Effects of Feedback and Collaboration on Changes in Children's Use of Mathematical Rules

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> Collaborative learning in schoolage children . . . is a well-established phenomenon. Studies have demonstrated repeatedly that for children of this age problem solving with a peer, even one who is no more knowledgeable than oneself, often leads to greater task understanding than problem-solving alone or even, in many cases, to problem solving in the context of instruction (Tomasello, Kruger, & Ratner, in press, p. 11).

If the opinion expressed by Tomasello and his colleagues is accurate, one might wonder why we need yet one more study to assess the effectiveness of peer collaboration. In fact, the evidence used to support claims about the efficacy of peer interaction--especially interaction between novices--for promoting cognitive growth is rather weak.

Much of the research on peer collaboration has been based on the idea that children make advances in their reasoning when they recognize conflicts between their understanding of a problem and the perspective of another (Piaget, 1932). It is widely believed that social (or socio-cognitive) conflict is more conducive to learning than conflict between a child's understanding of a problem and task materials (Azmitia & Perlmutter, 1989). However, the results of studies designed to assess the efficacy of socio-cognitive conflict in promoting learning are far from conclusive.

Perhaps the most widely cited study used to support the view that interaction between novices with different, but equally wrong, points of view can lead to cognitive growth is that conducted by Ames and Murray (1982). In this study, children who failed to conserve on a series of conservation of length tasks were assigned to one of a number of conflict conditions, including a social interaction condition in which they were paired with another nonconserver who had disagreed with them on the pretest conservation tasks. Ames and Murray found that

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nonconservers in the social interaction condition had higher conservation scores on both immediate and delayed posttests than children in any of the other conditions, and that advancement to a conservation response on the posttests required exposure to a conservation explanation during the social interaction. However, it must be noted that, when conservation judgements were given during the social interaction session, they were almost always given on the first problem and, once made, were the only judgement made about the item. This suggests that some of the children who were nonconservers on the pretest became conservers by the first problem in the social interaction session.

Thus, while the Ames and Murray study replicated earlier findings that nonconservers paired with conservers often show progress in their reasoning (Miller & Brownell, 1975), the findings cannot be used to support the claim that pairing children with equally unskilled peers can lead to increased task understanding. While some researchers have had greater success in showing that pairs of nonconservers benefit from collaboration (Mugny & Doise, 1978), these demonstrations are offset by other, better controlled studies in which such benefits have not accrued (Russell, 1982; Russell, Mills, & Reiff-Musgrove, 1990). Do children benefit from interacting with others no more knowledgeable than themselves? At this point, the data are inconclusive, at least with respect to Piagetian tasks.

A weakness of many peer collaboration studies is the lack of precise assessments of children's thinking before, during, and after interaction. As suggested by the Ames and Murray study, it is impossible to examine whether social interaction between novices leads to learning when the relative expertise of partners is unknown, changes suddenly, or is poorly specified from the outset. Perhaps this is why the strongest conclusion that can be drawn about peer collaboration is that much of the success can be attributed to the positive influence of a relative expert on the performance of a less competent child (Azmitia & Perlmutter, 1989).

The advantages of using precise measures of children's thinking for disentangling the benefits of interaction from that of expertise is illustrated by a series of studies examining peer collaboration on mathematical balance beam problems (Tudge, 1991; 1992). Tudge used Siegler's (1976, 1981) rule assessment method to classify children according to one of six rules used to solve balance beam problems. This approach allowed Tudge to pair children who used the same, partially incorrect rule to solve the balance scale problems and to compare the processes and outcomes of their collaboration with that of dyads composed of relative experts and novices and the performance of children working alone. He found that dyads were no more likely to progress to an advanced rule than children working alone, regardless of whether the children were paired with a partner who used the same incorrect rule or a more advanced rule. In fact, while some expert-novice pairs advanced, regression on this task was as likely as improvement unless children were provided feedback from either the experimenter or the materials as to the correctness of their predictions.

The lack of precise measures of children's thinking also limits what can be said about the mechanisms by which social interaction promotes learning. So, while peer collaboration has been shown to enhance performance on a variety of reasoning tasks, (Gauvain & Rogoff, 1989; Kruger, 1992; Phelps & Damon, 1989; Teasley, 1993; Webb, 1991), our understanding of the processes by which social interaction led to improved performance on these tasks remains quite vague.

The goal of the present study was to specify more exactly the mechanisms by which collaboration leads to advancement by examining collaboration in a domain where we could (a) obtain accurate assessments of individual performance prior to collaboration; (b) chart changes in individual strategy use during collaboration and independent performance; and (c) examine whether changes observed during collaboration are maintained in subsequent individual performance. We were especially interested in assessing the benefits of collaboration between peers who were equally competent, yet not expert on a task because it is this particular arrangement that is least understood. Finally, it seemed worthwhile to pursue this issue on academic tasks, since collaborative problem solving has become such a popular feature in classroom instruction (Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz, 1991; Saxe & Guberman, 1993; Webb, 1991).

Rule Use in the Domain of Decimal Fractions

Children learning decimal fractions often misapply mathematical rules that they have acquired in the course of learning about whole numbers or common fractions. (Resnick, Nesher, Leonard, Magone, Omanson, & Peled, 1989; Sackur-Grisvard & Leonard, 1985). Sackur-Grisvard and Leonard (1985) detected the use of incorrect rules by asking children to order decimal fractions. Resnick et al (1989) contrasted the rates of use of these incorrect rules among children in France, Israel, and the United States, and linked cross-national differences to differences in the order in which children in the three countries were exposed to common and decimal fractions in the school curriculum. The incorrect rules used by the children in both the Sackur-Grisvard and Leonard study and the study conducted by Resnick and her colleagues were all variants of one of two general rules Resnick labelled the *Whole Number Rule* and the *Fraction Rule*. According to the *Whole Number Rule*, the more digits there are to the right of the decimal point, the larger the number (e.g. 0.37 > 0.4). This rule reflects children's experience with whole numbers and may arise from classroom practice with decimal numbers that are always the same length where treating decimals like whole numbers always leads to the correct answer.

In contrast, children who adhere to the *Fraction Rule*, believe that the fewer digits to the right of the decimal point, the larger the number (0.7 > 0.94). The fraction rule may arise when children attempt to apply their understanding of the relationship between the size and number of parts denoted by the numerator and denominator in regular fractions to their as yet incomplete understanding of decimal notation. Children who adhere to the fraction rule may argue that three digit decimals are smaller than two digit decimals because thousandths are smaller units than hundredths. Other fraction rule users translate the decimal number into a regular fraction, but treat the digits to the right of the decimal point as the denominator rather than the numerator (e.g., 0.7 > 0.94 because 1/7 > 1/94). Resnick et al observed use of both the fraction rule and the whole number rule in each of their three samples. However, the rates with which the rules were used differed across the samples. The fraction rule was used by less than 10% of the French children, 33% of the Israeli children, and 18% of the American children (although 35% of children in the U.S. sample used the whole number rule and 29% could not be classified as to which rule they used to compare the sizes of decimal fractions). Resnick et al attribute the different rates of use of the two rules to the fact that students in the U.S. and Israel typically receive more in-depth instruction on common fractions before they are introduced to decimal fractions; in France, decimal instruction precedes instruction on common fractions by a substantial period of time.

