Keep it stupid simple.

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Abstract

Deep reinforcement learning can match and exceed human performance, but if even minor changes are introduced to the environment artificial networks often can't adapt. Humans meanwhile are quite adaptable. We hypothesize that this is partly because of how humans use heuristics, and partly because humans can imagine new and more challenging environments to learn from. We've developed a model of hierarchical reinforcement learning that combines both these elements into a *stumbler-strategist* network. We test transfer performance of this network using Wythoff's game, a gridworld environment with a known optimal strategy. We show that combining imagined play with a heuristic–labeling each position as "good" or "bad"–both accelerates learning and promotes transfer to novel games, while also improving model interpretability.

Deep reinforcement learning can rival human performance on games of strategy like chess (Silver et al. 2017) and Go (Silver et al. 2016), as well as less structured games like classic Atari video games (Minh 2015). Unlike humans, artificial networks are often unable to transfer performance to new situations, even in environments with relatively trivial changes (Zhang et al. 2018; Zhang, Ballas, and Pineau 2018). We hypothesize that part of the human capacity for transfer may come from our ability to imagine new environments and learn from them, a kind of counterfactual reasoning (Pearl 2017). Human success may also stem from our ability to develop and exploit simple heuristics (Hart 2005; Gigerenzer 2014). To try and improve transfer learning in artificial networks, we combined these two elements in a novel variation of hierarchical reinforcement learning, which we call the *stumbler-strategist* architecture. The intuition behind this is simple: it is easier to imagine new environments using simple stupid ideas over to complex ones.

Stumbler-strategist networks

We introduce two-layer *stumbler-strategist* networks (Figure 1), a variation on the DYNA-Q models (Sutton and Barto 2018). Stumblers are model-free reinforcement learning agents that observe and act in the environment. Like all model-free systems they can only discover the value of specific input-output associations, without any appreciation for the organizing features of the game that govern these associations (Sutton and Barto 2018).

Strategist layers are model-based agents that *can only* observe what the stumbler layers learn, *sampling* and *aggregating* and *classifying* information about the stumblers' actions. Strategists don't act directly but can, when confidence is high, bias the stumbler's actions. Similar to hypothesized role of rostral areas of the human prefrontal cortex (Frank and Badre 2011; Badre and Frank 2012), strategists hold abstract task representations, neither directly observing or acting on the world, but reasoning by heuristics and rules. Here we explore the simple heuristic: "every state is either *bad* or *good*".

Our key innovation is in how strategist-layers use information: they try and predict the heuristic class of game states in *new and more challenging* environments–a kind of imagination (Weber et al. 2017). That is, to learn to transfer knowledge, they imagine and play new games based on their observations of the *stumbler* layer's activity. Learning in the *strategist* layer can succeed in these new, harder, environments because it is asked only to predict the heuristic label rather then the complex action-value space present in the *stumbler*.

Wythoff's game

Despite their simplicity, "gridworld" environments can provide a challenging but controllable test-bed for transfer learning. In previous work, two state-of-the-art deep reinforcement learning networks could not adapt to subtle differences in the training and testing grid-world environments (Leike et al. 2017) suggesting they are reasonable–perhaps even ideal given their simplicity–test bed to study transfer.

To isolate strategy learning and transfer, our agents play an impartial board game called Wythoff's game. The game is played on a two dimensional grid in which players alternate turns to move an object that is initially placed randomly on the board. To win, players take turns moving the single game piece to the top-left corner. Every turn, the game piece can be moved horizontally, vertically, or diagonally. Moves must be made "upward" toward the final winning position, as shown in Figure 2**a**. Each player must move when it is their turn. The game ends when one player must take the final position (i.e., top-left position). The player in this position wins the game.

