Improving the Numerical Understanding of Children From Low-Income Families

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ABSTRACT—***Children from low-income backgrounds enter school with much less mathematical knowledge than their more affluent peers. These early deficits have long-term consequences; children who start behind generally stay behind. This article describes how a theoretical analysis of the development of number sense gave rise to an intervention that reduces this gap by producing large, rapid, and broad improvements in the mathematical competence of low-income preschoolers. Roughly, an hour of playing a simple, inexpensive, linear number board game produces gains in numerical magnitude comparison, number line estimation, counting, and numeral identification. Reasons for these large gains are discussed.***

KEYWORDS—*number; low income; intervention; mathematics; board games*

When children begin school, their mathematical knowledge already varies greatly. These early differences have long-term consequences. Children's mathematical knowledge in kindergarten predicts their math achievement test scores in elementary school, middle school, and even high school (Duncan et al., 2007; Stevenson & Newman, 1986). The relationship between early and later mathematical knowledge is roughly twice as strong as that between early and later reading achievement (Duncan et al., 2007).

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The stability of individual differences in mathematical knowledge makes it especially unfortunate that children from low-income families begin school with much less mathematical knowledge than their wealthier peers. This early achievement gap is evident on a wide range of foundational tasks: counting from one, counting up or down from numbers other than one, recognizing written numerals, adding, subtracting, and comparing numerical magnitudes (Ginsburg & Russell, 1981; Jordan, Kaplan, Olah, & Locuniak, 2006; Jordan, Levine, & Huttenlocher, 1994; Starkey, Klein, & Wakeley, 2004).

These differences in mathematical knowledge between preschoolers from more and less affluent backgrounds reflect differences in environmental support for learning. Middle-income parents engage in a broader range of explicitly mathematical activities with their children, and engage in these activities more frequently, than do low-income parents (Clements & Sarama, 2007; Starkey et al., 2004). Parents who engage in more numerical activities generally have children with greater mathematical knowledge (Blevins-Knabe & Musun-Miller, 1996).

These findings highlight the importance of identifying activities that can substantially improve the numerical knowledge of preschoolers from low-income backgrounds and that can be promoted for widespread use. Over the past few years, Geetha Ramani and I have applied a theoretical analysis of the development of understanding of numerical magnitudes to generate and test the effects of one such activity. The activity is simple and inexpensive, yet it produces large improvements on a broad range of numerical skills and knowledge in roughly 1 hr. I first describe the theoretical analysis that led to the intervention, and then examine evidence for its effectiveness.

THEORETICAL BACKGROUND

Number sense is an important part of mathematical competence (e.g., National Mathematics Advisory Panel, 2008). However, there is little agreement on what number sense is—as with

Justice Stewart's famous comment about pornography, it is easier to recognize than to define.

Reflecting this difficulty, most efforts to define number sense have cited broad and varied types of knowledge. For example, the National Mathematics Advisory Panel (2008) defined number sense as including skill at immediately identifying the numerical value associated with small quantities (e.g., 3 pennies); facility with basic counting; proficiency in approximating numerical magnitudes; principled understanding of place values and of how to compose and decompose whole numbers; knowledge of the commutative, associative, and distributive laws; and ability to apply those laws to solve problems. This definition reflects the diverse ways in which people use the term *number sense*, but it encompasses such a wide range of processes and knowledge that it makes number sense hard to conceptualize, investigate, and improve.

To address this problem, Siegler and Booth (2005) proposed an alternative definition that focuses on a single particularly important process: number sense is the ability to approximate numerical magnitudes. The approximations can involve results of numerical operations ("About how much is 97 × 38?") or attributes of objects, events, or sets ("About how much does a Prius weigh?" "About how many people attended the play?"). The definition emphasizes that number sense is about numerical *magnitudes*—being able to choose numbers whose magnitudes are close to the correct values.