The present study employs a rule assessment approach like that used by Tudge and Siegler on the balance beam to determine the strategies children use to solve problems involving decimal fractions. It is especially interesting to examine the processes of collaboration in children's explanations of decimal fractions, as children can bring very different, but equally wrong, misconceptions to the domain. The present study examines (1) whether working on decimal fraction problems with a partner helps fifth grade children overcome misconceptions found to be quite stable when children work alone; (2) whether social interaction is sufficient for children to show improvement in this domain, or if feedback is also necessary; (3) whether dyads composed of children who hold different misconceptions are more successful than those composed of children who share the same misconception.

Method

Subjects

The children in our study came from ten schools serving ethnically-mixed, middle-income communities. A sample of 517 fifth graders were presented an 80 problem pretest to identify the rules they used to compare the relative sizes of decimal fractions. Ninety-three percent of the fifth graders screened could be classified as using a *fraction rule*, *whole number rule*, or *correct rule* by a criterion under which at least 80% of their 80 pretest responses needed to conform to the prediction of that rule. One hundred and seventy-seven children who consistently followed either the fraction or the whole number rule were assigned to solve a new set of decimal problems alone or with a same-sex partner. Of these, 36 (20%) switched the rule they employed to solve the decimal problems between the pretest and a set of 20 warmup problems given at the beginning of the experimental session. Children who did not meet the 80% criterion for the original rule on the warmup problems or who were paired with partners who did not meet the criterion were dropped from these analyses. The data from an additional 17 children were excluded from the analyses because they were assigned to conditions dropped from the study, or because of problems with the procedure or equipment. The data to be reported here are based on 124 children (68 girls and 56 boys) who could be classified as using either the *whole number rule* or *fraction rule* on both the pretest and warmup problems.

Materials

While previous studies suggest that the errors children make when comparing decimal fractions can be classified according to several distinct rules derived from prior knowledge about whole numbers and common fractions (Resnick et al, 1989; Sackur-Grisvard & Leonard, 1985), these findings are based on children's performance on a small number of problems. We employed the rule assessment method formulated by Siegler (1976, 1981) to generate a set of 22 different types of problems which could be used as a more rigorous test of the stability of children's rule use when comparing decimal fractions (see Appendix A for a breakdown of problem types). The rule assessment approach allows for the classification of children's rules according to specific patterns of correct answers and errors. Children were classified as using one of the target rules if 80% or more of their responses corresponded to the pattern predicted by the rule.

Children's understanding of the relative sizes of decimal fractions was assessed by asking them to circle which of two decimal fractions was bigger (e.g., 0.19 versus 0.147). The pre- and posttests consisted of 80 of these problems; children were also presented 20 similar problems as "warmup" problems at the beginning of the experimental session. During the experimental session, children were asked to solve 12 more problems, each one representing a different problem type. The 12 problems were presented on two sheets of paper, with 6 problems to a page. In the feedback condition, correct answers were indicated by arrows placed under the larger number. The arrows were hidden by removable stickers, which children peeled off after they had given their response. In the no feedback conditions, the children were presented the same problems without the arrows and stickers (see Table 1).

Assignment to Rule

The 80 problem pretest revealed several variants of the whole number and fraction rules, each reflected by different patterns of errors. For the analyses reported here, we have combined all whole number rules and all fraction rules together. On the pretest, 63% of the children used a *whole number rule*, 14% used a *fraction rule*, and 15% used a *correct rule* for comparing decimal fractions (see Appendix B for a list of rules).

Procedure

Pretest. Fifth graders were presented the 80 problem pretest in their classrooms. We told the students that we were interested in how fifth graders think about decimal fractions because we wanted to discover better ways to teach students. Because the students were sometimes anxious that they did not know how to work with decimal fractions, we emphasized that we did not expect them to know how to do these problems, but wanted them to give it their best guess. On average, the fifth graders completed the pretest in less than ten minutes. The pretests were then scored to determine which rule children used to compare decimal fractions.

Design. Children were assigned to work by themselves ("alones") or with a partner of the same sex ("dyads"). Children were either paired with a partner who used the same rule ("whole/whole" dyads or "fraction/fraction" dyads) or with a child who used a different rule ("fraction/whole" dyads) on the pretest. The subjects were then assigned to either a "feedback" or "no feedback" condition. Because it was difficult to secure a sufficient number of fraction rule users to fill all the cells in a complete factorial design, we eliminated two cells that required large numbers of fraction rule users and did not look to yield very interesting results--the fraction/fraction without feedback condition and the fraction alone without feedback condition (see Figure 1).

Experimental Session. At the beginning of the experimental session, subjects were asked to solve 20 problems individually. This phase served as a warmup for subsequent problems and provided a means to assess whether children were using the same rule that day as they had on the pretest. "Alones" or "dyads" in which one or both children used a different rule than they had on the pretest were run in their assigned conditions; however, their data are not included in the analyses to be reported here.

The children were then presented with twelve decimal fraction problems to work on either alone or with a partner. Dyads were instructed to solve one problem at a time following this procedure: The children were first asked to independently determine which of two decimal fractions they believed to be bigger. Then, they were to compare their answers, and to explain why they believed the number they chose was bigger. Dyads in the feedback condition were then allowed to peel up the stickers to reveal the correct answer. Children in the alone condition followed the same procedure, except they were not asked to explain their answers. All children were told that there were some ways to solve decimal problems that led to the right answer on some problems, but to the wrong answer on others and also a way that got all the problems right. The children were encouraged to work [together] to find a way that got all the problems correct, and were informed that they would be asked to individually solve another set of problems where they would not find out if they were correct at the end of the session. After children completed problems 6 and 12, they were reminded that the goal of the task was to find one way to get all the problems right. In cases where both members of a dyad solved a problem incorrectly and received feedback that they were wrong, they were asked if they could figure out how the other number could be larger. The dyadic interactions were videotaped.

Posttest. The posttest was identical to the pretest. All children completed the posttest at the end of the experimental session.

Coding

The explanations provided by the partners during the experimental session were transcribed and coded by two raters. Explanations were coded as reflecting a *fraction rule*, *whole number rule*, or *correct rule* to compare the decimal numbers. Explanations that did not fit any of these three categories are combined here for the purposes of analyses as *other* explanations. The explanations provided by children included those that emphasized the notational features of the numbers (e.g., the whole number explanation, "0.94 is bigger than 0.7 because there are three numbers in it and only two in the other") as well as explanations that reflect children's efforts to think about the number in terms of the quantity the number represents (e.g., the whole number explanation, "0.94 is bigger than 0.7 because 94 is bigger than 7--there is like more stuff in 94"). Children were credited as using a correct rule when they provided explanations that were mathematically valid. Correct explanations included statements such as "if there is a 0 right next to the decimal point, it means that that number is smaller (comparing 0.64 and 0.029);" "you can see this one is bigger because if you add a zero, .40 is bigger than .37, or "0.536 is bigger than 0.27 because it has 5 tenths and 0.27 only has 2 tenths and 5 tenths is bigger than 2 tenths." Further examples of these explanation categories as well as those included in the category of "other" explanations are provided in Table 2. Inter-rater reliability, computed by exact agreement, exceeded 90%. Disagreements were resolved through discussion.

