Span and Rate of Apprehension in Children and Adults

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Children and adults quantified random patterns of dots, under unlimited exposure duration. For adults and children two distinct processes appear to operate. For adults the quantification of collections of from one to three dots is essentially errorless, and proceeds at the rate of 46 msec per item, while the quantification rate for from 4 to 10 dots is 307 msec per dot. For children the same operating ranges appear to hold, however children are much slower. The lower slope is 195 msec per dot, while the upper is 1049. Although the results for adults and children are similar except for the overall rates, the nature of the isomorphism between children and adults is unclear.

Early conceptions of the span of apprehension (e.g., Jevons, 1871) assumed that there was some number, N, of discrete objects that the mind could immediately perceive, apprehend, or recognize. That is, the time to determine exactly how many such objects were present in a stimulus containing n objects was presumed to be a constant, independent of n, for all $n \leq N$, and the empirical question was the value of N.

A series of studies by various investigators has demonstrated fairly conclusively that such a view of the "span of apprehension" is erroneous (Averbach, 1963; Jensen, Reese, & Reese, 1950; Kaufman, Lord, Reese, & Volkman, 1949; Saltzman & Garner, 1948; Von Szeliski, 1924). Two findings, suggested by several of the studies cited, and confirmed by our own investigations, (Chi, 1973; Klahr, 1973a) emerge. The first is that reaction time is always an increasing function of the number of items: it takes longer to "immediately apprehend" $n + 1$ items than n items for all values of n. The second finding is that a break in the curve relating RT to number of items occurs between three to five items.

This work was supported in part by Research Grant MH-07722 from the National Institute of Mental Health, and in part by a Grant from the Spencer Foundation. We wish to express our thanks to the children and parents of the Carnegie-Mellon Children's School, and to their teachers—Dr. Ann Taylor and Mrs. Joanne Pohlman—for their cooperation. Thanks also to Patricia Carpenter, William Chase, and Marcel Just for comments and suggestions. Requests for reprints should be addressed to David Klahr, Department of Psychology, Carnegie-Mellon University, Pittsburgh, Pa. 15213.

[1] Portions of this work were reported at the 46th Annual Meeting of the Midwestern Psychological Association, Chicago, Ill., May 1974.

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These results suggest that adults use two distinct processes for determining quantity. One process, called subitizing, is used to quantify the first three or four items. It operates at approximately 50 msec per item. The other process, called counting, is used for more than three or four items. It operates at approximimetely 300 msec per item.

The existence of these two distinct processes for quantification, and their central relation to more general quantitative concepts (Klahr & Wallace, 1973) raises the question of their ontogeny. The purposes of this study are to compare the quantification processes in adults and young children, to see if two such distinct processes also emerge for children, and to determine the rates and ranges of these processes.

Results of previous investigations suggested that several procedural variations might affect the functional relation between reaction time and n. The experiments to be reported here were part of a series in which effects of visual angle, eye movements, numeral naming time, and range of n were investigated with adult subjects (Chi, 1973; Klahr, 1973a). In this paper, we compare the results of only one of the adult experiments with the results of a replication on 5-year-old children.

EXPERIMENT I

Method

Subjects. Twelve adults, students and faculty from the Psychology Department, participated as volunteers. Three of the twelve subjects had previously participated in a pilot study.

Materials and apparatus. The materials consisted of random patterns of one to ten dots presented on a standard video monitor controlled by a DDP-116 real-time digital computer. Subjects responded by saying the number of dots they saw, thus triggering a voice actuated relay which was sensed by the computer. Latencies were measured by the computer to the nearest millisecond from the time the dot pattern appeared on the screen until the relay was actuated.

Procedure and design. Subjects were seated 60 in. in front of the video monitor, placed at eye level, with a voice actuated microphone 1 ft off to the right, and a response button at a comfortable distance to the left. Subjects were informed of the range of n. At the start of each trial, the word "READY" was displayed in the center of the screen. The subjects were instructed to fixate the "A" in "READY" and then to press the response button whenever they felt ready. After a 1.5-sec delay the dot pattern appeared on the screen. The subjects were instructed to respond by saying as quickly and as accurately as possible, the number of dots they saw. (The word "count" was carefully avoided in the instructions.) The subject's response actuated the relay and removed the pattern from the screen. The message "ENTER #" was next displayed and the subject's reported number was then entered through a keyboard connected to the computer. Then "READY" reappeared.

