

Tutorial:

**Causal** Model Search

Richard Scheines

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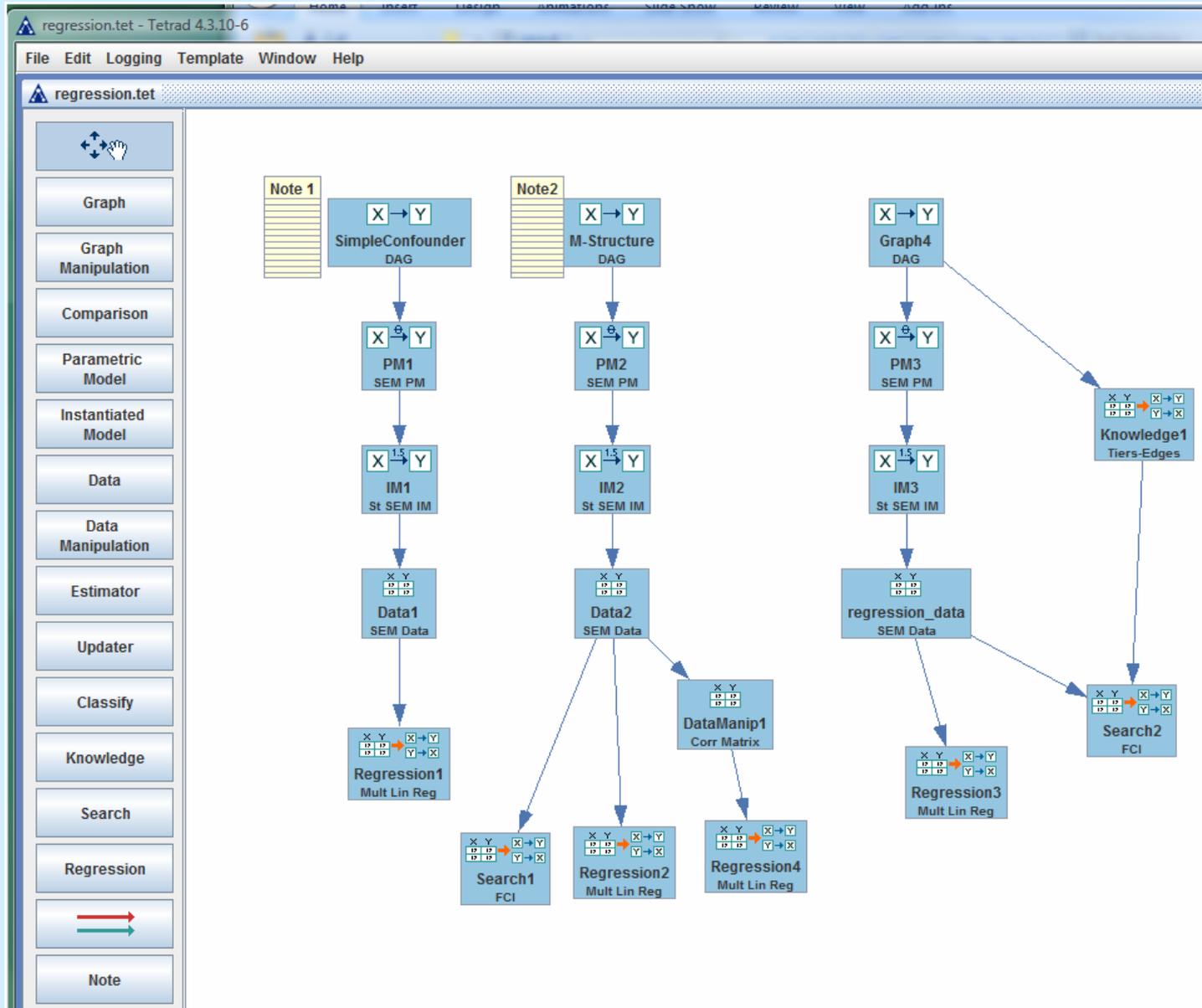
Peter Spirtes, Clark Glymour, Joe Ramsey,

others

# Goals

- 1) Convey rudiments of graphical causal models
- 2) Basic working knowledge of Tetrad IV

# Tetrad: Complete Causal Modeling Tool



# Tetrad

- 1) Main website: <http://www.phil.cmu.edu/projects/tetrad/>
- 2) Download: <http://www.phil.cmu.edu/projects/tetrad/current.html>
- 3) Data files: workshop.new.files.zip in [www.phil.cmu.edu/projects/tetrad\\_download/download/workshop/Data/](http://www.phil.cmu.edu/projects/tetrad_download/download/workshop/Data/)
- 4) Download from Data directory:
  - tw.txt
  - Charity.txt
  - Optional:
    - estimation1.tet, estimation2.tet
    - search1.tet, search2.tet, search3.tet

# Outline

- 1) Motivation
- 2) Representing/Modeling **Causal** Systems
- 3) **Estimation and** Model fit
- 4) **Causal** Model Search

# Statistical Causal Models: Goals

- 1) Policy, Law, and Science: How can we use data to answer
  - a) *subjunctive* questions (effects of future policy interventions), or
  - b) *counterfactual* questions (what would have happened had things been done differently (law)?
  - c) *scientific* questions (what mechanisms run the world)
  
- 2) Rumsfeld Problem: Do we know what we do and don't know: Can we tell when there is or is not enough information in the **data** to answer **causal** questions?

# Causal Inference Requires More than Probability

Prediction from Observation  $\neq$  Prediction from Intervention

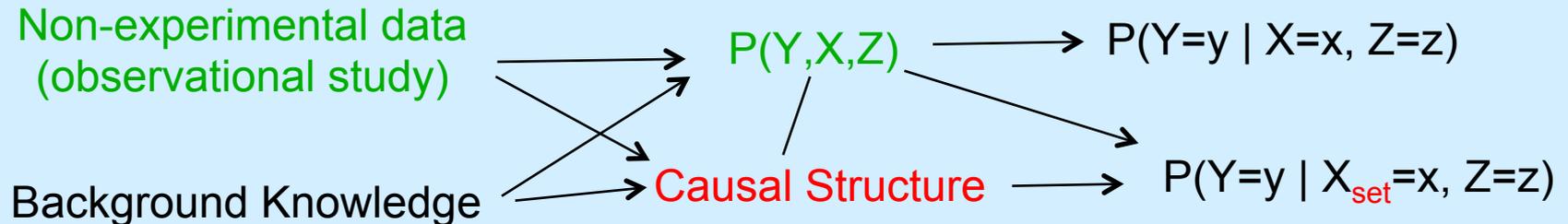
$P(\text{Lung Cancer 1960} = y \mid \text{Tar-stained fingers 1950} = \text{no})$

$\neq$

$P(\text{Lung Cancer 1960} = y \mid \text{Tar-stained fingers 1950}_{\text{set}} = \text{no})$

In general:  $P(Y=y \mid X=x, Z=z) \neq P(Y=y \mid X_{\text{set}}=x, Z=z)$

Causal Prediction vs. Statistical Prediction:



# Causal Search

Causal Search:

1. Find/compute *all* the **causal models** that are indistinguishable given background knowledge and **data**
2. Represent features common to all such models

**Multiple Regression** is often the *wrong* tool for **Causal** Search:

Example: Foreign Investment & Democracy

# Foreign Investment

## *Does Foreign Investment in 3<sup>rd</sup> World Countries inhibit Democracy?*

Timberlake, M. and Williams, K. (1984). Dependence, political exclusion, and government repression: Some cross-national evidence. *American Sociological Review* 49, 141-146.

N = 72

- PO degree of political exclusivity
- CV lack of civil liberties
- EN energy consumption per capita (economic development)
- FI level of foreign investment

# Foreign Investment

## Correlations

	po	fi	en	cv
po	1.0			
fi	<span style="border: 1px solid black; padding: 2px;">-.175</span>	1.0		
en	-.480	0.330	1.0	
cv	0.868	-.391	-.430	1.0

# Case Study: Foreign Investment

## Regression Results

$$po = .227*fi - .176*en + .880*cv$$

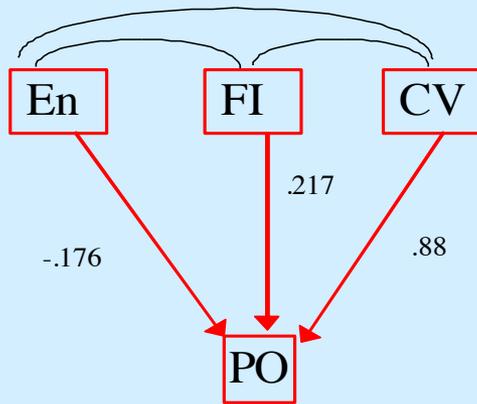
SE	(.058)	(.059)	(.060)
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t	3.941	-2.99	14.6
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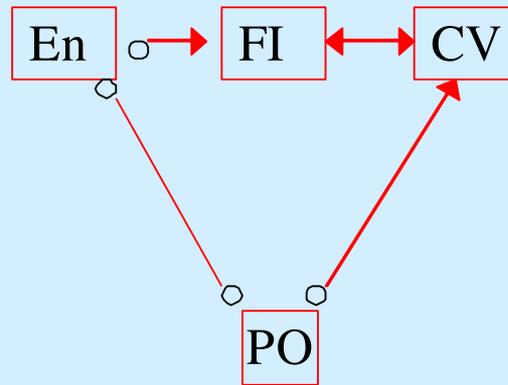
Interpretation: foreign investment **increases** political repression

# Case Study: Foreign Investment

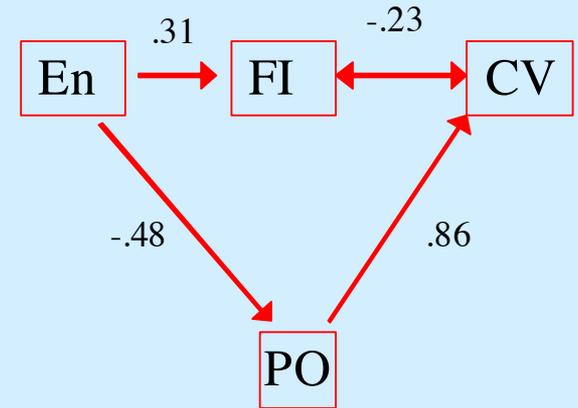
## Alternatives



Regression



Tetrad - FCI



Fit:  $df=2$ ,  $\chi^2=0.12$ ,  
p-value = .94

There is no model with testable constraints ( $df > 0$ ) that is not rejected by the data, in which FI has a positive effect on PO.

# Tetrad Demo

1. Load tw.txt data
2. Estimate regression
3. Search for alternatives
4. Estimate alternative

# Tetrad Hands-On

1. Load tw.txt data
2. Estimate regression

# Outline

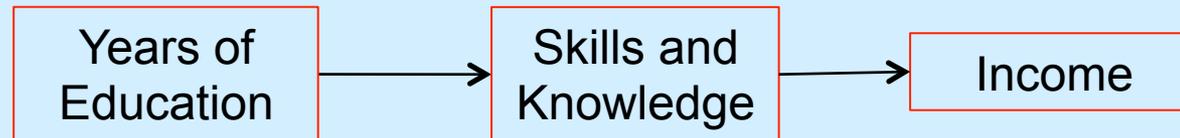
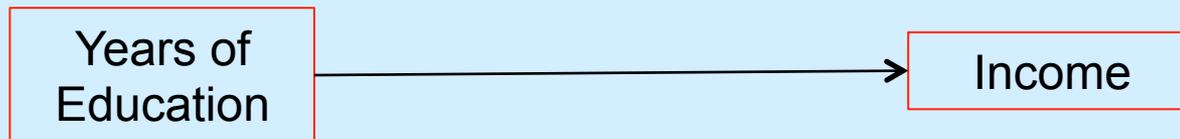
- 1) Motivation
- 2) Representing/Modeling **Causal** Systems
  - 1) **Causal Graphs**
  - 2) Standard Parametric Models
    - 1) Bayes Nets
    - 2) Structural Equation Models
  - 3) Other Parametric Models
    - 1) Generalized SEMs
    - 2) Time Lag models

# Causal Graphs

Causal Graph  $G = \{\mathbf{V}, \mathbf{E}\}$

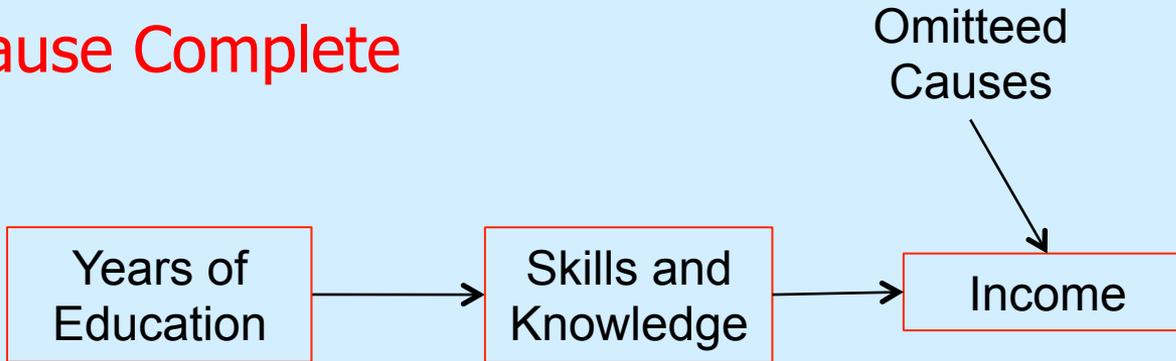
Each edge  $X \rightarrow Y$  represents a direct **causal** claim:

$X$  is a **direct cause** of  $Y$  relative to  $\mathbf{V}$

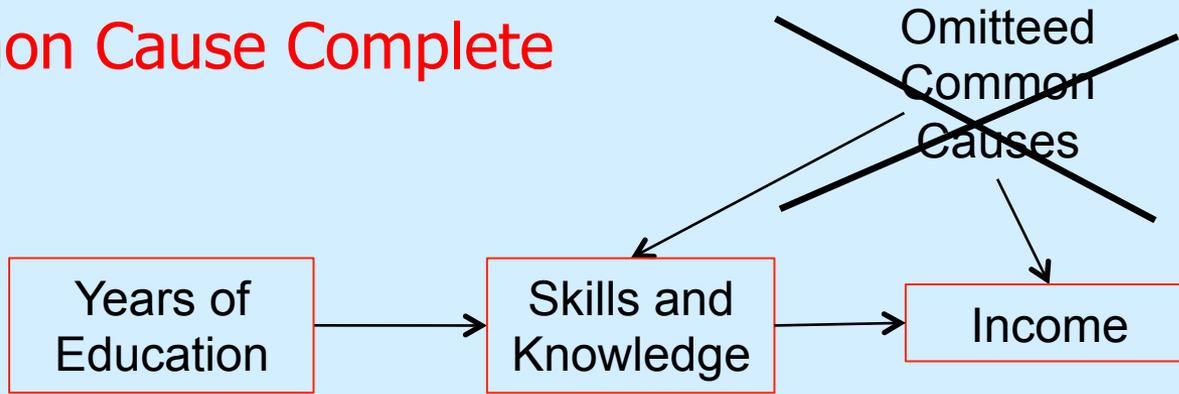


# Causal Graphs

*Not Cause Complete*



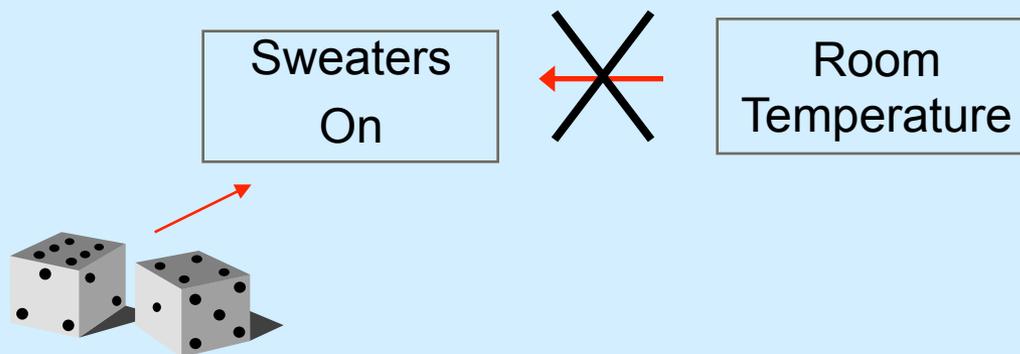
*Common Cause Complete*



# Modeling **Ideal Interventions**

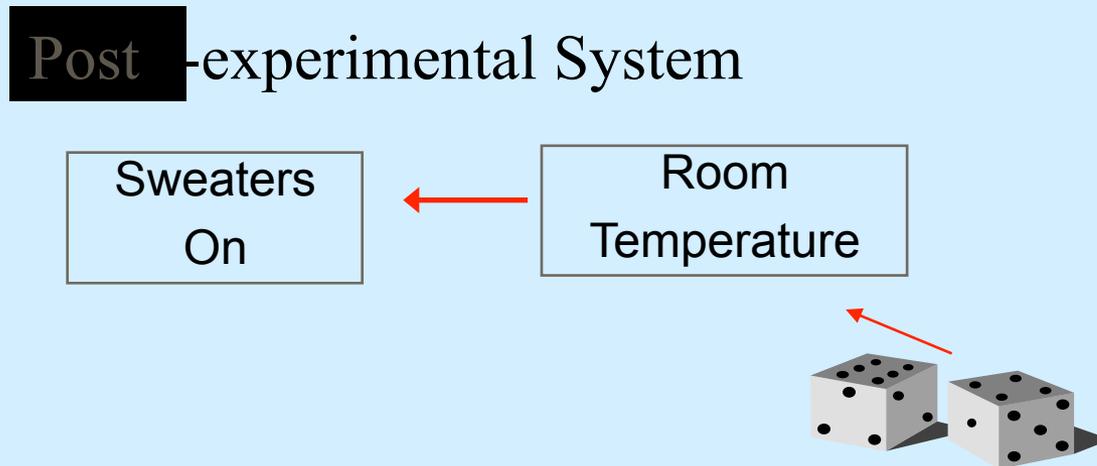
## Interventions on the Effect

**Post** experimental System



# Modeling **Ideal Interventions**

## **Interventions on the Cause**



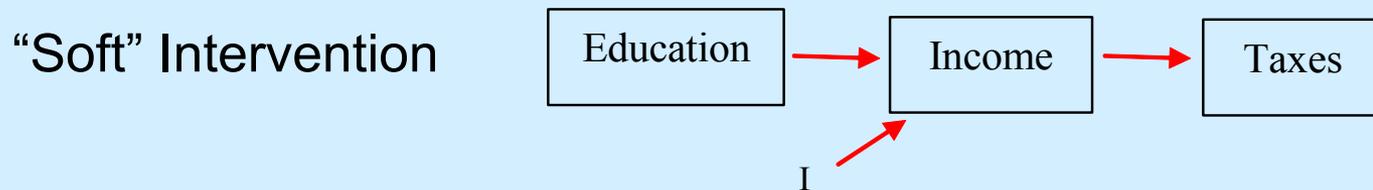
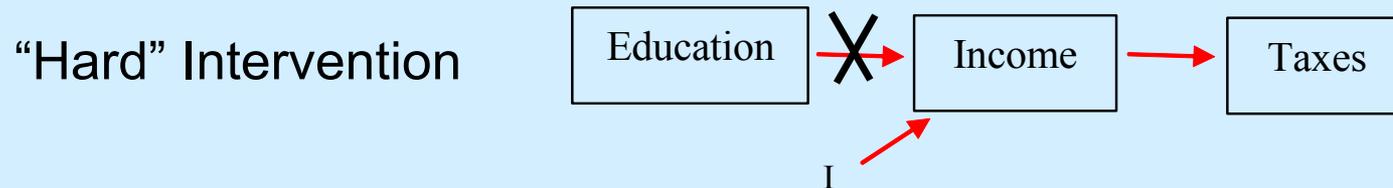
# Interventions & Causal Graphs

Model an **ideal intervention** by adding an “intervention” variable outside the original system as a direct cause of its target.