Results

The questions we hoped to address with this study were: (1) Will children who have the opportunity to collaborate on a series of decimal problems acquire a better understanding of decimal numbers than children who solve the problems alone as measured by performance on an individual posttest? (2) Is social interaction sufficient for children to show improved performance on this task, or is feedback also necessary for progress to occur? (3) Are dyads composed of children with different misconceptions more likely to promote understanding of the task than dyads composed of children who share the same misconception? (4) Do interactions between children who have different misconceptions differ from those where children share the same misconception?

Impact of Collaboration and Feedback on Posttest Performance

As can be seen in Table 3, the only children to become expert on the posttest were those who received feedback about the correctness of their answers during the experimental session. Regardless of their dyadic status, none of the children in the no feedback condition progressed to expert status on the posttest, while 49% of those in the feedback condition did, ${}^2(1)=39.36$, $p < .001$.

Since no children in the no feedback condition progressed to expert status on the posttest, the discussion of the impact of social interaction on posttest performance will be limited to children in the feedback condition. Children who worked with a partner during the experimental session and received feedback were more than twice as likely to use a correct rule on the posttest as children who received feedback but worked alone. Fifty-seven percent of the children who worked with a partner progressed to expert status on the posttest, whereas only 22% of the children who worked alone did so, $\chi^2(1)= 6.66$, $p < .01$. In addition, it was more likely for both memebers of a dyad to become expert than one member of a dyad to become expert and the other to use an incorrect rule on the posttest (41% versus 28% of the dyads, respectively).

Impact of Dyad Type on Posttest Performance

The percentage of children from the different dyad types who moved to expert status on the posttest ranged from 45% (whole number rule users paired with other whole number rule users) to 67% (fraction rule users paired with whole number rule users). Fraction rule users appeared slightly more likely to adopt the correct rule on the posttest than whole number rule users, regardless of whether they were paired with other whole number rule users or other fraction rule users (see Table 3). However, these differences are not statistically significant. In addition, in the feedback condition, fraction rule users who worked alone were no more likely to progress to expert status than were whole number rule users who worked alone.

Impact of Dyad Type on Collaboration

While there were no significant differences in the number of children from each dyad type who became expert on the posttest, we were also interested in whether there were differences in the nature of the collaborations observed among the different kinds of dyads. In this paper, we will focus our attention on changes in the kinds of explanations children in the different dyads offered over the course of the experimental session. One way to examine these changes is to compare each dyad type at the point during the experimental session where the children begin to consistently offer correct explanations for ways to compare decimal fractions. We scored a child as consistently offering a correct explanation at the point in the session where they gave a correct explanation and never again offered an incorrect explanation (e.g., a fraction or whole number explanation). (About 60% of the children who ever offered a correct explanation offered an incorrect explanation on a subsequent problem, while 40% of the children never offered an incorrect explanation once they gave a correct explanation. They did, however, sometimes offer explanations that we could not code as either correct or incorrect).

In the feedback conditions, there were differences between the different dyads with respect to the point in the session when correct explanations began to be advanced consistently. Dyads composed of two fraction rule users and one fraction rule user paired with a whole number rule user were very similar in both the rate at which children begin to offer correct explanations for comparing decimal numbers, and the proportion of children in each dyad type who provided correct explanations on all trials by the end of the experimental session. In both fraction/fraction and fraction/whole dyads, about 20% of the children offered correct explanations by the fourth problem in the session, while 60% provided correct explanations by the tenth problem and on all subsequent problems (see Figures 2 and 3). In contrast, only 5% of children in the whole/whole dyads provided correct explanations by the fourth problem, while 25% offered correct explanations by problem 10 and on all subsequent problems. The percentage of children in whole/whole dyads offering correct explanations by the end of the session did not exceed 45%, while almost 70% of the children in the whole/fraction dyads and 85% of the children in the fraction/fraction dyads were offering correct explanations by the end of the session. Chi square analyses revealed these differences to be significant, χ^2 (2)=13.87, p < .001; specific comparisons using Ryan's procedure showed the significant difference to be between the whole/whole and fraction/fraction dyads. The similarities between the fraction/fraction and fraction/whole dyads and the differences between these dyad types and whole/whole dyads is even more striking when one compares the point in the session where both members of the dyads begin to consistently offer correct rules for comparing decimal numbers (see Figure 4). Note that the point in the session where partners consistently begin to offer the correct explanation is more highly correlated in the whole/fraction and fraction/fraction dyads than in the whole/whole dyads (r=.74, .71, and .54, respectively) as well (see Figures 6, 7, and 8).

While is clear from the previous analyses that children in the fraction/fraction and fraction/whole dyads increased their use of a correct rule for comparing decimal numbers over the course of the session when given feedback, it is not clear whether children in the whole/whole feedback condition also shifted away from their original rule but failed to replace it with a correct rule, or whether they continued to rely on the whole number rule exclusively throughout the session. Figure 9 illustrates changes in the proportions of correct, fraction, whole number, and other kinds of explanations given by children in the whole/whole dyads in the feedback conditions over the course of the session.

While whole number children in the no feedback condition showed little change in the frequency with which they offer whole number explanations from the first four to the last four problems (see Figure 3), whole number rule users in the feedback condition show a steady and significant decrease in the number of whole number explanations given over the time, F (2,38) = 12.37, p < .01. These children also showed a slight increase in the number of correct explanations offered, F(2,38) = 3.41, p < .05; post hoc Newman Keuls comparisons revealed that the significant increase occurred between the first four problems and problems 5 through 8. They also showed an increase in the number of fraction rule explanations offered, $F(2,38) = 3.28$, $p < .05$; post hoc comparisons showed the number of fraction rule explanations given on the last four problems to be significantly higher than those given on problems 1 through 4 and 5 through 8. The same pattern was found for explanations that fell into our "other" category, $F(2,38) = 4.79$, $p < .01$. It is notable that, by the last four problems, over 50% of the explanations provided by children in the whole/whole with feedback condition were classified as "other". It appears that feedback informed the children in the whole/whole dyads that the whole number rule was wrong, but it did not enable most children to discover the correct rule. Unable to figure out a correct way to solve the problems, these children often resorted to guessing or established idiosyncratic solutions such as comparing the right most digits in each number, or identifying a pattern in the way correct answers appeared in the right or left columns on the page. These idiosyncratic solutions would not be supported if the children would test their rule on all problems, but as studies of children's scientific reasoning have shown, (Klahr, Fay, & Dunbar, in press; Kuhn, 1989), children often neglect to try their rule out on a sufficient number of problems, or when they do, they ignore problems that disconfirm their hypothesis.