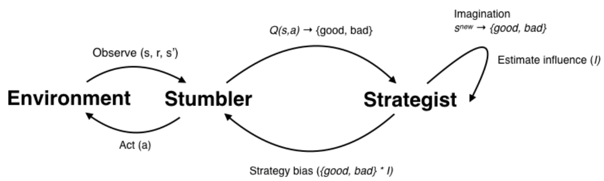

Figure 1: Diagram of the two-layer stumbler-strategist architecture. Stumblers are our name for model-free reinforcement learning agents, driven here by a version of the Q-learning rule and a ϵ-greedy action selection policy. As is typical for such agents the stumblers interact directly with the game environment, learning the state-action values ($Q(s, a)$). The strategist layer learns only with data extracted from the stumbler layer. To learn, the strategist first projects $Q(s, a)$ values sampled from the stumbler into one of two classes $\{\text{good}, \text{bad}\}$ and tries to predict the appearance of these classes in a new, larger, game board. If these predictions are successful, the influence of the strategist layer is incrementally increased (*Alg. 3*).

Impartial games offer a unique advantage for testing heuristic strategies. The difference between an impartial game, like Wythoff's, and a strategy game like Go, is that an impartial game has a single *ground truth* solution. Every position in an impartial game is either *hot*, meaning that there exists a winning strategy for the player about to make a move, or *cold*, meaning that under optimal play, the player about to make a move will always lose. The optimal strategy is always to move from a hot position to cold one. The distribution of *hot* and *cold* positions across the state space in an impartial game usually comes with inherent mathematical structure: $n = m\phi$,$m = n\phi$ where ϕ is the golden ratio and m and n are indices on the game board. This structure is visualized in Figure 2**b**.

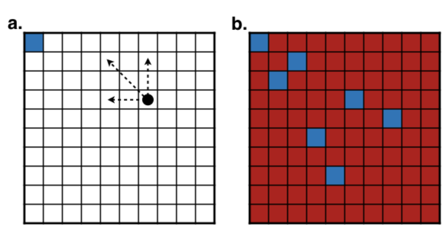

Figure 2: Wythoff's game play. **a.** Valid moves in Wythoff's are same a Queen's moves in chess. A piece (depicted here as a black circle) can be move laterally, vertically, or diagonally. The goal of the game it to move the piece to the origin of board, shown here as a blue square. **b,** Optimal play. Optimal moves are to "cold" positions (blue). Hot positions are in red. No cold position is accessible from any other cold position, however the winning position is cold. This means the optimal way to play is to always move *to* a cold position forcing your opponent to move *from* a cold position, from which they cannot win.

In a previous work (Raghu et al. 2017) studied Erdos-Selfridge-Spencer games, which also have an optimal strategy. The goals of our two papers are different. Like *Raghu et al*, we focus on studying transfer compared to an optimal player, move-by-move. Unlike *Raghu et al*, we try and transfer learning to new environments, rather than focusing only on transferring between changes in opponent strategy.

A good and bad heuristic

The heuristic used by the strategist is "positions are either good or bad". To classify every position on the board, first $Q(s, a)$ values from the stumbler are converted to expected values, where $V(s) = \text{max}Q(s, a)$. $V(s)$ for every s is classified as *bad* if $V(s) < V_{\text{bad}}$ and *good* if $V(s) > V_{\text{good}}$, where the thresholds V_{bad} and V_{good} are hyperparameters of the model.

Methods

We've designed a new two-level network architecture that combines traditional Q-learning, which we call *stumbling*, with a new *strategist* layer. The strategist layer has two key elements: an experimenter provided heuristic, and a input representation that allows this deep network to imagine, or extrapolate, the value of unseen board positions. These two elements work in combination, and allow our network to *directly* transfer it's knowledge to new–never before experienced–game boards, and rapidly learn new changes to the rules of the game. The heuristic–which projects $Q(s, a)$ values to $\{good, bad\}$ classes–is described in detail in the main text. In this section we describe the learning algorithms in the stumbler and strategist layers, as well as the the algorithm we use to control how much influence the strategist layer has over the stumbler. We conclude this section with a description of our approach to hyper-parameter tuning.