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Intervene on *Income*

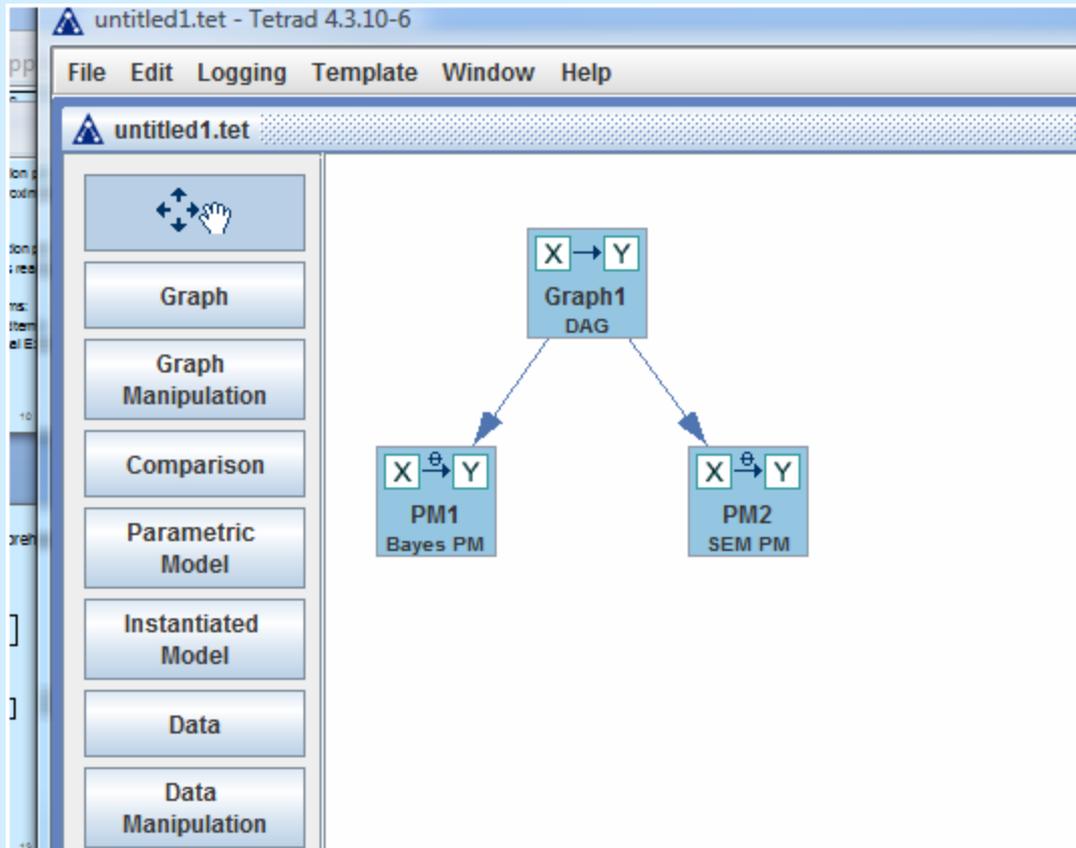


# Tetrad Demo & Hands-On

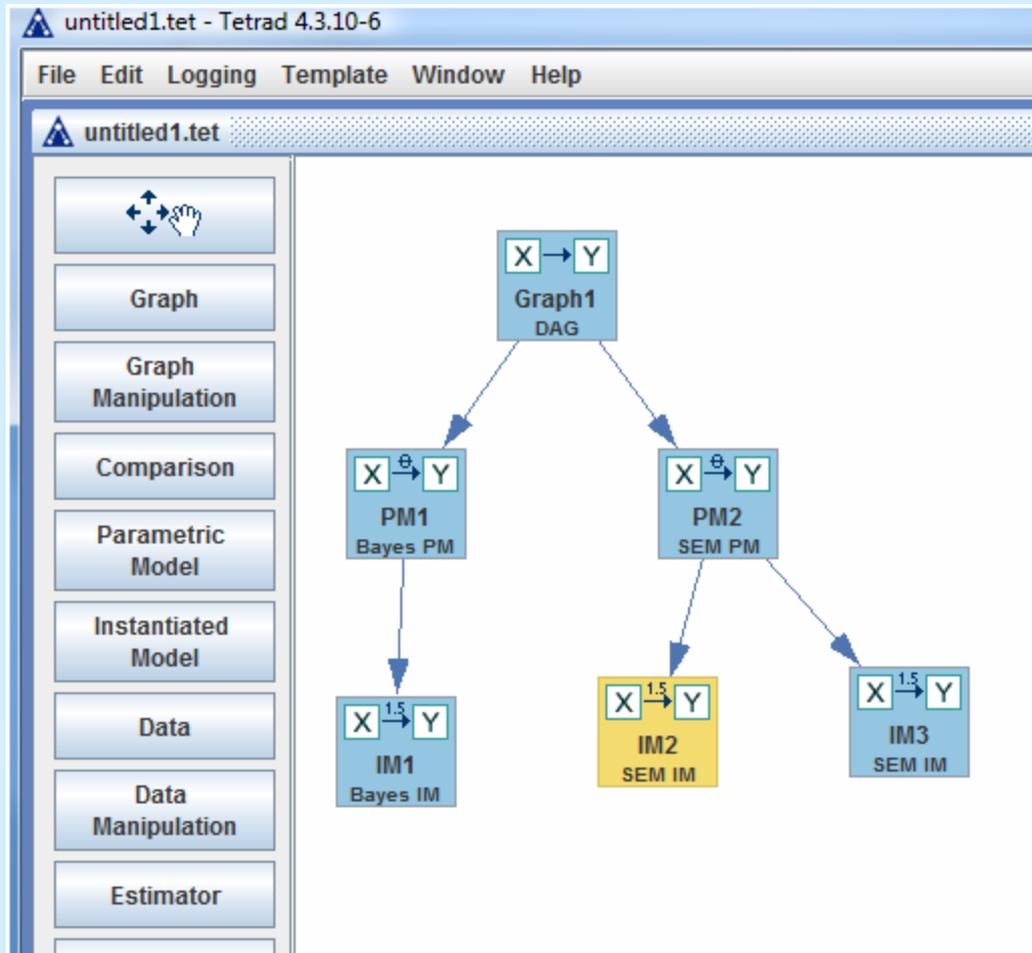
Build and Save an acyclic causal graph:

- 1) with 3 measured variables, no latents
- 2) with 5 variables, and at least 1 latent

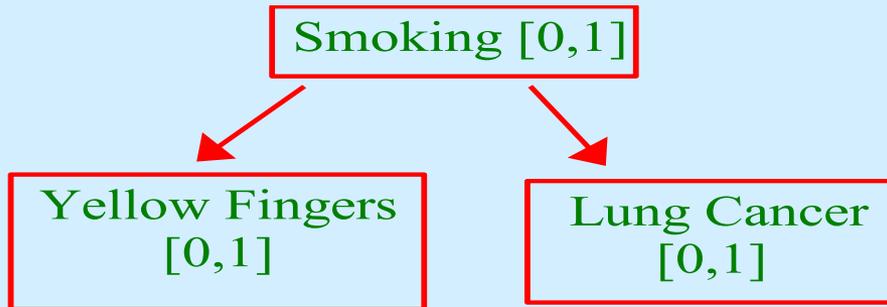
# Parametric Models



# Instantiated Models



# Causal Bayes Networks

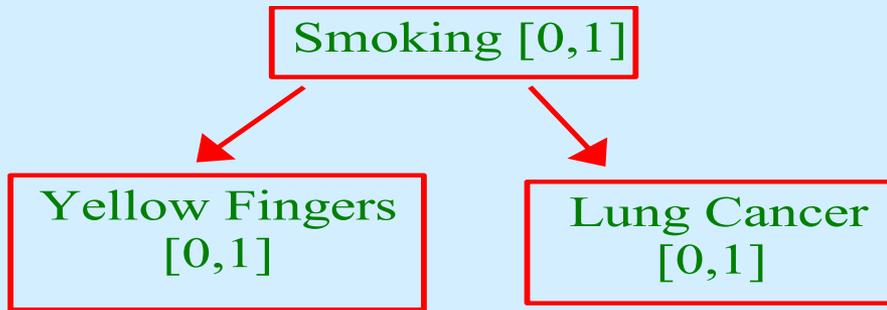


The Joint Distribution Factors  
According to the Causal Graph,

$$P(V) = \prod_{x \in V} \mathbf{P}(X \mid \text{Direct\_causes}(X))$$

$$P(S, YF, L) = P(S) P(YF \mid S) P(LC \mid S)$$

# Causal Bayes Networks



The Joint Distribution Factors

According to the Causal Graph,

$$P(V) = \prod_{x \in V} \mathbf{P}(X \mid \text{Direct\_causes}(X))$$

$$P(S) P(YF \mid S) P(LC \mid S) = f(\theta)$$

All variables binary [0,1]:

$$\theta = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}$$

$$P(S = 0) = \theta_1$$

$$P(S = 1) = 1 - \theta_1$$

$$P(YF = 0 \mid S = 0) = \theta_2$$

$$P(YF = 1 \mid S = 0) = 1 - \theta_2$$

$$P(YF = 0 \mid S = 1) = \theta_3$$

$$P(YF = 1 \mid S = 1) = 1 - \theta_3$$

$$P(LC = 0 \mid S = 0) = \theta_4$$

$$P(LC = 1 \mid S = 0) = 1 - \theta_4$$

$$P(LC = 0 \mid S = 1) = \theta_5$$

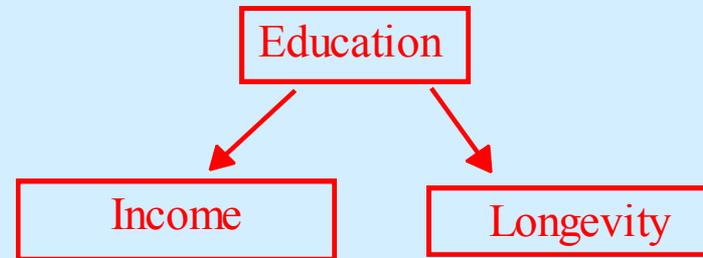
$$P(LC = 1 \mid S = 1) = 1 - \theta_5$$

# Tetrad Demo & Hands-On

- 1) Attach a Bayes PM to your 3-variable graph
- 2) Define the Bayes PM (# and values of categories for each variable)
- 3) Attach an IM to the Bayes PM
- 4) Fill in the Conditional Probability Tables.

# Structural Equation Models

Causal Graph



## ■ Structural Equations

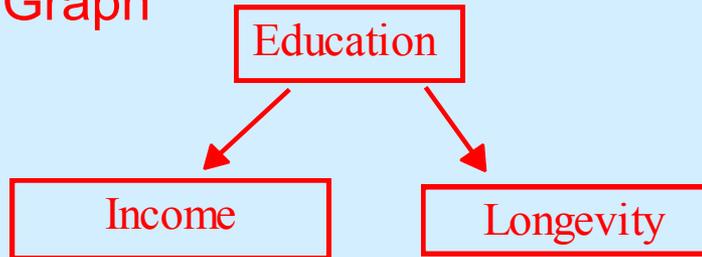
For each variable  $X \in \mathbf{V}$ , an *assignment* equation:

$$X := f_X(\text{immediate-causes}(X), \varepsilon_X)$$

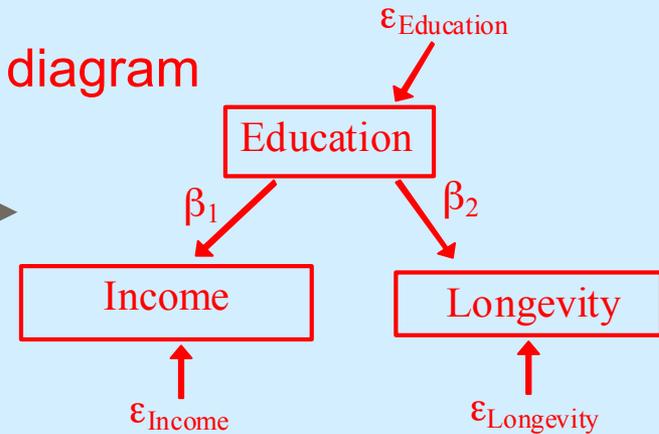
## ■ Exogenous Distribution: Joint distribution over the exogenous vars : $P(\varepsilon)$

# Linear Structural Equation Models

Causal Graph



Path diagram



Equations:

$$\text{Education} := \varepsilon_{\text{Education}}$$

$$\text{Income} := \beta_1 \text{Education} + \varepsilon_{\text{Income}}$$

$$\text{Longevity} := \beta_2 \text{Education} + \varepsilon_{\text{Longevity}}$$

Structural Equation Model:

$$\mathbf{V} = \mathbf{BV} + \mathbf{E}$$

Exogenous Distribution:

$$P(\varepsilon_{\text{ed}}, \varepsilon_{\text{Income}}, \varepsilon_{\text{Longevity}})$$

-  $\forall i \neq j \varepsilon_i \perp \varepsilon_j$  (pairwise independence)

- no variance is zero

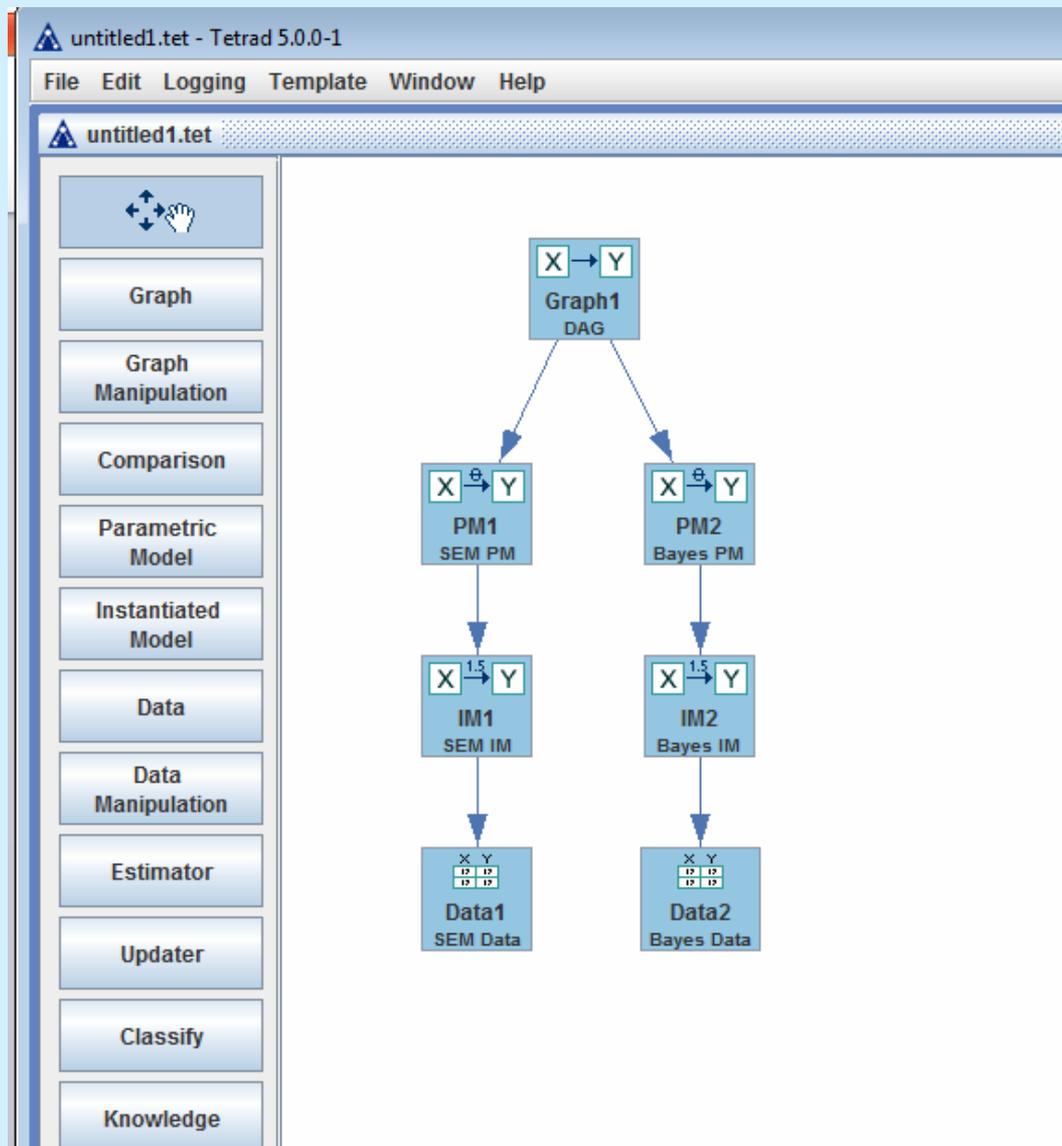
E.g.

$$(\varepsilon_{\text{ed}}, \varepsilon_{\text{Income}}, \varepsilon_{\text{Longevity}}) \sim N(0, \Sigma^2)$$