This perspective suggested that studying numerical estimation might provide a useful means for learning about number sense. Estimation and number sense are inherently related, in that both involve approximating magnitudes. The two are not identical—some estimation tasks can be performed through stereotyped procedures, such as rounding to the nearest 10, without any deeper sense of the magnitudes that the estimates should yield. When executed through means other than these routine procedures, however, accurate numerical estimation, like number sense, involves the ability to approximate numerical magnitudes.

The number line estimation task has proven especially useful for investigating number sense. This task involves presenting participants with lines with a number at each end (e.g., 0 and 100) and no other numbers or marks in between, and asking them to locate a third number on each line (e.g., "Where does 74 go?"). Participants estimate the locations of different specific numbers, one per number line, presented in random order, until they have estimated locations of numbers from all parts of the numerical range.

Number line estimation has several important advantages for investigating children's sense of numerical magnitudes. It can be used with numbers of any size and with fractions as well as whole numbers. It also transparently reflects ratios among numbers. Just as 60 is twice as large as 30, the distance of the estimated position of 60 from 0 should be twice as large as the distance of the estimated position of 30 from 0. More generally, estimated magnitude (y) should increase linearly with actual magnitude (x), with a slope of 1.00, as in the equation $y = x$.

Early in development, however, children's number line estimates rarely increase linearly with numerical magnitude. Many preschoolers, even ones who can count perfectly from 1 to 10, do not understand the rank order of the numbers' magnitudes. These children's number line estimates correlate only minimally with the magnitudes of the numbers they are estimating (Ramani & Siegler, 2008; Whyte & Bull, 2008).

Even after children learn the rank order of numbers' magnitudes, they still do not immediately represent the magnitudes as increasing linearly. Instead, their number line estimates often increase logarithmically with the size of the number being estimated (Figure 1A). Finally, after children gain experience with numbers throughout the range, their magnitude estimates increase linearly (Figure 1B).

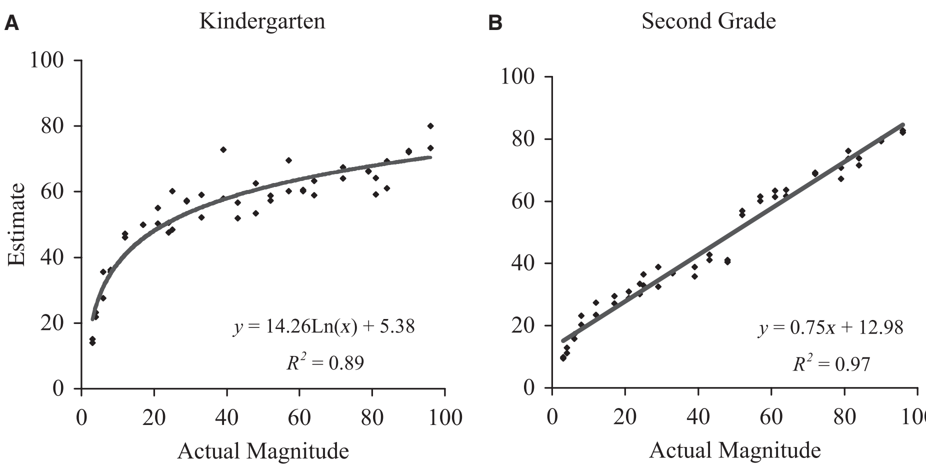

Figure 1. **Kindergartners' logarithmic patterns of number line estimates and second graders' linear patterns of estimates (data from Siegler & Booth, 2004).**

The transition from logarithmic to linear representations of numerical magnitudes occurs at different times for different numerical ranges. On 0–100 number lines, kindergartners consistently produce logarithmically increasing estimates, whereas second graders produce linearly increasing estimates (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler & Booth, 2004). On 0–1,000 number lines, second graders' estimates increase logarithmically, whereas fourth graders' and older children's estimates increase linearly (Booth & Siegler, 2006; Opfer & Siegler, 2007). The same child often produces linearly increasing estimates on smaller numerical scales (e.g., 0–100) and logarithmically increasing ones on larger scales (e.g., 0–1,000; Siegler & Opfer, 2003).

