

Expectations and Aspirations: Explaining Ambitious Goal-setting and Nonconvex Preferences

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Abstract

Research in psychology typically distinguishes between expectations and aspirations, yet economic models that incorporate reference dependence generally assume a single reference point. We propose a dual reference point model in which loss aversion is determined by an expectations-based reference point and value function curvature is anchored relative to an aspiration level or goal. The model provides an explanation of why people often set ambitious goals for themselves, a pattern that, we show, traditional single reference-point models have difficulty accounting for. In addition, the model predicts that people with aspirations above their current attainments will in some cases display nonconvex preferences, and this may help explain empirical observations of such patterns of behavior.

1 Introduction

One of the most significant insights that economics has adopted from psychology is the idea that preferences are reference-dependent - i.e., that utility does not depend on absolute levels of attainments but on attainments relative to salient points of comparison. The assumption of reference-dependence has proven useful in explaining a wide range of economic phenomena, from the disposition effect and the equity premium puzzle in finance to patterns of managed earnings in publicly traded companies' financial reports, and even to labor supply effects of changes in wages.

One of the criticisms of reference dependence that has often been leveled by its critics is the idea that many reference points are possible, such as prior levels of attainments, expected levels and those of other people. While such a multiplicity of reference points may be psychologically realistic, critics complain it introduces almost infinite degrees of freedom to explain phenomena *ex post*.

Addressing this critique, Köszegi and Rabin [41] have proposed a framework in which there is a single reference point defined by an individual's (rational) expectations. Assuming that other possible points of reference operate through expectations, they provide an integrated account of a wide range of phenomena, including some not addressed by prior models (such as the apparent lack of endowment effect exhibited by seasoned traders), albeit at the cost of some additional complexity caused by the endogeneity introduced by the fact that expectations influence behaviors that change expectations. There is empirical evidence coming from both laboratory studies [1, 21] and field studies [15] that people do exhibit loss aversion with respect to an expectations-based reference point.

While applauding the advance in precision introduced by the expectations-based reference point framework, we argue that a single reference point is insufficient to

capture a range of phenomena of great interest to economists and others who are interested in understanding human motivation and behavior. Specifically, we argue that, in addition to expectations, *aspirations* determine a second, distinct, kind of reference point with diverse and significant implications for human behavior.¹

The distinction between expectations and aspirations is an old one in psychology. Almost like Yin and Yang, the two concepts have historically been discussed as if, while distinct, they were inseparable.² Consistent with the dictionary definition, psychologists treat aspirations as a “hope or ambition of achieving something,” as well as “the object of such an ambition; a goal.” The key insight enabled by distinguishing between the two concepts is that one can expect to attain certain positive outcomes without aspiring to those outcomes, and can aspire to positive outcomes while believing one has little, if any, chance of actually achieving them - a possibility that is inherent in the commonly used expression “unrealistic aspirations.” Indeed, part of the goal of our paper is to propose when and why an individual might choose aspirations that are unrealistic, and to trace out the broader ramifications of such a decision, e.g., on motivation.

We present a model in which expectations and aspirations are not only separable, but have distinct consequences for the shape of the utility function. Specifically, separating the two assumed properties of the Prospect Theory value function, we propose a model in which gains and losses are defined relative to exogenous expectations (which often corresponds to the status quo), and diminishing sensitivity is defined relative to an aspiration level (i.e., goal). Following Koszegi and Rabin’s framework [41], there is both intrinsic “consumption utility,” as in standard economic theory,

¹There is a long record of evidence in the literature supporting the argument that aspirations often serve as reference points [56, 57, 72].

²In much of the literature establishing reference-dependent effects (see [52] for example), it is almost impossible to distinguish whether a proposed reference point is an expectation or an aspiration.

and “contrast utility” that incorporates loss aversion and diminishing sensitivity over consumption utilities. Informally, loss aversion is relative to “where you are (or expect to be)” while diminishing sensitivity is relative to “where you feel you should be (or would ideally like to be).” There is, in fact, considerable empirical evidence that multiple reference points operate simultaneously [50, 55, 54, 74] and particularly that individuals utilize both the status quo and an aspiration level within a single decision setting [67, 40, 33], but the role of each reference point has not been distinguished in previous work.

Our model has two major sets of implications. The first, already alluded to, is its ability to explain common patterns of goal-setting, most importantly the common tendency to set ‘unrealistic’ goals that are above one’s expectations. There is a large literature in psychology documenting patterns of goal-setting, almost all of it finding that people set goals above their current level of attainment, and often set goals that are unrealistically ambitious. Most recently, Heath et al. use an aspirational reference point to explain empirical results in the goals literature, e.g., predicting that people will work harder and perform better when they set specific, challenging goals [32].

When examined carefully, however, it can be seen that reference-dependent models incorporating a single reference point have difficulty explaining the ubiquity of ambitious goal-setting. The problem is that, with a single reference point, raising one’s reference point to motivate oneself would result in an immediate decline of utility which would be exacerbated by loss aversion. Such an immediate loss of utility is unlikely to be compensated for by any increase in subsequent attainment, particularly if such an increase leads to further upward adjustments of the reference point.

By decoupling aspirations from loss aversion, in contrast, our model allows an individual to raise their aspiration level without experiencing an immediate, commensurate decline in well-being. The idea that aspirations change value function cur-

vature rather than setting a level that distinguishes between gains and losses means that changes in aspirations will change an individual’s level of motivation, risk aversion and intertemporal substitution, apart from any potential impact on immediate wellbeing, and thus helps to explain the ubiquity of ambitious, and even unrealistic, goal-setting.

The model’s second major implication is that it predicts and explains patterns of nonconvex preferences. We generally assume that people have convex preferences, but in real life we see many behaviors that suggest otherwise. Specifically, we observe gambling, e.g. when people buy lottery tickets at a price well above their expected value; bingeing, e.g. when people immediately blow through an unexpected tax refund with a shopping spree; and unbalanced consumption, e.g. the stereotypical Cadillacs in the ghetto. Economists, and behavioral economists in particular, have proposed mechanisms that explain each of these effects: nonlinear probability weighting can explain gambling; hyperbolic time discounting can predict bingeing; and positional goods and Veblen effects can rationalize the purchase of affordable luxuries and the neglect of other goods. However, no simple model can explain these three behavioral patterns that all can be viewed as manifestations of nonconvex references.³ It is of course possible that these patterns are unrelated, but as Karelis argues in a provocative recent book [38], they tend to go hand in hand – all are disproportionately observed among low income people in affluent societies.

The model presented in this paper can explain these patterns and make predictions about when they occur. Gambling, bingeing, and unbalanced consumption all may arise as utility maximizing choices in certain situations, specifically when feasible

³Unbalanced consumption, simply for the sake of being unbalanced, is the canonical form of nonconvex preferences. We can think of bingeing as a manifestation of nonconvex preferences if we treat consumption in different time periods as independent dimensions of a consumption bundle. Similarly, we can think of risk-seeking as a manifestation of non-convex preferences if we treat the payoffs in different states of the world as independent dimensions of a consumption bundle.

consumption lies below a person’s aspiration level. Traditional models of reference-dependent utility, such as prospect theory, predict nonconvex preferences (arising from a convex utility function) over losses, but standard convex preferences over gains (due to a concave utility function) and rejection of fair gambles (due to loss aversion) [37]. When the status quo is the sole reference point, these predictions are independent of income and wealth. However, empirical evidence suggests that poor people are disproportionately inclined towards risk and uneven consumption. They are more likely to play the lottery [28], adopt unhealthier, riskier diets and smoking habits [48, 14], overspend and consume unsustainably upon paycheck receipt [65, 35, 53], and have lower savings rates [18, 10]. Karelis identifies five behavioral patterns common among the poor and contributing to the poverty cycle: not working much for pay; not getting much education; not saving for a rainy day; abusing alcohol; and taking risks with the law [38]. The source of such behavior, he argues, is that poor people tend to have increasing marginal utility of consumption, as their consumption is relieving misery rather than creating pleasure, and people exhibit diminishing marginal sensitivity to both misery and pleasure [38]. Debraj Ray independently argues that poor people with a “window” into the lives of the affluent often have large *aspirations gaps* – i.e., large distances between their current standard of living and their aspirations, which tend to cause them to fail to realize their aspirations [59]. Our model links Karelis’ insight – increasing marginal utility of consumption – with Ray’s insight – a large aspirations gap. This helps to explain why poor people are more likely than others to display nonconvex preferences, materializing as risk-seeking behavior, bingeing, and unbalanced consumption patterns.

The paper is organized as follows. Section 2 lays out the proposed modified utility function with aspirations and the status quo each serving as a distinct kind of reference point. We obtain general results about the utility function in Section 3,

describing its shape and establishing that preferences exhibit a status quo bias. In Section 4 we show that nonconvex preferences may arise when feasible consumption is below the aspiration level, so that people in certain situations (often poor people) prefer unbalanced consumption, bingeing at various points in time, and playing the lottery. In Section 5 we show that endogenous goal setting above one’s current level of attainment is significantly more robust with the distinction between aspirations and expectations than in a traditional prospect theory account. Section 6 concludes. All proofs are contained in the Appendix.

2 Reference Dependence

Let x be a multidimensional consumption bundle. Let r^0 be the status quo consumption bundle, and let r^* be an aspiration level (on each dimension) that the decision maker uses as a point of comparison.

As in [41, 39], a person has a “consumption utility” function $m(x)$ that corresponds to the traditional notion of outcome-based utility and also has “contrast utility” $n(x|r^0, r^*)$ (analogous to “gain-loss utility” in [41]), so that overall utility is given by $u(x|r^0, r^*) = m(x) + n(x|r^0, r^*)$. Both components of utility are separable across dimensions, $m(x) = \sum_h m_h(x_h)$ and $n(x|r^0, r^*) = \sum_h n_h(x_h|r_h^0, r_h^*)$.⁴ Along each dimension, contrast utility depends on the consumption utility of bundle x relative to the consumption utility of r^0 and r^* . Gains are distinguished from losses by comparing consumption utility to that of the status quo. Differences in consumption utility are

⁴The assumption of additively separable utility, as in [41], is for simplicity and to prevent the model from having too many degrees of freedom. It may be more palatable to think of the separable dimensions as hedonic dimensions representing attributes that people care about. In this case, consumption bundles (x , r^0 , and r^*) are presumably transformed by a linear operator into these vectors of attributes that people care about. More generally, if we are willing to accept the extra degrees of freedom, our model could just as well be formulated with $m(x) = \sum_h m_h(x)$, where $m_h(\cdot)$ is a function of the entire consumption bundle and represents the component of consumption utility along hedonic dimension h .

perceived relative to the aspiration level. Thus,

$$n_h(x_h|r_h^0, r_h^*) = \mu \left(v(m_h(x_h) - m_h(r_h^*)) - v(m_h(r_h^0) - m_h(r_h^*)) \right), \quad (1)$$

where

1. $v'(\cdot) > 0$;
2. for $s > 0$, $v''(-s) > 0$ and $v''(s) < 0$;
3. $\mu(s) = \begin{cases} s & \text{if } s \geq 0 \\ \lambda s & \text{if } s < 0 \end{cases}$ and $\lambda \geq 1$.

The function v captures diminishing sensitivity relative to the aspiration level r^* (on each dimension). It is increasing in the consumption utility of the bundle being compared to r^* , convex for consumption utility below that of r^* and concave for consumption utility above that of r^* . The function μ captures (constant) loss aversion relative to the status quo r^0 (on each dimension) [70]. Whenever the consumption utility of the bundle is less than that of the status quo, and thus contrast utility is negative, the weight on this contrast utility increases by a factor of λ , the coefficient of loss aversion.

Figure 1 below, showing an example of the full utility function with various aspiration levels, illustrates three major features of our formulation. First, in the special case that $r^* = r^0$, Equation 1 reduces to traditional reference dependence [37], as with the prospect theory value function acting on consumption utility as in Köszegi and Rabin's model [41]. If, additionally, consumption utility $m(\cdot)$ is linear, (a good approximation when stakes are low), then taking $r^* = r^0$ aligns our utility function with the standard prospect theory value function defined directly on outcomes [41]. Second, changes in the aspiration level r^* affect the utility of departures from

the status quo, but do not affect the utility of being at the status quo itself. This distinction turns out to be critical in Section 5, which lays out implications of this model for endogenous reference point selection. In traditional models, while raising the reference point generally increases an individual’s motivation, it also leads to an abrupt decrease in present utility. Because raising one’s aspiration level does not affect present utility but does affect the utility of alternative levels of consumption, this model predicts aspiration levels elevated for motivational purposes in a much wider range of situations. Third, changes in the aspiration level r^* do not result in simple monotonic transformations of the utility function. Reflecting the intuition and common finding in psychology that insufficiently or excessively ambitious goals can be demotivating, in our formulation intermediate aspiration levels will lead to higher marginal utility than aspiration levels at either extreme. The reach of aspirations extends beyond determining an individual’s level of motivation to domains of risk-taking, intertemporal consumption and, more generally, any decision likely to result in changes from the status quo.

Observe that Equation 1 is invariant under constant shifts in v . The magnitude of v determines the relative importance of contrast utility as compared with direct consumption utility. This can be made explicit by taking a factor of $\frac{\alpha}{1-\alpha}$ out of v , for $0 < \alpha < 1$. As overall utility may be rescaled by a constant factor, we can formulate normalized overall utility as

$$u(x|r^0, r^*) = (1 - \alpha)m(x) + \alpha \sum_h \mu \left(v (m_h(x_h) - m_h(r_h^*)) - v (m_h(r_h^0) - m_h(r_h^*)) \right). \quad (2)$$

The parameter α allows us to capture the salience of one’s aspirations, which we should carefully distinguish from the levels of these aspirations. We can imagine two people, both with high aspiration levels, but for one it’s an abstraction while for the

other it has deep significance, and we'd expect these people to behave quite differently. We consider α to be a constant across all dimensions (for parsimony), but an extension of the model would allow a vector α with heterogeneous elements to distinguish the salience of different aspirations. Besides capturing individual differences in the prominence of one's aspirations, introduction of the parameter α makes transparent the fact that our formulation reduces to standard consumer choice at the extreme ($\alpha = 0$). Similar to many behavioral economic theories, ours is an extension of traditional theory, not a rejection of it.