-  $\Sigma^2$  diagonal,

- no variance is zero

# Simulated Data



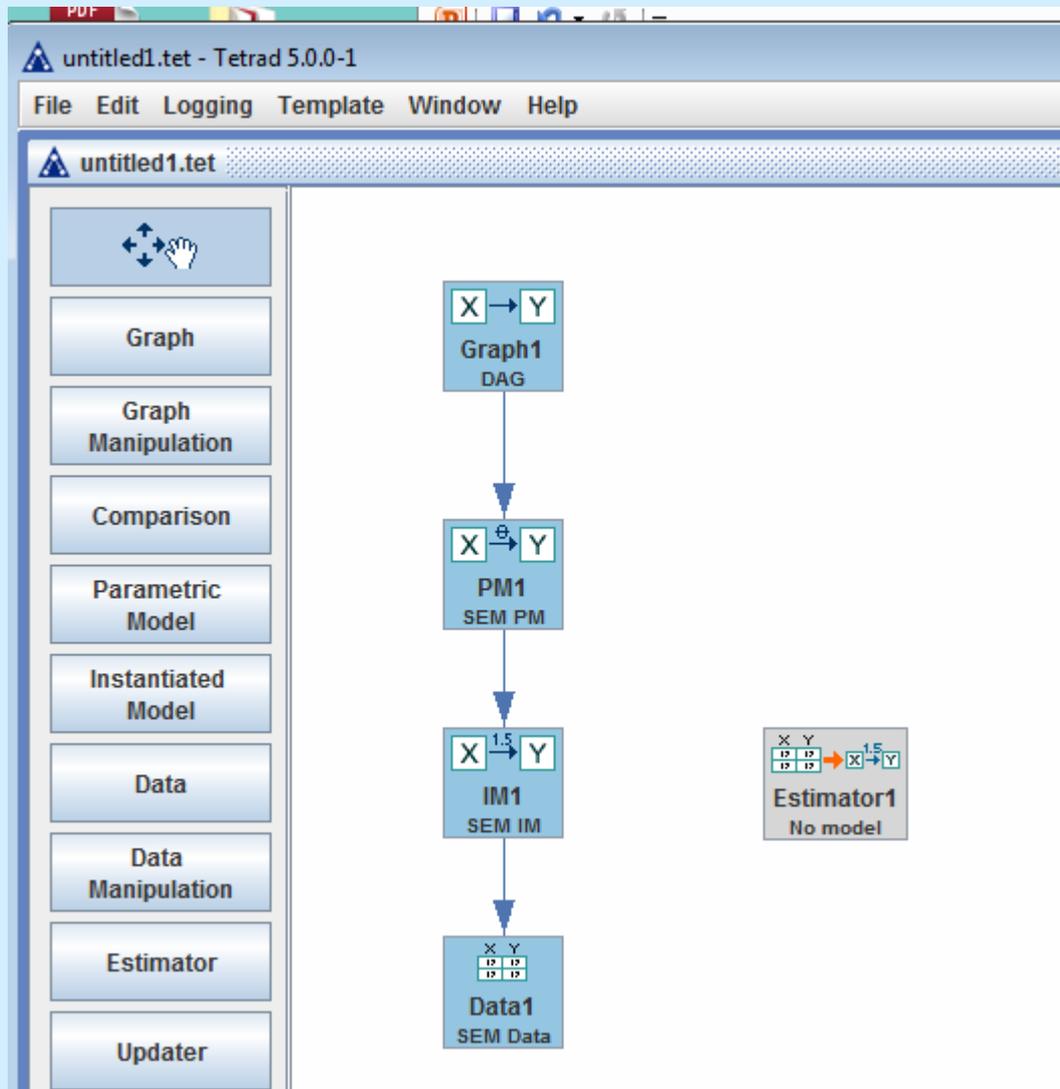
# Tetrad Demo & Hands-On

- 1) Attach a SEM PM to your 3-variable graph
- 2) Attach a SEM IM to the SEM PM
- 3) Change the coefficient values.
- 4) Simulate Data from both your SEM IM and your Bayes IM

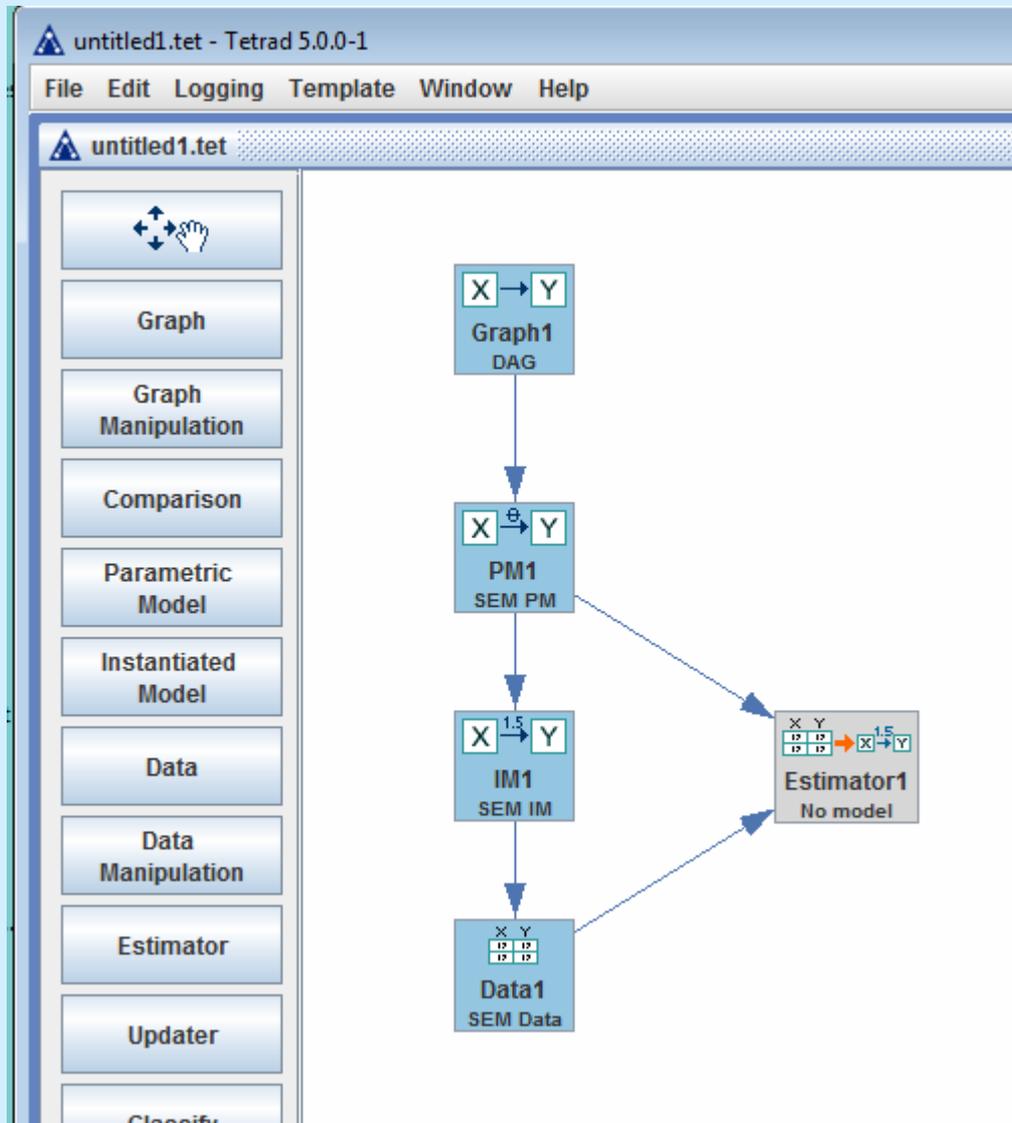
# Outline

- 1) Motivation
- 2) Representing/Modeling Causal Systems
- 3) Estimation and Model fit
- 4) Model Search

# Estimation

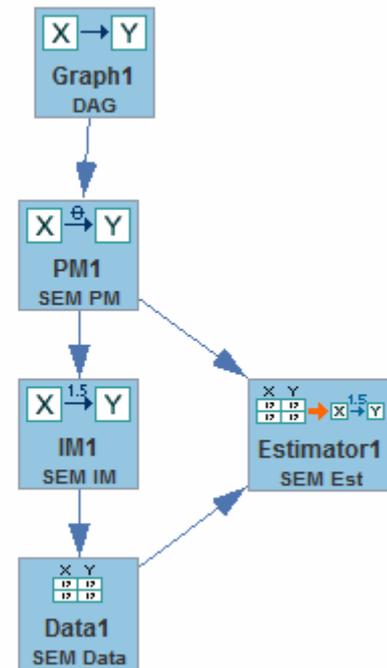
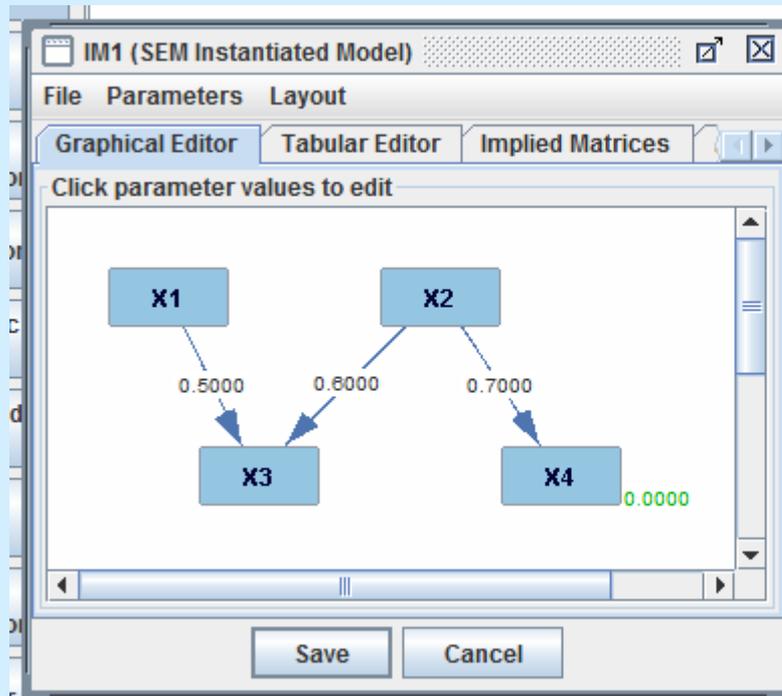


# Estimation

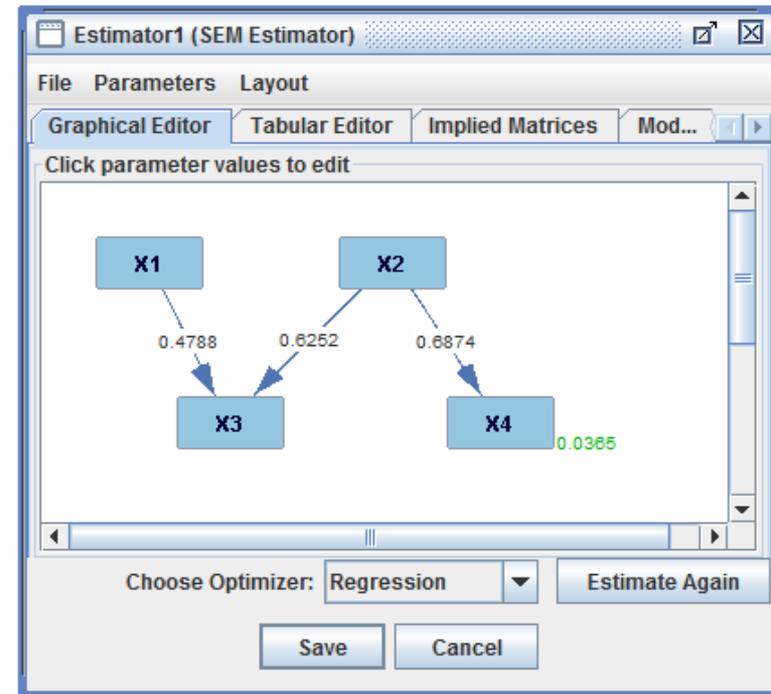
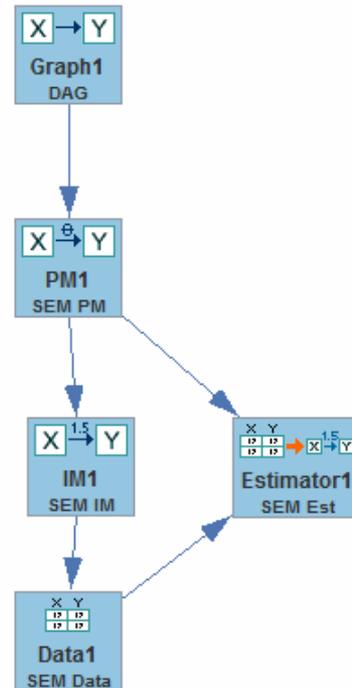
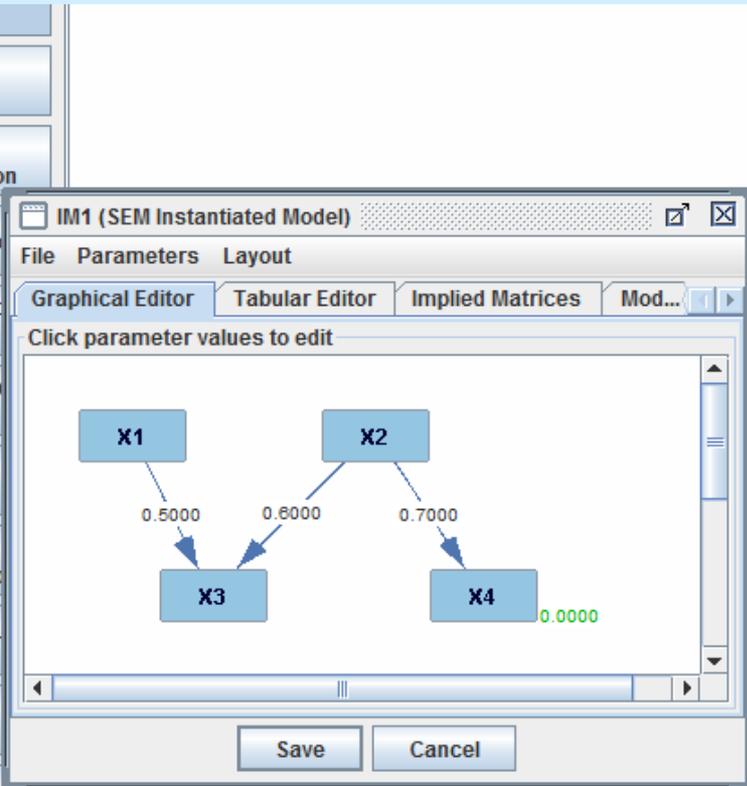


# Tetrad Demo and Hands-on

- 1) Select Template: “Estimate from Simulated Data”
- 2) Build the SEM shown below – all error standard deviations = 1.0 (go into the Tabular Editor)
- 3) Generate simulated data N=1000
- 4) Estimate model.
- 5) Save session as “Estimate1”



# Estimation



# Coefficient inference vs. Model Fit

Coefficient Inference: Null: coefficient = 0

p-value =  $p(\text{Estimated value } \hat{\beta}_{X1 \rightarrow X3} \geq .4788 \mid \beta_{X1 \rightarrow X3} = 0 \ \& \ \textit{rest of model correct})$

Reject null (coefficient is “significant”) when p-value <  $\alpha$ ,  $\alpha$  usually = .05

The image displays three windows from a structural equation modeling (SEM) software interface:

- IM1 (SEM Instantiated Model) - Graphical Editor:** Shows a path diagram with variables X1, X2, X3, and X4. Path coefficients are: X1 to X3 (0.5000), X2 to X3 (0.6000), and X2 to X4 (0.7000). The coefficient for X4 is 0.0000.
- Graph1 DAG (Directed Acyclic Graph):** A flowchart showing the relationships between variables: X1 and X2 point to X3, and X2 points to X4.
- Estimator1 (SEM Estimator) - Model Statistics:** A table showing the null hypothesis test results for the parameter from X1 to X3.

From	To	Type	Value	SE	T	P
X1	X3	Edge Coef.	0.4788	0.0334	14.3388	0.0000
X2	X4	Edge Coef.	0.6874	0.0303	22.7157	0.0000
X2	X3	Edge Coef.	0.6252	0.0316	19.7897	0.0000
X1	X1	Std. Dev.	0.9769	0.0427	22.3513	0.0000
X2	X2	Std. Dev.	1.0326	0.0477	22.3510	0.0000
X3	X3	Std. Dev.	1.0302	0.0475	22.3510	0.0000
X4	X4	Std. Dev.	0.9877	0.0436	22.3513	0.0000
X1	X1	Mean	-0.0320	0.0309	-1.0375	0.2998
X2	X2	Mean	-0.0233	0.0326	-0.7143	0.4752
X3	X3	Mean	0.0070	0.0415	0.1696	0.8654
X4	X4	Mean	0.0365	0.0384	0.9486	0.3431

# Coefficient inference vs. Model Fit

Coefficient Inference: Null: coefficient = 0

p-value =  $p(\text{Estimated value } \hat{\beta}_{X_1 \rightarrow X_3} \geq .4788 \mid \beta_{X_1 \rightarrow X_3} = 0 \ \& \ \textit{rest of model correct})$

Reject null (coefficient is “significant”) when p-value  $\ll \alpha$ ,  $\alpha$  usually = .05,

Model fit: Null: Model is correctly specified (constraints true in population)

p-value =  $p(f(\text{Deviation}(\Sigma_{ml}, S)) \geq 5.7137 \mid \text{Model correctly specified})$

The image displays a screenshot of a Structural Equation Modeling (SEM) software interface, likely LISREL, showing the model specification and fit statistics.

**Model Specification (IM1 SEM Instantiated Model):**

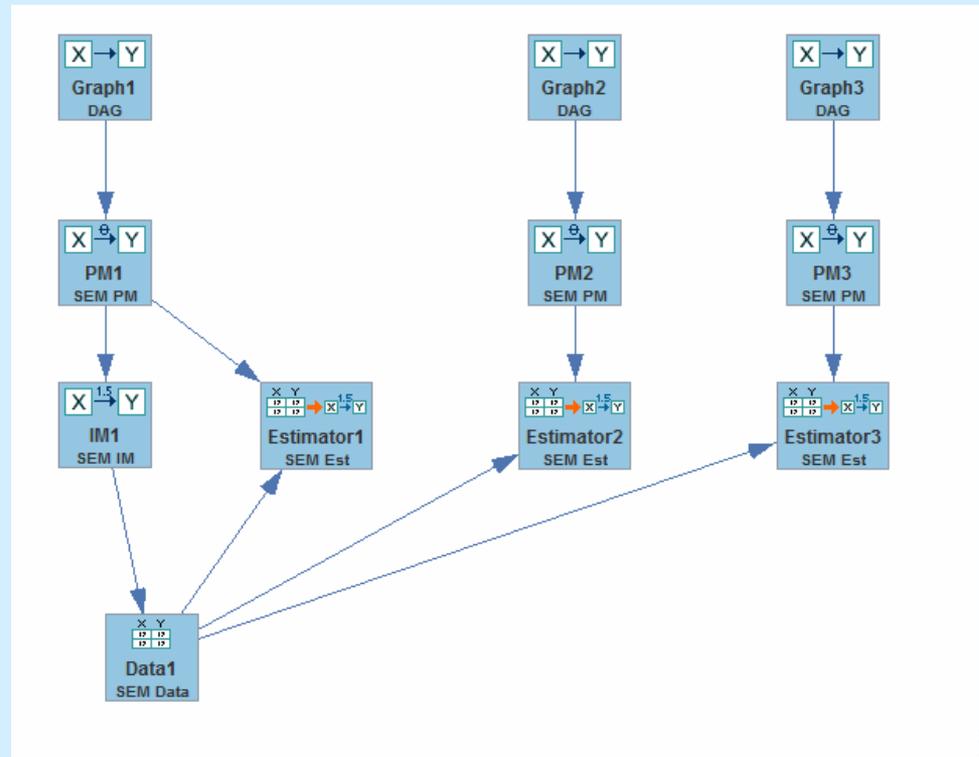
- Graphical Editor: Shows a path diagram with variables X1, X2, X3, and X4. Path coefficients are: X1 to X3 (0.5000), X2 to X3 (0.6000), X2 to X4 (0.7000), and X4 to X4 (0.0000).
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**Estimator1 (SEM Estimator):**

- Model Statistics: Degrees of Freedom = 3, Chi Square = 5.7137, P Value = 0.1264, BIC Score = -15.0095.
- The above chi square test assumes that the maximum likelihood function over the measured variables has been minimized. Under that assumption, the null hypothesis for the test is that the population covariance matrix over all of the measured variables is equal to the estimated covariance matrix over all of the measured variables written as a function of the free model parameters—that is, the unfixed parameters for each directed edge (the linear coefficient for that edge), each exogenous variable (the variance for the error term for that variable), and each bidirected edge (the covariance for the exogenous variables it connects). The model is explained in Bollen, Structural Equations with Latent Variable, 110. Degrees of freedom are calculated as  $m(m+1)/2 - d$ , where  $d$  is the number of linear coefficients, variance terms, and error covariance terms that are not fixed in the model. For latent models, the degrees of freedom are termed 'estimated' since extra constraints (e.g. pentad constraints) are not taken into account.