Discussion

The results of this study demonstrate that children who do not yet use a correct rule to compare decimal fractions are more likely to construct such a rule in collaboration with a partner than when working alone. However, the ability to construct such a rule appears to depend on receiving feedback as to whether the answers that result from proposed rules are correct or not. We did not find that pairing children with partners who held a different misconception regarding the relative sizes of decimal fractions promoted a greater understanding of decimal fractions than pairing children with others who shared the same misconception, although we did discover that, under feedback conditions, the processes of collaboration differed depending on the composition of the dyad. Under feedback conditions, dyads composed of two fraction rule users and those composed of one fraction rule user and one whole number rule user were very similar in the rate at which children abandoned incorrect rules and adopted correct rules for comparing decimal problems. In this section, we will discuss three issues raised by these findings: (1) Why was feedback necessary for the construction of a correct rule, even for dyads with different initial rules? (2) What aspects of the social interaction promoted children's understanding on this task? (3) Why was collaboration between children who shared the same misconception as effective in facilitating the use of a new strategy as collaboration between children who shared different misconceptions?

The Role of Feedback in Promoting Understanding

In order to use the correct rule on the posttest, children in this study first had to generate the rule, then adopt it. Did feedback primarily influence the initial generation of a correct explanation, or was generation rather commonplace and feedback more important in helping children decide which of a number of possible rules was the best one to use for comparing the relative sizes of decimal fractions?

The idea that feedback influenced the generation of a correct explanation is supported by the fact that the number of correct explanations advanced in the no feedback conditions was extremely low, only 4 instances overall. Does this mean that the effects of feedback were limited to the generation of correct rules? It is possible that, once generated, the correct ways of comparing the relative sizes of decimal fractions would seem eminently more credible than any of the alternatives. If this were the case, we would expect the correct rules to be adopted even without feedback. However, none of the children who advanced the correct explanations in the no feedback condition nor their partners adopted the correct rule on the posttest. Further, an average of 30% of children in each of the dyadic feedback conditions generated a correct explanation at least once during the interactive session, but failed to adopt it on the posttest.

Preliminary data from a follow-up study examining the interactions of experts and novices on this same task without the benefit of feedback further suggests that mere exposure to a correct explanation does not guarantee adoption. In this study, novices were paired with "stable" experts (children who used the correct rule on both the pretest and warm-up problems) or "new" experts (children who had used an incorrect rule on the pretest, but switched to a correct rule on the warmup problems) and assigned to no feedback conditions. Two-thirds of the novices working with "stable" experts but only 17% of those working with "new' experts adopted the correct rule on the posttest. We have not yet compared the quality of explanations offered by "stable" and "new" experts, although it seems unlikely that the quality of correct explanations offered by the "new" experts is, on average, significantly worse than that offered by novices in the dyadic conditions studied here. It should also be noted that about 15% of both stable and new experts regressed to an incorrect rule on the posttest in the absence of feedback.

These data suggest that feedback served two roles in the study: First, it provided compelling evidence to children that their old rules were not working and encouraged them to generate new rules. Secondly, it did provide a means by which children could judge the quality of the rules that were generated.

However, if feedback, and feedback alone, served these functions, why did so few children who worked alone and recieved feedback adopt the correct rule on the posttest? Because children in the alone conditions in this study did not provide explanations for their judgements on each problem, we cannot determine whether they ever generated a correct explanation or not. However, we conducted a follow-up study in which whole number rule users working alone and receiving feedback were asked to explain their reasoning on each problem to the experimenter. In this study, children working alone, receiving feedback, and giving explanations were half as likely to generate a correct explanation than whole number rule users paired with other whole number rule users (36% as opposed to 72%). Does this mean that the increase in correct explanations generated in the dyadic conditions was simply due to an increase in the likelihood that a correct explanation would be advanced by chance? Using the alone correct explanation generation rate of 36% as a base, only 59% of dyads including a whole number rule user would be expected to include at least one child who generates a correct explanation at some point in the session. Why was feedback combined with social interaction more powerful than feedback alone in promoting the generation of correct explanations?

Aspects of Social Interaction that Promote Cognitive Change

One important difference between the collaborative and alone conditions used in this study is that children provided explanations in the collaborative conditions but were discouraged from talking about the problems in the alone conditions. It is often believed that social interaction promotes better performance because children benefit when they verbalize their reasoning and make it explicit, or because making statements publicly encourages children to be more reflective and to solve problems more slowly. It is possible that the benefits accrued in the social interaction condition were due to the fact that collaborating children verbalized the strategies they were using on the task, but children working alone did not. However, as discussed above, children in the follow-up study who provided explanations as they worked on the problems alone were less likely to generate correct explanations during the experimental session than children working with a partner. Further, they were no more likely to adopt the correct rule on the posttest than children who worked alone and did not provide explanations, even though both groups received feedback; only 2 of the 12 whole number rule users who gave explanations adopted a correct rule on the posttest, as compared to 3 of the 12 whole number rule users who did not.

It is clear that collaboration on this task was beneficial beyond the verbalization of strategies or rules. We are currently analyzing the collaborative interactions in order to determine how the opportunity to collaborate impacted on children's generation and adoption of correct rules.

Shared versus Conflicting Misconceptions

Researchers working from a Piagetian perspective have long argued that social interaction among peers promotes cognitive development because it forces children to consider problems from multiple perspectives. According to this view, children paired with children who hold different misconceptions about a problem would experience socio-cognitive conflict and, in an effort to reduce that conflict, would show advances in their reasoning. In this study, dyads composed of children with disparate misconceptions were no more likely to advance than dyads composed of children who reasoned similarly. An implicit assumption of the socio-cognitive model of cognitive change is that children adhere strongly to one perspective on a problem and experience difficulty thinking about the problem from other perspectives. However, as in other areas of mathematics (Siegler and Shrager, 1984), some of the children in this study appeared to have several ways of approaching these problems. This suggests that the opportunity to debate the strengths and weaknesses of different approaches does not require two people with different perspectives, but two people who can think about different perspectives. While the members of the whole/whole with feedback dyads were as likely to be experts on the posttest as children from the other dyadic conditions, these children appeared to have greater difficulty considering alternative ways of solving the problems once they discovered their old rule did not work. As a result, these children tended to make greater progress later in the interactive session than did children from the other dyads, and it was more likely that one child rather than both children in a dyad had mastered a correct rule than was the case for the other dyad types.

Conclusion

Children who had the opportunity to collaborate with a partner on a series of decimal problems were more likely to use a correct rule on a posttest than children who worked alone, but only if they were given feedback as to whether the answers resulting from proposed rules were correct or not. Collaboration appeared to affect both the generation of correct rules and the adoption of those rules. Collaborating groups generated correct explanations at a rate higher than would be expected by chance. However, once generated, correct explanations were not always adopted. Since it is clear that mere exposure to a correct explanation does not ensure immediate adoption, it appears that collaboration also influenced whether the correct rules were adopted or not.

