Director's Corner: Math Matters

What pops into your mind when I invite you to **think about math**? *Yippee! Huh? Oh no!* Are you dreading the thought of helping your child with math throughout the school years? Are you poised for an intriguing puzzle? If so, there's a "chocolate math" puzzle included just for you. But keep reading before you try it!

Whether math excites or exhausts you, it's important to help foster positive attitudes and strong conceptual foundations for young children's mathematical thinking while they are in preschool and kindergarten. It's also important to reinforce the value of children's efforts so that they develop a **growth mindset**, rather than emphasizing their talents or intelligence. In studies where adults commented that children's success meant that they were smart vs. that they had worked really hard, those given "intelligence" feedback subsequently chose easier tasks than those given "effort" feedback. They were also more likely to lie about their performance when asked to share their scores with others. This year in my monthly articles, I'll share ways that families can engage in ***enjoyable math explorations starting now*** so that all of our children grow up with positive attitudes toward math and growth mindsets regarding future math learning.

At the Children's School, we introduce children to five conceptual areas within math, each of which corresponds to math topics they will encounter later in school.

1) **Number & Operations** (Arithmetic)
2) **Patterns & Functions** (Algebra)
3) **Shapes & Spatial Arrangements** (Geometry)
4) **Measurement**
5) **Data Analysis & Probability** (Statistics)

I'll elaborate on each of these in a subsequent article and share how our educators integrate math naturally into the classroom routines and activities. Here I'll start with two suggestions for families to more intentionally focus children's attention on the mathematics of their daily lives.

• Involving children in household work reinforces their place in the family, and you can highlight mathematical concepts at the same time. For example, when children **help with folding the laundry**, they can 1) Count the items, add how many are sorted or subtract to see how many are left to sort. 2) Notice patterns in the fabric. 3) Identify shapes in the designs. 4) Arrange items by size (which may also identify whose they are). 5) Sort items by type and compare quantities.

• Fall is a great time to **start or expand a collection**, whether it be rocks, sticks, leaves, acorns, pine cones, etc. 1) Count the items in your collection. 2) Arrange the items to make patterns that repeat or grow. 3) Identify the shapes in your collection. 4) Arrange the items in order of size, thickness, weight, etc. 5) Sort the items in many different ways and record which groups have more, fewer, or the same number. To give a personal example, my granddaughter Lucia has a "rock box" outside. Though she is not yet 2, she already gets the idea of simple patterns like rock, pine cone, rock, pine cone (which is a precursor to odd, even, odd, even). As we play, I elaborate the patterns to show the possibilities but follow her lead and interest regarding how far to go.

In both of these ways, and many others, we help children notice that math is everywhere to explore!

Director's Corner: How Many? (Part 1)

As we work together to build our children's positive attitudes and foundational math concepts, we naturally start with explorations that involve **number**. Throughout daily life, we find ourselves asking the question, "**How many**?" We may recognize the number or count to determine the amount, and then we compare to determine which set has more or fewer. With many sets, we can arrange them in order from the biggest to the smallest or vice versa.

Parents and educators can facilitate the developmental process by drawing children's attention to the quantities in their world. At school, we count children, snack items, days of school, steps and laps around the gym, claps and drumbeats, tablespoons of ingredients, etc. While counting, it's important for children to match each number word with exactly one item or event in "*one to one correspondence*". Gradually, children learn to link the **number** (•••) with both the **number word** (three) and the **numeral** (3), as well as with three fingers for younger children and three tally marks for older ones. Usually, the preschoolers can visually recognize small numbers up to 3 or 4 without counting, and those familiar with dice and dominoes can recognize even larger quantities that are arranged in standard formations. With the 3's, we count by 1's, but we start counting by 10's in the 4's and by 2's and 5's in the kindergarten. In all of these ways, we are helping the children to build a **mental number line** that will be the central conceptual structure for all of their arithmetic learning.

At school, we embed counting into our daily routines, such as the calendar and weather, and we frequently check children's skill levels with simple activities such as making a bracelet with 10 beads (4's) or putting the right number of acorns in numbered spaces on the light table (K). The 4's and kindergartners also count the number of days we've been in school to build excitement for celebrating 100 Day in February.