# Tetrad Demo and Hands-on

- 1) Create two DAGs with the same variables – each with one edge flipped, and attach a SEM PM to each new graph (copy and paste by selecting nodes, Ctl-C to copy, and then Ctl-V to paste)
- 2) Estimate each new model on the data produced by original graph
- 3) Check p-values of:
  - a) Edge coefficients
  - b) Model fit
- 4) Save session as:  
“session2”



# Charitable Giving

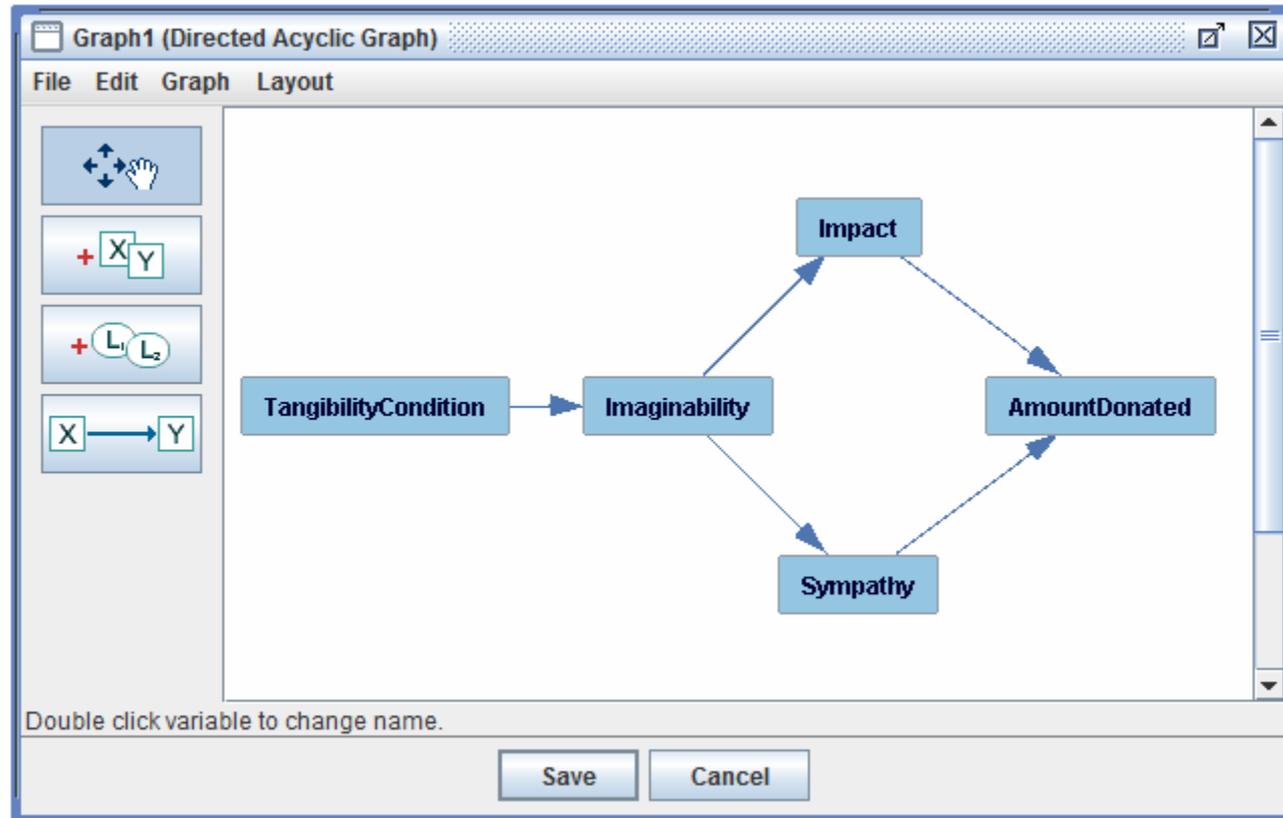
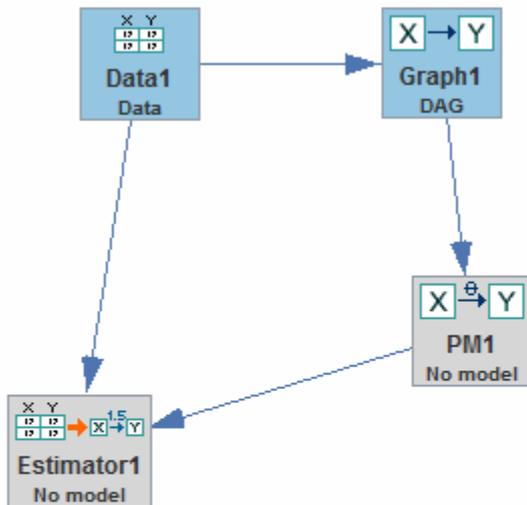
*What influences giving? Sympathy? Impact?*

*"The Donor is in the Details", Organizational Behavior and Human Decision Processes, Issue 1, 15-23, with G. Loewenstein, R. Scheines.*

N = 94

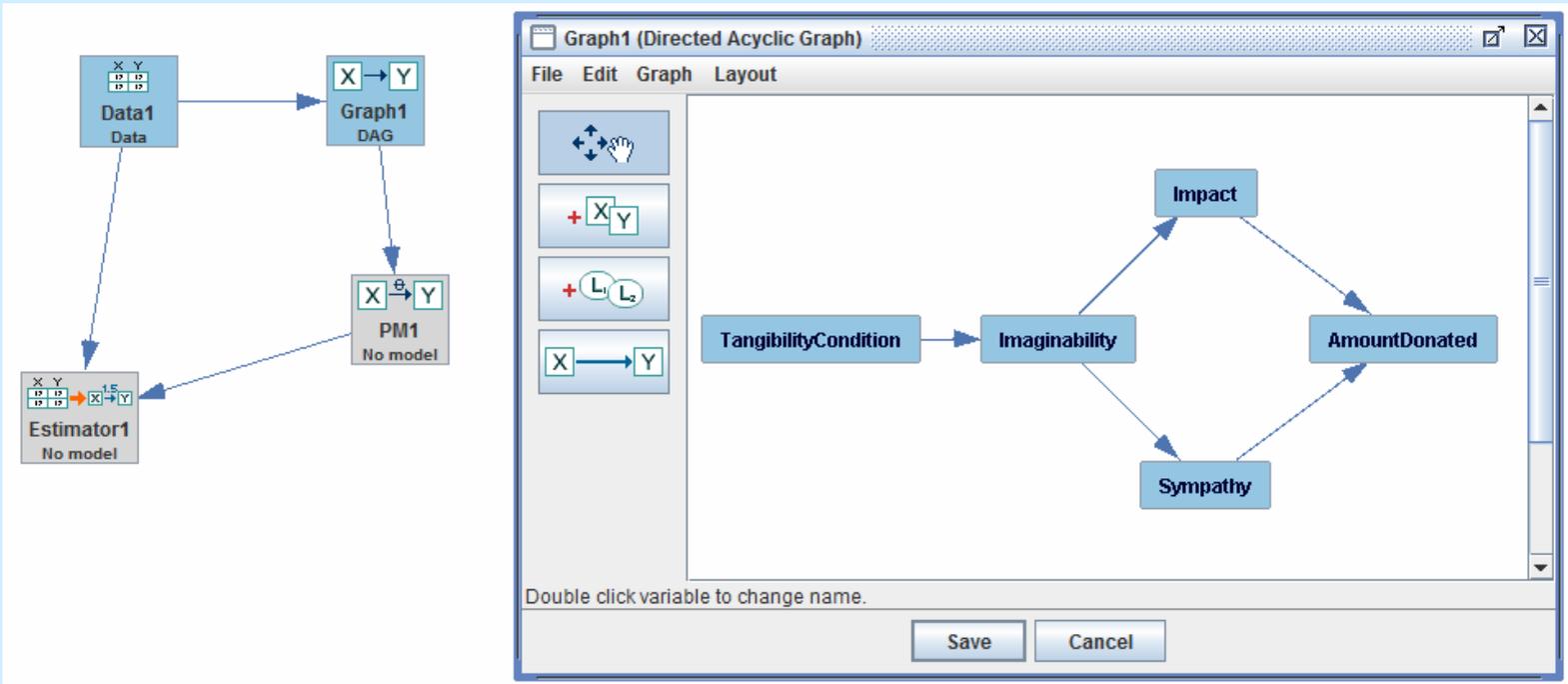
TangibilityCondition	[1,0]	Randomly assigned experimental condition
Imaginability	[1..7]	How concrete scenario I
Sympathy	[1..7]	How much sympathy for target
Impact	[1..7]	How much impact will my donation have
AmountDonated	[0..5]	How much actually donated

# Theoretical Hypothesis



# Tetrad Demo and Hands-on

- 1) Load charity.txt (tabular – not covariance data)
- 2) Build graph of theoretical hypothesis
- 3) Build SEM PM from graph
- 4) Estimate PM, check results

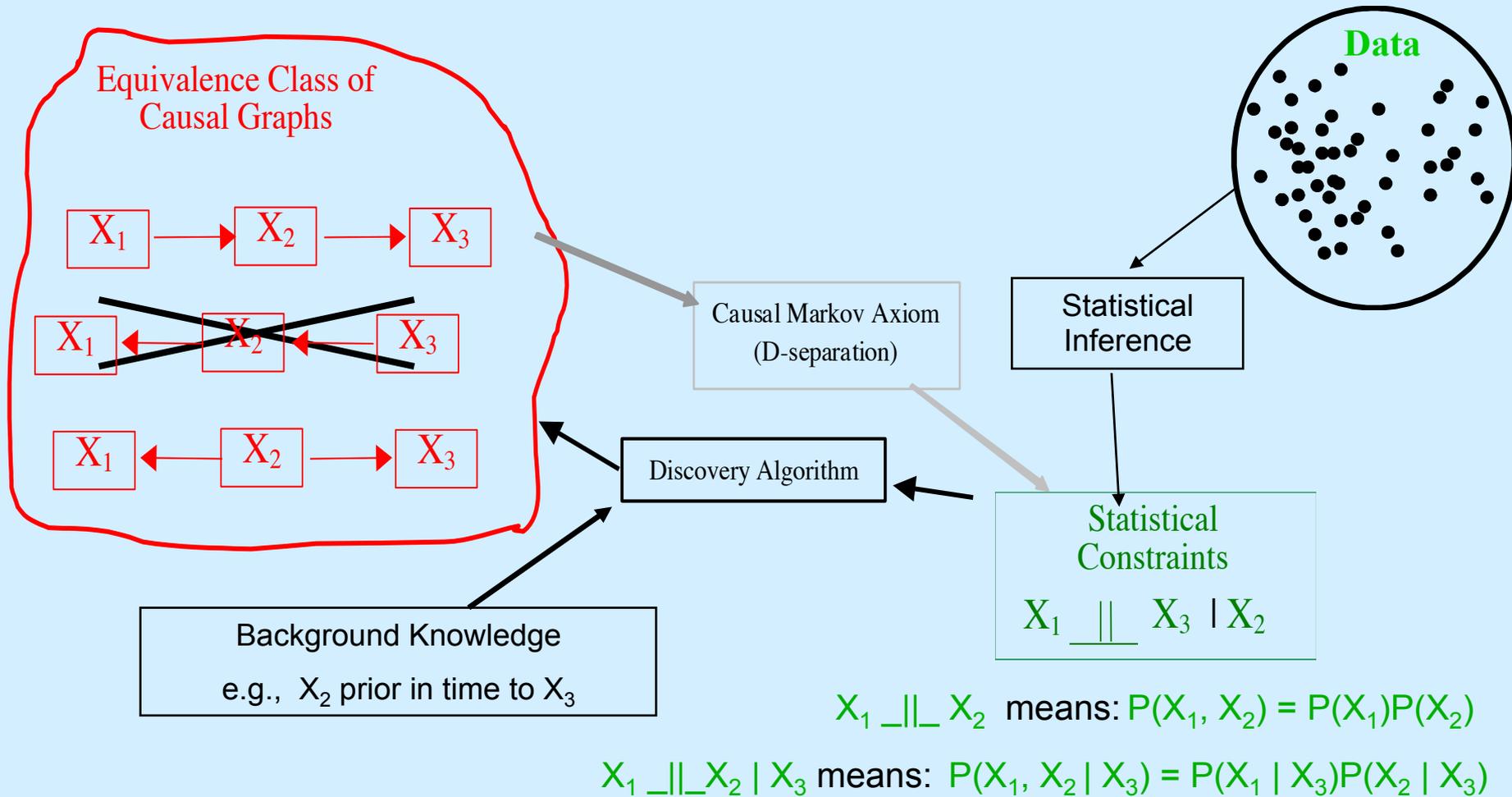


**10 Minute  
Break**

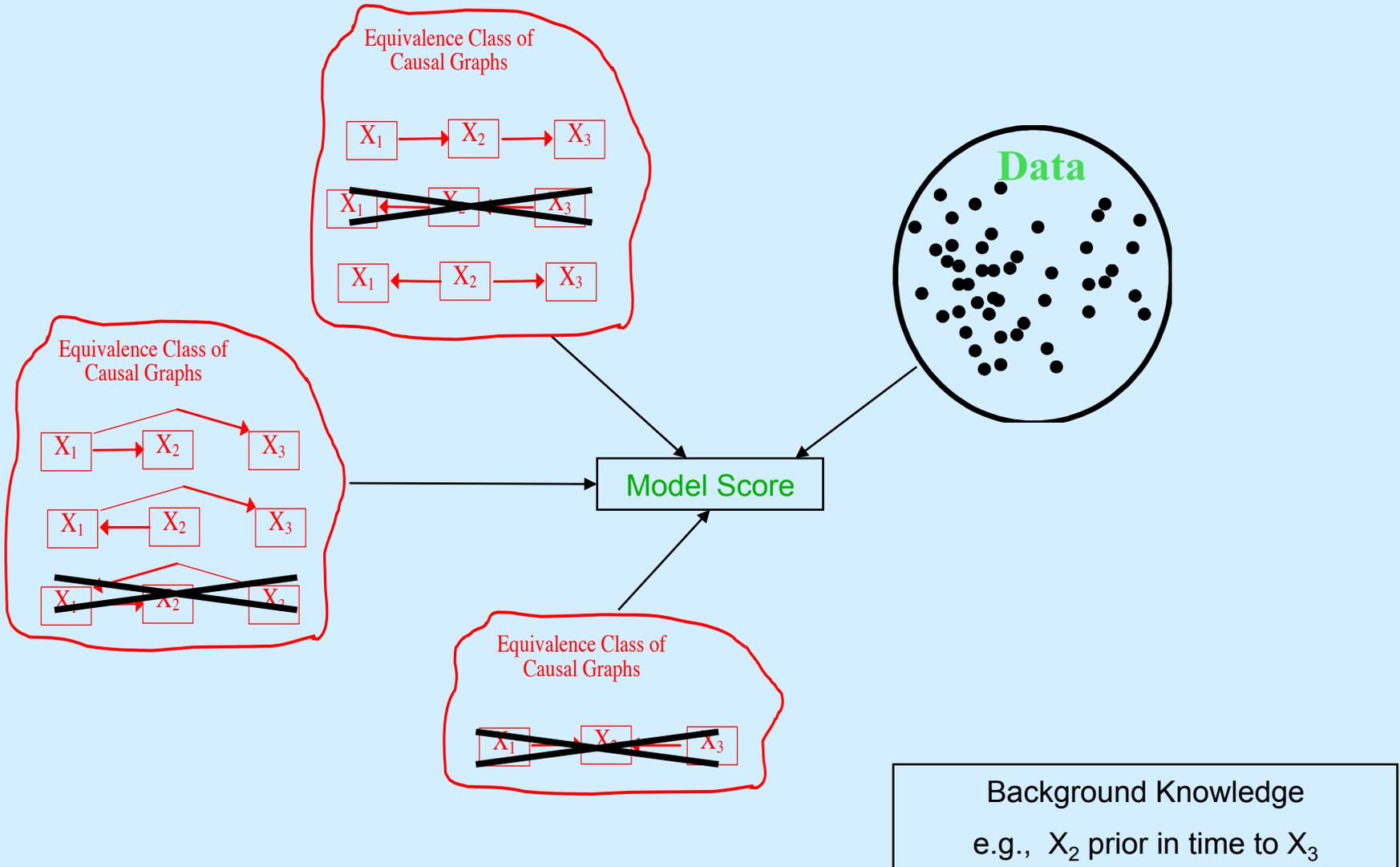
# Outline

- 1) Motivation
- 2) Representing/Modeling Causal Systems
- 3) Estimation and Model fit
- 4) Model Search
  - 1) Bridge Principles (**Causal Graphs**  $\Leftrightarrow$  **Probability Constraints**):
    - a) Markov assumption
    - b) Faithfulness assumption
    - c) D-separation
  - 2) Equivalence classes
  - 3) Search

# Constraint Based Search



# Score Based Search

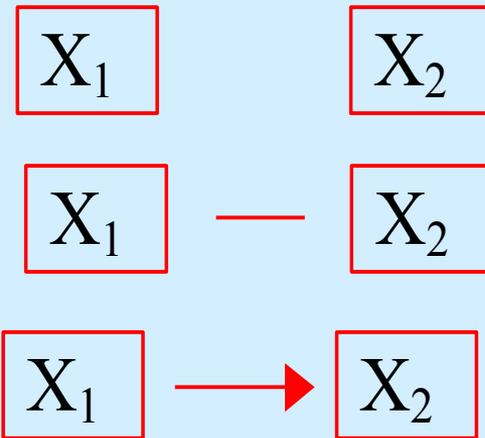


# Independence Equivalence Classes: Patterns & PAGs

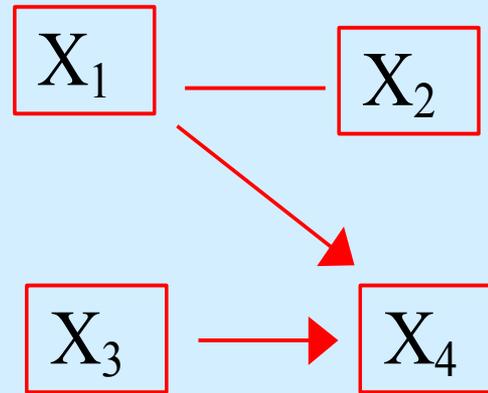
- Patterns (Verma and Pearl, 1990): graphical representation of d-separation equivalence among models with no latent common causes
- PAGs: (Richardson 1994) graphical representation of a d-separation equivalence class that includes models with *latent common causes* and *sample selection bias* that are d-separation equivalent over a set of measured variables  $\mathbf{X}$

# Patterns

Possible Edges



Example



# Patterns: What the Edges Mean

$X_1$

$X_2$

$X_1$  and  $X_2$  are not **adjacent** in any member of the equivalence class

---

$X_1$



$X_2$

$X_1 \rightarrow X_2$  ( $X_1$  is a **cause** of  $X_2$ )  
in every member of the  
equivalence class.