Figure 1 aids in interpreting Equation 2. It shows utility as a function of consumption on a single dimension while varying the aspiration level and adopting the following specifications: concave direct consumption utility ($m(x) = x + \ln x$), heavy weight on the contrast utility component ($\alpha = .8$), slightly exaggerated loss aversion ($\lambda = 5$), and diminishing sensitivity taking the form of a logistic function $v(s) = \frac{10}{1+e^{-.5s}}$. Later we return to Figure 1 to illustrate how risk seeking emerges from regions of convex utility given aspiration levels moderately above the status quo.

Applying the dual reference point utility model to each of the three domains that are our central focus – tradeoffs among multidimensional bundles, intertemporal consumption, and risky decisions – is straightforward. An intertemporal consumption profile $\mathbf{x} = (x(0), \dots, x(t), \dots)$ is evaluated according to its aggregate discounted utility relative to a sequence of aspiration levels and a status quo that is updated every period:

$$\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*) = u(x(0)|r^0, r^*(0)) + \sum_{t=1}^{\infty} D(t)u(x(t)|x(t-1), r^*(t)) \quad (3)$$

where $D(t)$ is the person's discount function.⁵ A stochastic outcome F is evaluated

⁵Intertemporal choice with reference-dependent utility is modeled somewhat differently in [42].

Reference-Dependent Utility:
Varying Aspiration Level Above and Below Status Quo

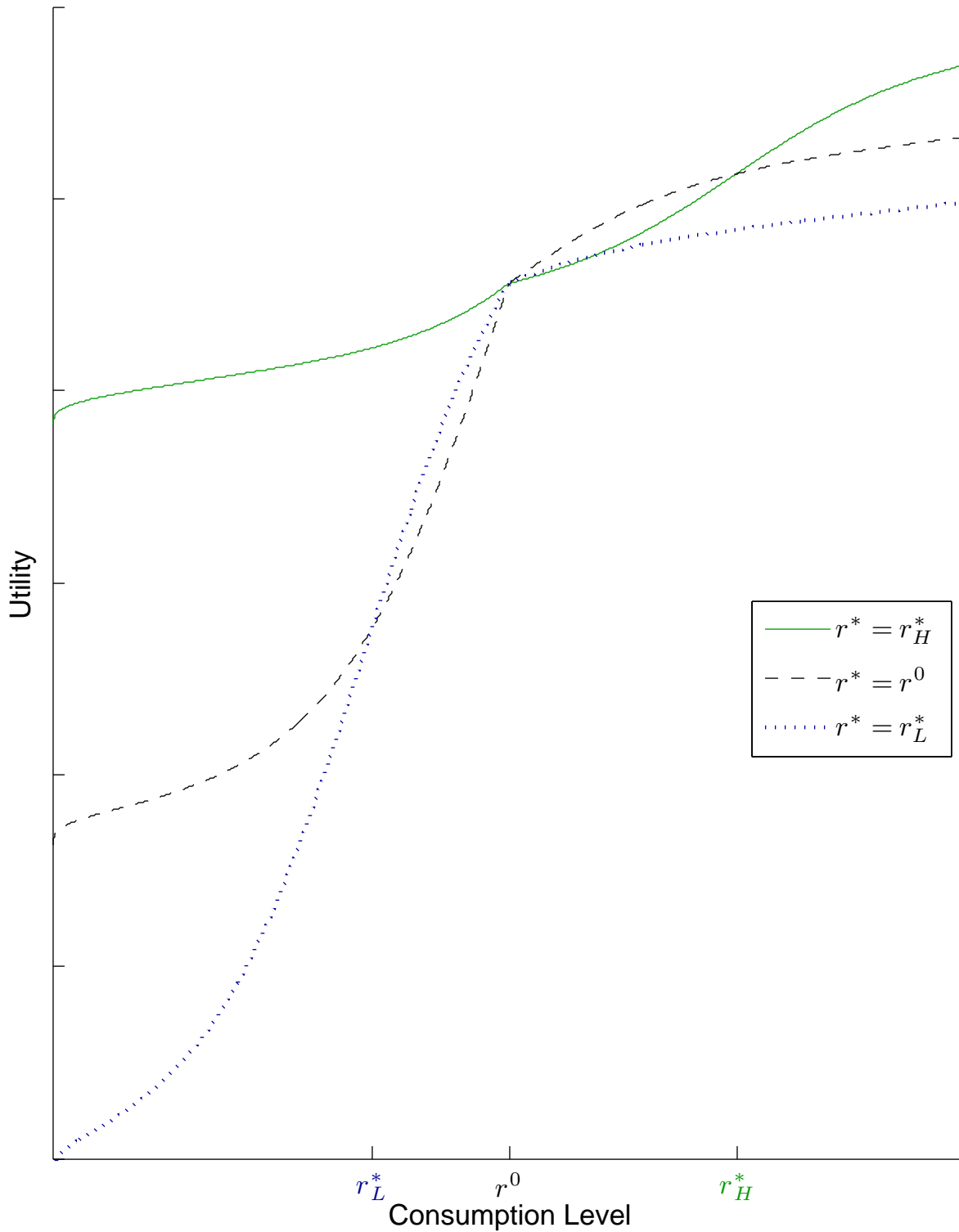


Figure 1: Utility as a function of consumption, adopting concave direct consumption utility and varying the aspiration level above and below the status quo.

according to its expected utility,

$$U(F|r^0, r^*) = \int u(x|r^0, r^*) dF(x). \quad (4)$$

A straightforward extension could accommodate subjective non-additive probabilities or decision weights as in cumulative prospect theory [71, 62, 27, 11].⁶

In applications prior to Section 5 (in which the aspiration level itself is a choice variable), the status quo r^0 and the aspiration level r^* are both given exogenously. To apply our model, these reference points must be observed, inferred, or induced. Presumably, the status quo is readily apparent in most applications. The aspiration level could be elicited directly (i.e., asking subjects for a self-reported goal) or induced by exposing subjects to a role model with observable performance or consumption (and perhaps checking through direct elicitation that this social comparison serves as the subjects' aspiration level).⁷

3 Comparative Statics of Reference Points

Following Köszegi and Rabin [41], we can examine the effects of varying reference points. Changing the status quo affects utility unambiguously, but changing the aspiration level has more nuanced effects. Proposition 1 establishes that holding outcomes fixed, people always prefer a poorer status quo reference point and (ignoring the motivational effects introduced in Section 5) prefer a lower aspiration level when moving toward this goal, but a higher aspiration level for gains when above this

⁶Stochastic reference points could be incorporated just like stochastic outcomes, as in [41], but our results do not require this additional complication.

⁷A revealed preference approach could be used to infer the aspiration level if consumption utility is assumed to be linear [63], but we cannot allow an extra degree of freedom in choosing this reference point when testing the model in the first place.

level or for losses when below it.⁸ Intuitively, a poorer status quo makes gains more attainable. Additionally, people prefer a more distant aspiration level when faced with losses and a nearer aspiration level when they can gain because closer to the aspiration level they are more sensitive to changes. Thus, whether a person wants to raise her aspiration level or not depends a lot on whether she can shift consumption to it.

Proposition 1

1. For all x , r^* , r^0 , and \tilde{r}^0 such that $m_h(\tilde{r}_h^0) \geq m_h(r_h^0)$ for all h ,

$$u(x|r^0, r^*) \geq u(x|\tilde{r}^0, r^*).$$

2. For any x , r^0 , r^* , and \tilde{r}^* such that for all h , either

- (a) $m_h(x_h) \geq m_h(r_h^0) \geq m_h(r_h^*) \geq m_h(\tilde{r}_h^*)$;

- (b) $m_h(r_h^*) \geq m_h(\tilde{r}_h^*) \geq m_h(r_h^0) \geq m_h(x_h)$;

- (c) $m_h(r_h^0) \geq m_h(x_h) \geq m_h(\tilde{r}_h^*) \geq m_h(r_h^*)$; or

- (d) $m_h(\tilde{r}_h^*) \geq m_h(r_h^*) \geq m_h(x_h) \geq m_h(r_h^0)$,

$$u(x|r^0, r^*) \geq u(x|r^0, \tilde{r}^*).$$

In part 2a, the attainable outcome is a gain, and among aspiration levels below the status quo, the higher (nearer) aspiration level is preferred to the lower (more distant) one. Thus, for example, a “B” student aspiring to maintain her grades above a C average would derive more utility from an A grade than would the student if she were only aspiring to maintain her grades above a D. In part 2b, the attainable outcome

⁸We might think of aspiration levels below the status quo as avoidance goals, whereas aspiration levels above the status quo represent approach goals [20].

is a loss, and among aspiration levels above the status quo, the higher (more distant) aspiration level is preferred to the lower (nearer) one. Returning to the student example, a “C” student who gets a D would feel worse if she was aiming for a B than if she was trying for an A. This “what-the-heck” effect [47] has been documented in psychology with the observation that dieters more easily accept falling far short of their aspirations than just missing them [60]. In part 2c, again in the loss domain but with aspiration levels below the status quo, the more distant aspiration level is still preferred, but now that is the lower one. An “A” student trying merely to maintain grades above a D average would be less disappointed with a B than had she been aspiring to stay above a C. Lastly, in part 2d, with a gain approaching an aspiration level above it, the lower (nearer) aspiration level is preferred, e.g., a “D” student who receives a C would be more pleased with this progress if she was aiming for a B than if she was trying for an A.

Proposition 1 tells us that the status quo, as an expectations-based reference point, has a direct impact on well-being. A lower status quo makes any outcome look better in comparison, and the opposite goes for a higher status quo. Proposition 1 also tells us that aspirations, on the other hand, do not impact static well-being, but rather affect the utility associated with changes in consumption. Thus, as we suggested in the introduction, high aspirations can motivate a person without making her miserable. This insight underlies our finding in Section 5 that endogenous goal setting above one’s current level of attainment is significantly more robust given our separate treatment of aspirations and expectations than in a traditional prospect theory account.

Despite the additional complexity of having an aspirational reference point that may depart from the status quo, we still obtain the core implication that preferences exhibit a status quo bias. Proposition 2 establishes this status quo bias.

Proposition 2 *Assume $\lambda > 1$. For any consumption bundles x, x' , and r^* , $u(x|x', r^*) \geq$*

$u(x'|x', r^*) \implies u(x|x, r^*) \geq u(x'|x, r^*)$, with equality if and only if $m_h(x_h) = m_h(x'_h)$ for all h .

In the presence of loss aversion, $\lambda > 1$, a person who is willing to abandon the status quo to move to an alternative (materially different) consumption bundle must strictly prefer this alternative bundle when it becomes the new status quo.

Proposition 3 describes the shape of the utility function. When consumption utility is an increasing affine function of the consumption bundle, overall utility on each dimension is increasing, concave for consumption above the aspiration level and locally convex for consumption below this level, but with a kink at the status quo that disrupts convexity there. Recall Figure 1 and see Figures 2 and 3 below, which show overall utility when the consumption utility function is slightly concave. The graphs are very similar in shape to the overall utility curves when consumption utility is linear.

Proposition 3 *If each m_h is a strictly increasing affine function, then overall utility $u(x|r^0, r^*)$ as a function of the consumption bundle, holding fixed the status quo and the aspiration level, satisfies the following properties:*

1. Monotonicity: *if $x_h \geq x'_h$ for all h with the inequality strict for some h , then $u(x|r^0, r^*) > u(x'|r^0, r^*)$.*
2. Slope discontinuity: $\lim_{\Delta x \searrow 0} \frac{u_{x_h}(r^0 - \Delta x \hat{\mathbf{e}}_{\mathbf{h}}|r^0, r^*)}{u_{x_h}(r^0 + \Delta x \hat{\mathbf{e}}_{\mathbf{h}}|r^0, r^*)} = \tilde{\lambda}$ for some $\tilde{\lambda} \in (1, \lambda)$ if $\lambda > 1$ and $\tilde{\lambda} = 1$ if $\lambda = 1$, using the subscript x_h to denote a partial derivative with respect to this variable and $\hat{\mathbf{e}}_1, \dots, \hat{\mathbf{e}}_{\mathbf{H}}$ to denote the unit basis vectors. *As in conventional prospect theory, there is a kink at the status quo.*
3. Local curvature: *$u(x|r^0, r^*)$ is locally convex on $\{x : x_h < r_h^* \text{ and } x_h \neq r_h^0 \text{ for all } h\}$ and is concave on $\{x : x_h > r_h^* \text{ for all } h\}$. (Note that this local convexity does*

not imply global convexity of the utility function over $\{x : x_h < r_h^ \text{ and } x_h \neq r_h^0 \text{ for all } h\}$ because this domain is not a convex region. We can only guarantee global convexity of $u(x|r^0, r^*)$ on convex subsets of this region.)*

In general, $u(x|r^0, r^*)$ could be either convex or concave on a given domain. If marginal consumption utility is diminishing in the consumption level, i.e., if $m(\cdot)$ is concave, then $u(x|r^0, r^*)$ must be concave on $\{x : x_h > r_h^* \text{ for all } h\}$ as composition of functions preserves concavity here, but $u(x|r^0, r^*)$ could still be convex or concave on other domains. Consider Figure 2 below, which adopts the same specification as Figure 1 previously. Utility is concave for low consumption levels because of the strong concavity of the direct consumption utility component in this domain. The curvature of the contrast utility function soon begins to dominate and there is a region of convex utility. There is always a kink at the status quo, which guarantees risk aversion over small scales there. And then utility becomes concave again at some consumption level just short of the aspiration level.⁹ Note that adopting an alternative specification with the direct consumption utility function more strongly concave might eliminate the region of convex utility altogether. Later, in Figure 3, we see that when the weight on direct consumption utility is high, its concavity may always dominate.

Figure 1 (introduced earlier) as well as Figures 2 and 3 illustrate the impact of varying the model's parameters. Figure 1 shows utility, varying the aspiration level above and below the status quo. We see that the utility function with an aspiration level below the status quo is not that different from traditional reference-dependent utility with the reference points for loss aversion and for diminishing sensitivity coinciding. Interesting effects arise when the aspiration level is above the status quo

⁹Concave direct consumption utility lowers the inflection point of the overall utility function somewhat below the aspiration level.