This growing reliance on linear representations for number line estimation is not an isolated phenomenon. Children undergo parallel changes from logarithmic to linear patterns at the same grades and ages on at least two other estimation tasks: numerosity estimation (generating approximately *N* dots on a computer screen where 0 and 1,000 dots are shown) and measurement estimation (drawing a line of approximately *N* units where lines of 1 and 1,000 units are shown; Booth & Siegler, 2006). Most children produce either linear estimation patterns on all three tasks or logarithmic estimation patterns on all of them. Number line estimates also correlate substantially with other measures of numerical magnitude knowledge, such as magnitude comparison and numerical categorization (Laski & Siegler, 2007). Perhaps most striking, linearity of estimates correlates substantially—typically between $r = .50$ and $r = .60$—with overall math achievement test scores at all grade levels between kindergarten and fourth grade (Booth & Siegler, 2006; Geary et al., 2007; Siegler & Booth, 2004).

These findings raise the question, What experiences lead children to first represent the magnitudes of small, verbally stated or written numerals as increasing linearly? Counting experience during the preschool period probably contributes, but appears insufficient to create a linear representation. Children often count perfectly in a numerical range a year or more before they generate linear representations of numerical magnitudes in that range (Le Corre, Van de Walle, Brannon, & Carey, 2006; Schaeffer, Eggleston, & Scott, 1974).

If counting experience is insufficient, what other experiences might contribute? One common activity that seems ideally designed for producing such representations is playing linear, numerical, board games—that is, board games with linearly arranged, consecutively numbered, equal-size spaces. For example, in the popular game Chutes and Ladders, the numbers 1–100 are arranged in a 10×10 grid, with each number in a separate square. The numbers 1–10 are ordered from left to right in the first row, the numbers 11–20 are ordered from right to left in the second row, the numbers 21–30 are ordered from left to right in the third row, and so on. Such board games provide multiple cues to both the order and the magnitude of numbers. The greater the number in a square of the game, the greater (a) the number of discrete movements of the token the child has made, (b) the number of number names the child has spoken, (c) the number of number names the child has heard, (d) the distance the child has moved the token, and (e) the amount of time that has passed since the game began. The linear relationships between numerical magnitudes and these kinesthetic, auditory, visuospatial, and temporal cues provide a broadly based, multimodal foundation for a linear representation of numerical magnitudes. Linear number board games also provide children with practice at counting and numeral identification. Thus, we expected that playing such games would improve counting and numeral identification, as well as understanding of numerical magnitudes.

EFFECTS OF PLAYING LINEAR NUMERICAL BOARD GAMES

An Initial Demonstration

To test this theoretical analysis, Siegler and Ramani (2008) randomly assigned thirty-six 4- and 5-year-olds from Head Start centers, the majority African American and the rest non-Hispanic White, to play either a number board game or a color board game. As Figure 2 shows, the number board and the color board were highly similar. Both included 10 horizontally arranged, different colored squares of equal size. The only difference was that the board used in the number version of the game had the numbers 1–10 listed consecutively from left to right, and the board used in the color version of the game did not. Children were presented with the number line estimation task with numbers 1–10 both before and after they played the game. For purposes of comparison, we also presented the number line estimation task to 22 children from middle-income backgrounds who had not played the game.

On each trial, children spun a spinner and moved their token the indicated number of spaces (one or two). The children needed to say the numbers or colors on the spaces through which they moved. Thus, preschoolers in the number board condition who were on a 4 and spun a 2 would say, "5, 6" as they moved, whereas peers in the color board condition who spun a "blue" would say "red, blue." If a child erred or could not name the numbers or colors, the experimenter correctly named them and had the child repeat the names while moving the token. Children participated in four 15- to 20-min sessions (including pretest and posttest) over a 2-week period; each game lasted 2–4 min. Thus, the preschoolers received roughly 1 hr of game-playing experience.