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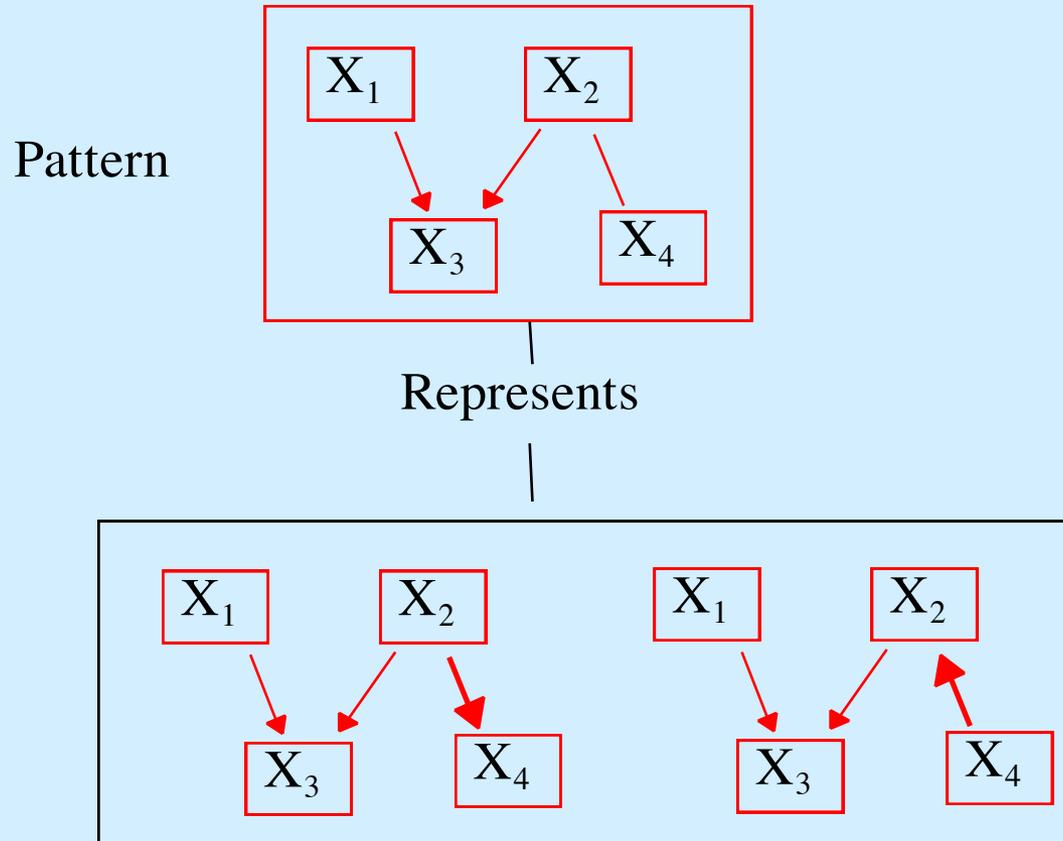
$X_1$



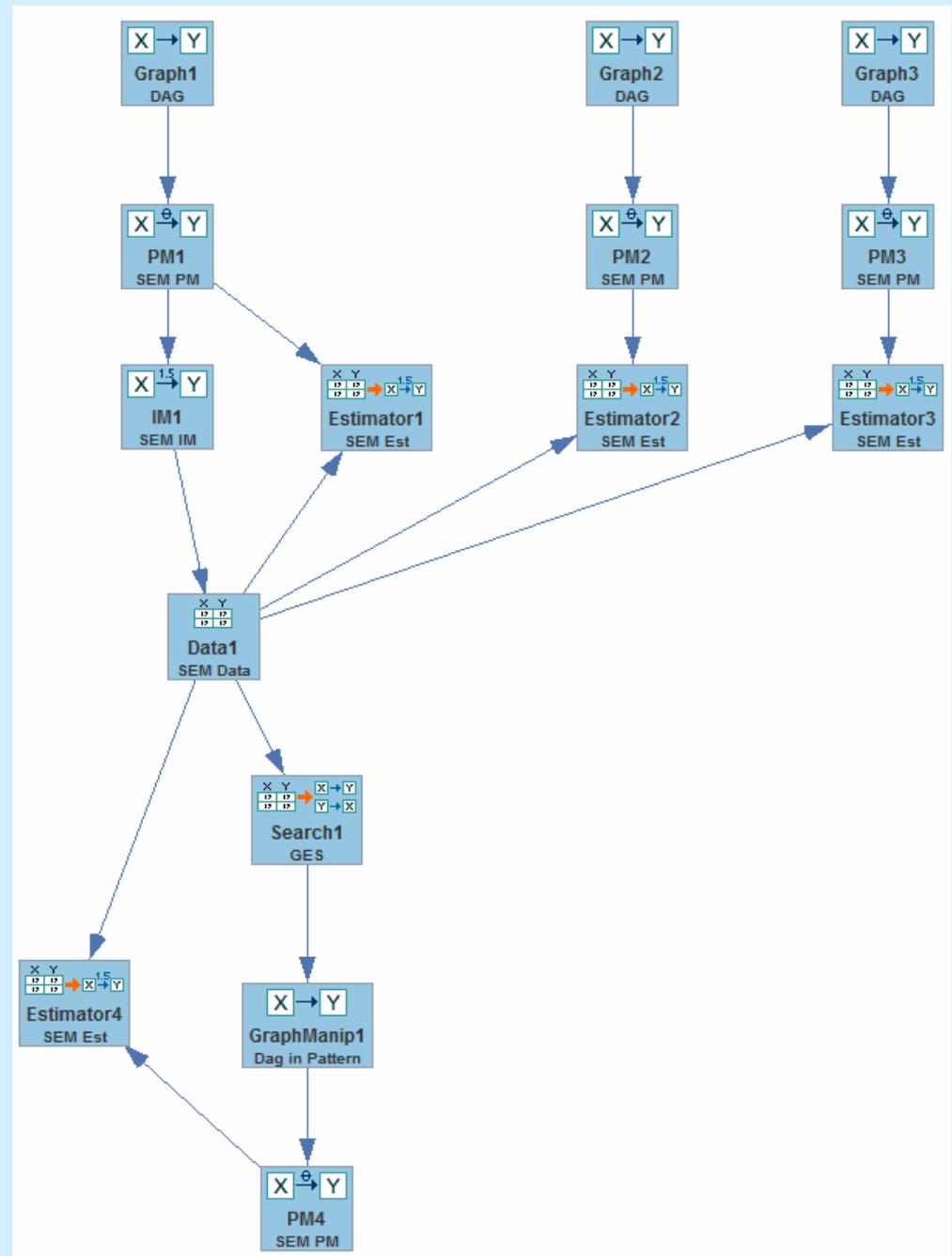
$X_2$

$X_1 \rightarrow X_2$  in some members of the  
equivalence class, and  $X_2 \rightarrow X_1$  in  
others.

# Patterns

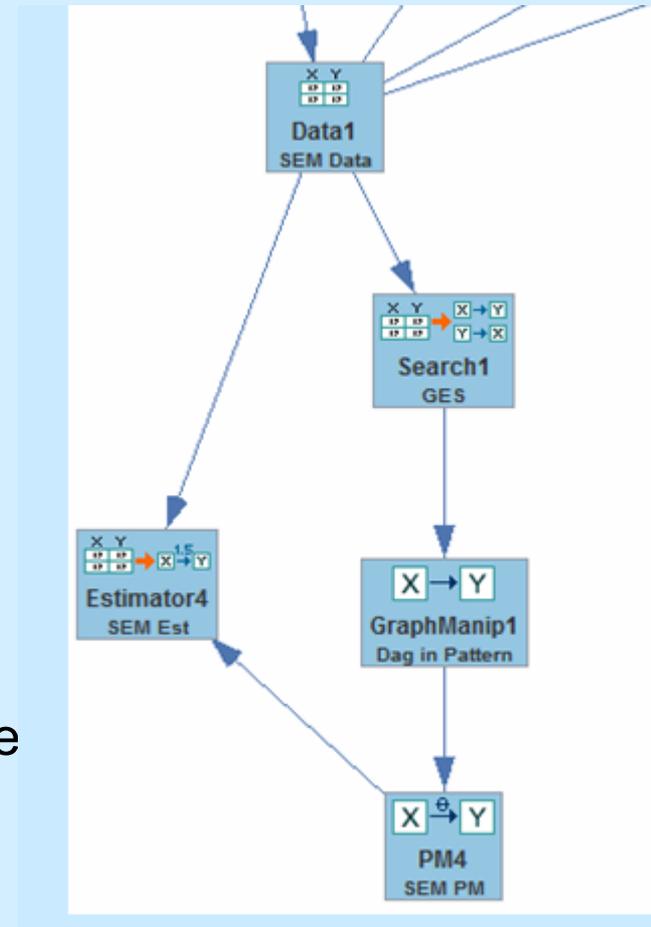


# Tetrad Demo and Hands On



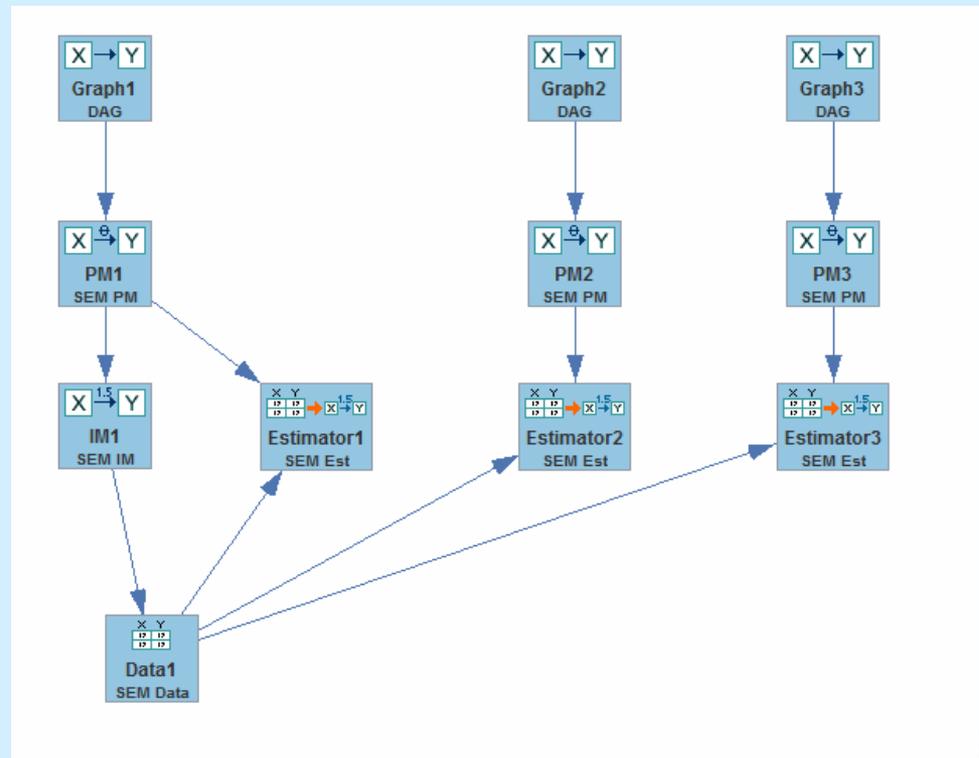
# Tetrad Demo and Hands-on

- 1) Go to “session2”
- 2) Add Search node (from Data1)
  - Choose and execute one of the “Pattern searches”
- 3) Add a “Graph Manipulation” node to search result: “choose Dag in Pattern”
- 4) Add a PM to GraphManip
- 5) Estimate the PM on the data
- 6) Compare model-fit to model fit for true mode



# Graphical Characterization of Model Equivalence

Why do some changes to the true model result in an equivalent model, but some do not?



# d-separation/Independence Equivalence

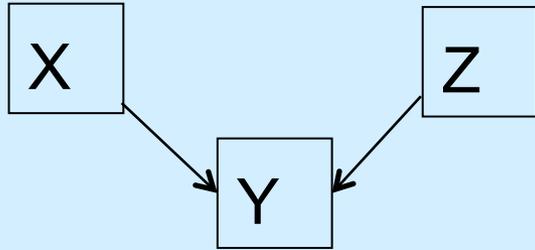
D-separation Equivalence Theorem (Verma and Pearl, 1988)

Two acyclic graphs over the same set of variables are d-separation equivalent iff they have:

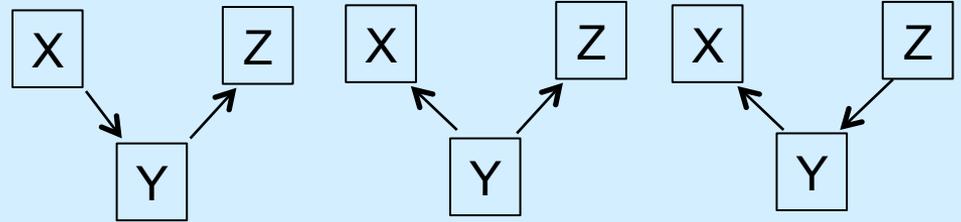
- the same adjacencies
- the same unshielded colliders

# Colliders

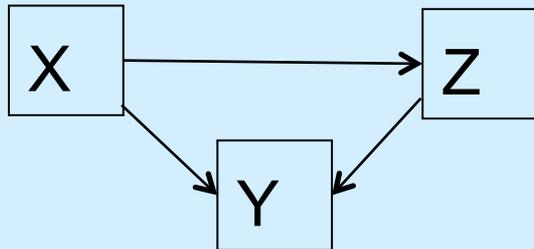
*Y: Collider*



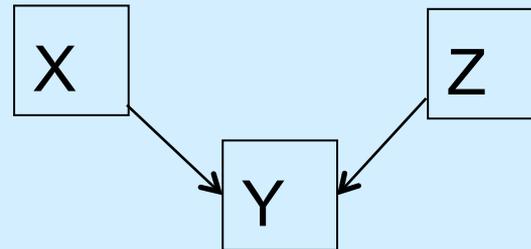
*Y: Non-Collider*



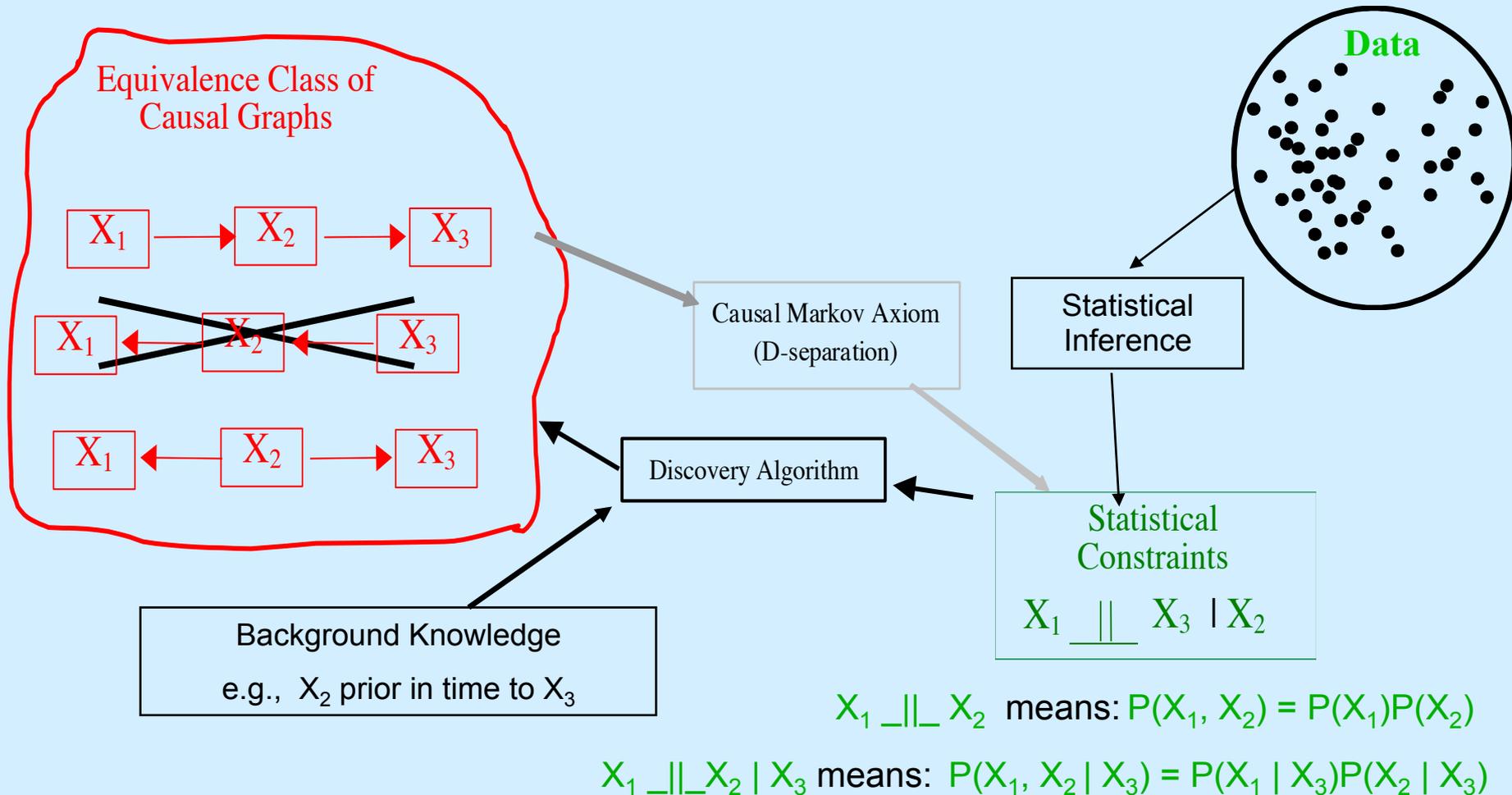
Shielded



Unshielded



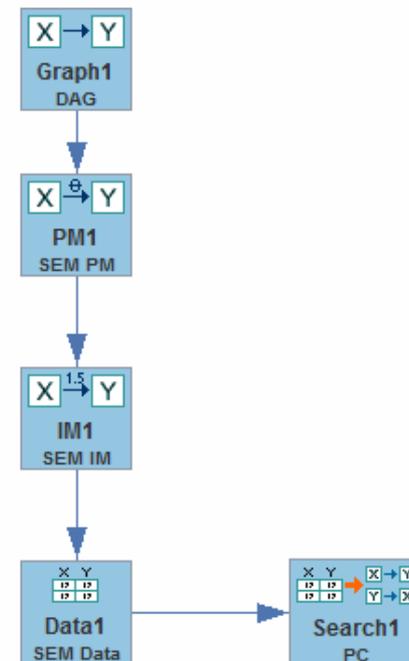
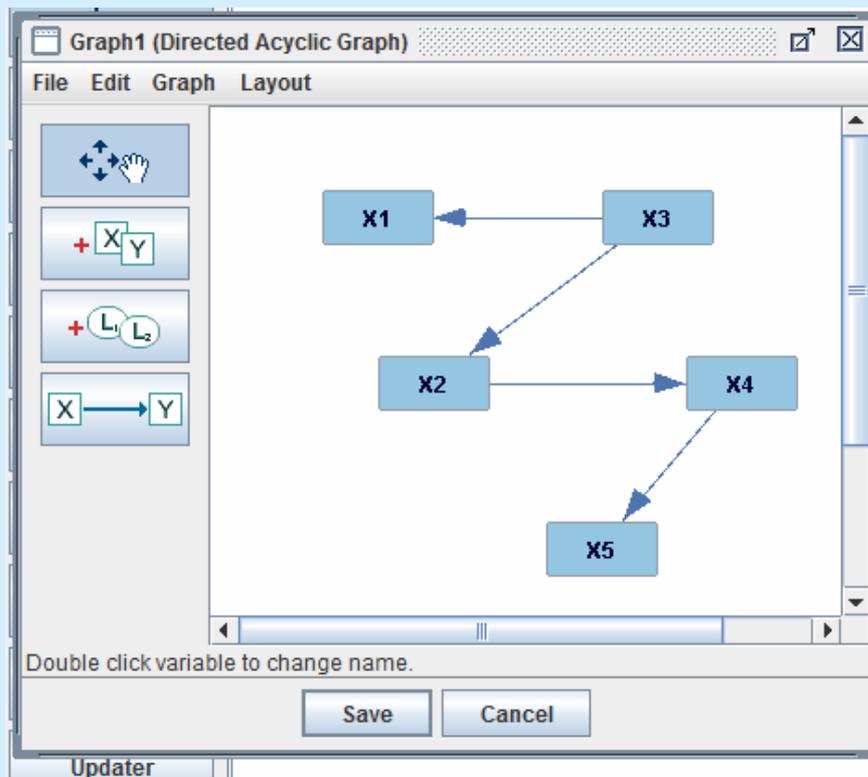
# Constraint Based Search