The findings of this study also underscore the importance of obtaining precise measures of children's thinking. The orignal forumlations of socio-cognitive conflict acknowledged that children may reason about a problem in a number of ways, that some of those ways may prove to be inconsistent, and that discussions among equals would might reveal those inconsistencies. In practice, however, contemporary researchers tend to use rough assessments of children's reasoning, and classify children as belonging to broadly defined groups (e.g., conservers and nonconservers). In order to disentangle the effects of social interaction from that of expertise, it is critical that researchers consider the nature of what children are thinking about as carefully as they think about the processes of interaction.

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TABLE 1

PROBLEMS USED IN EXPERIMENTAL SESSION

(1)	0.94	0.7
(2)	0.24	0.178
(3)	0.0	0.07
(4)	0.64	0.029
(5)	0.06	0.089
(6)	0.37	0.4
(7)	0.628	0.64
(8)	0.157	0.02
(9)	0.9	0.03
(10)	0.07	0.036
(11)	0.27	0.536
(12)	0.71	0.792

TABLE 2

EXPLANATIONS

WHOLE NUMBER RULE

I circled 94 because it is bigger than 7 (comparing .94 and .7)

Because, um, there's two numbers after point zero. (comparing .94 and .7).

.94 is bigger because the hundredths column will be bigger than the tenths column (comparing .94 and .7)

FRACTION RULE

I mean the lower number is always higher than the higher number.

I picked that one because, like I said before, the bigger the number, the smaller the fraction

'cause there's only two numbers after zero.. (comparing 0.628 and 0.64).

You see, this is 100ths. Oh, this would make a bigger fraction (explaining why 0.7 is larger than 0 94)

If you divided a pie into 7 pieces, you would have bigger pieces than if you divided a pie into 94 pieces. (explaining why .7 is bigger than .94).

CORRECT RULE

CORRECT UNDERSTANDING OF 0

Zero is nothing--there is nothing there.

'cause it doesn't have two zeros...it only had one zero. (comparing 0.157 and 0.02).

CORRECT/ELIMINATE RAGGEDNESS

I got 64 because it would be six hundred and forty (comparing 0.64 and 0.029).

Oh, maybe you take the first two numbers again and seventy-nine is higher than seventy-one. (comparing 0.71 and 0.792).

I just picked it because if you rounded, that would be 1.0 and it would still be bigger than this (pointing to 0.03) comparing 0.9 and 0.03).

CORRECT/UNDERSTAND COLUMN VALUES

I go by this number 0.9. (comparing 0.94 and 0.7).

All right, I think that because 2/10ths is bigger than that 1/10th, I don't know. (comparing 0.24 and 0.178).

Because of the six tenths of 100 and 6 is bigger than 0. (comparing 0.64 and *cause a thousandths - a hundred and seventy-eight thousandths is one block out of a whole thing, out of a whole thousandths blocks, and then twenty-four hundredths would be higher than a hundred seventy-eight thousandths because hundredths are bigger than thousandths.* (comparing 0.178 and .24).

OTHER

INCORRECT UNDERSTANDING OF ZERO

I mean sixty-four hundredths is less than twenty-nine thousandths but since it has a zero its um just like twenty-nine hundredths (comparing0.64 and 0.029)

Ok, oh, oh, twenty-nine, because it got two zeros (comparing 0.64 and 0.029)

IDIOSYNCRATIC WRONG REASONING

This category includes explanations that use numbers but are not grounded in mathematics (e.g., numbers with two digits are larger) as well as attempts to figure out the answers by "psyching" me out (e.g., the correct answers alternate from the right to the left side of the page).

Something that it has with two digits. It has to have something with two digits.

I found a pattern--its left, left, right, left, right, right on the first page and left, left, left, right, right, right on the second!

INCOMPLETE OR INCOHERENT EXPLANATIONS

Um, I just put because, I don't know, I heard something like this being like this not being where they'd be asking 9/10ths of a hundred-

REITERATE ANSWER

This category includes answers that are non-explanations; that is, the child provides a formal reply (often a complete sentence), but does not actually explain the reasoning behind the answer (and typically cannot once prompted).

GUESS

I don't know..I think I just guessed.

Table 3. Posttest Rule Status as a Function of Pretest Rule and Condition

	FEEDBACK			NO FEEDBACK		
	POSTTEST					
PRETEST	ORIGINAL RULE	CORRECT RULE	OTHER RULE	ORIGINAL RULE	CORRECT RULE	OTHER RULE
ALONE						
Whole	.58	.25	.17	.83	.00	.16
Fraction	.33	.17	.50	---	---	---
DYADS						
Whole	.28	.52	.20	.97	.00	.03
Fraction	.07	.63	.30	.43	.00	.57

Other Rule=rule other than a correct rule or the rule used on the pretest

Figure 1. Design

	DYADIC CONDITIONS			ALONE CONDITIONS	
	Fraction/Whole	Fraction/Fraction	Whole/Whole	Fraction	Whole
FEEDBACK					
NO FEEDBACK					