At home, families have many opportunities for counting and comparing numbers in enjoyable ways. Perhaps the best come during free play and family time with games. Playing with blocks or other construction & craft materials offers many opportunities for noticing, counting and comparing quantities. Card games, like Uno and Go Fish, and various number bingo games highlight quantities and numerals, while board games highlight the one to one correspondence of moving a marker along a track and the fact that higher numbers take more moves and more time to count.

Another great way to support children's developing number concepts is through reading stories that focus on quantities, such as "Feast for 10", "Two Ways to Count to 10", "How Many Legs in All?", and "How Many is a Million?" Don't be afraid to talk about high numbers, even if the children have trouble imagining the scale. It takes lots of practice to develop a robust mental number line, with young children doing best with 0 to 10 or 20, early elementary children extending to 0-100, and later elementary children advancing to 0-1,000. Children's interest in practicing will result in developing a strong number sense as the basis for other math concepts.

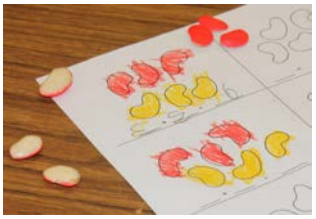
Remember to tailor these suggestions to your family's unique home life and interests. Whether you are counting ball catches or worms or cookies or coins, your interest in the mathematics of life will be a great model for your children!

Director's Corner: How Many? (Part 2)

Once children begin to understand small numbers and how they are organized into a mental number line, they can explore simple **arithmetic** for answering "**How many**?" Arithmetic involves composing and decomposing numbers in a variety of ways so that we come to understand five as composed of two and three, one and four, or even zero and five. Similarly, one hundred is ten 10s, five 20s, four 25s, or two 50s, etc. The operations of addition, subtraction, multiplication, and division are the formal procedures we use for arithmetic, and children will learn formal symbols for writing arithmetic equations in elementary school, but we begin with learning how to compose and decompose numbers in everyday life.

Parents and educators can facilitate the developmental process by highlighting **numerical combinations**, such as our family of 3 inviting another family of 4 for dinner, which means we'll have 7 people all together. We'll need 7 cups and 7 plates, etc. but we might need 14 cookies for dessert. At school, if each group plants some flowers, we can add the amounts to see how many flowers were planted all together. Children being absent and snack getting eaten is natural subtraction. Needing 10 markers for each of five tables means we need 50 total; whereas, sharing a container of blocks means that we will have more for each with just a few children playing than with many children playing. At school, we typically use some type of **physical counters** that children can manipulate (hence called "manipulatives") to help make the problems concrete. Research shows that children attend best with manipulatives that are simple and uniform, such as chips, rather than objects that vary in color, shape, or kind, all of which might just distract them from the numerical features. Here again, the developing **mental number line** supports understanding because addition is counting up the number line, subtraction counting down, and multiplication skip counting. Division requires partitioning the line into so many equal sections from a starting point.

At home and school, **diverse practice** in the context of free play, routines, games, and stories is what gradually builds positive attitudes and strong math concepts. Painting lima or kidney beans on one side makes them great for practicing combinations by shaking and rolling to see "What makes 6?" (or whatever number of beans you use). Simple games with two dice (even ones with just 1-3 pips) introduce addition. Songs like "Five Little Monkeys Jumping on the Bed" introduce subtraction. One fun "division" story called "The Doorbell Rang" explores the challenge of sharing cookies among children when more people keep ringing the doorbell and joining the group.



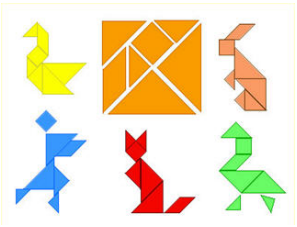
What matters most is that children and adults engage in enjoyable, developmentally appropriate math explorations together. Enclosed with this newsletter is a recent research report from *Science*, entitled "Math at home adds up to achievement in school". The extensive, well-controlled study shows that weekly parent-child interactions using an iPad application with numerical stories led to an additional 3-month gain in 1st graders' math achievement compared to interactions with a similar reading application. The effect was strongest for families in which parents had identified themselves as more math anxious. Note that the point isn't the technology, though that was essential for tracking the family usage patterns, but rather the enjoyable interactions about meaningful math. Look for natural opportunities to explore math with your children. You'll be amazed!