# Background Knowledge

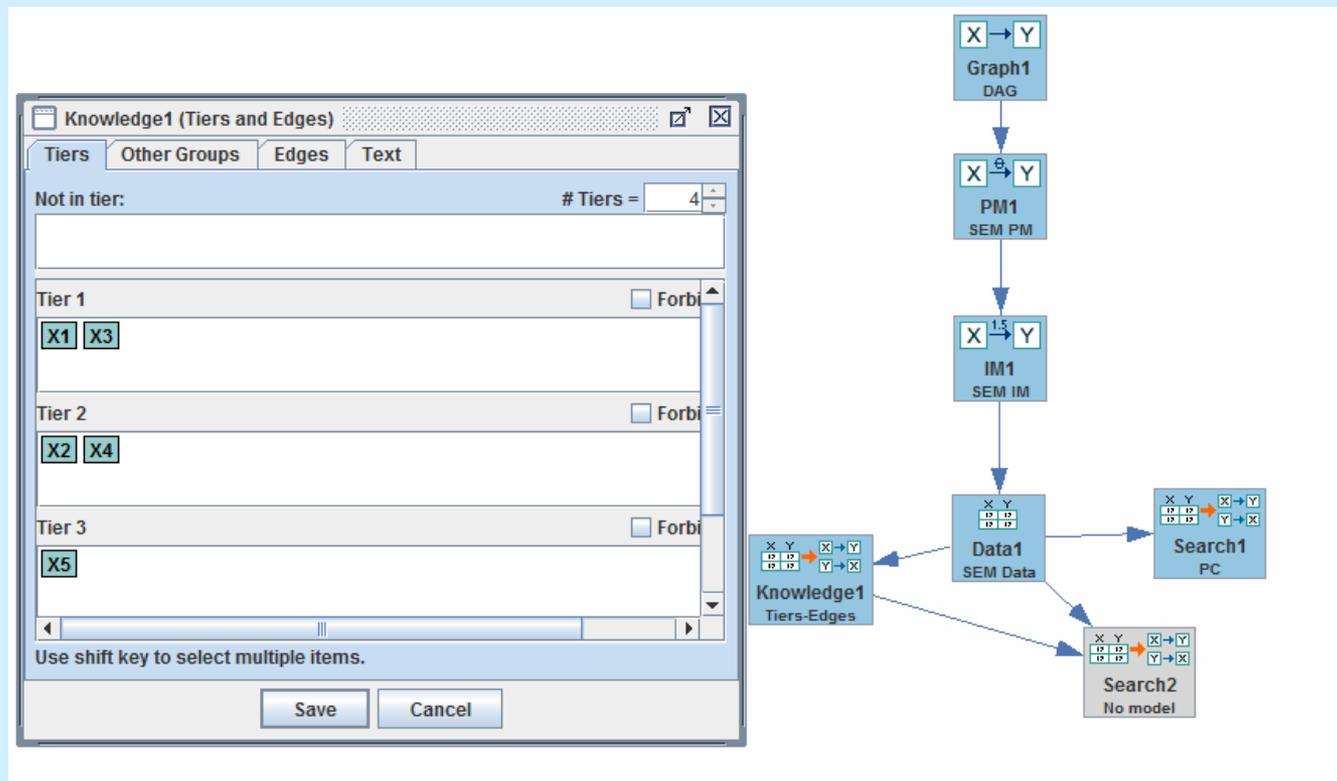
## Tetrad Demo and Hands-on

- 1) Create new session
- 2) Select "Search from Simulated Data" from Template menu
- 3) Build graph below, PM, IM, and generate sample data N=1,000.
- 4) Execute PC search,  $\alpha = .05$



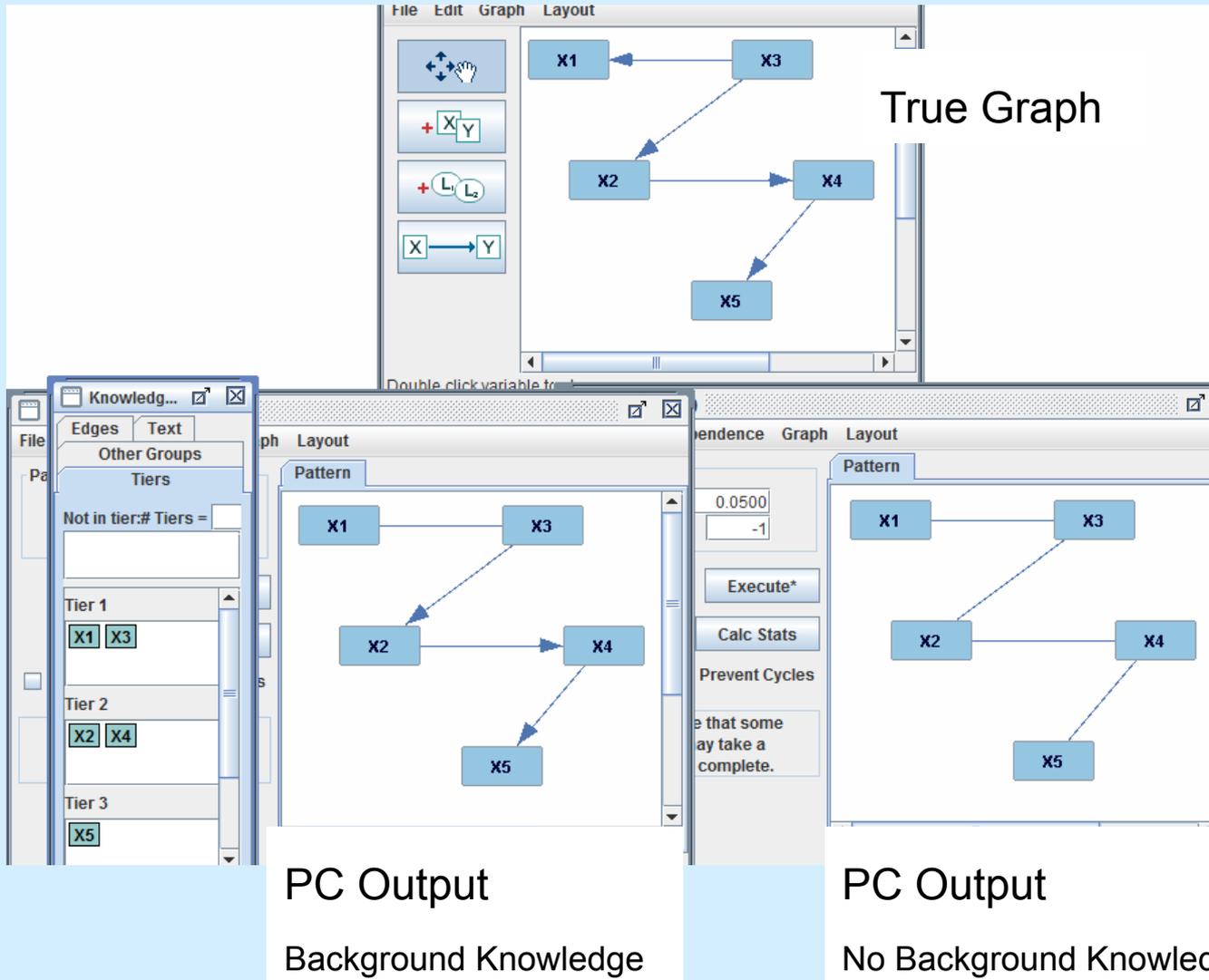
# Background Knowledge Tetrad Demo and Hands-on

- 1) Add “Knowledge” node – as below
- 2) Create “Tiers” as shown below.
- 3) Execute PC search again,  $\alpha = .05$
- 4) Compare results (Search2) to previous search (Search1)



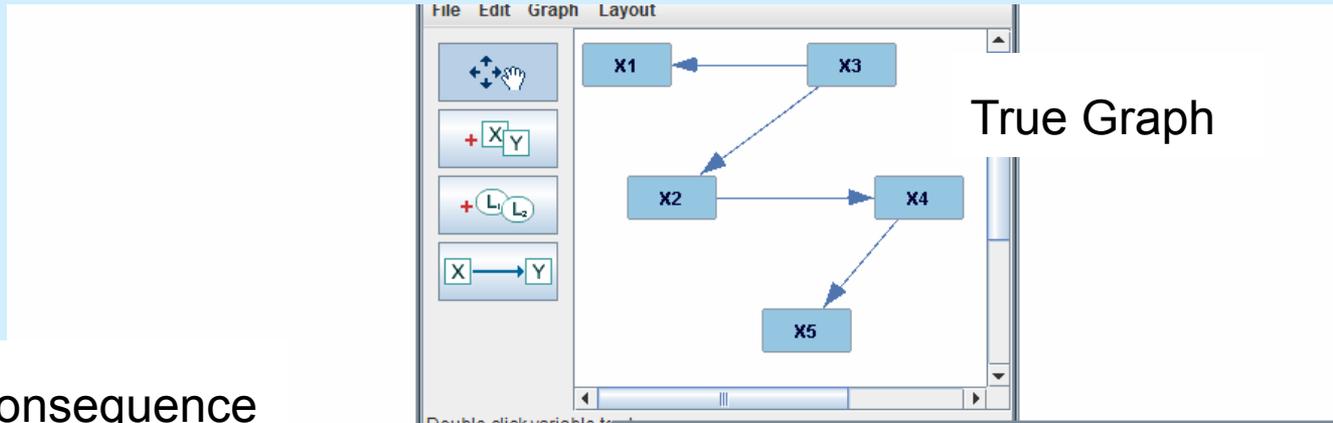
# Background Knowledge

## Direct and Indirect Consequences

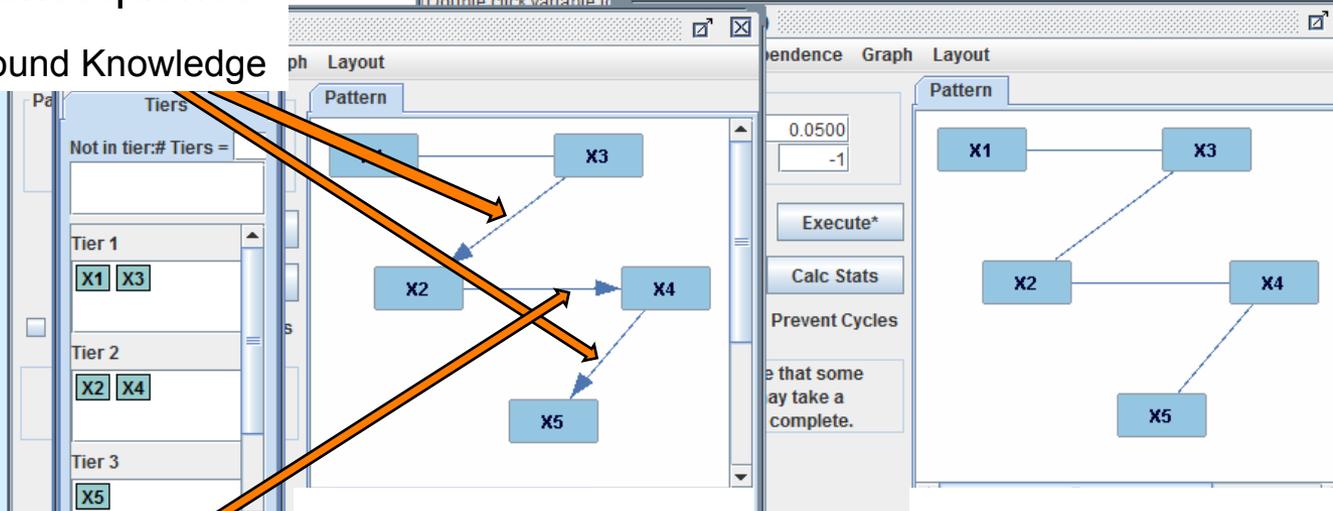


# Background Knowledge

## Direct and Indirect Consequences



Direct Consequence  
Of Background Knowledge



Indirect Consequence  
Of Background Knowledge

PC Output  
Background Knowledge

PC Output  
No Background Knowledge

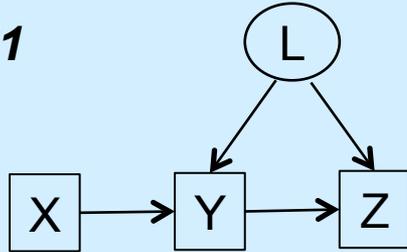
# Independence Equivalence Classes: Patterns & PAGs

- Patterns (Verma and Pearl, 1990): graphical representation of d-separation equivalence among models with no latent common causes

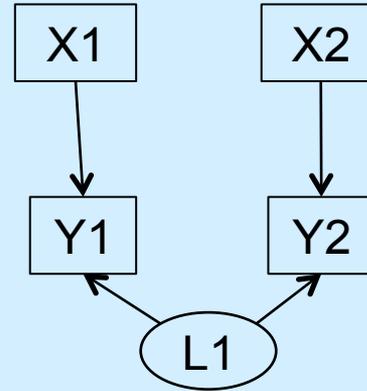
- PAGs: (Richardson 1994) graphical representation of a d-separation equivalence class that includes models with *latent common causes* and *sample selection bias* that are d-separation equivalent over a set of measured variables  $\mathbf{X}$

