Literature Review of Optimal Transport and Riemannian Geometry

Qiran Sun

Department of Mathematical Sciences, Mellon College of Science, Carnegie Mellon University, USA

qirans@andrew.cmu.edu

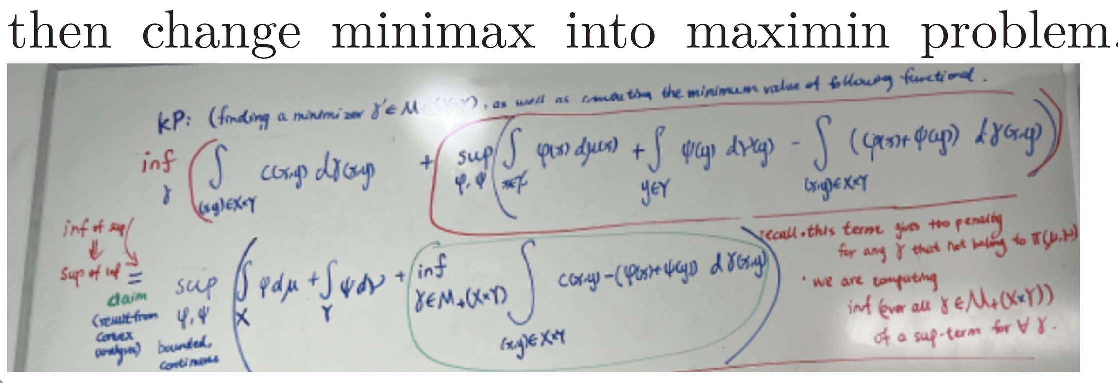
Supervisor: Professor Jonathan Simone, CMU Department of Mathematical Sciences

Introduction

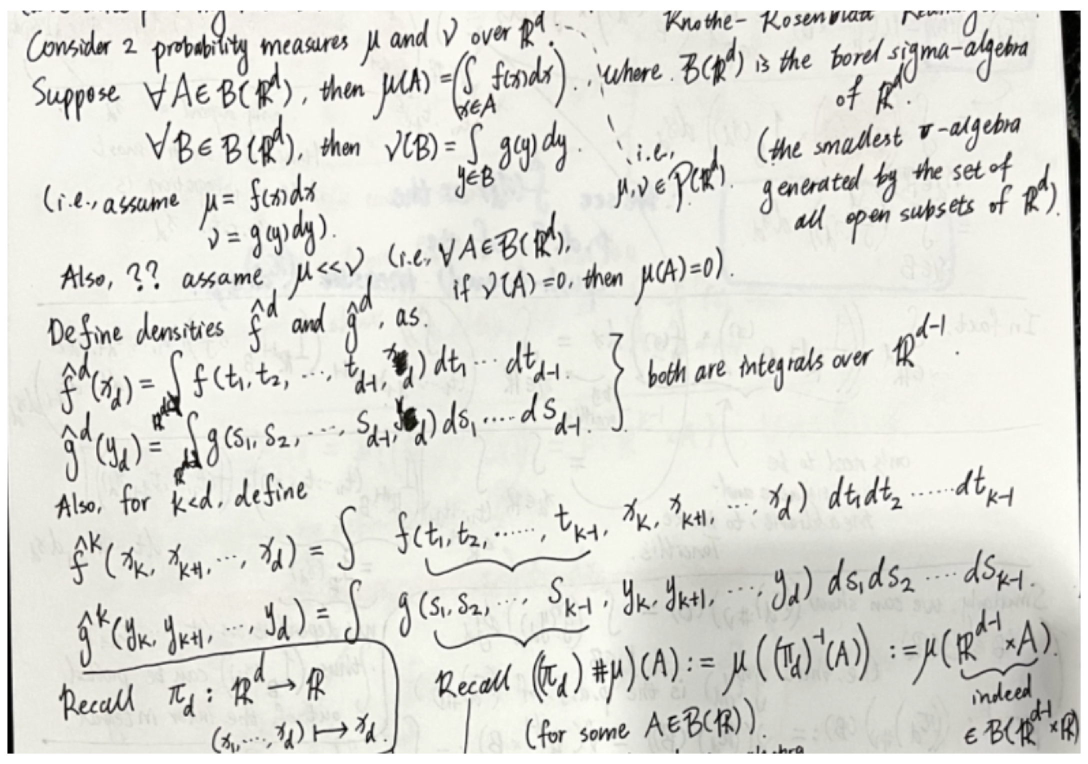
Based on 2 semesters of advanced undergraduate direct reading courses on Optimal Transport and Differential Geometry, I have culminated with a series of notes on respective theories as well as their connections. Main topics include the formulation of Monge Problem, Kantorovich Problem, Dual Problem, as well as a variational formulation of Dual Problem, and the Semi-Dual objective of optimal transport.