Reference-Dependent Utility: Varying Aspiration Level Above Status Quo

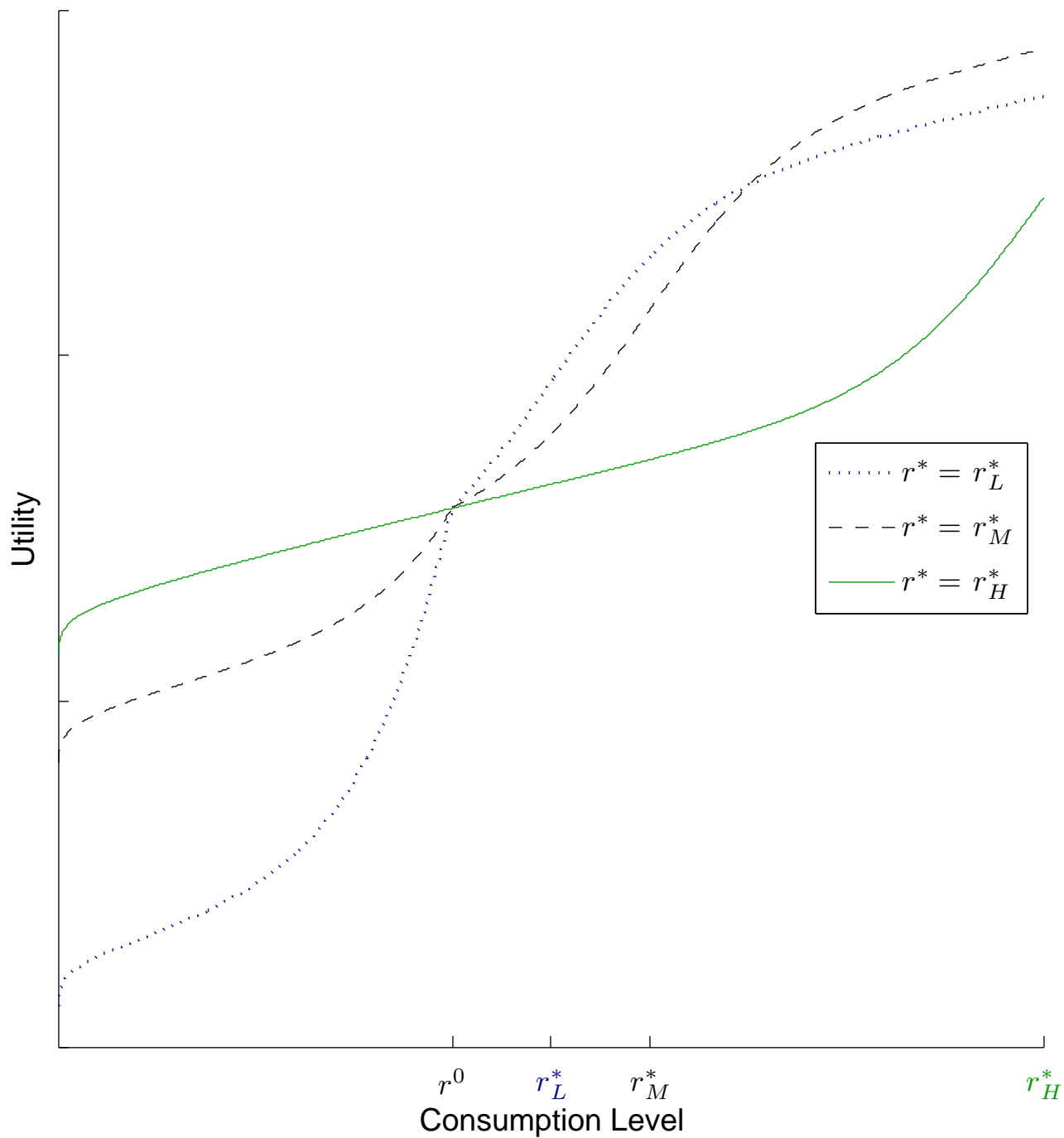


Figure 2: Utility as a function of consumption, adopting concave direct consumption utility and varying the height of the aspiration level above the status quo.

because now we have a region of convex utility over gains. Thus, a necessary, but not sufficient, condition for some fair gambles to be accepted is that the aspiration level must be above the status quo.¹⁰ Even with an aspiration level above the status quo, however, gambles at low enough stakes are rejected because of loss aversion and large enough gambles are rejected because the utility function is concave both at very high and at very low levels of consumption.

Figure 2 shows utility varying how far above the status quo the aspiration level is. There is a sweet spot for aspiration levels above the status quo conducive to gambling: when the aspiration level is only slightly above the status quo, local convexity of the contrast utility function cannot overcome loss aversion; when the aspiration level is too far above the status quo, the contrast utility function is almost flat, and only gambles that promise a chance of unreasonably large gains would be accepted; but, when the aspiration level is significantly, yet attainably, above the status quo, some fair gambles appear quite attractive. The utility function with an aspiration level above the status quo has different implications depending on how high above the status quo the aspiration level is. A moderately high aspiration level might be conducive to playing the stock market, whereas an extremely high aspiration level might be more attuned to the lottery. Following Ray [59], we expect the latter pattern to be especially representative of poor people.

To understand why low income individuals should be more prone to holding aspirations that are above their current level of attainment, it is useful to think of where aspirations come from. Undoubtedly the most important determinant of aspirations is social comparison; people tend to aspire to what others possess or achieve, regardless of whether they think it is likely that they themselves will achieve possess or

¹⁰When the status quo meets one's aspirations, rather than falling short, we might reasonably think that the individual is (or has been) more satisfied, and our prediction of greater risk aversion in this context conforms with the empirical finding that individuals in a good mood tend to reject as too risky gambles that are acceptable to subjects in a bad mood [36].

achieve those things. This is, of course, the key insight of Duesenberry's [17] Relative Income Hypothesis, Leibenstein's [45] concept of "Veblen effects" (drawing on the earlier work of Veblen [73]), diverse empirical and theoretical work in economics (e.g., [19, 9, 23, 12]), and numerous empirical and theoretical papers in psychology (e.g., [22, 16]). Resulting in part from the fact that poorer people are more likely than more affluent people to encounter people who are wealthier than they are (e.g., in the work-place) and are more likely to encounter media depictions and news accounts of people who are wealthier than they are, it is natural that poorer people would tend to have relatively high aspiration levels. Another important determinant of aspirations is past attainments. Once again, as a result of regression to the mean, people who are relatively poor are likely to have been more affluent in the past, while people who are relatively affluent are likely to have been poorer in the past. This (as well as a similar effect applying to expectations) is, of course, the main assumption in Milton Friedman's Permanent Income Hypothesis that causes lower income individuals to have higher marginal propensities to consume [26]. Lastly, one can think of aspirations arising, in part, from where one thinks one should be, based on personal characteristics such as gender, level of education, race, etc. Thus, for example, one would expect, and indeed Clark and Oswald [12] find evidence supporting, that people who are more educated have higher income aspirations (though Clark and Oswald also observe lower levels of well-being, which would also indicate expectations of higher salaries commensurate with higher education). Again, based simply on regression to the mean, we would expect that people with low incomes will on average have characteristics that would predict incomes higher than what they actually earn.

The utility curves in Figure 2 with the aspiration level above the status quo bear a striking resemblance to a utility function proposed early on by Markowitz [51]. Both exhibit risk seeking for small gains and risk aversion for large gains, but the attraction

to risk is interrupted at the origin (i.e., the point of “customary wealth” or the status quo) due to loss aversion. Markowitz proposed such utility curvature more than half a century ago to provide an explanation for observed patterns of risk preference. Here, we construct very similar utility curves based on psychological insights into the distinct effects produced by aspirations and status quo reference dependence.

Figure 3 varies the weight on the contrast utility component (either $\alpha = .2$ or $\alpha = .8$) while retaining the specification of Figures 1 and 2 in other respects. Figure 3 shows that even with an aspiration level above the status quo, nonconvex preferences arise only for salient aspiration levels. To some degree we might expect high aspiration levels across the income spectrum, but their salience to be higher for poor people. This would also lead to the prediction that poor people are more likely to exhibit nonconvex preferences.

While it is difficult to make blanket statements characterizing decision makers in our model as risk seeking or risk averse, there are certain situations in which we can identify risk preferences. For example, the presence of loss aversion in our model leads to a prediction of risk aversion for small enough gambles relative to the status quo. Concavity of both the contrast utility and consumption utility functions for consumption levels above the aspiration level also produces convex preferences (i.e., risk aversion) in this domain. In the following section, we identify situations in which nonconvex preferences (i.e., risk attraction) arise.

4 Nonconvex Preferences

Nonconvex preferences underlie our results in each of the next three subsections, in which we identify conditions under which we can expect to see unbalanced consumption, bingeing, and lottery playing, respectively. According to our model, these three

Reference-Dependent Utility:
Varying Weights on Consumption Utility and Contrast Utility

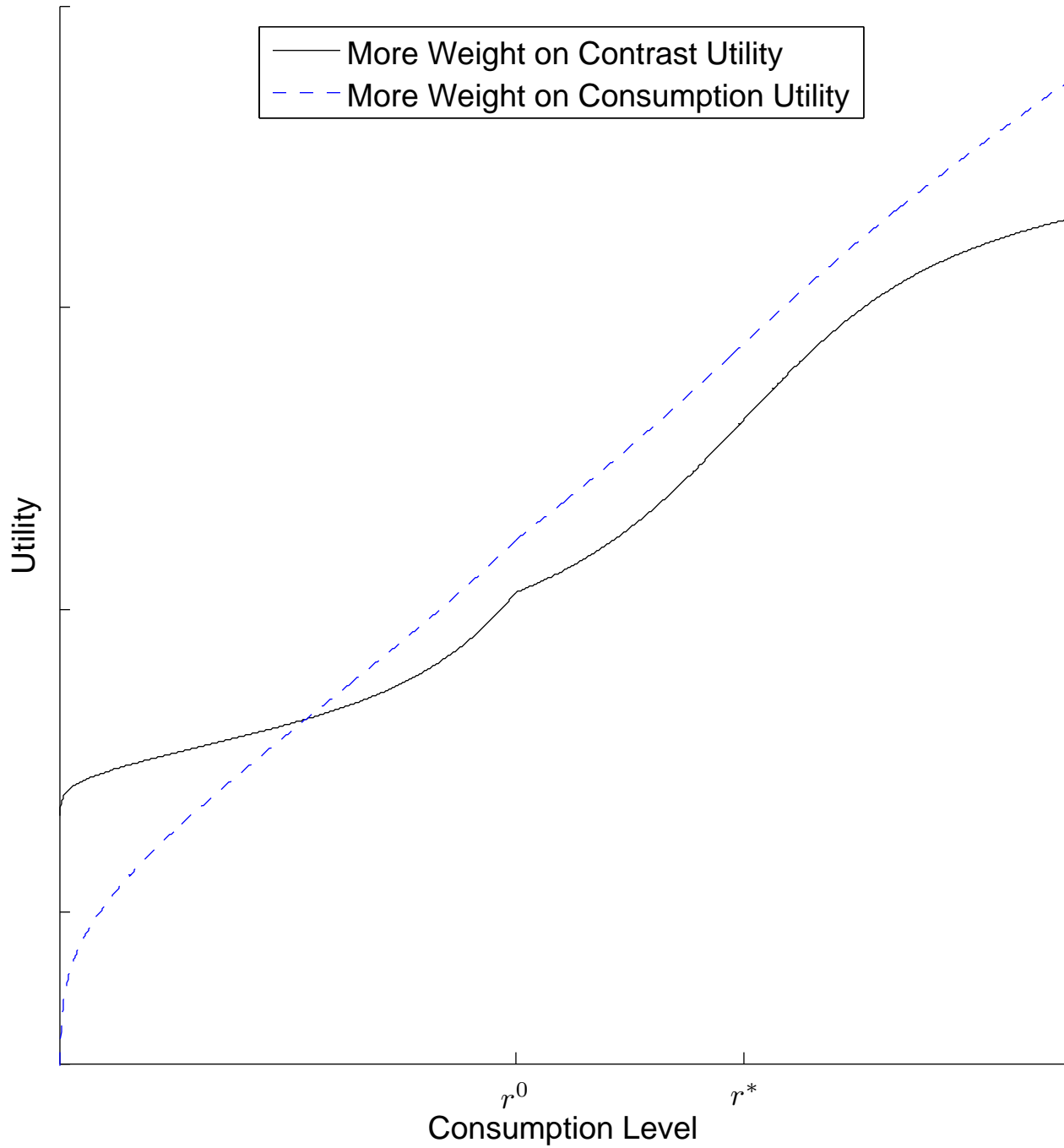


Figure 3: Utility as a function of consumption, adopting concave direct consumption utility and an aspiration level above the status quo and varying the relative weights on consumption utility and contrast utility.

forms of nonconvex preferences all arise from convexity in the utility function that can occur (depending on parameter values) when there is a potential region of consumption that is above the individual's current level, but below the aspiration level. The model helps to explain why poor people, for whom actual consumption cannot reach their aspirations, are most susceptible to these perverse patterns of behavior. Moreover, it allows us to predict that, specifically, poor people living in more economically integrated communities (or simply in richer communities) should display nonconvex preferences more often than similarly poor people in economically segregated communities (or in homogeneously poor communities). Exposure to better-off individuals would raise one's aspiration levels. Poor people with windows into the lives of the affluent tend to see little marginal value in an incremental change in consumption, but larger marginal benefits from a bigger increase. They gain more from a small chance of consumption that realizes their aspirations, or from temporarily realizing them, or from realizing just a single aspiration on one dimension of consumption, than from boosting consumption more evenly. Nonconvex preferences, not over losses relative to the status quo, but over outcomes that are all well below a person's aspirations, leads to risk-seeking behavior, bingeing, and unbalanced consumption patterns.

4.1 Unbalanced Consumption

Astute observers may have noticed the prevalence of billboards advertising fancy cognacs in poor urban areas, but assumed that the placement was simply an error made by executives headquartered at distant locations. After all, a luxury good has lower demand in low income areas, and cognac certainly seems like a luxury. In fact, such placements are quite deliberate and reflect an important insight known well to marketers: people at all income levels desire luxury; those at lower income levels, however, are more likely to seek out what are sometimes labeled "affordable

luxuries” [5]. Affordable luxuries tend to be luxury goods in the technical sense (of having high income elasticity of demand) only at low income levels. Of course, the existence of substitute goods is one reason an affordable luxury may cease to be a luxury good at higher income levels. But for broader categories of goods that are independent of each other, as we are implicitly dealing with when utility is separable across different dimensions of consumption, the existence of affordable luxuries must be a feature of utility function curvature. Our model helps to explain the prevalence of affordable luxuries, even in situations when, though affordable in some sense, their purchase in fact places severe pressures on other dimensions of consumption. The different dimensions of consumption can be substitutes, not because of interdependent consumption utility, but because it may be impossible to simultaneously attain the aspiration level on every dimension.