Director's Corner: What Shape?

Though counting and simple arithmetic are the most natural ways to begin engaging children in everyday math, there are four additional domains of math to consider as we aim to foster positive attitudes and build strong conceptual foundations. This semester, I'll encourage you to go beyond asking How Many? to ask What Shape? What Patterns? How Big? and How Frequent?

Geometry is the domain of mathematics related to the question **What Shape?** It involves skills of recognizing and forming shapes (individually and in combination), working with spatial arrangements and symmetry, and other types of spatial reasoning. As with arithmetic, we tend to limit ourselves to simple discussions of regular shapes with young children (e.g., circle, square, and triangle), but they are curious about and capable of both noticing and creating much more interesting geometric arrangements. Consider the bilateral symmetry in the bug below and how a kindergartner might describe the shapes used to create the bug or those created by the black lines in the cool / warm color painting. As children indicate interest, they can certainly acquire a broad vocabulary of lines, two-dimensional, and three-dimensional shapes, such as zigzag, spiral, oval, trapezoid, cube, and cylinder. As we describe arrangements, we can introduce 3's to spatial terms such as above, below, beside, behind, etc. and progress to more sophisticated vocabulary, such as horizontal, vertical, and diagonal, for kindergartners.

At school and home, we can offer **a variety of enjoyable geometry activities** from open-ended building explorations (e.g., with blocks, cups, etc.), to assembling puzzles, to solving tangram challenges (see image to the right). Using cookie cutters, shape stencils, and geo-boards (see image below) allows children to begin creating simple shapes that their fine motor skills might not be ready to make independently. We also practice spatial reasoning with children when putting away items and packing, since we determine placement based on the match between an object's size and shape and the space available. Modeling the use of maps and helping children to begin mapping the spaces most familiar to them highlights the usefulness of spatial representations for navigation. The same is true for assembly diagrams, which children start using for Lego creations and other building toys.



As with arithmetic, **diverse practice** in the context of daily living and free play is what gradually builds positive attitudes and strong math concepts. Begin by noticing the geometry around you, both indoors and outdoors. You may observe more rectilinear (straight line) shapes with manmade objects, buildings, etc. and more curvilinear ones in nature.

Remember that the aim is to find what fascinates you and your children so that you will have fun doing developmentally appropriate spatial explorations together with increasing complexity over time.

Director's Corner: Textile Math

Our whole school study of Textiles affords abundant opportunities to deepen children's foundational math concepts in fun ways. With the diversity of fabrics around us, we can easily ask questions to help us explore all five domains of math: How Many? What Shape? What Patterns? How Big? How Frequent? As I've mentioned in previous articles, the process begins with simply noticing that math is everywhere, just ready for us to explore. For example, as I sit in my favorite spot in my den at home to write this article, there are 5 pillows within easy reach, each with a different textile cover (and most likely varied textile fills).





Here are some ideas about how to engage children in conversations about the math that is literally woven or sewn into the fabrics:

1) **Number & Operations** (Arithmetic) – Certainly, there's lots to count here, but also think about the ways that counting one part of a regular arrangement (such as the center squares) can be used to also tell you the number of dots, rings, and outer squares without having to count. Also consider whether there are more birds, flowers, or leaves, and for older children how many more.

2) **Patterns & Functions** (Algebra) – Patterns are most obvious horizontally and vertically on one pillow but diagonally on another. On closer inspection, one can see the in and out pattern of embroidery on the pinecone pillow, the warp and weft on the woven pillows, and the up and down of zigzag stitches on the metallic circles design.

3) **Shapes & Spatial Arrangements** (Geometry) – Here the circles and squares are the most obvious and easily named shapes, so start with those, add new terms like "concentric", and note that some shapes are created by lines while others are created by arrangements of smaller shapes. Challenge more advanced children with naming and identifying symmetry in less regular shapes, like the leaves, berries, flowers, house, etc.

4) **Measurement** – Begin with comparative size by arranging the pillows from biggest to smallest or thickest to thinnest. For older children, discuss the problem of the rectangular pillow being longer but narrower than the largest square one. Which IS biggest? Maybe prove your point by measuring.