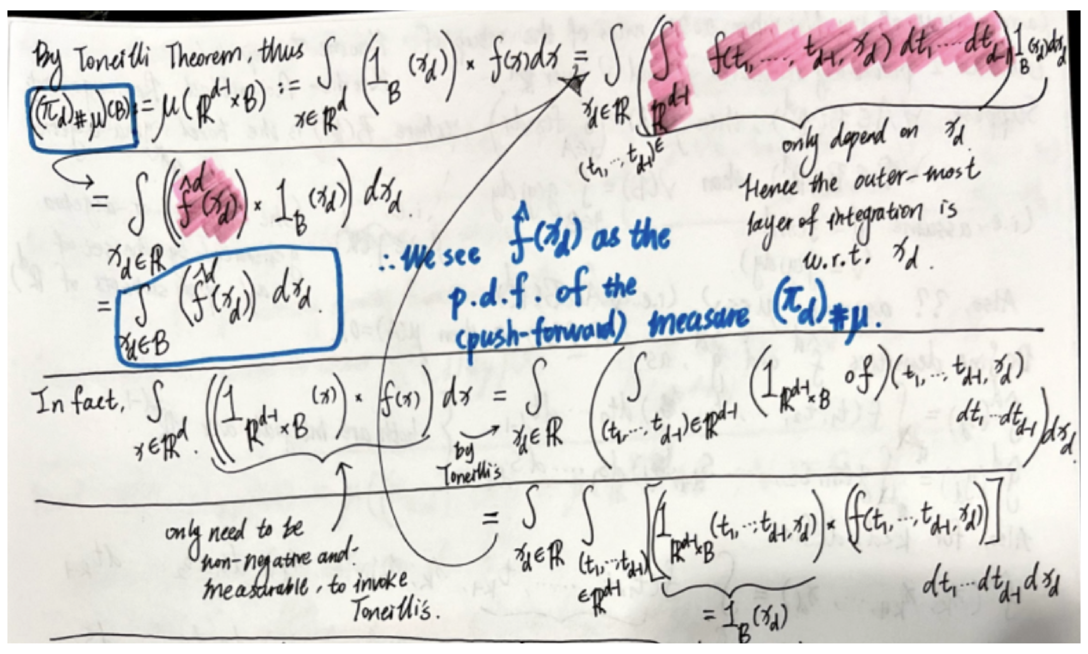
Dual Problem

We rewrite marginal-density constraint on a transference plan into a penalty term, then change minimax into maximin problem.