Code for all experiments is available at `https://github.com/CoAxLab/azad`

Stumbler learning

Stumbler learning is governed by Q-Learning, extended to allow for "top-down" strategist feedback and for joint-action observations, where learning updates happen over player-opponent joint-action pairs. Here, the stumbler was implemented as a look-up table.

Strategist layer learning

The strategist layer is itself a two-layer deep neural network, trained on input coordinates (i, j) and output values derived using the *good-bad* heuristic described in the main text.

Error backpropagation relied on stochastic gradient descent, with a learning rate of α_r. Training set batch sizes were 1/2 the size of the dataset and were sampled with replacement.

Strategist layer influence

To judge how much influence the *strategist* should have over the the *stumbler*, the two layers play a single game of Wythoff's using a purely greedy strategy. They play on a game board larger than the one the stumbler layer was

Algorithm 1 Learning algorithm used by stumbler

```
procedure STUMBLE(Q)
    n ← Learning episode
    ε_0 ← Max exploration-exploitation
    γ ← Value bias
    α_s ← Stumbler learning rate
    I ← Strategist influence
    B ← Strategy bias, given Strategist
    G ← Initialize Wythoff's game
    s ← A position in G
    while G continues do
        ε ← ε_0 / (log n + e)                          ▷ Anneal
        action ← ε-greedy(Q(s, action) + I * B(s), ε)
        do action on G, update s
        if G ends then
            reward ← 1                                 ▷ Winning move
        else                                           ▷ Opponent plays
            do action on G, update s
            if G ends then
                reward ← −1                            ▷ Opponent wins
            else
                reward ← 0
        Q' ← max Q-value from s'                       ▷ Joint-action update
        Q(s, action) ← α(reward + γQ' − Q(s, action))
        I ← Influence(Stumbler, Strategist)
    return Q
```

Algorithm 2 Learning algorithm used by the Strategist

```
procedure STRATEGIST(Stumbler)
    α_r ← Strategist learning rate
    n_r ← Number of training episodes
    n ← Episode counter
    B ← Strategist bias
    Q ← Complete Q-table from Stumbler
    dataset ← GoodBadHeuristic(Q)                      ▷ See main text
    model ← initialized neural network with default set-
tings
    while n < n_r do
        trainset ← sample(dataset)
        backpropagate model with trainset through cost
func
    B ← model for all s
    return B
```

trained on. The intuition behind this approach is that if the strategist layer has useful transferable knowledge it should soundly defeat the stumbler. Every time the *strategist* layer wins, it's influence I over the *stumbler* increases by α_I. Every time the *strategist* looses its influence declines by the same amount.

Algorithm 3 Strategist influence algorithm

```
procedure INFLUENCE(Stumbler, Strategist)
    α_I ← Influence learning rate
    I ← Influence
    win ← Strategist score
    G ← Initialize Wythoff's game
    s ← A position in G
    while G continues do
        action ← greedy(Stumbler(s))
        do action on G, update s
        if G ends then
            win ← 0
        action ← greedy(Strategist(s))
        do action on G, update s
        if G ends then
            win ← 1
    if win > 0 then
        I ← I + α_I
    else
        I ← I − α_I
    I ← clip(I, −1, 1)                                 ▷ I is limited
    return I
```

Network training

All networks were trained on a fixed number of 75,000 episodes, which preliminary runs showed were well past the learning plateau. Stumbler-only training used this iteration count directly. The *stumbler-strategist* network required a nesting the training procedure, which worked as follows. To begin training first the *stumbler* would play n_s games and learn from the each game independently (by rollout) (*Alg* 1.). Then the strategist takes in the stumbler extracts its value estimates, applies the heuristic (see *A good and bad heuristic* above) generating a dataset, and would train on n_r samples of this dataset (*Alg* 2.). Finally the influence of the strategist was estimated (*Alg* 3.). This overall pattern then repeats for n iterations, but the total number of training iterations $n * n_r * n_r$ was constrained to be 75,000.