# Interesting Cases

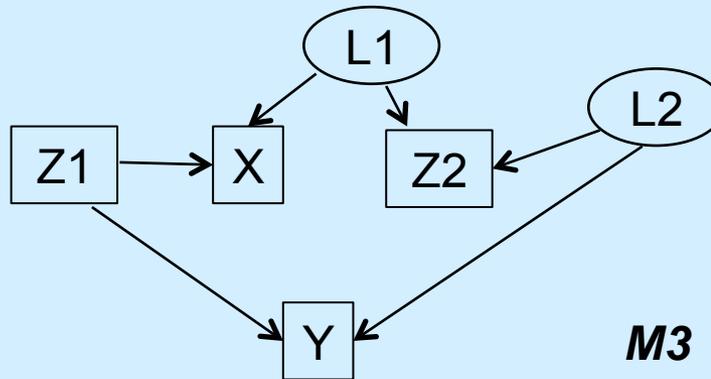
**M1**



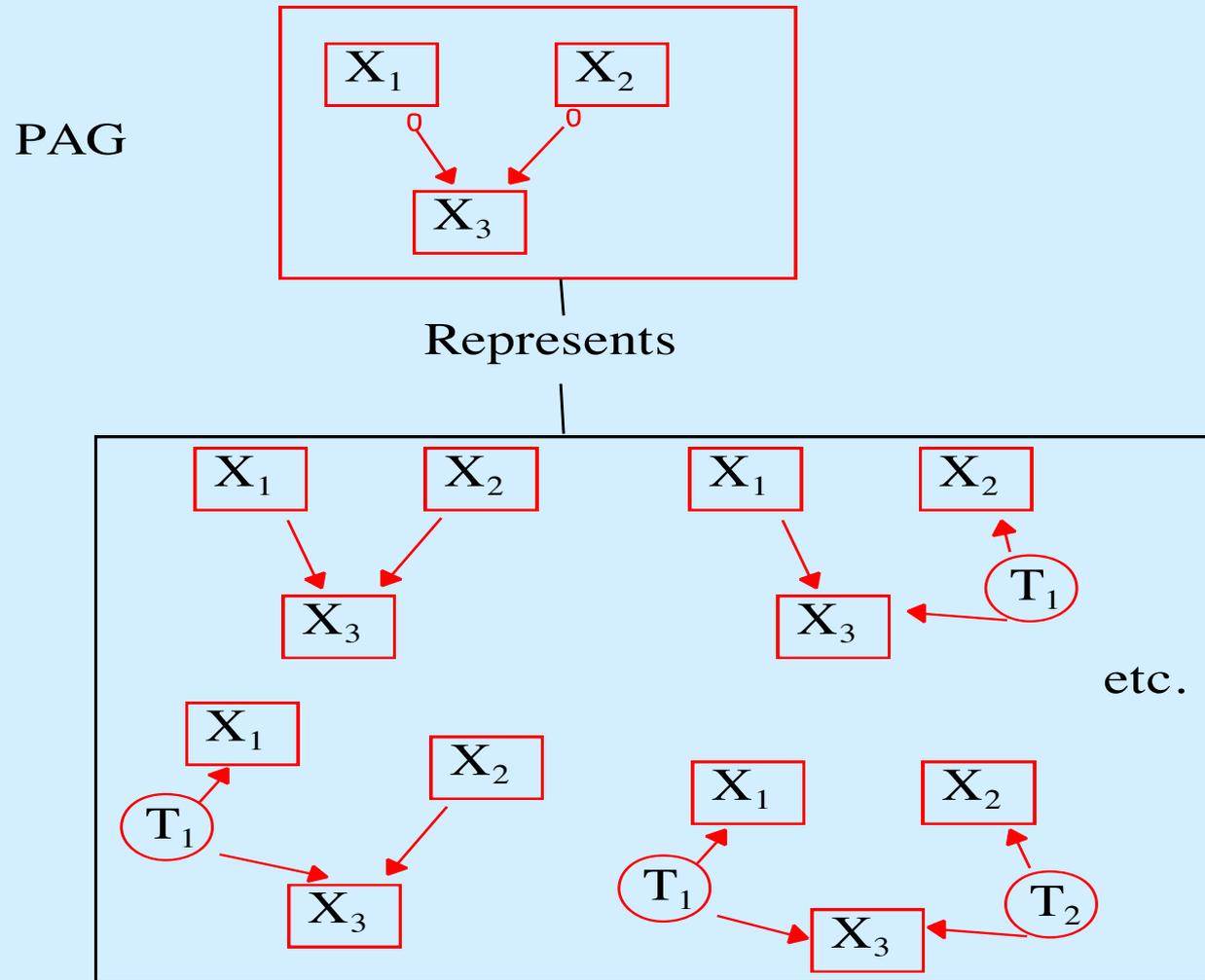
**M2**



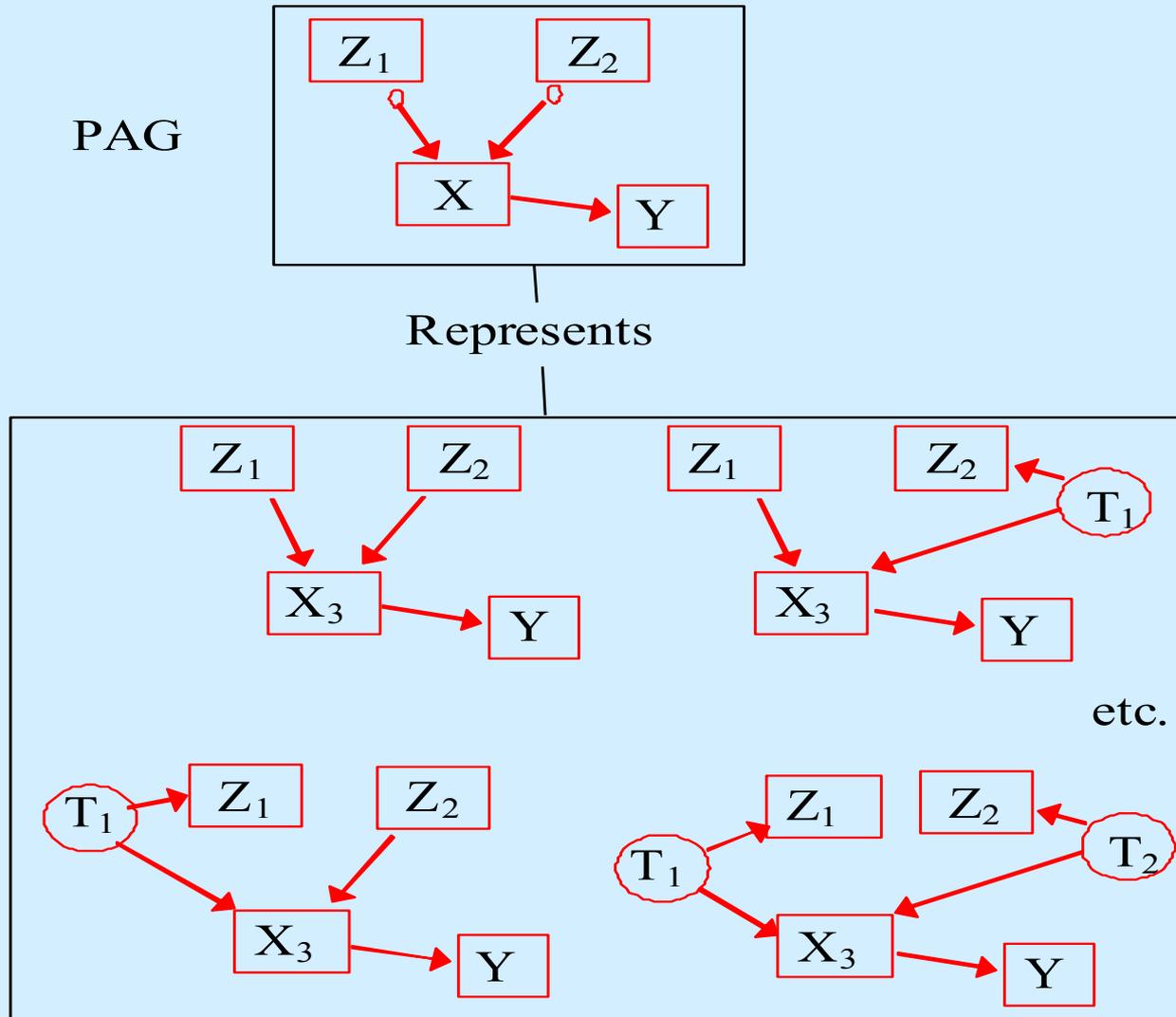
**M3**



# PAGs: Partial Ancestral Graphs

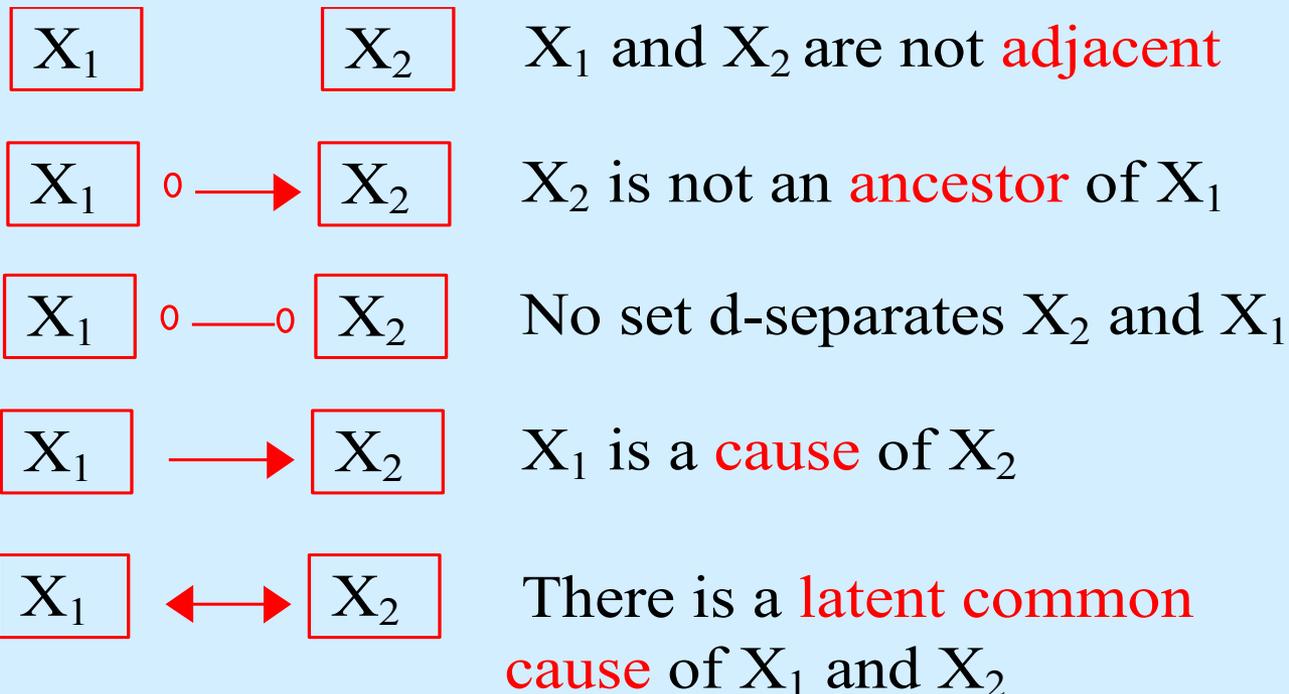


# PAGs: Partial Ancestral Graphs



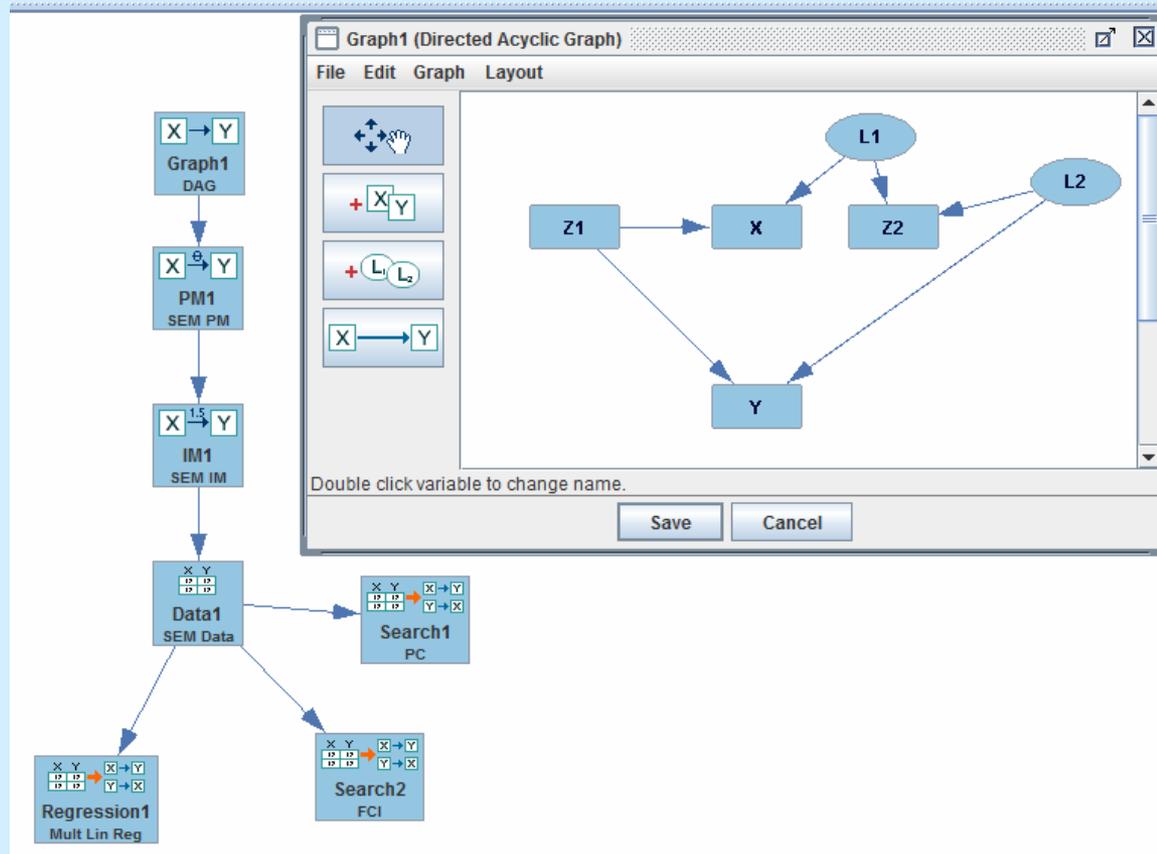
# PAGs: Partial Ancestral Graphs

What PAG edges mean.



# Tetrad Demo and Hands-on

- 1) Create new session
- 2) Select “Search from Simulated Data” from Template menu
- 3) Build graph below, SEM PM, IM, and generate sample data N=1,000.
- 4) Execute PC search,  $\alpha = .05$
- 5) Execute FCI search,  $\alpha = .05$
- 6) Estimate multiple regression, Y as response, Z1, X, Z2 as Predictors

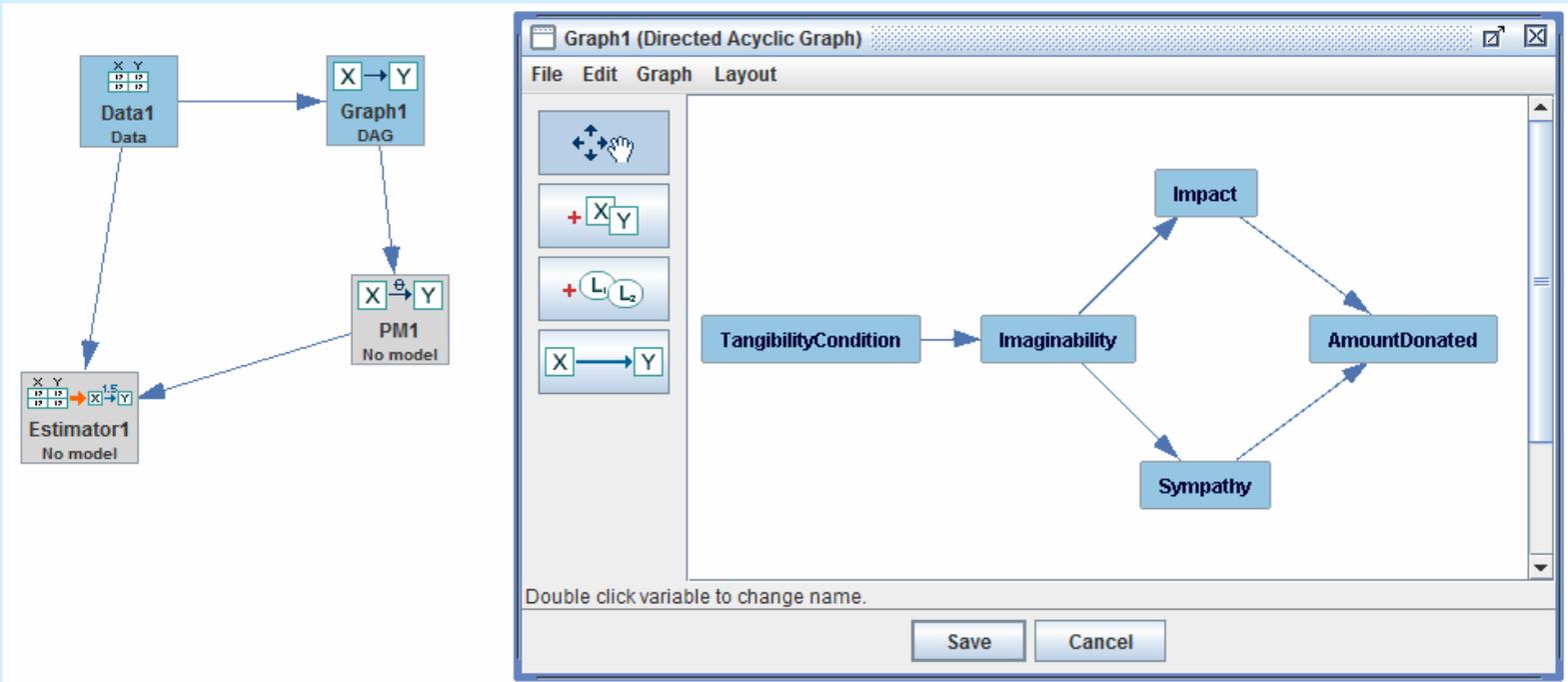


# Search Methods

- Constraint Based Searches
  - PC, FCI
  - Very fast – capable of handling >5,000 variables
  - Pointwise, but not uniformly consistent
- Scoring Searches
  - Scores: BIC, AIC, etc.
  - Search: Hill Climb, Genetic Alg., Simulated Annealing
  - Difficult to extend to latent variable models
  - Meek and Chickering Greedy Equivalence Class (GES)
  - Slower than constraint based searches – but now capable of 1,000 vars
  - Pointwise, but not uniformly consistent
- Latent Variable Psychometric Model Search
  - BPC, MIMbuild, etc.
- Linear non-Gaussian models (Lingam)
- Models with cycles
- And more!!!

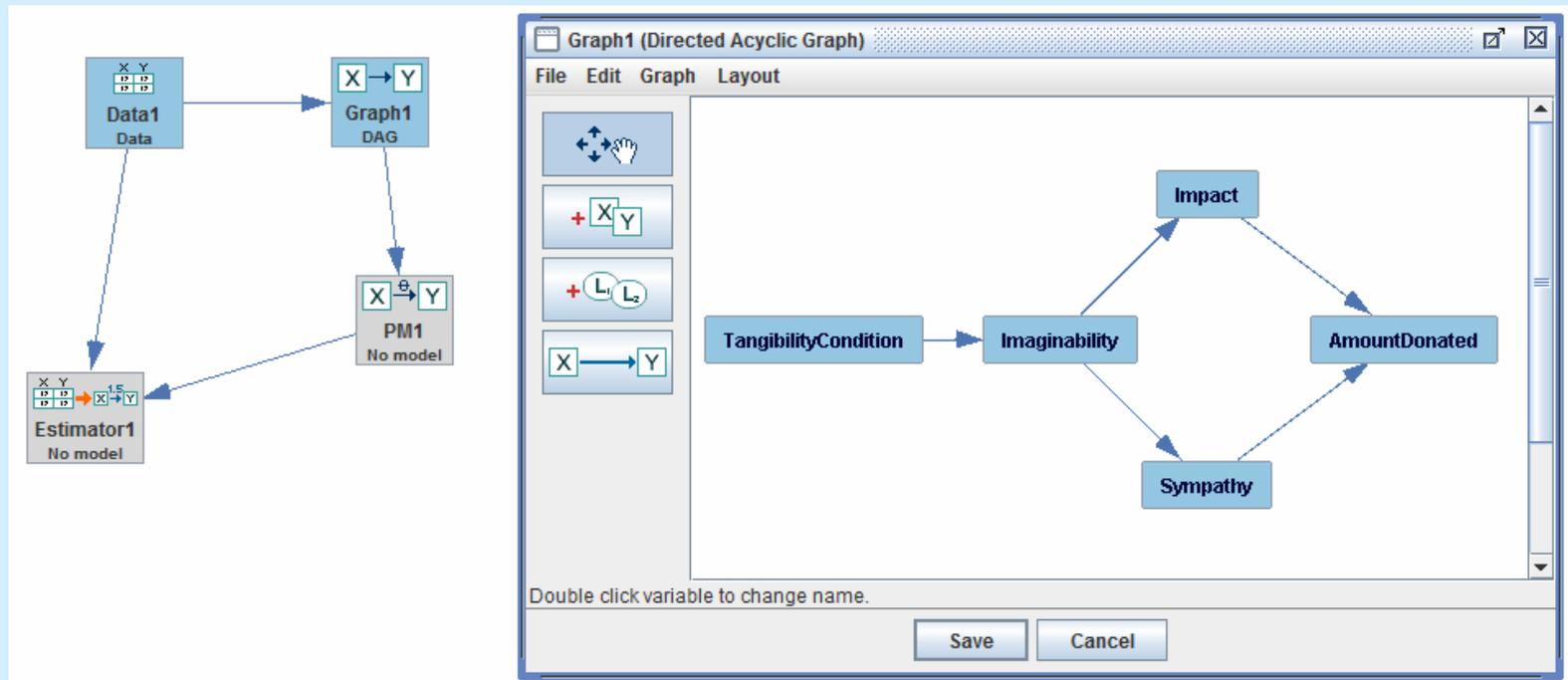
# Tetrad Demo and Hands-on

- 1) Load charity.txt (tabular – not covariance data)
- 2) Build graph of theoretical hypothesis
- 3) Build SEM PM from graph
- 4) Estimate PM, check results