PERCENT OF CHILDREN ADVANCING CORRECT EXPLANATIONS ON THAT PROBLEM AND ALL LATER PROBLEMS

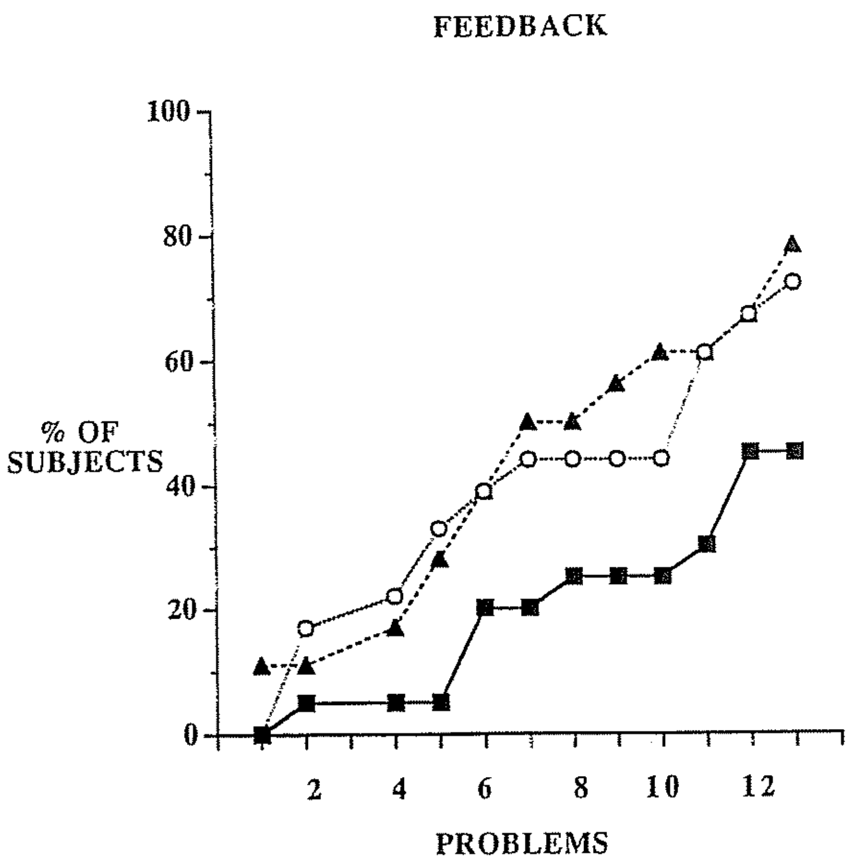


FIG. 2

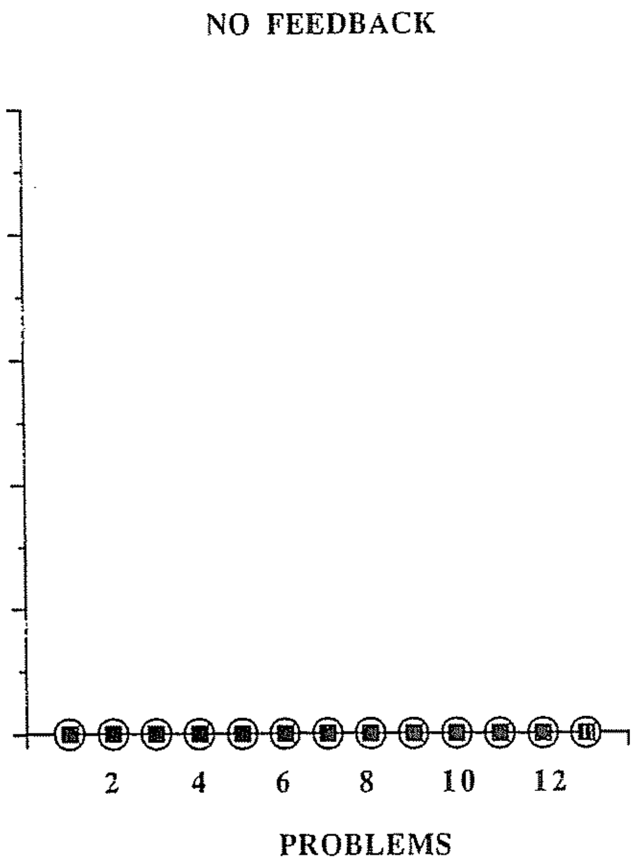


FIG. 3

WHOLE/WHOLE

FRACTION/WHOLE

FRACTION/FRACTION

PERCENT OF DYADS IN WHICH BOTH PARTNERS ADVANCED CORRECT EXPLANATIONS ON THAT PROBLEM AND ALL LATER PROBLEMS

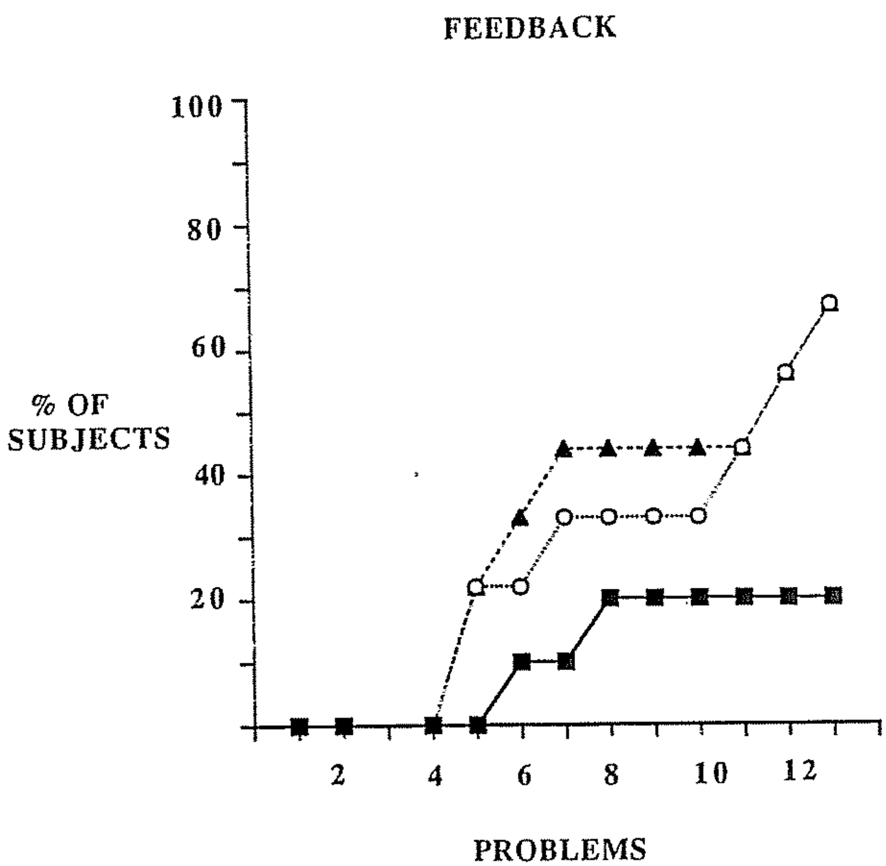


FIG. 4