This brief experience had dramatic effects. Before playing the number board game, the best fitting linear function accounted for an average of 15% of the variance in individual children's number line estimates. After playing the game, the best fitting linear function accounted for an average of 61% of the variance. This was as high as the percentage of variance accounted for by the best fitting linear function among peers from middle-income

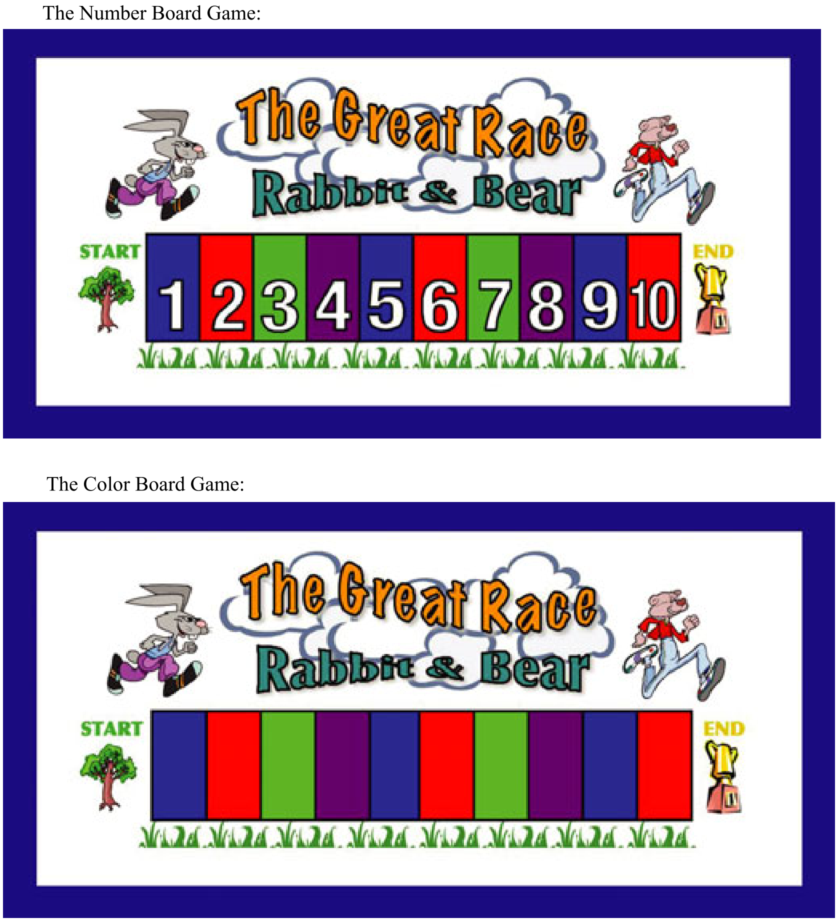

Figure 2. **The number board and color board used in the interventions.**

backgrounds who did not play the game (60%). In contrast, playing the color board game did not affect the low-income children's number line estimates; the best fitting linear function accounted for an average of 18% of the variance in their estimates on both pretest and posttest. Thus, playing the number board game improved the low-income preschoolers' number line estimates and presumably their sense of numerical magnitudes.

Generality Over Time, Tasks, and Types of Knowledge

Ramani and Siegler (2008) tested the generality of the benefits of playing the number board game, in terms of both the range of numerical knowledge that children acquire and the stability of learning over time. The investigators compared effects of playing the number and color board games on preschoolers' understanding of the numbers 1–10 on four tasks: number line estimation, magnitude comparison ("Which number is bigger, *N* or *M*?"), numeral identification ("What number is on this card?"), and counting ("Can you count from 1 to 10?"). Playing the numerical board game was expected to produce gains on the magnitude comparison task for the same reason as on the number line task—improved understanding of numerical magnitudes. Playing the number board game also was expected to improve counting and numeral identification, because it provides practice and feedback on those skills too.