Our next theorem identifies conditions under which extremes of consumption on different dimensions is preferred to moderate consumption on all dimensions. Once again, this occurs when the aspiration level is better than either outcome in the choice set on every dimension. We also assume that outcomes either are always in the domain of gains or always in the domain of losses.

For ease of notation, let $\Delta_h(x|r^*) = m_h(x_h) - m_h(r_h^*)$. Given consumption bundles x^1 and x^2 and aspiration level r^* , define the permutations σ^1 and σ^2 of $(1 \dots H)$ so that both $\Delta_{\sigma^1(h)}(x^1|r^*)$ and $\Delta_{\sigma^2(h)}(x^2|r^*)$ are weakly increasing in h .

Theorem 1 *Suppose $m(x^1) = m(x^2)$, and for $x \in \{x^1, x^2\}$, either:*

1. $m_h(x_h) \leq m_h(r_h^0)$ and $m_h(x_h) \leq m_h(r_h^*)$ for all h ; or
2. $m_h(r_h^0) \leq m_h(x_h) \leq m_h(r_h^*)$ for all h .

Then $u(x^1|r^0, r^*) > u(x^2|r^0, r^*)$ if

$$\sum_{h=1}^{\tilde{h}} \Delta_{\sigma^1(h)}(x^1|r^*) \leq \sum_{h=1}^{\tilde{h}} \Delta_{\sigma^2(h)}(x^2|r^*) \quad (5)$$

for all $\tilde{h} \in \{1 \dots H\}$ with the inequality strict for some \tilde{h} .¹¹

Equation (5) in Theorem 1 corresponds to the preferred outcome x^1 having more extremes of consumption than x^2 because x^1 has consumption on some dimension that is more disappointing than x^2 ever is ($\min_h \{m_h(x_h^1) - m_h(r_h^*)\} \leq \min_h \{m_h(x_h^2) - m_h(r_h^*)\}$) and also has consumption on some other dimension that is more pleasing than x^2 ever is ($\max_h \{m_h(x_h^1) - m_h(r_h^*)\} \geq \max_h \{m_h(x_h^2) - m_h(r_h^*)\}$). This notion of having more extremes of consumption (formalized by Equation (5)) in the domain of multidimensional bundles is analogous to the concept of a mean-preserving spread in the domain of stochastic outcomes.¹²

Even if from a pure consumption point of view it makes sense to prefer moderate consumption across dimensions, once we take account of aspirations, it may be perfectly rational to, in effect, specialize on certain dimensions. While previous analysis invoking Veblen effects has acknowledged increasing marginal utility of expenditure on a single dimension and attributed it to status-seeking, our analysis suggests it could reflect a desire to live up to an internal standard on at least one dimension.

¹¹For $\tilde{h} = H$, (5) holds as an equality because we recover $m(x) - m(r^*)$ on both sides. (The theorem assumes that $m(x^1) = m(x^2)$, so we denote it simply as $m(x)$.)

¹²The concept of a mean-preserving spread allows us to make comparisons of second-order stochastic dominance, though of course second-order stochastic dominance favors means over extremes, whereas Theorem 1 here describes scenarios in which the more extreme outcome is preferred to the more moderate one.

4.2 Bingeing

There is a long literature under the heading of *payday effects* describing the prevalence of unsustainable consumption immediately following paycheck receipt, especially among poor people [65, 35, 53, 31]. The traditional account for this behavior is based on present-biased preferences. To the degree there is a bias, this account suggests payday effects result in a decrease in welfare. Our framework, in contrast, suggests bingeing may be a perfectly rational phenomena that improves welfare in certain situations. Despite the fact that bingeing is often observed immediately following payday, it is also possible people might save up to binge. When bingeing occurs depends on the relationship between the subjective discount rate and the real interest rate.

Intertemporal consumption in our model is subject to competing influences. Naturally, subjective discounting at a rate greater than the real interest rate has the effect of encouraging consumption sooner rather than later and vice versa. A concave consumption utility function, satisfying diminishing marginal utility of consumption, moderates this effect. Loss aversion works in the opposite direction, favoring steady consumption over decreasing patterns of consumption. Lastly, diminishing sensitivity often pushes consumption in particular periods closer to the aspiration level.

When feasible consumption is well below the status quo and aspirational reference points, diminishing sensitivity of the contrast utility function along with subjective discounting more than the real interest rate promotes heavy early consumption, i.e. bingeing, over and beyond that resulting from discounting alone. That is, the imbalanced intertemporal consumption produced in our model can be characterized as bingeing and not merely impatience. Analogously, when subjective discounting is less than the real interest rate over a finite horizon and the decision maker gains a windfall pushing feasible consumption above a very low status quo but not within reach of the aspiration level, our model predicts saving up to binge in the final pe-

riods of the time horizon. When the subjective discount rate is the real interest rate, steady consumption, prudent saving, and sporadic bingeing are all possible in different situations.

For a benchmark comparison, we examine the effect of discounting in the absence of loss aversion and diminishing sensitivity by considering $\alpha = 0$, eliminating contrast utility. The discount function $D(t)$ could be exponential or may exhibit declining impatience with $\frac{D(t+1)}{D(t)}$ increasing in t (sometimes called a declining rate of time preference [25]). The only requirement is that $D(t)$ is a strictly decreasing function with $D(0) = 1$. We assume $m(\cdot)$ is continuously differentiable, strictly increasing, and weakly concave. For simplicity, consider one-dimensional consumption bundles, $x(t) \in \mathbb{R}_+$. Given real interest rate $\rho > 0$, the budget constraint is

$$\sum_{t=0}^{\infty} \frac{x(t)}{(1+\rho)^t} = Y, \quad (6)$$

where Y is the present endowment.

With $\alpha = 0$, the first order conditions for the optimal intertemporal consumption profile are

$$m'(x(t)) D(t) - \bar{K} \frac{1}{(1+\rho)^t} = 0 \quad (7)$$

for all t , where \bar{K} is the Lagrangian multiplier on the budget constraint. If $D(t)(1+\rho)^t$ is increasing in t , there is a danger that the model predicts saving up forever as an artifact of our simplifying assumption that there are infinitely many periods. We do want to allow for the possibility that $D(t) \cdot (1+\rho)^t$ is increasing in t , and consequently that saving is preferred to borrowing, but in this case we truncate the time horizon by assuming that there exists a time period T such that $D(t) = 0$ for $t > T$. We may have a boundary solution in which $x(t') = 0$ for some t' if $m'(0) D(t')(1+\rho)^{t'} \leq \bar{K}$ with \bar{K} defined by the first order condition (7) binding for some t in conjunction with

the budget constraint (6). If $m'(s)$ grows arbitrarily large as s goes to 0, then we have an interior solution in which equation (7) binds for all t . We then obtain

$$x(t) = (m')^{-1} \left(\frac{\bar{K}}{D(t)(1 + \rho)^t} \right) \quad (8)$$

for all t . Observe that we get a decreasing (increasing) pattern of consumption $x(t + 1) < (>) x(t)$ when $D(t)(1 + \rho)^t$ is decreasing (increasing) in t .

Let $\mathbf{x}_{(\alpha=0)}^*$ denote the optimal intertemporal consumption profile subject to the budget constraint (6) for the special case when $\alpha = 0$. Let $\mathbf{x}_{(\alpha>0)}^*$ denote the optimal intertemporal consumption profile in general, allowing $\alpha > 0$, given the same budget constraint. For simplicity we consider fixed aspiration levels, $r^*(t) = r^*$ for all t .

In Theorem 2 we compare the optimal intertemporal consumption profile $\mathbf{x}_{(\alpha>0)}^*$ determined by our full utility function with $\alpha > 0$ against the benchmark of equation (8) describing intertemporal consumption $\mathbf{x}_{(\alpha=0)}^*$ in the absence of contrast utility. We identify conditions under which intertemporal consumption becomes more extreme. Bingeing may occur right away in the initial periods or alternatively in the final periods after an interval of saving up. The conditions are technical and are stated formally in the appendix only.

Theorem 2 *Assume consumption utility is continuously differentiable, strictly increasing, and weakly concave, and is becoming infinitely steep at low levels of consumption. The optimal consumption profile for $\alpha = 0$, $\mathbf{x}_{(\alpha=0)}^*$, is determined by equations (8) and (6).*

1. *Suppose the subjective discount rate is at all times sufficiently greater than the real interest rate so as to compensate for an overall factor that measures how convex the utility function is. (This is made precise in the appendix.) Suppose also the present endowment is less than (or equal to) both the status quo and*

the aspiration level, and that, as a function of consumption at a given particular time, overall utility has at most one inflection point over consumption levels below the endowment. Allow the discount function to be exponential or to exhibit declining impatience. Then the optimal intertemporal consumption profile features decreasing consumption over time, and taking into account contrast utility makes early consumption even heavier and later consumption even lighter. That is, relative to the effect of high subjective discounting in isolation, the contrast utility function generates more bingeing in the initial periods.

- 2. Suppose the discount function is exponential with a rate below the real interest rate over a finite time horizon and with no regard for a future beyond that horizon. Suppose the future value of the endowment will not surpass the aspiration level within this time horizon, that the status quo is sufficiently low, and that as a function of consumption at a given particular time, overall utility has at most one inflection point over consumption levels affordable within the time horizon. Then the optimal intertemporal consumption profile features increasing consumption over time, and taking into account contrast utility makes early consumption even lighter and later consumption even heavier. That is, relative to the effect of low subjective discounting in isolation, the contrast utility function generates more bingeing in the final periods on the horizon.*

Part 1 of the theorem describes unsustainably high consumption in initial periods. This behavior would be consistent with the literature on payday effects if we were to impose credit constraints so that the endowment is periodically replenished. Part 2 of the theorem describes saving up to binge. With the introduction of naivete and an infinite time horizon, this behavior would be consistent with miserliness, a cycle of continual deferral of consumption with the prospect of a big binge looming at

an always future date. Thus, part 2 of Theorem 2 could help to explain the miser's behavior of constantly saving for the future but never enjoying the fruits of this saving. Either a real rate of interest higher than the subjective discount rate, a desire for improvement, or pleasure from anticipation would all have similar consequences contributing to miserliness.

In Theorem 3 we consider subjective discounting at the real interest rate. We identify conditions in which the optimal intertemporal consumption profile features steady consumption, prudent saving, or sporadic bingeing. The consumption profile depends on the curvature of both pieces of the utility function and on the degree of loss aversion.

Theorem 3 *Assume $m(\cdot)$ is continuously differentiable, strictly increasing, and weakly concave, and $\lim_{s \rightarrow 0} m'(s) = \infty$. Suppose $D(t) = \frac{1}{(1+\rho)^t}$. The optimal consumption profile for $\alpha = 0$ is $x_{(\alpha=0)}^*(t) = Y \frac{\rho}{1+\rho}$ for all t .*

1. *Suppose $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ is concave everywhere. Then $x_{(\alpha>0)}^*(t) = Y \frac{\rho}{1+\rho}$ for all t .*
2. *Suppose $r^0 < Y \frac{\rho}{1+\rho}$ and $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ is concave when $x(t) > \xi$ for all t but is locally convex on $\{\mathbf{x} : r^0 < x(t) < \xi \text{ and } x(t) \neq x(t+1) \text{ for all } t\}$ for some ξ . When λ is sufficiently large, there exists $\tilde{t} \geq 0$ and $x_H^* \geq \xi$ such that optimal consumption initially remains at the status quo up until time \tilde{t} and then jumps to x_H^* afterwards:*

$$x_{(\alpha>0)}^*(t) = \begin{cases} r^0 & \text{for } t < \tilde{t} \\ x_H^* & \text{for } t > \tilde{t} \end{cases}$$

and $x_{(\alpha>0)}^*(\tilde{t}) \in [r^0, x_H^*]$.

3. *Suppose $\lambda = 1$ and $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ is concave when $x(t) < \omega$ for all t and when $x(t) > \xi$ for all t , but is convex when $\omega < x(t) < \xi$ for all t , for some $\omega <$*

$Y \frac{\rho}{1+\rho} < \xi$. Suppose as well that $\rho \leq 1$. Then there exists $x_L^* \leq \omega$ and $x_H^* \geq \xi$ and $\mathcal{T} \subset \mathbb{Z}^+$ such that optimal consumption alternates irregularly between x_L^* and x_H^* as indicated by \mathcal{T} :

$$x_{(\alpha>0)}^*(t) = \begin{cases} x_H^* & \text{for } t \in \mathcal{T} \\ x_L^* & \text{for } t \notin \mathcal{T}. \end{cases}$$

The set \mathcal{T} will not in general be an interval.

In part 1 of the theorem, the optimal consumption profile remains steady consumption. In part 2 we get prudent saving in the sense that consumption reaches a sustainably high level. In part 3, sporadic bingeing occurs. Consumption switches back and forth between high and low levels. The elements of \mathcal{T} depend sensitively on the endowment Y , so the periods of higher consumption appear sporadically. This sporadic bingeing is an optimal choice in the absence of loss aversion, given discrete time and a utility function with a convex region. This convex utility is only possible when the feasible steady consumption level would be below the aspiration level.

4.3 The Lottery

Research consistently finds that low-income individuals spend a higher percentage of their income, or even absolutely more, on lottery tickets than do wealthier individuals (see, e.g., [13, 30, 29]). The high rate of lottery play among the poor is unfortunate. Low income individuals can less afford the approximately 50% implicit tax in part because poverty establishes narrower “margins of error” for financial behavior [7]. Thus, lottery tickets can be considered one of a number of behaviors (with payday loans, instant tax refunds and rent-to-own being others) that collectively contribute to “poverty traps” - patterns of inefficient behavior that prevent low-income individuals

from improving their financial situations. Our model can help to explain why, given its manifest pitfalls, lottery play is so prevalent among low-income individuals.

Recall Figure 2, which suggested that when the aspiration level is far above the status quo, skewed gambles with high upside, as with a lottery, are particularly attractive. Poorer people, who by definition have a status quo of meager wealth, tend to have aspirations well above this status quo [59]. We now formalize this intuition. We begin with a more careful examination of conditions that contribute to playing the lottery.