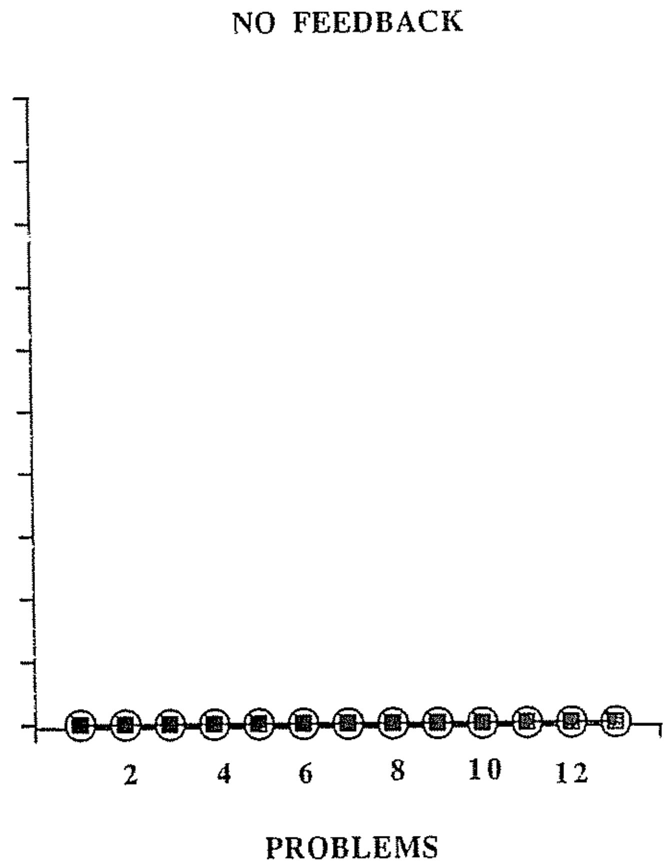


FIG. 5

WHOLE/WHOLE

FRACTION/WHOLE

FRACTION/FRACTION

PROBLEMS ON WHICH DYAD MEMBERS BEGAN TO ADVANCE CORRECT EXPLANATIONS CONSISTENTLY: FEEDBACK CONDITIONS

WHOLE/FRACTION

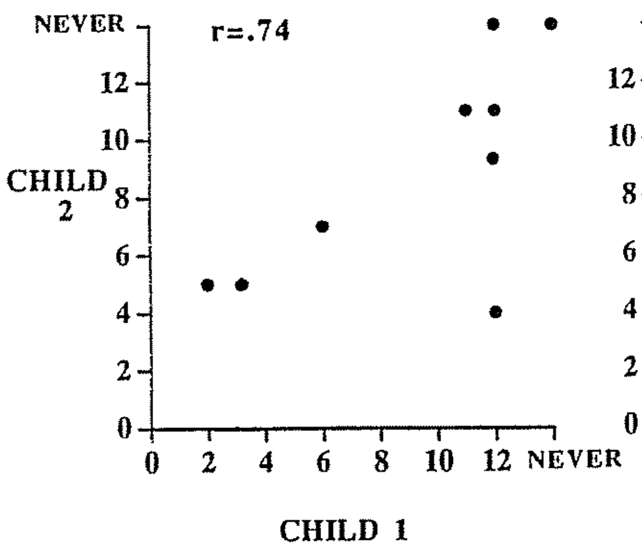


FIG. 6

FRACTION/FRACTION

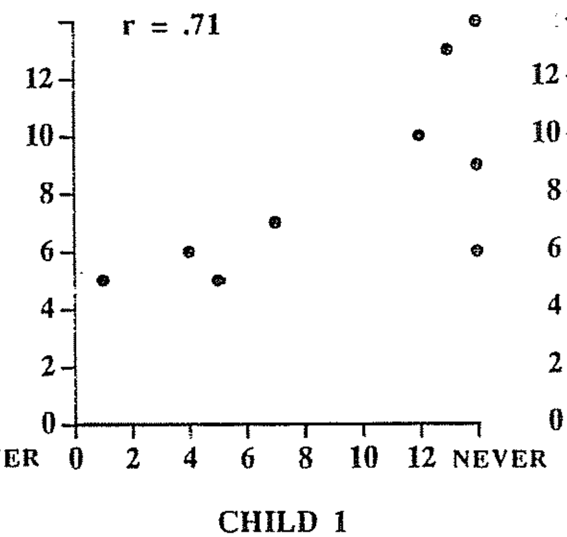


FIG. 7

WHOLE/WHOLE

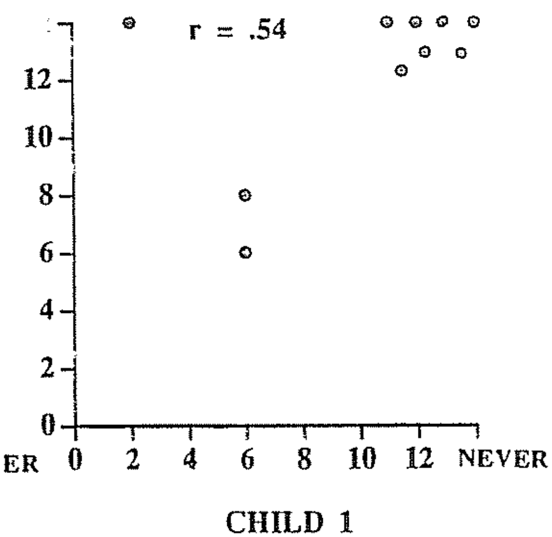
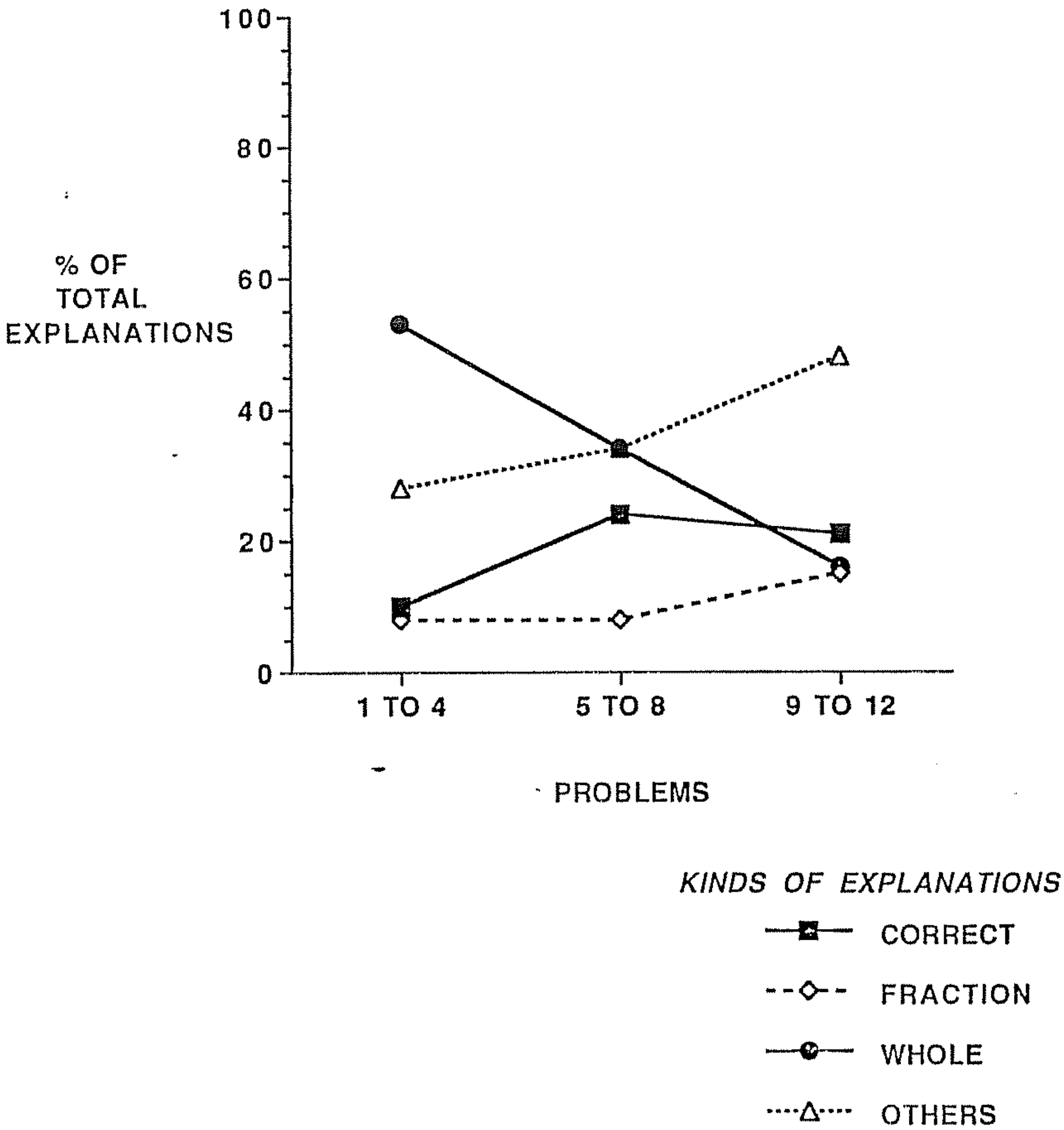