Note: in all training, we independently model the player and their opponent as *stumblers*. Only the player can have a *strategist* layer. The opponent is always stumbling around.

Parameter tuning

Overall, network performance was robust to a wide range of hyperparameter settings. Parameter tuning was done via grid search, carried out piece-wise. First, the stumbler layer was tuned. Second, the strategist layer's learning and heuristic parameters were tuned. Next V_{bad} and V_{good} were optimized followed by the influence rate (α_I). Finally, the depth and unit number of the strategist layer was optimized. Each stage

Table 1: Network hyperparameters

Meaning	Symbol	Value
Stumbler learning rate	α_s	0.4
Strategist learning rate	α_r	0.025
Influence learning rate	α_I	0.2
Exploration-exploitation	ϵ	0.4
Value bias	γ	0.5
Stumbler iterations (strategist only)	n_s	500
Strategy iterations (strategist only)	n_r	500
Good threshold	V_{good}	0.5
Bad threshold	V_{bad}	-0.5
Influence (initial)	I	0.0
Strategist bias (initial)	B	0.0

relied, in part, on the previous tuning stages. The stumbler was implemented as simple one-hot look-up table, and so had no internal parameters.

The optimal hyperparameter configuration we arrives at is found in *Table 1*.

Stumbler tuning In tuning the stumbler we explored ϵ from $(0.1 - 1)$, α_s from $(0.01, 1)$, and γ from $(0.1 - 1)$. In taking 10 samples from each range and searching the full permutation space, we sampled a 1000 hyperparameter combinations in tuning the *stumbler*,

Stumbler-strategist tuning The stumbler-strategist tuning stage one explored learning rate α_r and the training iteration numbers n_s and n_r, over the following respective ranges, (0.001-0.1), (100,1000), and (100,1000) forming a 10x3x3 sampling space (Figure 3**b**). Stage two searched V_{bad} and V_{good} from $(-1, 0)$ and $(0, 1)$ forming a 10x10 sampling space. Stage three tuned the influence rate α_I $(0.01 - 1.0)$ over 20 samples. The final stage explored the depth and width of the two-layer *strategist* from n_{hidden1} from $(15, 500)$ and n_{hidden2} from $(0, 50)$, in 10x10 sampling space (not shown). As is clear in Figure 3, there is substantial degree of slackness or robustness in parameter choices. As such we hand picked middle values from each "good" region (Table 1). For more information in the meaning of these parameters see *Network training*.

Note: annealing of ϵ (Alg 1.) is required for convergence to optimal play.

Results

During learning the strategist layer never directly observes or acts on the game, instead it tries to extrapolate, or imagine, the $\{good, bad\}$ values on a larger game board that the stumbler layer never encounters. This extrapolation is possible because the complex $Q(s, a)$ value structure is reduced to a binary $\{good, bad\}$ representation, which is turn based on the heuristic that each position can be exclusively either good or bad. Our hope was that our heuristic would naturally map onto the *hot/cold* board structure of Wythoff's game. (See the *Wythoff's* section above for more).

Figure 4 shows how the two layers of the network value board positions at different stages of learning. Models that are developed in the earlier stages of training remain mostly