For simplicity, consider consumption bundles consisting of just a single dimension, wealth (i.e., $x \in \mathbb{R}$). Let the probability measure F_{z,p,r^0} denote the fair gamble relative to the status quo with a chance p of winning a payout of $z - r^0$. Specifying outcomes in terms of total wealth, we have $F_{z,p,r^0}(\{z\}) = p$ and $F_{z,p,r^0}(\{\frac{r^0 - pz}{1-p}\}) = 1 - p$. We can represent a fair lottery by taking $p \ll 1$ and $z - r^0$ sufficiently large. We describe a sufficient condition to accept a fair lottery in Theorem 4. First, we introduce notation for the average value of the function v' over an interval $[s_1, s_2]$:

$$\bar{v}'(s_1, s_2) = \frac{\int_{s_1}^{s_2} v'(t) dt}{s_2 - s_1}$$

and similarly for m' .

Theorem 4 *Suppose $m' > 0$ and $r^0 < r^*$. Then, $U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*) \geq 0$ if*

$$\alpha + (1 - \alpha) \bar{v}'(m(r^0) - m(r^*), m(z) - m(r^*)) \geq \frac{\bar{m}'\left(\frac{r^0 - pz}{1-p}, r^0\right)}{\bar{m}'(r^0, z)} [\alpha + (1 - \alpha) \lambda v'(m(r^0) - m(r^*))]. \quad (9)$$

Theorem 4 describes a sufficient condition to accept a fair lottery. If the fair lottery is

strictly preferred, i.e., if equation (9) is a strict inequality, then continuity of the utility function implies that even slightly unfair lotteries with negative expected values will be accepted as well.

Equation (9) in the theorem is technical, but permits some interpretation. The ratio $\frac{\overline{m}'\left(\frac{r^0 - pz}{1-p}, r^0\right)}{\overline{m}'(r^0, z)}$ depends on the concavity of m . The utility function for wealth is generally thought to be weakly concave, consistent with universally diminishing marginal utility over levels of wealth. However, the concavity should only become noticeable over large scales; the utility of wealth is approximately linear for small stakes [2] and even for moderate stakes, lest we build up unrealistic degrees of risk aversion over large stakes [58]. Thus, $\frac{\overline{m}'\left(\frac{r^0 - pz}{1-p}, r^0\right)}{\overline{m}'(r^0, z)} > 1$, and while it is increasing in z , it should be doing so quite slowly. The possibility of satisfying equation (9) arises because

$$\overline{v}(m(r^0) - m(r^*), m(z) - m(r^*)) > v'(m(r^0) - m(r^*)) \quad (10)$$

by the convexity of v , as long as z is not too large. If this inequality is wide enough, and if enough weight is placed on contrast utility (i.e. $1 - \alpha$ large enough), then it may overcome the risk aversive effects of loss aversion and diminishing marginal utility for wealth.

Let us now examine the role that poverty plays in the decision whether to play the lottery. As we argue in Section 3, poor people (with low r^0) tend to have aspirations more highly above their pre-existing levels of attainment, i.e., are more prone to have $r^0 \ll r^*$. When $r^0 \ll r^*$, and consequently $m(r^0) \ll m(r^*)$, we find $v'(m(r^0) - m(r^*))$ is small because we are at a relatively flat part of the value function; we may even obtain $v'(m(r^0) - m(r^*)) \approx 0$ if the convexity of v is strong enough. On the other hand, $\overline{v}(m(r^0) - m(r^*), m(z) - m(r^*))$ may remain large because the interval contains the region in which v is steeply increasing. Having a high aspiration level relative to

the status quo, a circumstance we associate with a state of poverty, produces the best conditions for satisfying equation (9). Formally, we obtain the following corollary, which states that a lottery will become more attractive as the aspiration level departs farther from the status quo (as poverty becomes more acute), up to a certain point.

Corollary 1 *Assume m is continuously differentiable, strictly increasing and weakly concave. Suppose $r^0 < \underline{r}^* < \tilde{r}^*$ and $v'''(\cdot) \leq 0$ over $\left[m\left(\frac{r^0 - pz}{1-p}\right) - m(\tilde{r}^*), \min\{0, m(z) - m(\underline{r}^*)\} \right]$.¹³ Then $U(F_{z,p,r^0}|r^0, \underline{r}^*) \geq u(r^0|r^0, \underline{r}^*) \implies U(F_{z,p,r^0}|r^0, \tilde{r}^*) \geq u(r^0|r^0, \tilde{r}^*)$. The obtained inequality must be strict unless m is linear, $\lambda = 1$, $v''' = 0$ over the entire given interval, and equality holds in the antecedent.*

We should also note that more acute subjective poverty only suffices to make lotteries more attractive up to a point. If poverty is so acute that even winning the lottery would not bring the decision maker near his or her aspiration level, then the lottery is not desirable. The following corollary formalizes the intuition that if the aspiration level were to rise so far above the status quo that it becomes hopelessly unattainable (i.e., even winning the lottery would not bring the decision maker near this aspiration level), then the lottery would not be chosen.

Corollary 2 *Assume m is continuously differentiable, strictly increasing and weakly concave, with no asymptote at infinity. Fix r^0 and the fair lottery F_{z,p,r^0} . Then,*

$$\lim_{r^* \rightarrow \infty} U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*) \leq 0$$

with equality if and only if m is linear and either $\lambda = 1$ or $\lim_{s \rightarrow -\infty} v'(s) = 0$.

Our model sheds light on the phenomenon that poor people are disproportionately inclined to play the lottery. For some people (i.e., for people with reference points

¹³Given that v'' switches from positive to negative at 0, it is quite reasonable to assume $v''' \leq 0$ in this interval as long as \tilde{r}^* is not too large in relation to $r^0 - pz$.

that satisfy equation (9) in Theorem 4), the lottery is in fact utility-enhancing. The small chance of surpassing one's aspiration level may well be worth the small cost most likely incurred. Undoubtedly, additional factors such as nonlinear probability weighting and the peanuts effect could also contribute to the attractiveness of the lottery. However, the possibility of the lottery improving welfare would be mitigated if people adjust to the new status quo after winning the lottery so that contrast utility does not persist in contributing to welfare. There is strong evidence such hedonic adaptation is typical [24].

The lottery is commonly denounced as a regressive tax on poor people. In one respect our analysis calls into question this common view of the lottery because even an actuarially unfair lottery may be utility-maximizing for low income individuals. Still, we do not deny that such a lottery is unfair to poor people. Given their (perhaps entirely rational) attraction to skewed lotteries, the high rent the state extracts can be viewed precisely as a very high tax on a consumption good that poor people do find particularly valuable.

5 Endogenous Goal Setting

Until now, we have taken the aspiration level to be exogenous. Another possibility is that the aspiration level could be a choice variable. Setting one's own aspiration level, in a dynamic context, would affect incentives for choices about effort exertion (a tradeoff of consumption across different dimensions), saving or spending (intertemporal consumption), and risk taking. An attainably high aspiration level could be used to motivate oneself in the face of present-biased time preferences.

Psychologists know the important role goals play in helping people organize their lives [46, 3]. From the point of view of economic theory, however, goals must be ex-

plained in terms of preferences. Goals are not merely desired outcomes. Rather, they are used to motivate a person to achieve desired outcomes. But, if the outcomes are desired, why are goals necessary? The importance of goal setting becomes apparent when we consider three insights in conjunction: hyperbolic discounting (or cognitive biases more generally) distorts decision making [43, 25]; knowledge of this bias creates a principal-agent problem or intrapersonal game for a sophisticated decision maker [66, 61, 6]; and a goal may modify preferences by serving as a reference point in a person’s utility function [32]. Recent work has analyzed optimal goal setting in models in which the goal serves as a sole reference point [69, 34]. These analyses provide important insights into the goal setting process. However, because they rely on models with only one kind of reference point, a reference point that demoralizes as well as motivates, a problem often arises: either the decision maker must be restricted to setting only goals which she will then attain or else the decision maker will select the minimal possible goal. Distinguishing an aspirations-based reference point from an expectations-based (status quo) reference point allows us to overcome this problem.¹⁴

We apply both a traditional reference-dependent model and the modified dual reference point model to a simple example in which goals may be set endogenously. A decision maker faces a two-period consumption problem with a tradeoff between leisure and wealth. In period one she may choose to forego leisure and work and thus obtain additional wealth in period two. Knowing of her quasi-hyperbolic discounting, in period zero she chooses a goal which may or may not motivate her to be patient the following period. We impose no a priori restriction on the attainability or motivational content of this goal.

Consumption bundles are two-dimensional, representing leisure and wealth respec-

¹⁴The insight that optimal ambition not only affects performance (through motivation), but also affects satisfaction directly predates prospect theory even [49]. Indeed, it applies to our dual reference point model as well. Nevertheless, (unlike, say, prospect theory) we do not assume that achieving the aspired level of performance constitutes merely breaking even.

tively: $x = (x_l, x_w) \in \mathbb{R}_+ \times \mathbb{R}_+$ in each period t . There is an initial endowment of leisure and wealth, Y_l and Y_w , and the opportunity to work a chosen number of hours, up to some maximum Δ , at a fixed wage. Payment is not received until period two, so $x_w(1) = Y_w$, and maximal leisure is recovered in period two, so $x_l(2) = Y_l$. Without loss of generality we can normalize the units of leisure to the units of wealth in order to set the wage at 1. Feasible consumption must thus satisfy the budget constraint $x_l(1) + x_w(2) = Y_l + Y_w$ with $x_l(1) \in [Y_l - \Delta, Y_l]$ and $x_w(2) \in [Y_w, Y_w + \Delta]$.

In existing models of goal-setting, the goals chosen in period zero are the sole reference points, $r_l \in \mathbb{R}_+$ and $r_w \in \mathbb{R}_+$. The period one decision is then simply to choose $x_l(1)$ and $x_w(2)$, conditional on r_l and r_w , that satisfy the budget constraint and maximize

$$(1 - \alpha) m_l(x_l(1)) + \alpha \mu \circ v(m_l(x_l(1)) - m_l(r_l)) + \\ \beta \delta (1 - \alpha) m_w(x_w(2)) + \beta \delta \alpha \mu \circ v(m_w(x_w(2)) - m_w(r_w)),$$

where $\beta < 1$ and $\delta \leq 1$ are the hyperbolic discount parameters, and we have included both direct consumption utility and prospect theory's contrast utility, just as for the dual reference point model and as in recent literature [41, 69, 34]. The period zero decision is then to choose r_l and r_w to maximize

$$\beta \delta (1 - \alpha) m_l(x_l(1)) + \beta \delta \alpha \mu \circ v(m_l(x_l(1)) - m_l(r_l)) + \\ \beta \delta \alpha \mu \circ v(m_w(x_w(1)) - m_w(r_w)) + \beta \delta^2 \alpha \mu \circ v(m_l(x_l(2)) - m_l(r_l)) + \\ \beta \delta^2 (1 - \alpha) m_w(x_w(2)) + \beta \delta^2 \alpha \mu \circ v(m_w(x_w(2)) - m_w(r_w)),$$

where $x_l(1)$ and $x_w(2)$ are understood to be functions of r_l and r_w . This is a straightforward optimization problem, but we cannot express the solution in closed form

without making additional specifications.

With the dual reference point model, the goals chosen in period zero are the aspirational reference points, $r_l^* \in \mathbb{R}_+$ and $r_w^* \in \mathbb{R}_+$, and the status quo reference points are the initial endowments, $r_l^0 = Y_l$ and $r_w^0 = Y_w$. The period one decision is then to choose $x_l(1)$ and $x_w(2)$, conditional on r_l^* and r_w^* , that satisfy the budget constraint and maximize

$$\begin{aligned} & (1 - \alpha) m_l(x_l(1)) + \alpha \mu (v(m_l(x_l(1)) - m_l(r_l^*)) - v(m_l(r_l^0) - m_l(r_l^*))) + \\ & \quad \beta \delta \alpha \mu (v(m_l(x_l(2)) - m_l(r_l^*)) - v(m_l(x_l(1)) - m_l(r_l^*))) + \\ & \beta \delta (1 - \alpha) m_w(x_w(2)) + \beta \delta \alpha \mu (v(m_w(x_w(2)) - m_w(r_w^*)) - v(m_w(x_w(1)) - m_w(r_w^*))). \end{aligned}$$

And the period zero decision is to choose r_l^* and r_w^* to maximize

$$\begin{aligned} & \beta \delta (1 - \alpha) m_l(x_l(1)) + \beta \delta \alpha \mu (v(m_l(x_l(1)) - m_l(r_l^*)) - v(m_l(r_l^0) - m_l(r_l^*))) + \\ & \quad \beta \delta^2 \alpha \mu (v(m_l(x_l(2)) - m_l(r_l^*)) - v(m_l(x_l(1)) - m_l(r_l^*))) + \\ & \beta \delta^2 (1 - \alpha) m_w(x_w(2)) + \beta \delta^2 \alpha \mu (v(m_w(x_w(2)) - m_w(r_w^*)) - v(m_w(x_w(1)) - m_w(r_w^*))), \end{aligned}$$

where of course $x_l(1)$ and $x_w(2)$ are functions of r_l^* and r_w^* . This too is a straight-forward optimization problem, but again we require further specification to express a closed form solution.

Intuitively, depending on the specification, we can expect three kinds of equilibria to emerge. First, there is the possibility that leisure is so valued that it is preferred to wealth even by the period zero goal-setting self. In this case the individual would have no reason to set an elevated goal to motivate herself. The present-bias experienced at period one will only help the decision maker reach the desired outcome of not working at all. As this outcome does not entail a change from the status quo, a

wide range of aspiration levels will constitute equilibrium choices for the period zero self in the dual reference point model, and this choice have no effect on utility in equilibrium. The traditional model, on the other hand, makes a stark and rather unappealing prediction. The goals for both leisure and wealth would be zero, because in the traditional model higher goals directly lower utility. The traditional model's prediction should not be construed as the absence of a goal; the model does not admit such a choice. Rather, the traditional model predicts the decision maker sets a goal to avoid financial ruin and to not work all the time.