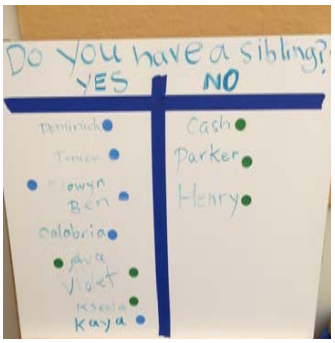
5) **Data Analysis & Probability** (Statistics) – With these pillows, or better yet with all the pillows in the house, we could create physical pillow bar graphs to learn the frequency of square vs. rectangular pillows (4 to 1), patterned vs. scenes (3 to 2), cotton vs. synthetic (1 to 4), etc.

Clearly, you can go as far as your imagination and the children's interest takes you. Enjoy!

Director's Corner: How Frequent?

Next for consideration in our series on ways to encourage children's interest in and exploration of foundational math concepts is learning about **how frequently** people, objects and events occur in the world. Humans naturally notice patterns of data in their lives and set expectations or choose actions based on the probabilities they discover. Basically, we do **statistics** all the time. Statistics is the domain of mathematics that includes classifying and organizing data via varied representations and then using the information to make decisions. Even 8-month-old infants express surprise when an adult repeatedly draws white balls from a container that they can see contains mostly red balls and 15 month olds use similar probabilities to form expectations about what type of actions to use with certain objects (see an interesting TED Talk by Laura Schultz at https://www.ted.com/talks/laura_schulz_the_surprisingly_logical_minds_of_babies - t-1057552).

In early childhood, we begin explorations by noticing properties of people, objects, and events in our everyday lives and then using those features to categorize them. The 3's sort toys during cleanup time, count friends' characteristics, notice when there are more sunny days than cloudy ones, etc. Preschool 4's begin to use graphing strategies to organize birth months, name length, family size, and other data about the friends in each class. By kindergarten, the entry routine includes a "Question of the Day", and the weekly "Clipboard Helper" takes a survey of the class, being careful to note who has already been asked, and then reports the results to the group. The kindergartners typically get so interested in the surveys that they start inventing their own by second semester!



At home, families can promote this early data collection and analysis in both **fun and useful ways** by sorting dishes, clothes, and recyclables, considering the chances of rolling certain numbers or drawing certain cards during games, comparing choices and preferences of friends and family members while taking beverage or dessert orders, etc. Even in the car, children can tally the color of vehicles they pass and use the data to predict what they are most likely to encounter next. Clipboards and graph paper with large squares enhance the intrigue for children. As with arithmetic and geometry, **varied opportunities for practice** helps solidify concepts, so offer lots of chances and follow the children's lead in terms of which ones to pursue in the most depth.

Remember to reinforce the effort required to complete a task and do it well, as well as the ways hard work helps us to improve and keep learning. Such a "**growth mindset**" prepares children to persevere through challenges and take in initiative to seek new learning opportunities. As with all aspects of math, the possibilities are endless so use whatever resources you and your family have to invent your own ways to explore statistics. I'd love to hear about your math adventures!

Director's Corner: What Patterns?

Noticing **patterns** in the world and describing the mathematics of change is the essence of **algebra**, which the Khan Academy describes as "the language through which we describe patterns" and a "simple way to express a repetitive process". According to Marshall Haith's research, an infant as young as 2-3 months old can anticipate where objects will appear on a screen if there is a systematic pattern, such as left – right – left – right or top – middle – bottom – top – middle – bottom. We can tell that infants learn the pattern and can predict the next location because their eyes move in advance of the object appearing.

By preschool, children can remember and follow many types of patterns, starting with routines in their home life, such as take a bath - brush teeth – hear a bedtime story or sing the hello song - do the calendar – check the weather, etc. Children's learning of such patterns is especially obvious in their protests that one parent or a substitute teacher isn't following the routine "correctly". Additional evidence comes from the popularity of songs with actions, such as "head, shoulders, knees, and toes", or repetitive stories, such as "Brown Bear, Brown Bear, What Do You See? I see a red bird looking at me. Red bird, red bird, what do you see? etc." As with other foundational math concepts, adults who draw children's attention to patterns in the world help them begin to notice them naturally.