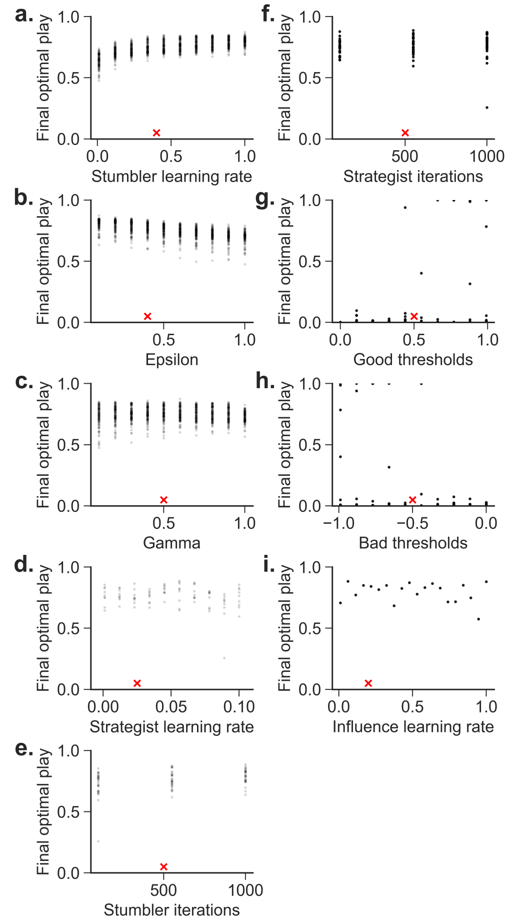

Figure 3: Hyperparameter tuning. **a.** Stumbler learning rate **b.** Epsilon **c.** Gamma **d.** Strategist learning rate **e.** Stumbler iterations **f.** Strategy iterations **g.** Good threshold **h.** Bad threshold **i.** Influence learning rate The red "x" in all figures is the optimal or reference parameter value used in all models, unless noted otherwise. The search strategy for these values is described in *Parameter tuning*. Reference values are in *Table 1*.

irrelevant to transfer; however, models that *meaningfully* generalize, although with low accuracy, begin to emerge soon after initial training. Such models are crucial for the learning process because they influence the way the stumbler chooses to explore different action spaces. Without such guidance, the stumbler explores actions without any overall purpose or insight. With the guidance from the strategist, the stumbler explores actions that would either contradict or confirm the strategist's imagined hypothesis about the nature of the learning environment it cannot directly learn from.

Stumbler-strategist performance

We compared the performance of the *stumbler* to the *stumbler-strategist* network. Figure 5**a** shows the fraction of moves that were optimal for both networks during learning. Initial learning in the stumbler-strategist is more than

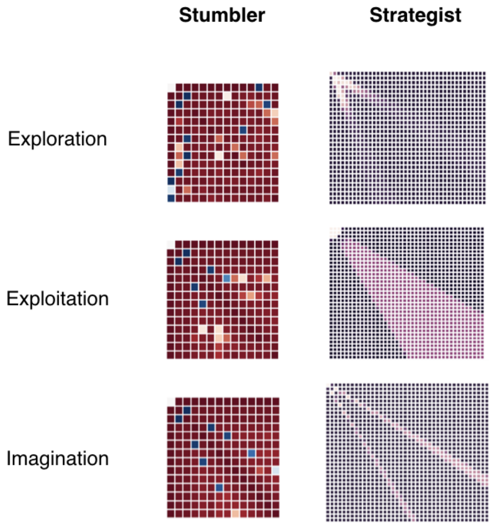

Figure 4: Learning in three stages of training on Wythoff's game. The early models (exploration) will be largely unsuccessful, while certain inaccurate transfers (exploitation) will supply reasonable strategies to the stumbler, allowing the provision of useful datasets into the network that translate into accurate and general models (imagination). In this example the stumbler trained on a 14 by 14 board for 2000 game-plays, across each strategist time-step. The strategist learned to play on a 50 by 50 game board.

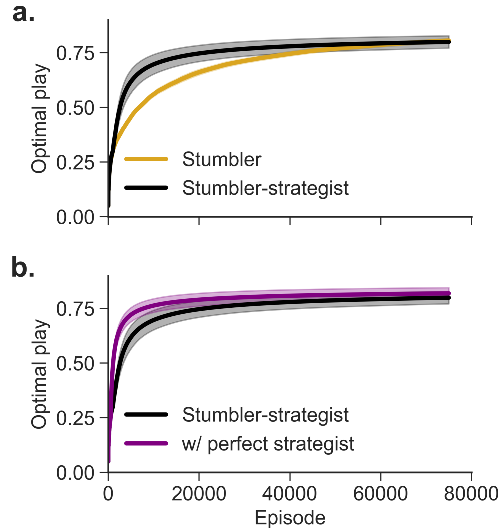

Figure 5: **a.** Learning optimal moves in Wythoff's game with (black) and without (gold) strategist layer. **b.** Learning when a stumbler layer is biased by a perfect strategist's (purple), compared to the learned strategist from *a.* (black). Error bars denote 2 * standard error, and were calculated from 20 unique random seeds.