Patterns were generated by randomly distributing dots on the intersections of a 5 × 5 grid. The grid lines were approximately .45 in. apart. The subjects sat 60 in. away from the display, so that the patterns were within foveal view, subtending a visual angle of less than 1.8°. Dot locations were determined prior to display, so that all dots in a pattern appeared simultaneously. For each value of n ($1 \leq n \leq 10$), 16 patterns were presented in a random sequence. Subjects took from 15 to 30 min to complete all 160 trials. There were three rest periods of at least 1 min each.

Results

Latencies. Mean error free RT's averaged over all 12 subjects are plotted in Fig 1. Due to the apparent slope discontinuity in the region of $3 \leq n \leq 5$, separate analyses of variance and trend analysis were performed for $n = 1 - 3$, $n = 1 - 4$, etc., until a significant quadratic component appeared. The subitizing range was defined by the first appearance of a quadratic trend. Then a linear regression was run independently over the upper and lower subranges of n.

A significant quadratic trend appeared for $n = 1 - 4$ ($F(1,33) = 15.9$, $p < .01$), but not for $n = 1 - 3$. Therefore, a least squares fit was

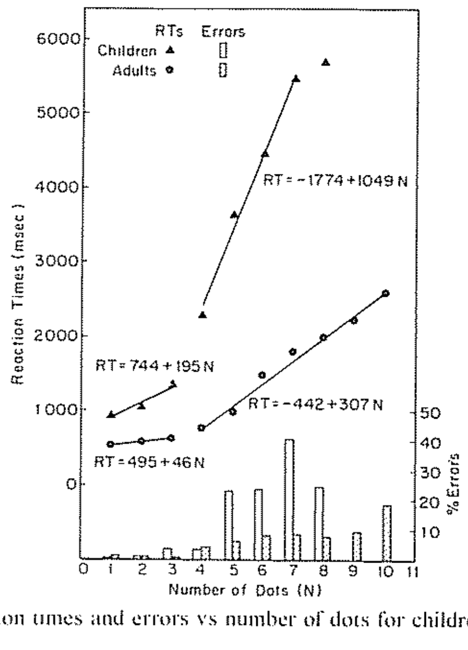

FIG. 1. Reaction times and errors vs number of dots for children and adults.

applied to the first 3 points, yielding a slope of 46 msec and an intercept of 495 msec (R^2 = 99.3%, RMSD = 5.4 msec).

For the upper curve, fit to the range $n = 4 - 10$, the slope and intercept are 307 msec, and −442 msec, respectively (R^2 = 98.9%, RMSD = 65.9 msec).

Errors. Errors are shown at the bottom of Fig 1. Within the subitizing range, the overall error rate is 0.87%, and there are no significant differences among error rates for $n = 1 - 3$. Similarly, in the counting range, there are no significant differences in errors except for $n = 10$; however, the overall error rate for counting (8.9%) is significantly higher than for subitizing ($t(11) = 3.6$, $p < .01$, 1-tailed).

EXPERIMENT II

Method

Subjects. Twelve children ranging in age from 5–0 to 6–5 (mean age: 5–8) participated in this experiment. All the children were either attending nursery school or had attended nursery school. A few days before the actual experiment was run, the children were brought to the lab to familiarize them with the apparatus and the procedure.

Stimulus materials and apparatus. The same computer-controlled apparatus was used. The stimulus materials were again dot patterns (n = 1 − 8) randomly displayed on the 25 possible locations of a 5 × 5 grid. The maximum angle subtended by the 5 × 5 grid was 1.5°.[2]

For each n, 5 patterns were displayed. Hence, there were a total of 40 trials. After every 10 trials, a "SMILE" was displayed in the center of the video monitor, at which time a break was introduced to reduce fatigue and boredom. Each S ran through the experiment twice or a total of 80 trials, which took approximately 30–40 min.

