathrm{t}, \mathbf{r}, \mathbf{v}, \Psi)$$

A single orbit is parameterized equivalently by

$$\underbrace{(a, e, i, \Omega, \omega, T_p)}_{\textit{Keplerian elements}} \iff \underbrace{\mathbf{x} = (\mathbf{r}_0, \mathbf{v}_0, t_0)}_{\textit{Cartesian state at epoch}}$$

Given N observations, a nonlinear solver finds a locally optimal $\mathbf{x}$ minimizing the cost

$$Q = \boldsymbol{\nu}^T W \boldsymbol{\nu}$$

where $\boldsymbol{\nu}$ is the residual vector and W is the diagonal matrix of inverse observation variances (assuming independence).

Uncertainty on the parameters is estimated from the quadratic expansion of the cost,

$$\Delta Q \approx \Delta \mathbf{x}^T C \, \Delta \mathbf{x} \qquad C = B^T W B \qquad B = \frac{\partial \boldsymbol{\nu}}{\partial \mathbf{x}}$$

Under Gaussian observation noise, ΔQ is chi-squared distributed with 6 degrees of freedom, defining confidence ellipsoids in parameter space.

Point-in-Time Prediction

The dynamics of the system determine how uncertainty evolves. Linearizing around a solution $\mathbf{x}_0$, we predict the state at time $t$via the equations of motion and propagate uncertainty through the Jacobian of the orbital dynamics — the **state transition matrix**:

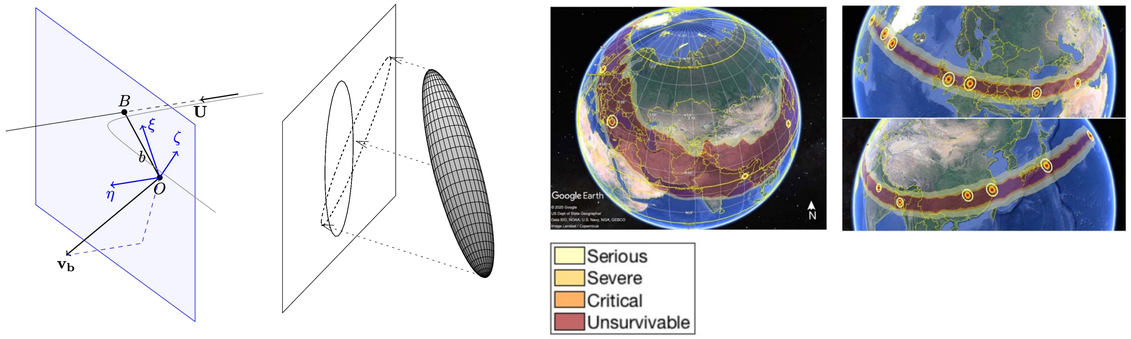
$$A(\mathrm{t}, \mathrm{t}_0) = \frac{\partial \mathbf{x}(t)}{\partial \mathbf{x}_0}$$

The predicted covariance at time t is

$$C_p(t) = A(t, t_0)\, C^{-1}\, A(t, t_0)^T$$

where $C^{-1} = (B^T W B)^{-1}$is the epoch covariance. Observation uncertainty enters via W; dynamical uncertainty propagation enters via A.

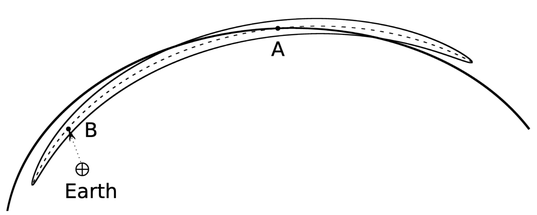
Impact Probabilities



Left: Impact risk is assessed on the **b-plane**, the plane through the target's center perpendicular to the incoming trajectory (F. Letizia et al. 2016). Middle: Uncertainty in the orbit is projected onto the b-plane (D.E. Vavilov 2025). Right: Hazard is assessed through detailed projection onto the surface of the Earth (V. Reddy et al. 2022).

Impact risk is assessed in a few ways:

- **Linear**: the point-in-time covariance is estimated at impact time and projected onto the b-plane. Impact probability is integrated over the intersection region.
- **Monte Carlo**: Synthetic datasets are generated from observational errors to estimate orbital prediction errors. Each orbit is propagated to the impact time and the probability of impact is estimate from the number of intersecting trajectories.
- **Systematic Ranging**: On-sky observations leave only asteroid range (distance at observation time) and range-rate unconstrained. Orbits are generated by densely sampling in range and range-rate. Orbits are weighted by a population model. Useful for short-time impact assessment.
- **Lines of Variations (LOV)**: Uncertainty grows along-track, leading to a 1-D parameterization of uncertainty. Reduces the sampling space and is useful for long-term monitoring.



A visualization of how uncertainty spreads along-track (D.E. Vavilov 2025).

Room for improvement?

- Fundamentally about non-linear transformation of PDFs
- How to obtain speedup and maintain accuracy?