The study assessed performance on the four tasks not only on a pretest and immediate posttest but also on a follow-up 9 weeks after the final game-playing session. The goal was to examine stability of learning over time—both whether pretest–posttest gains endured and whether individual differences in knowledge persisted. The participants were one hundred and twenty-four 4- and 5-year-olds from Head Start centers, slightly more than half of whom were African American and the rest predominantly non-Hispanic Whites.

As in the previous study, accuracy of number line estimation increased from pretest to posttest among children who played the numerical board game (Figure 3A). Gains remained present on the follow-up. In contrast, there was no change in the accuracy of estimates of children who played the color board game.

The same pattern was evident on the magnitude comparison, numeral identification, and counting tasks (Figures 3B to 3D). In all cases, preschoolers who played the number board game showed improvements that persisted over time, whereas peers who played the color board game showed neither immediate nor delayed improvements.

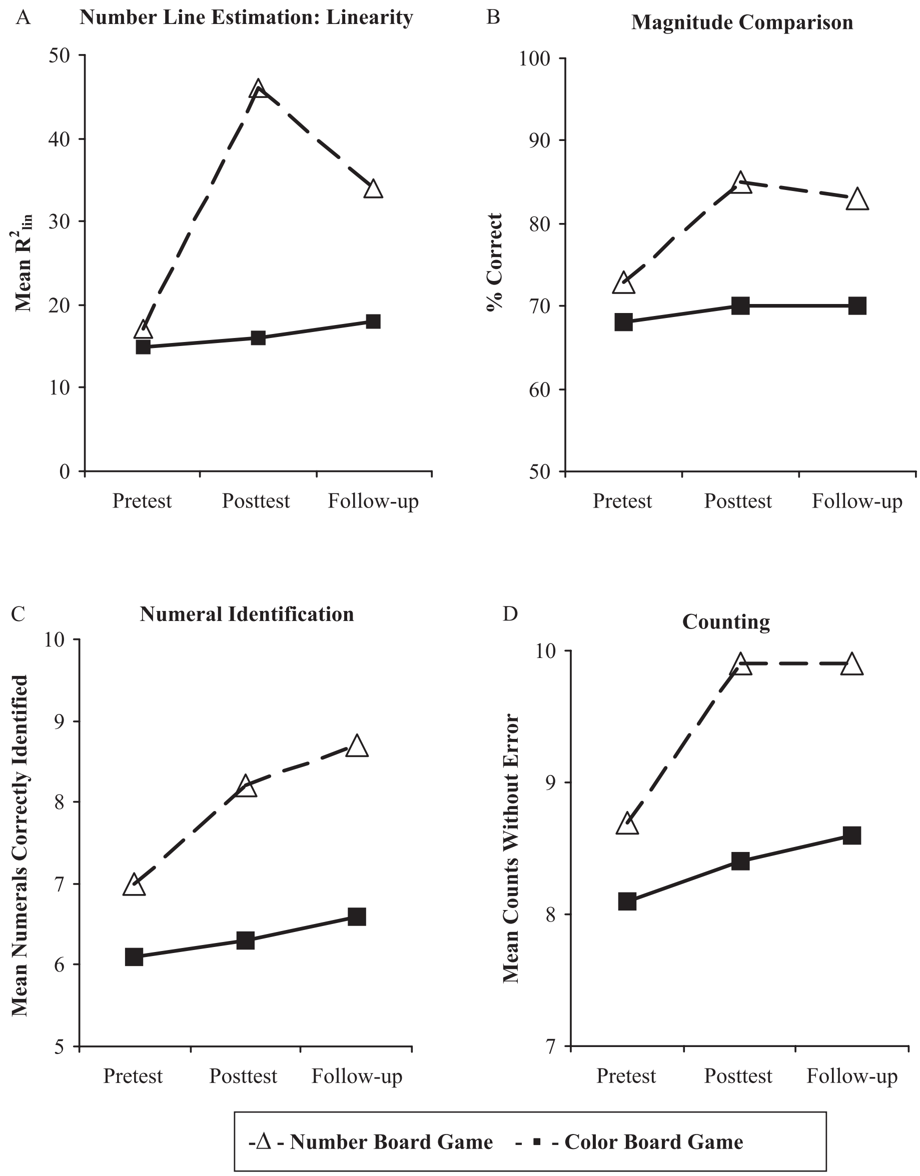

Figure 3. Effects of numerical board game experience on number line estimation, magnitude comparison, numeral identification, and counting (data from Ramani & Siegler, 2008).