At the other extreme, there is the possibility that wealth is sufficiently valued relative to leisure so that a motivational goal is unnecessary. Despite present-bias, the period one self will choose to work the maximum number of hours. Once again, the traditional model predicts goals of zero on both dimensions will be chosen. The dual reference point model handles goal setting differently here. The sophisticated decision maker is aware that choosing to work will feel like a loss on the leisure dimension (temporarily, anyway) and a gain on the wealth dimension. She will choose a goal for wealth that is above the status quo, yet attainable, because subsequently attaining that goal will add to her utility, and an infinite goal for leisure (given sufficient concavity of the contrast utility function), because this extreme goal will minimize the disappointment with the loss. Setting a goal of infinity can be interpreted as aiming to “do your best,” an instruction psychologists often give to the control group (with typically negative consequences for motivation) when testing the motivational power of specific, attainable goals [46].

A more interesting *motivational equilibrium* involves choice of a goal on the wealth dimension to motivate oneself to work. (If the goal motivates oneself to work the maximum number of hours Δ , we call it a *strong motivational equilibrium*.) A sophisticated decision maker may use such a goal to resolve a conflict with her future

self. In period one she will choose to work and in period zero she must set a goal on the wealth dimension that provides the right incentives for the next period. In the traditional model, this goal for wealth would be the minimal level consistent with overcoming present-bias. There is no guarantee, a priori, that the goal would even be above the status quo. In the modified model that permits dual reference points, however, the goal for wealth would certainly be above the status quo, but attainably so. Once again, the goal for leisure would be zero in the traditional model and infinity in the dual reference point model. Not only do the goals set in the dual reference point model seem more reasonable, but they are also more robust. In the traditional model, there is a direct utility cost to imposing a goal on oneself, and thus very often the added self control is not worth the cost. In the dual reference point model, there is no such drawback to using a goal for self control. To illustrate the robustness of motivational goal setting in the dual reference point model, as compared with the traditional prospect theory model, we examine the existence of a strong motivational equilibrium after further specification of these models.

We consider the following specification of our two models: linear consumption utility $m_w(x_w) = m'_w x_w$ and $m_l(x_l) = m'_l x_l$ and power law contrast utility $v(\cdot) = |\cdot|^p$ with $p < 1$. With this specification we can identify the region in parameter space that permits a motivational equilibrium, but the analytic solution is difficult to interpret. Choosing reasonable values for a few parameters helps us visualize the robustness of goal setting with each model. We take $\beta = .8$ and $\delta = 1$ as quasi-hyperbolic discount parameters. Loss aversion has been estimated as $\lambda = 2$ [70]. For the sake of this example, we suppose $Y_l = 1$, $Y_w = .8$, $\Delta = .9$, and $\alpha = .2$. In Figure 4 we let $p = .8$, and in Figure 5 we let $p = .2$.

For illustration, we explore the circumstances under which utility maximizers set elevated goals that motivate themselves to work as much as allowed. Figures 4 and 5

thus show values of the marginal consumption utilities for wealth and leisure, m'_w and m'_l , for which a strong motivational equilibrium exists, for the dual reference point model and for traditional prospect theory. The critical regions are striped. Vertical stripes indicate existence of a strong motivational equilibrium with the dual reference point model. Horizontal stripes are for traditional prospect theory. Observe that there are no horizontal stripes in Figure 4. This is because traditional prospect theory does not produce a strong motivational equilibrium for these parameter values. (That's why we show Figure 5 with $p = .2$ even though this parameter value is a bit outside the range of experimental estimates we have seen [71, 54, 8]). The dual reference point model, on the other hand, does permit a person to overcome present bias by setting an elevated goal to motivate working fully. In Figure 5, where prospect theory does produce a strong motivational equilibrium, the dual reference point model also does so in the same region and beyond. Below and to the right of the critical regions, a motivational goal is unnecessary; with trivial goals, the equilibrium level of work will nevertheless be maximal here. Above and to the left of the critical regions, working the maximum number of hours is undesirable.

Regions in which the single reference point model predicts that people cannot motivate themselves to work as much as possible and the dual reference point model predicts that people can use goals to do so arise from what could be called the “misery effect” in prospect theory. That is, the higher the goal in prospect theory, the more demoralizing it is. This misery effect is independent of whether the goal is subsequently attained or not. Just setting the goal would make one miserable. Distinguishing the role of goals from the role of expectations-based reference points eliminates this effect and makes goal setting a much more attractive endeavor. Considering the ubiquity of goals in people's lives, there is clearly a need for a mechanism, such as dual reference points, that can make sense of common observation.

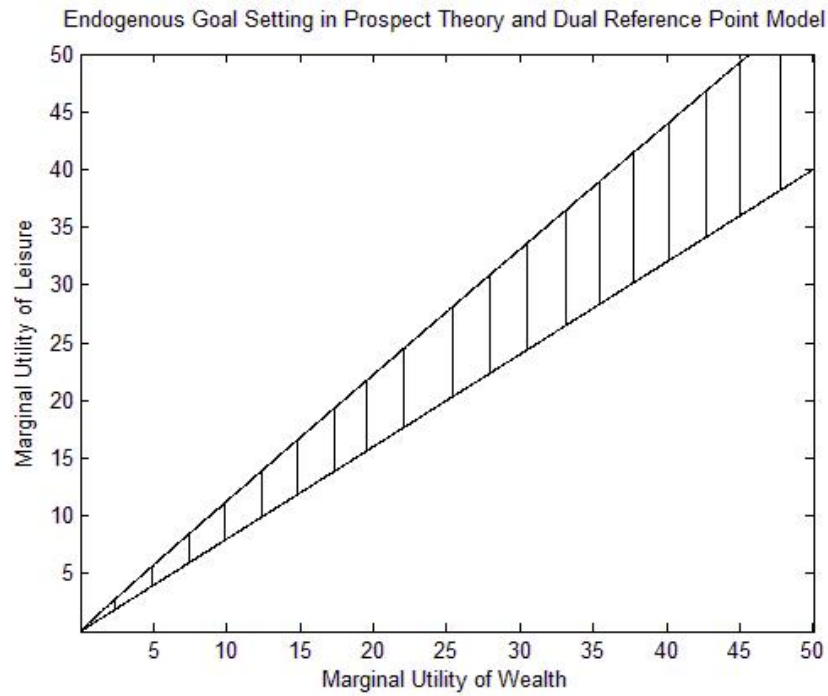


Figure 4: $p = .8$. The vertically striped region shows parameter values under the dual reference point model that permit a strong motivational equilibrium, one in which an elevated goal motivates the person to work as much as possible. Such an equilibrium does not exist under traditional prospect theory.

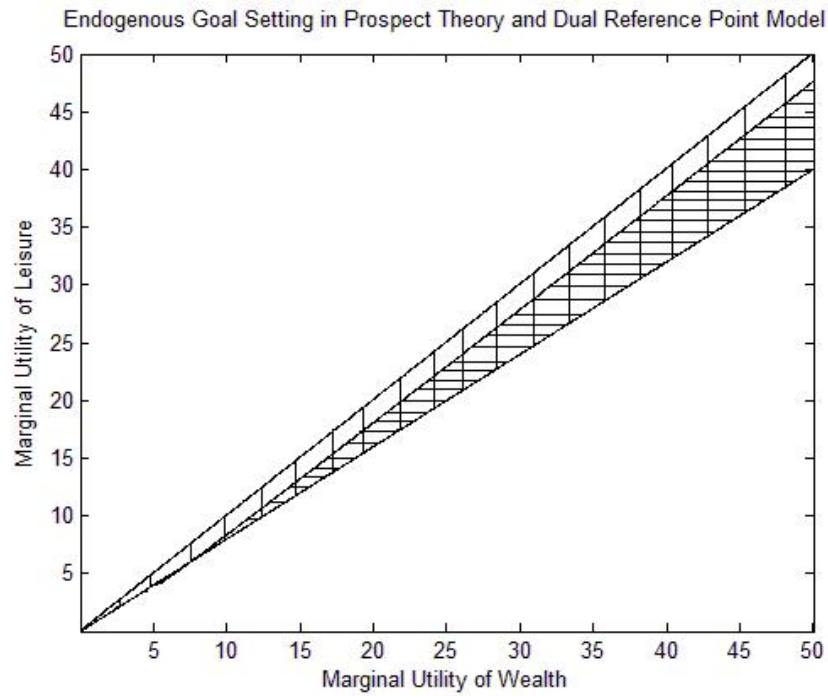


Figure 5: $p = .2$. The horizontally striped region shows parameter values that permit a strong motivational equilibrium under traditional prospect theory. This region is included in the larger vertically striped region that indicates existence of a strong motivational equilibrium under the dual reference point model.

6 Discussion

When consumption levels are inadequate relative to a person’s aspirations, nonconvex preferences make sense as a way to alleviate suffering. In our model, unbalanced consumption, bingeing, and gambling sometimes arise as utility-maximizing choices given a poor person’s convex contrast utility function. In the domain of risk, a skewed lottery is the mechanism by which nonconvex preferences manifest themselves. In the domain of intertemporal consumption, it is the ability to save up or to access credit that allows bingeing, and when it comes to multi-dimensional baskets of goods, it is the existence of goods that are luxurious but attainable that enables unbalanced consumption. We emphasize that these patterns of behavior are, in our model, the result of utility maximization, not a loss of self-control.¹⁵ Nevertheless, some, such as lottery purchases, are perverse in the sense that they tend to contribute to the state of poverty that helps produce them. Strangely, it is the ambitious aspirations – the same ones the goals literature says are the most motivating [46] – that are most likely to produce these patterns of behavior.

Disentangling loss aversion and diminishing sensitivity as separate effects brought upon by expectations and aspirations respectively has implications for a variety of common phenomena. For example, it is widely observed that during an asset boom, risk-taking becomes more prevalent. This has been attributed to rising levels of optimism, what Shiller calls “irrational exuberance” [64]. Our framework suggests an alternative explanation. To the degree that seeing other people’s success raises one’s own aspiration level, our model predicts that risk appetites should increase during an asset boom. A traditional form of prospect theory with aspirations as the reference point [32] could also produce this effect, but only at the expense of giving up the prediction of a status quo bias, as the reference point would need to be set very high.

¹⁵Of course, we do not rule out self-control problems in addition to nonconvex preferences [4, 31].

We retain the prediction of a status quo bias owing to loss aversion being relative to the status quo and simultaneously obtain the prediction of increasing risk appetites in boom times as a consequence of diminishing sensitivity around an aspiration level.

Economists have also suggested that the structure of executive compensation can be viewed as a series of tournaments among employees, with winners of the tournament in one round eligible to compete in the tournament one level up [44]. Compensation of the CEO represents the prize in the tournament that motivates lots of employees to work long hours at high effort costs without a guarantee of compensation. With rational expectations, however, each employee recognizes there is a small chance of winning the grand prize (i.e. becoming the CEO), and so this salary structure is no more motivational than a direct compensation scheme (though it may have other benefits, such as reduced monitoring costs). Our framework provides extra justification for this tournament theory of executive compensation. If aspirations are a source of diminishing sensitivity in the utility function, a tournament with a salient grand prize should be highly motivating. Employees with high aspirations would be more willing to choose extreme points on the income-leisure budget constraint, thus trading off additional leisure time for smaller gains in expected compensation. Once again, our model accounts for this behavior without forsaking the well-established finding of status quo bias.

We first took the aspiration level to be exogenous and then considered the possibility that it might be an endogenous choice variable. Other interesting extensions would involve describing alternative processes that could determine the aspiration level endogenously. The source of the aspiration level might be a person's past experiences or a social comparison. This would force us to consider dynamics. Aspirations could evolve over time depending on the choices a person makes. Once achieved, an aspiration would most likely be replaced with a new, more ambitious one. Alterna-

tively, aspirations could spread across a social network depending on the choices other people make. We might speculate that taking the aspiration level as the height of past experience could explain the archetypal pattern of drug use following a fall from grace, as an individual in such circumstances might desire a temporary high at any cost relative to a status quo so far below a salient reference point. Also speculative is the notion that sibling differentiation could be explained with aspirations based on a social comparison that is particularly salient due to sibling rivalry – if an older sibling is quite accomplished in some way and sets for the younger a high aspiration level on one dimension of success, the younger sibling may prefer pursuing other more attainable dimensions of success (cf. [68]). The dynamics of this process, and the possibility of reaching an equilibrium aspiration level, remain tantalizing open problems.

Future investigations of dual reference point models might also consider hedonic issues more deeply. Our model indicates that nonconvex preferences are optimal in the sense that they maximize utility. But this assumes that ex post utility (“experience utility” in Kahneman’s parlance) is equivalent to ex ante utility. Mixed findings concerning the happiness of lottery winners is only one of many findings that question the validity of this assumed equivalence. Achieving aspirations might well not be synonymous with achieving happiness.

Appendix

A Sufficient (But Not Necessary) Condition for Risk Seeking

We identify two straightforward conditions under which nonconvex preferences emerge and thus a gamble is preferred to a sure outcome that has the same expected consumption utility. Nonconvex preferences may arise when the aspiration level is better than all possible outcomes on all dimensions. Thus, when a decision maker has ambitious

aspirations, (s)he more often exhibits risk-seeking behavior.

Given a probability measure F over consumption bundles, let x^F be a sure outcome such that $m_h(x_h^F) = \int m_h(x_h)dF(x)$ for all h . That is, on each dimension the consumption utility of x^F is the expected consumption utility of F .

Theorem 5 *When on each dimension all outcomes are worse than the aspiration level and they are either all gains or all losses, a gamble over these outcomes is preferred to a sure outcome that nets the expected consumption utility of that gamble on each dimension. Suppose for all h , either:*

1. $m_h(x_h) \leq m_h(r_h^0)$ and $m_h(x_h) \leq m_h(r_h^*)$ for all x in the support of F ; or
2. $m_h(r_h^0) \leq m_h(x_h) \leq m_h(r_h^*)$ for all x in the support of F .

Then $U(F|r^0, r^*) \geq u(x^F|r^0, r^*)$ with equality if and only if $F(\{x^F\}) = 1$.