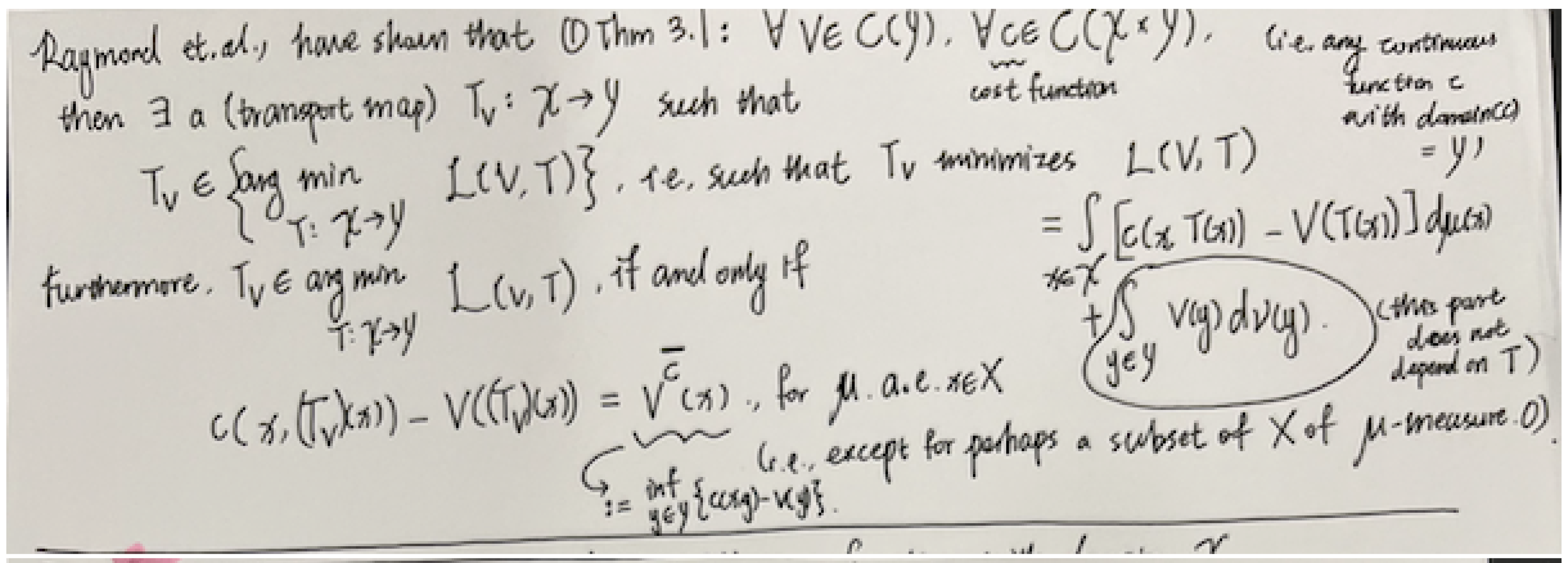
Knothe-Rosenblatt Rearrangement

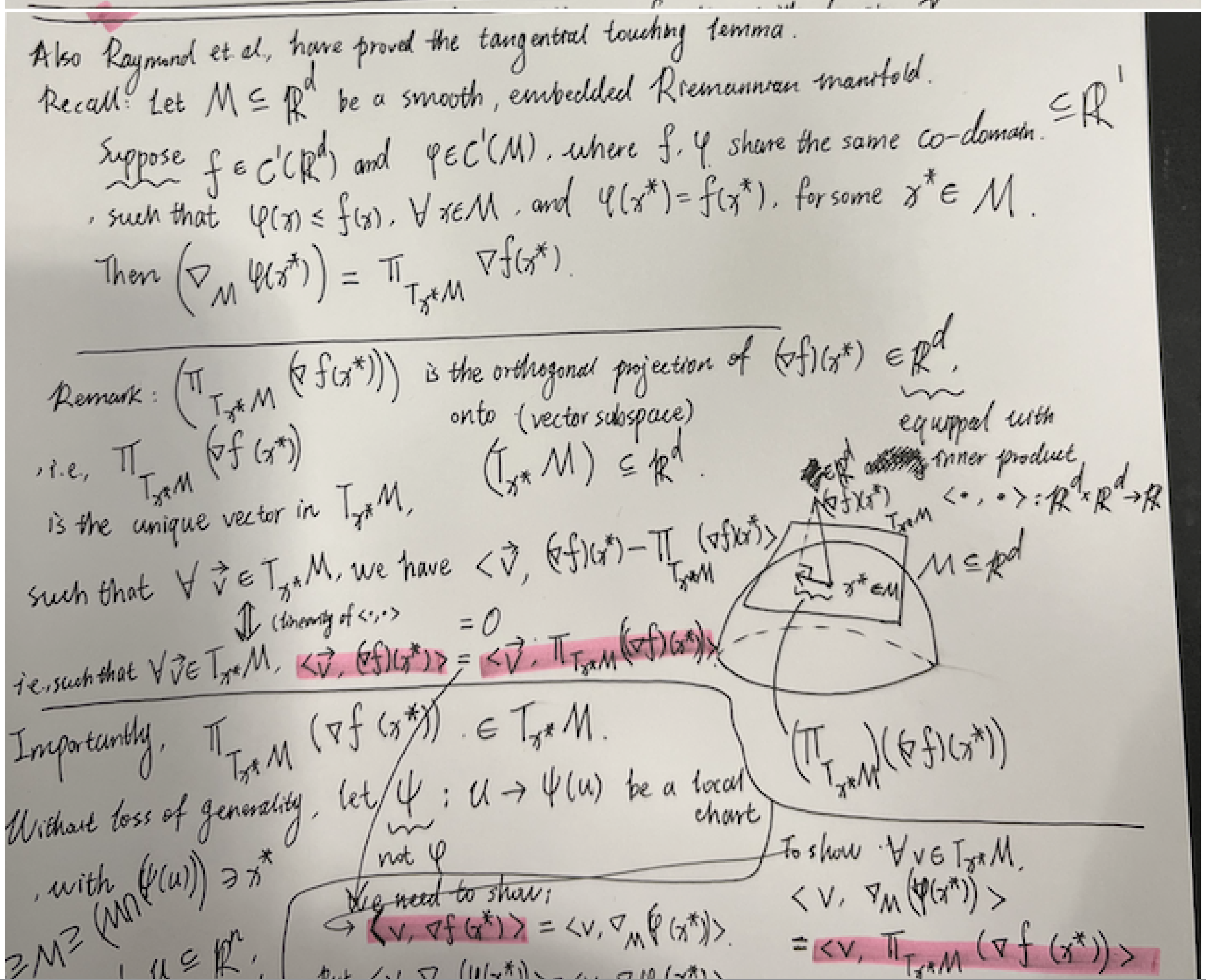




Semi-Dual Objective L(V,T) (Raymond Chu et al., 2026)

Equivalent condition for attaining arginf L(V,T):





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