twice as fast as the stumbler alone. However both networks plateau at the same level of final performance, indicated by the overlapping curves over the last few thousand games in Figure 5**a**.

To see how well the learned strategist in Figure 5 did in comparison to the best possible strategist, we compared learning between the learned strategist and a perfect oracle Figure 5**b**). As we noted above, there is a single optimal strategy for playing Wythoff's, which means we can hard code perfect play into the strategist layer. The learned strategist comes close to matching the perfect alternative, suggesting near-optimal accuracy in our learned network (Figure 5**b**).

Parameter sensitivity and heuristic design

To ensure the observed good performance was not specific to a particular set of parameters, we measure final optimal play performance over the last 100 episodes as parameters were independently perturbed. We compare these perturbations to a baseline run using standard parameters (*Table 1.*), but run with 20 different random seeds while parameter perturbations used a fixed seed. Intuitively, if the performance variability due to parameter changes is similar to random behavioural changes, we can consider that parameter choice to be robust.

We sampled a range of values ranging from 1/2 to 2 times the standard parameter values (*Table 1*). The variability in these perturbations were highly similar to that seen in the baseline (random) condition (Figure 6**a**.) suggesting our parameter selections were sufficiently robust.

We also consider two alternate heuristics–*good* only or *bad* only. Both these approaches prevented the strategist from develop any significant influence over the stumbler, learning to no improvement in training performance (Figure 6**b**.) compared to the stumbler alone.

Game board transfer

The strategist layer drives near optimal performance on never before seen game boards with no additional training. Unlike previous figures, here we examine performance of the *stumbler* and *strategist* layers independently as they are completely uncoupled. This is possible because for these experiments learning is turned off, and pre-trained models are used; the same models whose learning performance is shown in Figure 5-6.

We increased the board size from size the stumbler was originally trained on (15x15) up to 500x500, in 50 unit increments. As is typical, as soon as the board size increases beyond it's direct experience, optimal play for the stumbler plummets (Figure 7**a**.), quickly matching a the performance of random choice agent (Figure 7**b**.). However on the 15x15 board the stumbler model, with full access the $Q(s, a)$ outperforms the strategist, which can be seen by comparing the first points in both curves in Figure 7**a**. and in Figure 7**c**. The *strategist* on the other hand maintains a high level of performance over the entire range of novel game boards.