Proof By construction of x^F , consumption utility of the gamble equals consumption utility of the sure outcome, $\int m(x)dF(x) = m(x^F)$. The assumptions of the theorem guarantee that $n(x|r^0, r^*)$ is convex for all x in the support of F because the consumption utility of all possible outcomes is less than the consumption utility of the aspiration level, and outcomes are either all gains or all losses, so loss aversion plays no role. We can apply Jensen's Inequality to obtain the desired result. ■

Proof of Proposition 1

1. Obvious.
2. It suffices to show that for all h , $n_h(x_h|r_h^0, r_h^*) \geq n_h(x_h|r_h^0, \tilde{r}_h^*)$ because the intrinsic consumption utility terms from both sides cancel. Our claim reduces

to:

$$\begin{aligned} & v(m_h(x_h) - m_h(r_h^*)) - v(m_h(x_h) - m_h(\tilde{r}_h^*)) \\ & + v(m_h(r_h^0) - m_h(\tilde{r}_h^*)) - v(m_h(r_h^0) - m_h(r_h^*)) \geq 0. \end{aligned}$$

This follows from the convexity (concavity) of v over the negative (positive) half line, assuming any one of the conditions (2a) through (2d) of the proposition. ■

Proof of Proposition 2

Suppose $u(x|x', r^*) \geq u(x'|x', r^*)$. Then

$$(1 - \alpha)m(x) + \alpha \sum_h \mu(v(m_h(x_h) - m_h(r_h^*)) - v(m_h(x'_h) - m_h(r_h^*))) \geq (1 - \alpha)m(x')$$

because contrast utility on the right hand side is 0. We subtract the nonzero contrast utility from both sides and rewrite it as:

$$(1 - \alpha)m(x) \geq (1 - \alpha)m(x') + \alpha \sum_h \bar{\mu}(v(m_h(x'_h) - m_h(r_h^*)) - v(m_h(x_h) - m_h(r_h^*)))$$

$$\text{where } \bar{\mu}(s) = \begin{cases} \lambda s & \text{if } s \geq 0 \\ s & \text{if } s < 0. \end{cases}$$

With $\lambda > 1$, observe that $\bar{\mu}(s) > \mu(s)$ for any $s \neq 0$. (They are equal when $s = 0$.)

Thus,

$$(1 - \alpha)m(x) \geq (1 - \alpha)m(x') + \alpha \sum_h \mu(v(m_h(x'_h) - m_h(r_h^*)) - v(m_h(x_h) - m_h(r_h^*)))$$

with equality if and only if $m_h(x_h) = m_h(x'_h)$ for all h . This is our desired result. ■

Proof of Proposition 3

1. Straightforward consequence of m_h strictly increasing and $v'(\cdot) > 0$.
2. Take the derivative to obtain

$$u_{x_h}(x|r^0, r^*) = \alpha m'_h(x_h) + (1 - \alpha)v'(m_h(x_h) - m_h(r_h^*)) m'_h(x_h) \begin{cases} 1 & \text{if } x_h \geq r_h^0 \\ \lambda & \text{if } x_h < r_h^0. \end{cases}$$

So,

$$\lim_{\Delta x \searrow 0} \frac{u_{x_h}(r^0 - \Delta x \hat{\mathbf{e}}_h | r^0, r^*)}{u_{x_h}(r^0 + \Delta x \hat{\mathbf{e}}_h | r^0, r^*)} = \frac{m'_h(r_h^0) (\alpha + (1 - \alpha)\lambda v'(m_h(r_h^0) - m_h(r_h^*)))}{m'_h(r_h^0) (\alpha + (1 - \alpha)v'(m_h(r_h^0) - m_h(r_h^*)))}.$$

Cancel $m'_h(r_h^0)$ to obtain $\tilde{\lambda} = \frac{\alpha + (1 - \alpha)\lambda v'(m_h(r_h^0) - m_h(r_h^*))}{\alpha + (1 - \alpha)v'(m_h(r_h^0) - m_h(r_h^*))}$.

3. Composition of a convex (concave) function with an affine function preserves convexity (concavity), as does summing convex (concave) and affine functions. Thus, u inherits the local convexity (concavity) of v , except right when $x_h = r_h^0$ for some h , i.e. when μ introduces a kink as loss aversion kicks in on some dimension. The partial derivative u_{x_h} decreases across this discontinuity, so it has the effect of introducing a point of local concavity. ■

Proof of Theorem 1

An assumption of the theorem is that consumption utility is the same for x^1 and x^2 , so the comparison is in contrast utility $n(\cdot|r^0, r^*)$. The assumption that outcomes are either gains on all dimensions or losses on all dimensions means that loss aversion plays no role, i.e., μ can be disregarded. Thus, $u(x^1|r^0, r^*) > u(x^2|r^0, r^*)$ if and only

if

$$\sum_{h=1}^H v(m_h(x_h^1) - m_h(r_h^*)) > \sum_{h=1}^H v(m_h(x_h^2) - m_h(r_h^*)), \quad (11)$$

or, using our shorthand notation,

$$\sum_{h=1}^H v(\Delta_h(x^1|r^*)) > \sum_{h=1}^H v(\Delta_h(x^2|r^*)). \quad (12)$$

We expand $v(\cdot)$ using the first two terms of its Taylor series (about an arbitrary value s_0) and the corresponding Lagrange remainder:

$$v(s) = v(s_0) + v'(s_0)(s - s_0) + \int_{s_0}^s \int_{s_0}^t v''(\theta) d\theta dt. \quad (13)$$

We use this Taylor expansion to compute the sums in (12), letting $s = \Delta_h(x|r^*)$ for $x \in \{x^1, x^2\}$ and for every h . The constant terms cancel from both sides of (12). Moreover, the sum of the first order terms cancel from both sides because $\sum_h \Delta_h(x^1|r^*) = m(x^1) - m(r^*) = m(x^2) - m(r^*) = \sum_h \Delta_h(x^2|r^*)$. Thus, we have (12) if and only if

$$\sum_{h=1}^H \int_{s_0}^{\Delta_h(x^1|r^*)} \int_{s_0}^t v''(\theta) d\theta dt > \sum_{h=1}^H \int_{s_0}^{\Delta_h(x^2|r^*)} \int_{s_0}^t v''(\theta) d\theta dt. \quad (14)$$

Equation (14) is invariant if we permute the indices because we sum over all dimensions anyway. Bringing everything to the left side, (14) is equivalent to

$$\sum_{h=1}^H \int_{\Delta_{\sigma^2(h)}(x^2|r^*)}^{\Delta_{\sigma^1(h)}(x^1|r^*)} \int_{s_0}^t v''(\theta) d\theta dt > 0. \quad (15)$$

The inner integral in (15), a function of the dummy variable t , is an antiderivative of v'' , i.e., it differs from v' by a constant. The important thing to note is that it is increasing in t for $t < 0$ because of the convexity of v . We will denote the average

value of this function over an interval $[s_1, s_2]$ as

$$\overline{v'^+}(s_1, s_2) = \frac{\int_{s_1}^{s_2} \int_{s_0}^t v''(\theta) d\theta dt}{s_2 - s_1}.$$

Naturally, the average value of an increasing function over an interval is increasing in the endpoints of that interval, and it is straightforward to check by taking derivatives that $\overline{v'^+}(s_1, s_2)$ is increasing in both s_1 and s_2 when $s_1 < 0$ and $s_2 < 0$. (We can cover the case that $s_1 = s_2$ by defining $\overline{v'^+}(s_1, s_2)$ in this scenario as the limit as $s_1 \rightarrow s_2$.)

We can rewrite our desired inequality, Equation (15), as

$$\sum_{h=1}^H (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v'^+}(\Delta_{\sigma^2(h)}(x^2|r^*), \Delta_{\sigma^1(h)}(x^1|r^*)) > 0. \quad (16)$$

Note that the theorem assumes that for all h , $\Delta_h(x^1|r^*)$ and $\Delta_h(x^2|r^*)$ are negative, so we are always in the domain in which $\overline{v'^+}$ is increasing. As the permutations σ^1 and σ^2 are defined so that $\Delta_{\sigma^1(h)}(x^1|r^*)$ and $\Delta_{\sigma^2(h)}(x^2|r^*)$ are weakly increasing in h , we find that $\overline{v'^+}(\Delta_{\sigma^2(h)}(x^2|r^*), \Delta_{\sigma^1(h)}(x^1|r^*))$ is weakly increasing in h . As shorthand notation, we define $\overline{v'_h} = \overline{v'^+}(\Delta_{\sigma^2(h)}(x^2|r^*), \Delta_{\sigma^1(h)}(x^1|r^*))$.

By induction, we show that for $\tilde{h} \in \{1 \dots H\}$,

$$\sum_{h=1}^{\tilde{h}} (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v'_h} \geq \sum_{h=1}^{\tilde{h}} (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v'_h} \quad (17)$$

as long as the hypothesis of the theorem, Equation (5), holds. Moreover, the inequalities become strict at some point.

The base case of (17) with $\tilde{h} = 1$ holds trivially. Assume that (17) holds for $\tilde{h} \in \{1 \dots \tilde{H} - 1\}$. We claim that it then holds for $\tilde{h} = \tilde{H}$ as well. Using the induction

hypothesis, we have

$$\begin{aligned} & \sum_{h=1}^{\tilde{H}} (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v_h^+} \geq \\ & \sum_{h=1}^{\tilde{H}-1} (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v_{\tilde{H}-1}^+} + \left(\Delta_{\sigma^1(\tilde{H})}(x^1|r^*) - \Delta_{\sigma^2(\tilde{H})}(x^2|r^*) \right) \overline{v_{\tilde{H}}^+} \end{aligned}$$

with strictness in the hypothesized inequality implying strictness here as well. Equation (5) and the fact that $\overline{v_h^+}$ is weakly increasing in h give us

$$\sum_{h=1}^{\tilde{H}-1} (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v_{\tilde{H}-1}^+} \geq \sum_{h=1}^{\tilde{H}-1} (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v_{\tilde{H}}^+}$$

with the inequality strict if (5) is strict with $\tilde{h} = \tilde{H} - 1$ and either $\Delta_{\sigma^1(\tilde{H}-1)}(x^1|r^*) < \Delta_{\sigma^1(\tilde{H})}(x^1|r^*)$ or $\Delta_{\sigma^2(\tilde{H}-1)}(x^2|r^*) < \Delta_{\sigma^2(\tilde{H})}(x^2|r^*)$. For the largest value of \tilde{h} for which (5) is strict, it is necessarily the case that $\Delta_{\sigma^1(\tilde{h})}(x^1|r^*) < \Delta_{\sigma^1(\tilde{h}+1)}(x^1|r^*)$. Thus, we obtain (17), and in particular, a strict inequality for $\tilde{h} = H$:

$$\sum_{h=1}^H (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v_h^+} > \sum_{h=1}^H (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) \overline{v_H^+}. \quad (18)$$

Recall that $\sum_{h=1}^H (\Delta_{\sigma^1(h)}(x^1|r^*) - \Delta_{\sigma^2(h)}(x^2|r^*)) = 0$. Thus, the right side of inequality (18) is 0. This observation turns (18) into our desired result, inequality (16). ■

Formal Statement and Proof of Theorem 2

Theorem 2 *Assume $m(\cdot)$ is continuously differentiable, strictly increasing, and weakly concave, and $\lim_{s \rightarrow 0} m'(s) = \infty$. The optimal consumption profile for $\alpha = 0$, $\mathbf{x}_{(\alpha=0)}^*$, is determined by equations (8) and (6).*

1. *Suppose*

$$\Upsilon(t) := \sup_{x' \geq x} \left\{ \frac{(1 - \alpha)m'(x') + \alpha m'(x')v'(m(x') - m(r^*)) \left(1 - \frac{D(t+1)}{D(t)}\right)}{(1 - \alpha)m'(x) + \alpha m'(x)v'(m(x) - m(r^*)) \left(1 - \frac{D(t+1)}{D(t)}\right)} \right\} < \frac{D(t)}{D(t+1)} \frac{1}{1 + \rho}. \quad (19)$$

(If $\hat{U}(\cdot|0, \mathbf{r}^*)$ is concave everywhere, then $\Upsilon(t) = 1$ for all t , and equation (19) simply requires that $D(t)(1 + \rho)^t$ is strictly decreasing in t .) Suppose also $Y \leq r^0$ and $Y \leq r^*$ and that fixing $x(t') = r^0$ for $t' < t$ and $x(t') = 0$ for $t' > t$, $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ as a function of just $x(t)$ has at most one inflection point on $[0, Y]$. Allow the discount function to be exponential or to exhibit declining impatience, i.e., $\frac{D(t+1)}{D(t)}$ is increasing or flat. Then $x_{(\alpha=0)}^*(t)$ is decreasing in t and there exists \tilde{t} such that $(x_{(\alpha>0)}^*(t) - x_{(\alpha=0)}^*(t))$ is positive for $t < \tilde{t}$ and negative for $t > \tilde{t}$, i.e., the optimal consumption profile allowing $\alpha > 0$ features even heavier early consumption or bingeing in the initial periods.

2. *Suppose the discount function is exponential, $D(t) = \delta^t$ with $\frac{1}{1+\rho} < \delta < 1$, for $t \in [0, T]$ and as $D(t)(1 + \rho)^t$ is then strictly increasing in t , suppose $D(t) = 0$ for $t > T$, for some T . Additionally, suppose $Y(1 + \rho)^T \leq r^*$ and r^0 is sufficiently small,¹⁶ and that fixing $x(t') = 0$ for $t' < t$ and $x(t') = Y(1 + \rho)^T$ for $t' > t$, $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ as a function of just $x(t)$ has at most one inflection point on $[0, Y(1 + \rho)^T]$. Then $x_{(\alpha=0)}^*(t)$ is increasing for $t \leq T$ and there exists \tilde{t} such that $(x_{(\alpha>0)}^*(t) - x_{(\alpha=0)}^*(t))$ is negative for $t < \tilde{t}$ and positive for $\tilde{t} < t \leq T$, i.e., the optimal consumption profile allowing $\alpha > 0$ features even heavier delayed consumption or bingeing in the final periods.*

¹⁶Specifically, suppose $m'(r^0) > \delta^T (1 + \rho)^T m' \left(\frac{Y((1+\rho)^{T+1} - (1+\rho)^T)}{(1+\rho)^{T+1} - 1} \right)$.