In school, we start with **noticing** patterns and **predicting** what will come next, all using everyday objects and situations, such as clothing designs, repeated actions in setting the table for snack (e.g., napkin-cup-napkin-cup), or up-down-up-down, etc. The next challenge is to **copy and extend** patterns in contexts such as stringing beads, building with blocks, playing follow the leader, etc. We also begin to use letter names to describe patterns. For example, an AB pattern has two repeating elements, such as red-blue-red-blue or clap-snap-clap-snap or eventually odd-even-odd-even. By this time in the year, kindergartners can recognize, extend, and even **create** much more sophisticated patterns, such as ABBC or ABAC, etc.

At home, there are many common opportunities to promote children's interest in patterns. Both indoors and outdoors, pay attention to the designs on plants, furniture, buildings, etc. Cooking affords opportunities for repeated actions such as adding an ingredient and then stirring, and table-setting patterns have even more elements, especially for fancier meals. Card games involve patterns of numbers and suits, and most all games involve sequences of actions and turns. The enclosed "***Preschoolers as Pattern Sleuths***" article from Teaching Young Children offers even more ideas, as well as some children's books that emphasize patterns. As always, aim to **take the child's lead** regarding what is interesting and enjoyable, particularly in terms of using their favorite toys or pastimes as arenas for pattern play (e.g., patterns in block building, with stuffed animals, in art, with sports, etc.). Experiencing the same types of patterns in **diverse contexts and different modalities** (visual, tactile, auditory, etc.) will deepen children's understanding of the core concepts.

Working with patterns gives children opportunities to practice other concepts of math as well, starting with counting pattern elements, noticing their shapes and spatial arrangements, etc. Patterns become most useful when they help children make predictions about what comes next. Helping children identify predictability in the world alerts them to regularities that they can use to organize their thinking and behavior, which then makes them more confident and likely to engage more fully.

Director's Corner: How Big?

Our final domain of mathematics to consider is **measurement**, which involves assigning a number to a characteristic so as to more easily compare objects, events, etc. We all use measurement frequently in order to buy the right size clothes, add the correct amount of ingredients in a recipe, manage our time, watch the gas gauge to plan when to refuel our cars, and so on. Professionals in a wide range of fields create specialized measurements for hardness of rocks, intensity of odors, brightness of light, amount of pressure, etc., as needed to improve the precision of their comparisons.

We first use measurement when we begin to **compare attributes** of ourselves, such as when young children announce that they are taller, faster or stronger than someone else. Our initial lessons involve direct comparison, such as standing back to back or running a race. For measurement of weight or temperature, we can sometimes hold one object in each hand to estimate which one is heavier or warmer. Sometimes we compare whole sets of objects by arranging them in a series from thickest to thinnest, darkest to lightest, longest to shortest, etc.

Once children understand the idea of comparison, we introduce units of measurement so that we can measure without the constraint of having people or objects in the same place at the same time. We start informally with common units such as steps from one place to another or paper cups-full to serve an equal amount of snack. Soon, children notice the problem with such **informal measures** because individuals will get different answers when they measure distance or the piles of snack items will not appear even. For size measurement, we often start with something called "unifix cubes" which are a uniform size, easily snap together to reach the desired length, and can then be counted to compare length. Then, we introduce **formal measurement tools** and begin discussing how to use them accurately. Rulers, tape measures, scales, thermometers, measuring spoons and cups, timers, and clocks are commonplace in our science centers, kitchen, woodworking area, and other places within the school.

As with all aspects of math, teachers and parents can help children to **notice opportunities for measurement**, such as the growth of plants in our spring gardens, how much later it stays light in the evening, the temperature of the bath water, the weight of recycling we collect every week, etc. By following the progression outlined above, we ensure that children **build a strong conceptual foundation** for understanding that all measures are invented and need an objective standard for comparison. Young children will have the most **positive experiences** with measurement if we follow their interests, such as using measurement with their own growth, collections, hobbies, etc. As situations arise, you might challenge the children to invent ways to measure which joke is funnier, baby happier, cat friendlier, etc. Doing so will prompt them to learn that there are still measures yet to be invented. To keep everyone focused on having a **growth mindset**, emphasize measures of personal growth and improvement rather than taking a competitive stance. Summer is a wonderful time to use math skills to keep some records of your family fun, the number of books read or games played, the properties of objects collected, the distances traveled on foot or by car, etc. Consider documenting your math explorations and sharing them with us in the fall. Enjoy exploring math!