FIG. 8

FIGURE 9

CHANGES IN PROPORTIONS OF EACH KIND OF EXPLANATION OVER THE INTERACTION SESSION

WHOLE/WHOLE WITH FEEDBACK



APPENDIX A. PROBLEM TYPES

Pair Lengths	Examples	Pair Types	Incorrect Rule → Correct Answer
0.N - 0.NX[1]	0.0 - 0.03	0.0 - 0.0X	Whole number
	0.0 - 0.54	0.0 - $0.X_1X_2$	Whole number
	0.6 - 0.03	0.A - 0.0B	Fraction (*not fraction w/ 0 deletion*), Whole with 0 deletion[2]
	0.3 - 0.06	0.B - 0.0A	Fraction, Whole with 0 is less
	0.6 - 0.47	0.A - 0.BX	Fraction
	0.4 - 0.72	0.B - 0.AX	Whole number
	0.8 - 0.83	$0.X_1$ - $0.X_1X_2$	Whole number
0.N - 0.NXX	0.0 - 0.032	0.0 - $0.0X_1X_2$	Whole number
	0.0 - 0.546	0.0 - $0.X_1X_2X_3$	Whole number
	0.5 - 0.036	$0.X_1$ - $0.0X_2X_3$	Fraction
	0.6 - 0.478	0.A - $0.BX_1X_2$	Fraction
	0.4 - 0.726	0.B - $0.AX_1X_2$	Whole number
	0.8 - 0.836	$0.X_1$ - $0.X_1X_2X_3$	Whole number
0.NX - 0.NXX	0.03 - 0.073	0.0B - 0.0AX	Whole number
	0.07 - 0.054	0.0A - 0.0BX	Fraction
	0.05 - 0.643	$0.0X_1$ - $0.X_2X_3X_4$	Whole number
	0.72 - 0.063	$0.AX_1$ - $0.0BX_2$	Fraction (*not fraction w/ 0 deletion*), Whole with 0 deletion
	0.24 - 0.049	$0.BX_1$ - $0.0AX_2$	Fraction, Whole with 0 is less
	0.69 - 0.473	$0.AX_1$ - $0.BX_2X_3$	Fraction
	0.43 - 0.724	$0.BX_1$ - $0.AX_2X_3$	Whole number
	0.87 - 0.835	$0.X_1A$ - $0.X_1BX_2$	Fraction
	0.83 - 0.875	$0.X_1B$ - $0.X_1AX_2$	Whole number

KEY

LET N=ANY DIGIT ≤ 9

LET X=ANY DIGIT $1 \leq X \leq 9$

LET A=ANY DIGIT $1 \leq A \leq 9$, NOT EQUAL TO X_n

LET B=ANY DIGIT $1 \leq B \leq 9$, NOT EQUAL TO X_n AND $< A$

NOTES

[1] We pilot tested two forms of the problem set, one with nonzero integers in the ones column (e.g., 2.4) and one with zeros in the ones column (0.4). There were no differences in performance between the two sets.

[2]The original problem set included 20 types of problems. In the original problem set, we did not differentiate problems of the form 0.X - 0.0X and 0.0X - 0.0XX that were solved correctly using zero deletion and those for which zero deletion produced the wrong answer. Zero deletion is used when children drop or delete all the zeros in the numbers and compare what remains, e.g., 0.3 and 0.06 becomes a comparison of .3 and .6, or 3 and 6. We found zero deletion to be very common: about one-half of all whole number rule users and a small number of fraction rule users used zero deletion. Use of this rule reflects, at minimum, an incomplete understanding of place value so we thought it important to distinguish children who use this rule from those who use a "pure" whole number rule, or those who use "zero is less" (the number with the zero in the tenths position is smaller). In fact, the distinction proved important when comparing the posttest performance of children who used each of the three whole number rules. While over 80% of those whole number rule users who used "zero is less" used the correct rule on the posttest, only 25% of whole number rule users who used "zero deletion" adopted the correct rule on the posttest.

[3]The 80 problem pretest and posttest included 4 problems of each of the 18 problem types not affected by the use of zero deletion, and 2 problems of each of the 4 problem types used to detect zero deletion.

APPENDIX B

LIST OF RULES

WHOLE NUMBER RULES

Pure whole number rule - The longer the number, the larger it is (e.g., 0.345 > 0.9). Children who use this rule ignore the decimal points and treat the numbers as if they were whole numbers.

Whole number with zero deletion - When there is a zero in the tenths position, delete the zero and apply the whole number rule to what remains (e.g., 0.9 > 0.07 because 9 > 7, and 0.013 > 0.2 because 13 > 2).

Whole number with zero-is-less - The larger number is determined by the whole number rule, except when there is a zero in the tenths position in one of the numbers being compared. In this case, the number with the zero in the tenths position is always smaller (e.g., 0.4 > 0.092).

FRACTION RULES

Pure fraction rule - The shorter the number, the larger it is (e.g., 0.3 > 0.945). This rule is sometimes based on the notion that thousandths are smaller than hundredths, and hundredths are smaller than tenths, so the longer the number, the smaller it is. Children who adhere completely to this rule also believe that 0.0 is greater than any number with digits to the right of the decimal point. Other children misinterpret the decimal notation to mean that the numbers to the right of the decimal point refer to a denominator (e.g., 0.7 > 0.94 because 1/7 > 1/94). These children may argue that 0.0 is larger than any decimal fraction because 0.0 = 0/0 or 1/0 =1.

Fraction rule with correct 0 - The larger number is determined by the fraction rule, except when comparisons involve 0.0. In these cases, the children understand that any quantity is larger than 0.0. (0.234> 0.0).

Fraction rule with zero deletion - When there is a zero in the tenths position,delete the zero and apply the fraction rule to what remains (e.g., 0.03 0.6 because .3 > .6).

CORRECT RULE

Children are able to compare the relative sizes of decimal fractions accurately by (1) transforming the numbers so they have an equal number of digits to the right of the decimal point, either by adding zeros to the end of the shorter number (e.g., .0.94 > 0.7 because 0.94 > 0.70), dropping digits from the end of the longer number (e.g., 0.64 > 0.628 because 0.64 > 0.62), or rounding (e.g., 0.6 > 0.47 because 0.6 > 0.5); (2) translating decimal fractions into common fractions and comparing them accurately; or (3) comparing the face values of the digits beginning with the column with the greatest place value.