Proof As $m'(\cdot)$ (and hence its inverse as well) is decreasing, inspection of equation (8) reveals that when $D(t)(1 + \rho)^t$ is increasing (or decreasing) in t , $x_{(\alpha=0)}^*(t)$ is as well. The assumption that $\lim_{s \rightarrow 0} m'(s) = \infty$ ensures that $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ is concave for \mathbf{x} near $\mathbf{0}$. In both parts of the theorem, we assume there is at most one inflection point of $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ viewed as a function of just $x(t)$ over a domain that includes the values of $\mathbf{x}_{(\alpha=0)}^*$. So if $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ is concave in $x(t)$ at $x_{(\alpha=0)}^*(t)$ for some t , it is also concave in $x(t')$ at $x_{(\alpha=0)}^*(t')$, for all $t' > t$ under the conditions of part 1 and for all $t' < t$ under the conditions of part 2. To show that $x_{(\alpha>0)}^*(t)$ is more extreme than $x_{(\alpha=0)}^*(t)$, we take the partial derivatives of the full utility function $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ from equation (3) with respect to $x(t)$, evaluated at $\mathbf{x}_{(\alpha=0)}^*$. We will find that $(1 + \rho)^t \left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{\mathbf{x}_{(\alpha=0)}^*}$ is increasing (or decreasing) in t when $x_{(\alpha=0)}^*(t)$ is increasing (or decreasing, respectively). Given the concavity of $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$ at $x_{(\alpha=0)}^*(t)$, this means $x_{(\alpha>0)}^*(t)$ is pushed to be even more extreme than $x_{(\alpha=0)}^*(t)$.

Equation (19) implies at the very least that $D(t)(1 + \rho)^t$ is strictly decreasing in t . (When v is sufficiently convex, it implies that the rate of decrease is sufficiently great, but when $\frac{d^2 u(x|0, r^*)}{dx^2} + \frac{D(t+1)}{D(t)-D(t+1)}(1 - \alpha)m''(x) < 0$ for all x , we find that the maximum in equation (19) occurs when $x' = x$, so $\Upsilon(t) = 1$ and we have just the minimal requirement that $D(t)(1 + \rho)^t$ is strictly decreasing.) In part 1 of the theorem, then, $x_{(\alpha=0)}^*(t)$ is decreasing in t and $x_{(\alpha=0)}^*(0) < r^0$, so loss aversion is present every time period. In this domain we find

$$\begin{aligned} \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} &= (1 - \alpha)D(t)m'(x(t)) + \\ &\quad \alpha D(t)m'(x(t))v'(m(x(t)) - m(r^*)) \left(\lambda - \frac{D(t+1)}{D(t)}\lambda \right). \end{aligned} \quad (20)$$

Plugging in $x(t) = (m')^{-1} \left(\frac{\bar{K}}{D(t)(1+\rho)^t} \right)$, we get

$$\left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{\mathbf{x}_{(\alpha=0)}} = (1-\alpha) \frac{\bar{K}}{(1+\rho)^t} + \alpha \lambda \frac{\bar{K}}{(1+\rho)^t} v' \left(m \left((m')^{-1} \left(\frac{\bar{K}}{D(t)(1+\rho)^t} \right) \right) - m(r^*) \right) \left(1 - \frac{D(t+1)}{D(t)} \right).$$

We now observe that $(1+\rho)^t \left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{\mathbf{x}_{(\alpha=0)}}$ is decreasing in t , as $\left(1 - \frac{D(t+1)}{D(t)} \right)$ is decreasing or flat by assumption and $v' \left(m \left((m')^{-1} \left(\frac{\bar{K}}{D(t)(1+\rho)^t} \right) \right) - m(r^*) \right)$ is decreasing because the budget constraint ensures v is convex in this domain. Thus, $x_{(\alpha>0)}^*(t)$ is pushed up for small t and down for large t , relative to $x_{(\alpha=0)}^*(t)$. While moving in this direction is necessary to obtain a local maximum of $\hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)$, we must still show that a global maximum with $x_{(\alpha>0)}^*(t+1) > x_{(\alpha>0)}^*(t)$ for some t may not be attained. Equation (19) makes such a global maximum impossible, as we would have $\frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t+1)} (1+\rho)^{t+1} < \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} (1+\rho)^t$.

In part 2 of the theorem, $x_{(\alpha=0)}^*(t)$ is increasing in $t \leq T$ and $x_{(\alpha=0)}^*(0) > r^0$, so loss aversion plays no role. In this domain we find that for $t < T$,

$$\frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} = (1-\alpha) \delta^t m'(x(t)) + \alpha \delta^t m'(x(t)) v' (m(x(t)) - m(r^*)) (1-\delta),$$

but at $t = T$, the future no longer matters, and

$$\frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(T)} = (1-\alpha) \delta^T m'(x(T)) + \alpha \delta^T m'(x(T)) v' (m(x(T)) - m(r^*)).$$

Plugging in $x(t) = (m')^{-1} \left(\frac{\bar{K}}{\delta^t(1+\rho)^t} \right)$, we get for $t < T$:

$$\left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{\mathbf{x}_{(\alpha=0)}^*} = (1-\alpha) \frac{\bar{K}}{(1+\rho)^t} + \alpha \frac{\bar{K}}{(1+\rho)^t} v' \left(m \left((m')^{-1} \left(\frac{\bar{K}}{\delta^t(1+\rho)^t} \right) \right) - m(r^*) \right) (1-\delta); \quad (21)$$

at $t = T$ we get a similar expression, only missing the factor of $(1-\delta)$ from equation (21). Analogously with the proof of part 1, we observe that $(1+\rho)^t \left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{\mathbf{x}_{(\alpha=0)}^*}$ is increasing for $t \leq T$ because of the convexity of v and the fact that the $t = T$ term is missing a factor of $(1-\delta) < 1$. Thus, $x_{(\alpha>0)}^*(t)$ is pushed down for small t and up for larger t (not exceeding T), relative to $x_{(\alpha=0)}^*(t)$. ■

Proof of Theorem 3

When the subjective discount rate equals the real interest rate, there is no direct preference for saving or spending, but time preferences can arise from preferences over consumption levels and patterns. When the utility function is concave, the optimal consumption profile can be found with the method of Lagrangian multipliers. There is no time dependence in the first order equations, so we get a steady consumption pattern in which $\sum_{t=0}^{\infty} \frac{x^*}{(1+\rho)^t} = Y$. This implies $x^*(t) = Y \frac{\rho}{1+\rho}$ for all t .

When the utility function has a convex region, consumption levels are pushed out of the region by decreasing consumption in some periods and increasing consumption in others. When loss aversion is strong enough, saving so much that a loss is incurred is never worthwhile. If a feasible consumption profile involves more than one period of consumption in the convex region, a better consumption profile would save more in the earlier period and consume more in the later period until consumption in one of these periods moved beyond the convex region. To avoid losses, saving must occur

first and cannot push consumption below the status quo. Consumption in the later periods must be steady because utility becomes concave again.

When the utility function is convex at the feasible steady consumption profile and loss aversion is absent, we can use the method of Lagrangian multipliers to find stable consumption levels above and below the convex region. Because

$$\left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{x(t)=\omega} < \left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{x(t)=\xi},$$

it is possible to find many pairs $x_H^* \geq \xi$ and $x_L^* \leq \omega$ such that

$$\left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{x(t)=x_L^*} = \left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{x(t)=x_H^*}.^{17} \quad (22)$$

If feasible, the optimal x_H^* and x_L^* would be located such that the expressions in equation (22) also equal

$$\frac{\int_{x_L^*}^{x_H^*} \left. \frac{\partial \hat{U}(\mathbf{x}|r^0, \mathbf{r}^*)}{\partial x(t)} \right|_{x(t)=s} ds}{x_H^* - x_L^*}, \quad (23)$$

and thus no gain is possible from moving between these consumption levels.

Equations (22) and (23) determine the optimal consumption levels x_H^* and x_L^* , but it remains to show that it is feasible to allocate the entire budget at just these levels in discrete time periods. The set \mathcal{T} enumerates the time periods in which the higher consumption level x_H^* is chosen. In the absence of loss aversion, there is no cost to switching back and forth between higher and lower consumption, and this will in general be necessary to exhaust the budget. The budget constraint is

$$\frac{1}{1 - \frac{1}{1+\rho}} x_L^* + \sum_{t \in \mathcal{T}} \left(\frac{1}{1 + \rho} \right)^t (x_H^* - x_L^*) = Y,$$

¹⁷If v is not continuously differentiable at 0, we also consider the possibility that $x_H^* = r^*$ and the partial derivative at x_L^* is between the left hand and right hand partial derivatives at x_H^* .

or equivalently,

$$\sum_{t \in \mathcal{T}} \left(\frac{1}{1 + \rho} \right)^t = \frac{Y - \frac{1+\rho}{\rho} x_L^*}{x_H^* - x_L^*}. \quad (24)$$

The right hand side of equation (24) is necessarily strictly between 0 and $\frac{1+\rho}{\rho}$ by our assumption bounding the present endowment Y so that steady consumption would be in the region of convex utility. For $\rho \leq 1$, we can construct \mathcal{T} so that the left hand side of equation (24) converges to any value between 0 and $\frac{1+\rho}{\rho}$. ■

Proof of Theorem 4

$$\begin{aligned} U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*) &= \alpha \left(p m(z) + (1-p)m\left(\frac{r^0 - pz}{1-p}\right) - m(r^0) \right) + \\ &\quad (1-\alpha) \left[p [v(m(z) - m(r^*)) - v(m(r^0) - m(r^*))] + \right. \\ &\quad \left. (1-p)\lambda \left[v\left(m\left(\frac{r^0 - pz}{1-p}\right) - m(r^*)\right) - v(m(r^0) - m(r^*)) \right] \right]. \end{aligned}$$

We arrange terms and make use of our notation for the average value of v' , as follows:

$$\begin{aligned} U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*) &= \alpha \left(p(m(z) - m(r^0)) + (1-p) \left(m\left(\frac{r^0 - pz}{1-p}\right) - m(r^0) \right) \right) + \\ &\quad (1-\alpha) \left[p [m(z) - m(r^0)] \bar{v}'((m(r^0) - m(r^*)), m(z) - m(r^*)) \right. \\ &\quad \left. - (1-p)\lambda \left[m(r^0) - m\left(\frac{r^0 - pz}{1-p}\right) \right] \bar{v}'\left(m\left(\frac{r^0 - pz}{1-p}\right) - m(r^*), m(r^0) - m(r^*)\right) \right]. \end{aligned}$$

We arrange terms again by factoring:

$$\begin{aligned}
& U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*) = \\
& \quad p(m(z) - m(r^0)) [\alpha + (1 - \alpha) \bar{v}'((m(r^0) - m(r^*)), m(z) - m(r^*)))] \\
& - (1-p) \left(m(r^0) - m\left(\frac{r^0 - pz}{1-p}\right) \right) \left[\alpha + (1 - \alpha) \lambda \bar{v}'\left(m\left(\frac{r^0 - pz}{1-p}\right) - m(r^*), m(r^0) - m(r^*)\right) \right].
\end{aligned}$$

Now making use of the notation for the average value of m' , we get:

$$\begin{aligned}
& \frac{1}{p(z - r^0)} (U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*)) = \\
& \quad \bar{m}'(r^0, z) [\alpha + (1 - \alpha) \bar{v}'((m(r^0) - m(r^*)), m(z) - m(r^*)))] \\
& - \bar{m}'\left(\frac{r^0 - pz}{1-p}, r^0\right) \left[\alpha + (1 - \alpha) \lambda \bar{v}'\left(m\left(\frac{r^0 - pz}{1-p}\right) - m(r^*), m(r^0) - m(r^*)\right) \right].
\end{aligned}$$

So, $U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*) \geq 0$ if and only if

$$\begin{aligned}
& \alpha + (1 - \alpha) \bar{v}'((m(r^0) - m(r^*)), m(z) - m(r^*)) \geq \\
& \quad \frac{\bar{m}'\left(\frac{r^0 - pz}{1-p}, r^0\right)}{\bar{m}'(r^0, z)} \left[\alpha + (1 - \alpha) \lambda \bar{v}'\left(m\left(\frac{r^0 - pz}{1-p}\right) - m(r^*), m(r^0) - m(r^*)\right) \right]. \quad (25)
\end{aligned}$$

We obtain the desired sufficient inequality by using the convexity of v to place an upper bound on the right hand side. ■

Proof of Corollary 1

For any $r^* \in [r^*, \hat{r}^*]$, a necessary and sufficient condition for $U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*) \geq 0$ is given by equation (25) in the proof of Theorem 4. We take a

derivative of this inequality (25) with respect to r^* . On the left hand side we obtain

$$-(1 - \alpha) m'(r^*) \frac{v'(m(z) - m(r^*)) - v'((m(r^0) - m(r^*)))}{m(z) - m(r^0)}.$$

This can be rewritten as $-(1 - \alpha) m'(r^*) \bar{v}''((m(r^0) - m(r^*)), m(z) - m(r^*))$. Similarly, on the right hand side we obtain

$$-\frac{\bar{m}'\left(\frac{r^0 - pz}{1 - p}, r^0\right)}{\bar{m}'(r^0, z)} (1 - \alpha) \lambda m'(r^*) \bar{v}''\left(m\left(\frac{r^0 - pz}{1 - p}\right) - m(r^*), m(r^0) - m(r^*)\right).$$

Concavity of m implies $\frac{\bar{m}'\left(\frac{r^0 - pz}{1 - p}, r^0\right)}{\bar{m}'(r^0, z)} \geq 1$. The condition that $v''' \leq 0$ over the relevant interval implies that

$$\bar{v}''\left(m\left(\frac{r^0 - pz}{1 - p}\right) - m(r^*), m(r^0) - m(r^*)\right) \geq \bar{v}''((m(r^0) - m(r^*)), m(z) - m(r^*)).$$

Thus, the right hand side of inequality (25) is decreasing faster than the left hand side, as r^* increases. Our result follows. ■

Proof of Corollary 2

A necessary and sufficient condition for $U(F_{z,p,r^0}|r^0, r^*) - u(r^0|r^0, r^*) \geq 0$ is given by equation (25) in the proof of Theorem 4. In the high aspiration level limit we have

$$\begin{aligned} \lim_{r^* \rightarrow \infty} \bar{v}'((m(r^0) - m(r^*)), m(z) - m(r^*)) &= \lim_{r^* \rightarrow \infty} \bar{v}'\left(m\left(\frac{r^0 - pz}{1 - p}\right) - m(r^*), m(r^0) - m(r^*)\right) \\ &= \lim_{s \rightarrow -\infty} v'(s). \end{aligned}$$

Concavity of m implies $\frac{\bar{m}'\left(\frac{r^0 - pz}{1 - p}, r^0\right)}{\bar{m}'(r^0, z)} \geq 1$. Taking the limit of equation (25) as $r^* \rightarrow \infty$, it can only be satisfied at equality when m is linear and when either $\lambda = 1$ or

$\lim_{s \rightarrow -\infty} v'(s) = 0.$ ■

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