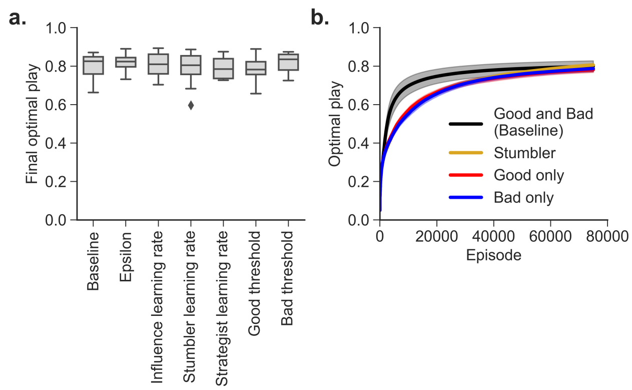

Figure 6: **a.** Hyper-parameter sensitivity. *Baseline* is 20 standard runs, randomly initialized, using optimal hyperparameters (whose selection is describe in the Methods section). These optimal values are found in Table 1. Epsilon (ϵ) is the exploration parameter in the standard ϵ-greedy algorithm (sampled here from $0.01 - 0.8$). *Influence learning rate* (α_I) controls quickly the strategists bias effects the stumblers actions $(0.01 - 0.4)$. *Stumbler learning rate* is the stumblers learning rate (α_s), which in typical Q-learning is often denoted by α $(0.2 - 0.6)$. The *strategist learning rate* (α_r) controls in the the deep strategist layer $(0.01 - 0.05$; values larger than 0.08 lead to catastrophic failure). The *good* and *bad threshold* are the value thresholds, V_{good} and V_{bad} (sampled from $0.0 - 0.5$ and $-0.5 - 0$ respectively). **b.** Effect of alternate heuristic choices. Our standard heuristic maps $Q(s, a)$ values to *good*/*bad* classes. Here we explored an alternate approach where the strategist layer predicts only *good* or only *bad*, rather than both. Error bars denote 2 * standard error.

Rule transfer

Next we consider transfer between different grid-world games. To do this we keep the game boards the same as out initial experiments (Figures 5-6 but alter the rules of play. Our agents play two new games: Nim and Euclid's. In Euclid's game the movement directions are the same Wythoff's game (up, left or diagonal) but in these directions during Euclid play only select positions are available. Euclid's game is a playable instantiation of Euclid's algorithm. So valid positions are those that could be obtained by taking differences between row m and column n when m and n are also multiples of greatest common denominator of m and n, (i.e, $(m, n) if (GDC(m, n)$.

In the game of Nim diagonal moves are forbidden, leaving only moves to the up and left. Like Wythoff's though, and unlike Euclid, as position in a valid direction is allowed. The simple removal of diagonal play profoundly alters optimal play. The optimal way to play a 2d game of Nim is to move such that row and column indices are equal, which amounts to moving to the diagonal positions (via up and left). This is profoundly different than the optimal play in Wythoff's, which centers around position that are multiples of the golden ratio (See the *Wythoff's game* section above).

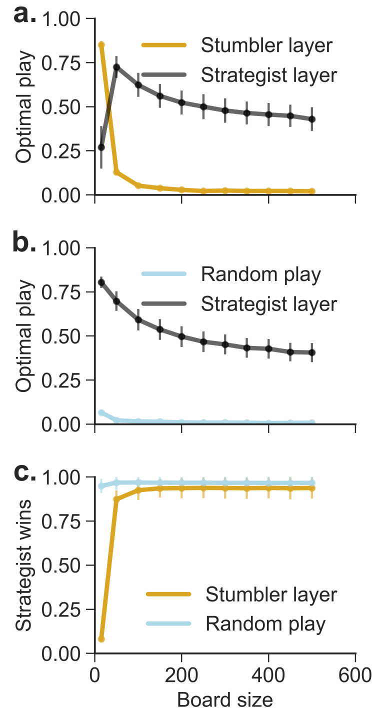

Figure 7: Transfer performance on new larger game boards, with frozen layers. **a.** Optimal play observed when the stumbler (goldenrod) and strategist (black) layers complete in several (1000) games of Wythoff's. The x-axis denotes increases in board size. Prior to this experiment, neither layer has experienced a board larger than 15x15 (for the stumbler) or 50x50 (for the strategist. **b.** The strategist layer versus a random agent (blue) who's choices were sampled uniformly from the available actions at each position, and could not learn **c.** Fraction of the games won by the strategist versus the stumbler (goldenrod) or the random player (blue). At each board size 1000 games were played, each begun with a unique random seed.

With changes to the rules of the game change, the stumbler layers for all models needed to be reset. We therefore studied how a transferring a Wythoff-trained strategy layer, to a re-initialized stumbler layer impacted learning in either Euclid's' game or Nim (Figure 8). To measure only the benefits of transfer we quantified the ratio of player/ opponent wins with and without a pre-trained layer. Player models without the pre-trained layer also featured a functioning strategist, though of course it began with a random initial configuration. As in all previous experiments the opponent agent doesn't have a strategist layer; It is left stumbling around.