Procedure. The same general procedure was followed as in Experiment I. Subjects were again informed of the range of n. The subjects were seated 60 in. in front of the monitor with the word READY displayed, and they were told to press a response button whenever they felt ready. (The children were instructed about the meaning of the word.) The READY sign would then disappear, followed 1.5 sec later by the dot pattern. The subjects quantified the dots and gave a verbal response, which was then entered through the keyboard by the experimenter. (Once again, the word "count" was not used.) Occasionally, some subjects requested that they enter the response numbers themselves, in which case the experimenter made sure that no errors were made at the keyboard.

[2] This different visual angle was the result of using a different model video monitor in Experiment II.

Results

Latencies. A plot of the mean RT's across 12 subjects as a function of n is shown in Fig. 1. An obvious break in the curve seems to occur between n = 3 and 4. This is supported by the fact that a significant quadratic component exists for $n = 1 - 4$ ($F(1,33) = 15.23$, $p < .01$), whereas no significant quadratic trend exists for $n = 1 - 3$. Hence, a linear regression was fit to the first 3 points, yielding a slope of 195.2 msec and intercept of 744.4 msec ($R^2 = 94.2\%$, RMSD = 39.4 msec). Least-square fit was also applied to the upper curve ($n = 4 - 7$) with $n = 8$ excluded, because subjects tended to guess or estimate that a pattern of dots was 8 whenever 7 or 8 dots appeared. This is supported in part by the observation that the error rate for 7 is much greater than for 8 (41.4% vs 29.1%). The slope and intercept for $n = 4 - 7$ are 1049 msec and −1773 msec ($R^2 = 99.1\%$, RMSD = 109.6 msec). (If $n = 8$ is included in the analysis, the results are: slope = 881 msec, intercept = −934 msec, $R^2 = 95.9\%$, RMSD = 256.8 msec.)

Errors. The overall error rate within the subitizing range ($n = 1 - 3$) is 1.57%. Error rates in the range ($n = 1 - 4$) are not significantly different from each other (although the RT's are much slower for $n = 4$). For $n = 4 - 7$ there is a significant increase in error rates ($F(1,33) = 31.9$, $p < .01$), and the overall error rate, 22.8%, is significantly higher than the error rate in the subitizing range ($t(11) = 5.8$, $p < .01$, 1-tailed).

DISCUSSION

In both adults and young children, there appear to be (at least) two distinct quantification processes. One process, operating almost errorlessly on the range below $n = 4$ is about 5 to 6 times as rapid as the other, which operates on the range above $n = 3$.

Because of the similar characteristics of quantification processes in children and adults (such as location of the break, ratio of the slopes, and error patterns) it is tempting to suggest that the same underlying processes are operating in both age groups, and that children are just slower by a constant ratio over the entire range. However such speculations are probably premature, for they are inconsistent with other estimates we have of processing rates in children and adults.

For example, we know from subjective reports, other experimental work (Beckwith & Restle, 1966), and some information processing models (Klahr, 1973b) that the adult upper slope probably represents the time to group and subitize subsets of dots while computing a running sum. However, most of our children could not yet add, and in most cases one could observe them overtly enumerating one-by-one. When asked what they did on the upper ranges, children usually reported that they counted. Thus the adults and children are likely to be utilizing different processes over the upper ranges.

As for the lower ranges, it is unclear how to interpret the children's data. Is it "slow" subitizing? Both adults and children report that they "just know" the lower responses ("My head told me" said one child.) However the 195 msec rate for children is comparable to the 250 msec rate of numeral recitation for 5-year-olds; for adults, recitation rate is about 200 msec (Landauer, 1962), four times the adult subitizing rate.

Our results indicate that in order to study the development of the span of apprehension, one should replace that venerable notion with the concept of *quantification processes*. Other investigators (Gelman, 1972; Schaeffer, Eggleston, & Scott, 1974) have begun to demonstrate the importance of distinguishing between large and small number processing capacity in young children. Regardless of what processes underlie these quantification operators, the results presented here warrant close examination by experimenters interested in pursuing developmental studies where quantities are involved, such as conservation and class inclusion.

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RECEIVED: JUNE 11, 1974; REVISED: September 5, 1974.