Individual differences within each experimental condition showed considerable stability. Individual children's pretest, posttest, and follow-up scores were highly correlated, despite many children in the number board group improving substantially on all measures. Consistent with the theoretical interpretation, a factor analysis revealed three clear factors underlying individual differences: a magnitude factor, on which both number line estimation and magnitude comparison loaded strongly; a numeral identification factor; and a counting factor.

Thus, playing a simple linear numerical board game produces large gains in varied types of numerical knowledge, gains that are stable over at least a 9-week period. An important goal for future research is to establish whether board game experience has even longer term effects, such as effects on mathematics achievement test performance in elementary school, and whether playing this game produces improvements on additional numerical tasks.

Board Game Play in the Everyday Environment

These results indicate that playing the linear numerical board game can improve preschoolers' mathematical knowledge. However, they do not indicate what role, if any, board games occupy in numerical development in the everyday environment. To address this issue, Ramani and Siegler (2008) obtained self-reports about preschoolers' experiences with board games, card games, and video games at home and in the homes of friends and relatives. The self-reports came from the low-income preschoolers who participated in the experiment described in the last section, and from age peers from middle-income backgrounds. We hypothesized that children from middle-income backgrounds would have greater experience with board games and that a child's amount of experience playing board games would correlate positively with that child's numerical knowledge.

The data were consistent with both hypotheses. Children from middle-income backgrounds reported twice as much experience with board games as children from low-income backgrounds. Interestingly, the children from middle-income backgrounds reported less video game experience than their peers from low-income backgrounds. Within the low-income sample (the only group for which we had obtained numerical proficiency data), amount of board game experience correlated positively with all four measures of numerical proficiency. Whether preschoolers reported having played Chutes and Ladders, the commercial game that seems closest to the present board game, also correlated positively with their performance on all four numerical tasks. In contrast, amount of experience with video games correlated with proficiency on only one of the four numerical tasks (number line estimation). Thus, both correlational and causal evidence points to a connection between playing numerical board games and development of numerical knowledge.

CONCLUSIONS

Playing the linear number board game described in this article improves a variety of aspects of early numerical understanding: number line estimation, magnitude comparison, counting, numeral identification, and (in a recently completed study) learning answers to arithmetic problems (Siegler & Ramani, in press). These findings add to an increasing body of evidence indicating that efforts to improve the numerical understanding of preschoolers from low-income backgrounds can yield large, broad, and rapid learning (Clements & Sarama, 2007; Griffin & Case, 1999; Starkey et al., 2004; Whyte & Bull, 2008). The first three of these articles reported the effects of preschool curricula that attempt to inculcate a wide range of mathematical skills, including counting, numeral identification, estimation, arithmetic, spatial sense, geometric reasoning, numerical reasoning, measurement, and logical relationships. The fourth article described a 2-week intervention highly similar to the present one.

These success stories raise the question of why relatively limited interventions can produce such large and broad improvements in low-income preschoolers' mathematical knowledge. One likely reason is that many of these children have had few explicitly mathematical experiences prior to the interventions. Systematic observations of home and preschool environments of young children from low-income backgrounds indicate that neither environment typically provides many experiences where the children's attention is directed to numerical magnitudes or other mathematical relations (Plewis, Mooney, & Creeser, 1990; Tudge & Doucet, 2004). Because of this limited numerical experience in the environments of many children from low-income backgrounds, because early differences in mathematical understanding tend to persist throughout schooling, and because of the large, broad, and rapid effects of early interventions documented here and elsewhere, increasing the number of preschoolers who receive interventions of documented effectiveness in improving mathematical knowledge seems a goal worth pursuing.

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