In Euclid's–whose optimal play is a restricted subset of Wythoff's making it more difficult to learn–strategy transfer accelerates learning (Figure 8**a.**). Meanwhile in Nim, strategy transfer does not improve performance (as would be expected given the different optimal strategies) but it also does not significantly hinder learning either. The influence algorithm we employ (*Alg.* 3) quickly limits bad strategies, leaving the network able to re-train itself.

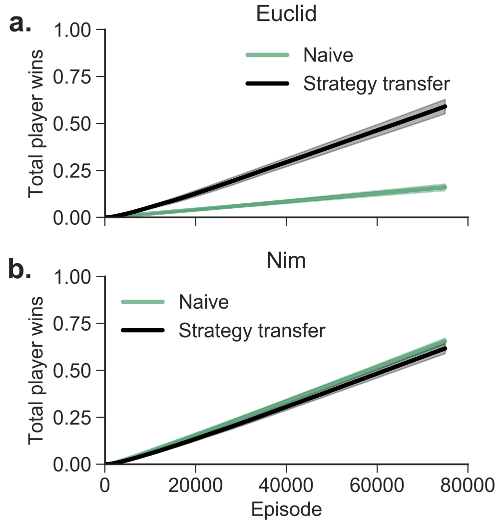

Figure 8: Rule transfer: learning with a pre-trained strategist layer. **a.** Player wins in Euclid's game, with and without strategy transfer. **b.** Player wins in Nim. In both models only the player could benefit from a strategist layer. As with all previous simulations, the opponent here was limited to traditional Q-learning.

Discussion

Our key innovation is the introduction of a *strategist* layer to more traditional RL networks, that imagines play in *new and more challenging* environments. The strategist layer however only succeeds in these new, harder, environments because it is learns to predict a simple, but useful, heuristic: "every state is good or bad".

Heuristics map complex contingencies to simple rules (Parpart 2017). Heuristics are widely used in human cognition, having been tuned by evolution and experience to support memory, and transfer knowledge (Gigerenzer 2014). A heuristic is easier to map between environmental states, as it is not dependent on a specific set of complex actions. Intuitively, heuristics also simplify imagining never before experienced outcomes. Though not put to use here, heuristics also ease the transfer of knowledge between agents. For example, it's intuitively easier to transfer knowledge between a student student and teacher, or to engage in cooperative inference, when using simple (appropriate) heuristics rather complex action-value tensors.

We studied our stumbler-strategist in a toy "gridworld" environment, with known optimal play (i.e. we studied Wythoff's game). While this allowed us to quantify the efficiency of strategy learning, it is a highly simplified environment by design. Moving from simple games such as this to open-ended strategy games like Go and to complex visual environments like classic Atari games will require at least two further innovations. First, gridworld games share common coordinates. It is therefore simple to move from smaller boards to larger and more challenging boards by just mapping between common coordinates. Using a stumbler-strategist network in visually complex environments, like classic Atari games, requires solving this projection problem, which is nontrivial. There is room for optimism though, as the response in higher-level layers in DQN networks is related to the perceptual similarity of the input images (Minh 2015), suggesting strategists could observe not only values, but also critical perceptual relationships by only studying stumblers' responses to the world.

Second, we studied a single strategic heuristic (good/bad). It is not reasonable to expect this heuristic to apply in all but a few select instances. However there exists a substantial literature in game theory representing a large pool of possible heuristics and strategies (Gigerenzer 2014; Parpart 2017; Hart 2005; Rieskamp and Otto 2006). While these will certainly not describe optimal performance in all situations, there are perhaps consistent moments to be found in complex games where simple strategies, and matched counterfactual simulations, will allow for information to be efficiently transferred between environments.

Acknowledgments.

The research was sponsored by the Army Research Laboratory and accomplished under Cooperative Agreement Number W911NF-10-2-0022. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. government.

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