# Tetrad Demo and Hands-on

- 1) Create background knowledge: Tangibility exogenous (uncaused)
- 2) Search for models
- 3) Estimate one model from the output of search
- 4) Check model fit, check parameter estimates, esp. their sign



**Thank You!**

# Additional Slides

# Constraint-based Search

- 1) Adjacency
- 2) Orientation

# Constraint-based Search: Adjacency

1.  $X$  and  $Y$  are adjacent if they are dependent conditional on all subsets that don't include them
2.  $X$  and  $Y$  are not adjacent if they are independent conditional on any subset that doesn't include them

# Search: Orientation

## *Patterns*

*Y Unshielded*

X ——— Y ——— Z

X ~~\_||\_~~ Z | Y

X ~~\_||\_~~ Z | Y

**Collider**

**Non-Collider**

X → Y ← Z

X ——— Y ——— Z

X ←—— Y ←—— Z

X ←—— Y →—— Z

X →—— Y →—— Z

# Search: Orientation

*PAGs*

*Y Unshielded*



~~X~~ \_||\_ Z | Y

X \_||\_ Z | Y

**Collider**

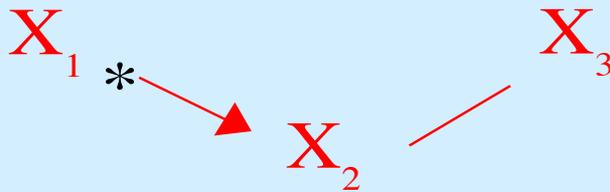
**Non-Collider**



# Search: Orientation

*Away from Collider*

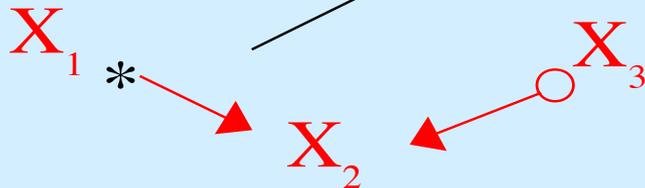
Test Conditions



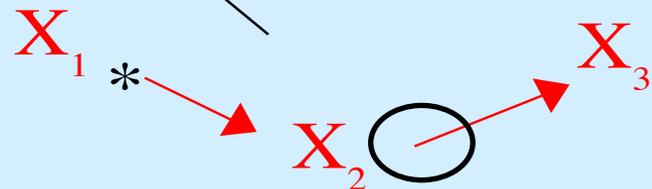
- 1)  $X_1 - X_2$  adjacent, and *into*  $X_2$ .
- 2)  $X_2 - X_3$  adjacent
- 3)  $X_1 - X_3$  not adjacent

Test  $X_1 \perp\!\!\!\perp X_3 \mid X_2$

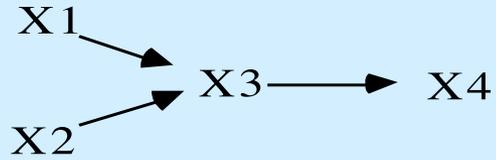
No



Yes



### Causal Graph



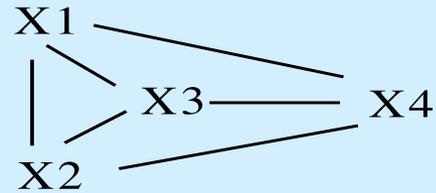
### Independencies

$$X1 \perp\!\!\!\perp X2$$

$$X1 \perp\!\!\!\perp X4 \mid \{X3\}$$

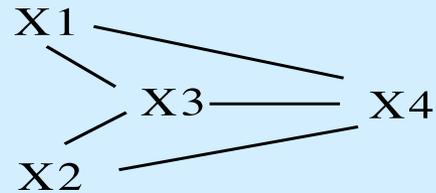
$$X2 \perp\!\!\!\perp X4 \mid \{X3\}$$

Begin with:



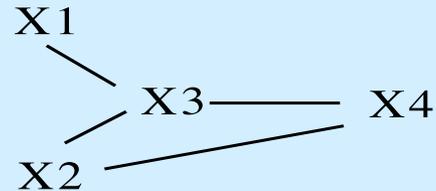
From

$$X1 \perp\!\!\!\perp X2$$



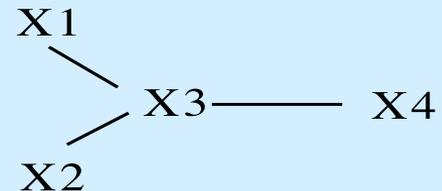
From

$$X1 \perp\!\!\!\perp X4 \mid \{X3\}$$



From

$$X2 \perp\!\!\!\perp X4 \mid \{X3\}$$

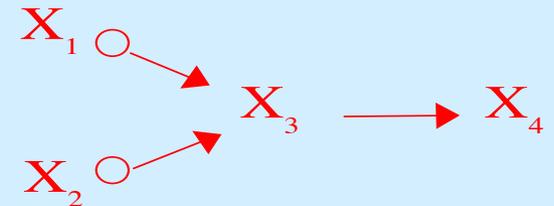
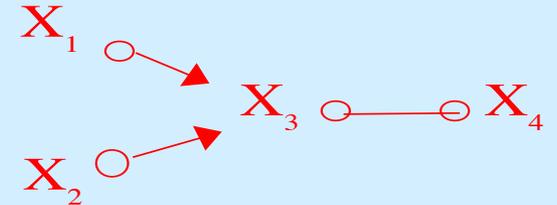
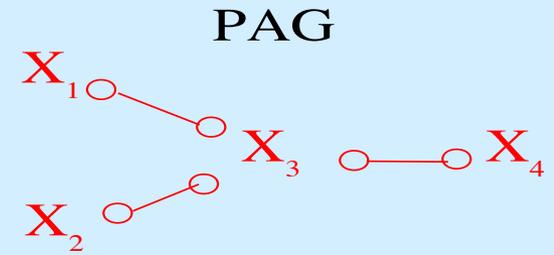
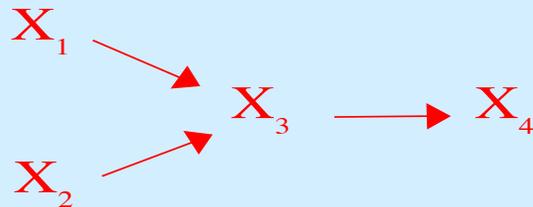
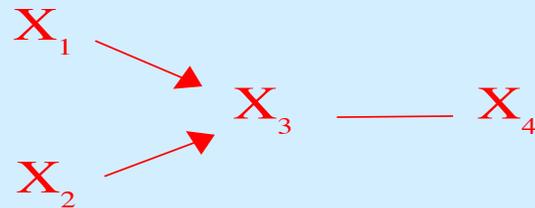
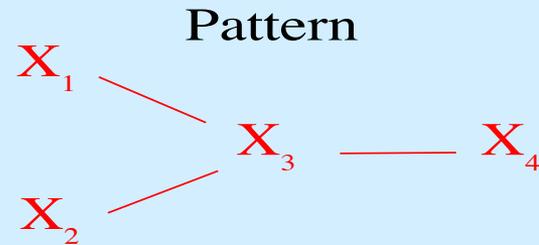


# Search: Orientation

After Orientation Phase

$X_1 \parallel X_2$   
—

$X_1 \parallel X_4 \mid X_3$   
 $X_2 \parallel X_4 \mid X_3$   
—



## Bridge Principles:

*Acyclic Causal Graph* over  $V \Rightarrow$  Constraints on  $P(V)$

Weak Causal Markov Assumption

$V_1, V_2$  **causally disconnected**  $\Rightarrow V_1 \perp\!\!\!\perp V_2$

$V_1, V_2$  **causally disconnected**  $\Leftrightarrow$

- i.  $V_1$  **not a cause of**  $V_2$ , and
- ii.  $V_2$  **not a cause of**  $V_1$ , and
- iii. No **common cause**  $Z$  of  $V_1$  and  $V_2$

$$V_1 \perp\!\!\!\perp V_2 \Leftrightarrow P(V_1, V_2) = P(V_1)P(V_2)$$

## Bridge Principles:

**Acyclic Causal Graph** over  $V \Rightarrow$  Constraints on  $P(V)$

Weak Causal Markov Assumption

$V_1, V_2$  **causally disconnected**  $\Rightarrow V_1 \perp\!\!\!\perp V_2$



Determinism

(Structural Equations)



Causal Markov Axiom

If  $G$  is a causal graph, and  $P$  a probability distribution over the variables in  $G$ , then in  $\langle G, P \rangle$  satisfy the Markov Axiom iff:

every variable  $V$  is **independent** of its **non-effects**,  
**conditional** on its **immediate causes**.

# Bridge Principles:

Acyclic Causal Graph over  $V \Rightarrow$  Constraints on  $P(V)$

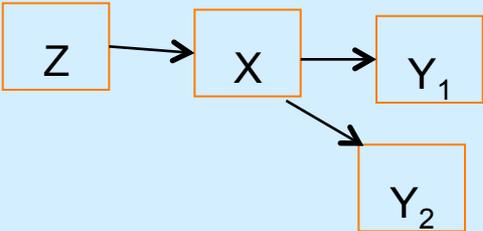
Causal Markov Axiom

Acyclicity

d-separation criterion

Causal Graph

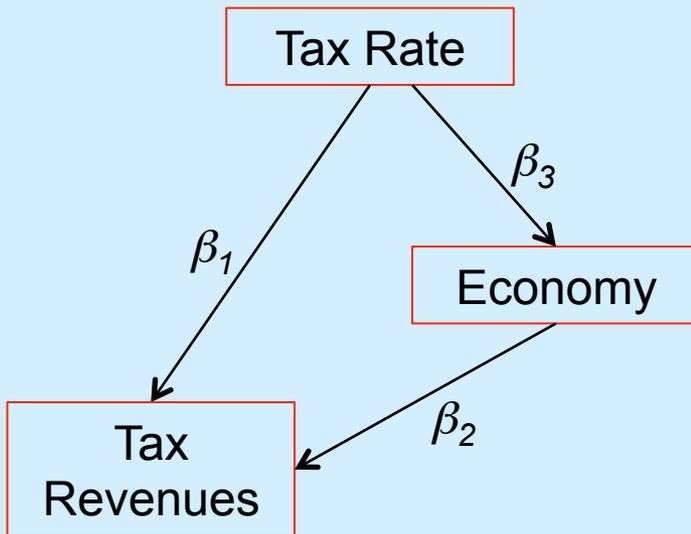
Independence Oracle



$Z \perp\!\!\!\perp Y_1 \mid X$	$Z \perp\!\!\!\perp Y_2 \mid X$
$Z \perp\!\!\!\perp Y_1 \mid X, Y_2$	$Z \perp\!\!\!\perp Y_2 \mid X, Y_1$
$Y_1 \perp\!\!\!\perp Y_2 \mid X$	$Y_1 \perp\!\!\!\perp Y_2 \mid X, Z$

# Faithfulness

Constraints on a **probability distribution P** generated by a **causal structure G** hold for all parameterizations of **G**.



$$Revenues := \beta_1 Rate + \beta_2 Economy + \varepsilon_{Rev}$$

$$Economy := \beta_3 Rate + \varepsilon_{Econ}$$

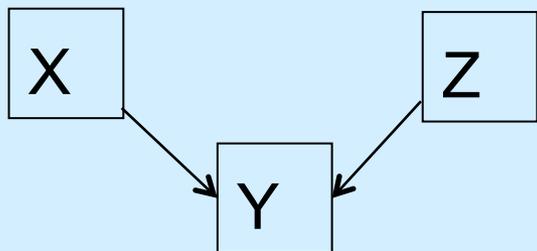
Faithfulness:

$$\beta_1 \neq -\beta_3\beta_2$$

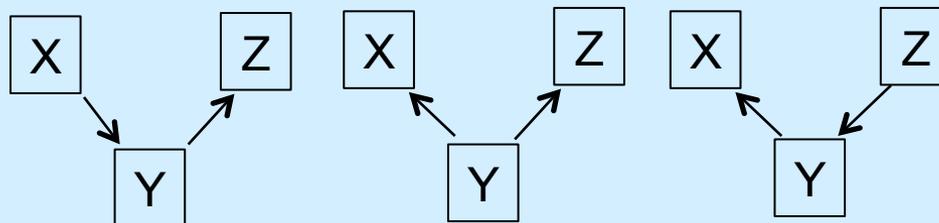
$$\beta_2 \neq -\beta_3\beta_1$$

# Colliders

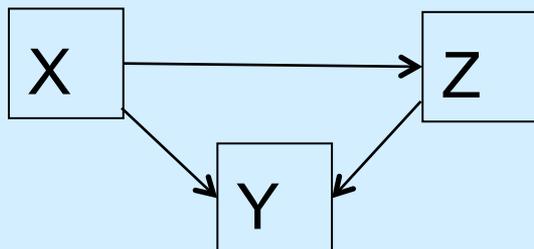
*Y: Collider*



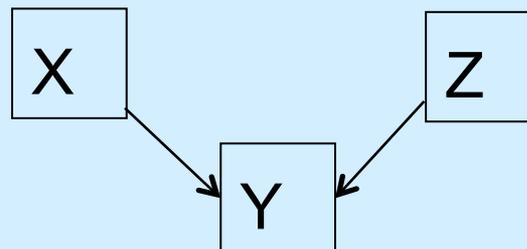
*Y: Non-Collider*



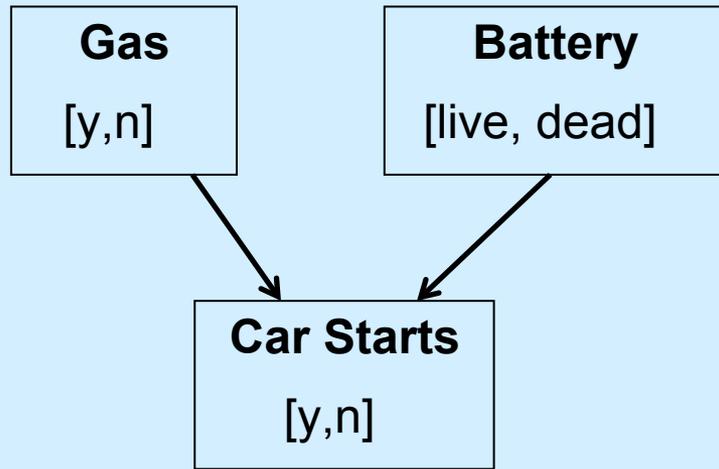
Shielded



Unshielded



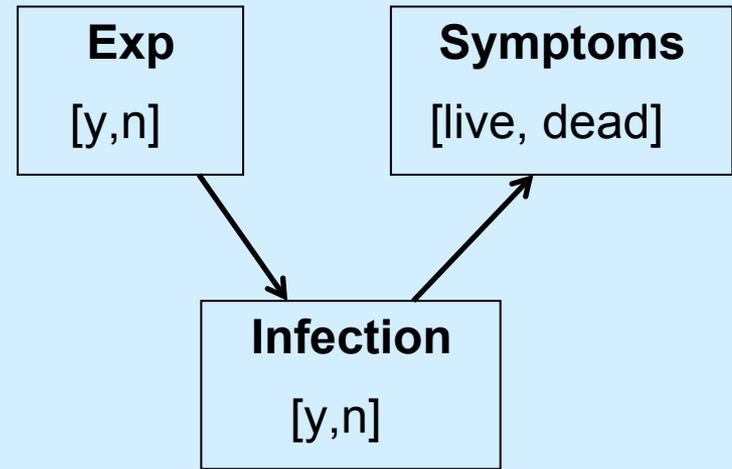
**Colliders** induce  
Association



Gas  $\perp\!\!\!\perp$  Battery

Gas  $\not\perp\!\!\!\perp$  Battery | Car starts = no

**Non-Colliders** screen-off  
Association



Exp  $\not\perp\!\!\!\perp$  Symptoms

Exp  $\perp\!\!\!\perp$  Symptoms | Infection

# D-separation

$X$  is *d-separated* from  $Y$  by  $Z$  in  $G$  iff

Every undirected path between  $X$  and  $Y$  in  $G$  is *inactive* relative to  $Z$

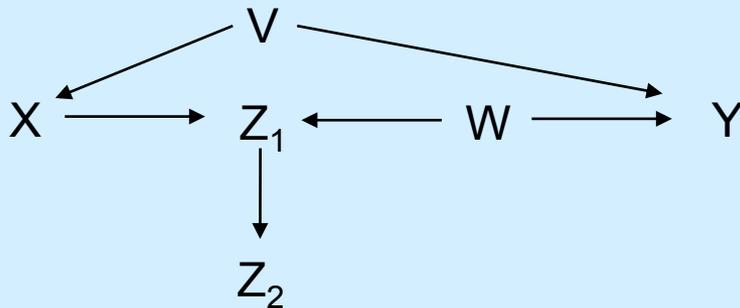
An undirected path is *inactive* relative to  $Z$  iff  
*any* node on the path is *inactive* relative to  $Z$

A node  $N$  (on a path) is *inactive* relative to  $Z$  iff

- a)  $N$  is a non-collider in  $Z$ , or
- b)  $N$  is a collider that is not in  $Z$ ,  
and has no descendant in  $Z$

A node  $N$  (on a path) is *active* relative to  $Z$  iff

- a)  $N$  is a non-collider not in  $Z$ , or
- b)  $N$  is a collider that is in  $Z$ ,  
or has a descendant in  $Z$



Undirected Paths between  $X$ ,  $Y$ :

1)  $X \text{ ---} Z_1 \text{ ---} W \text{ ---} Y$

2)  $X \text{ ---} V \text{ ---} Y$

# D-separation

$X$  is *d-separated* from  $Y$  by  $Z$  in  $G$  iff

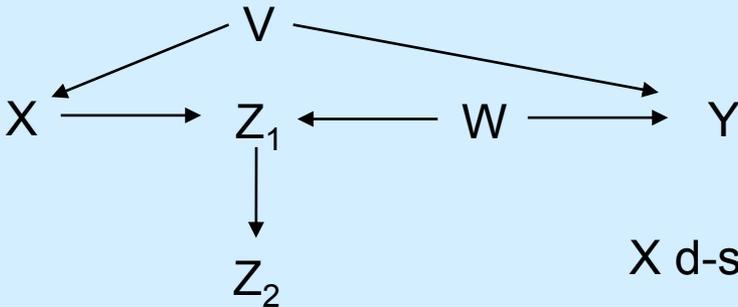
Every undirected path between  $X$  and  $Y$  in  $G$  is inactive relative to  $Z$

An undirected path is inactive relative to  $Z$  iff

any node on the path is inactive relative to  $Z$

A node  $N$  is inactive relative to  $Z$  iff

- a)  $N$  is a non-collider in  $Z$ , or
- b)  $N$  is a collider that is not in  $Z$ ,  
and has no descendant in  $Z$



Undirected Paths between  $X$ ,  $Y$ :

$$1) X \leftrightarrow Z_1 \leftarrow W \rightarrow Y$$

$$2) X \leftarrow V \rightarrow Y$$

$X$  d-sep  $Y$  relative to  $Z = \emptyset$  ?      *No*

$X$  d-sep  $Y$  relative to  $Z = \{V\}$  ?      *Yes*

$X$  d-sep  $Y$  relative to  $Z = \{V, Z_1\}$  ?      *No*

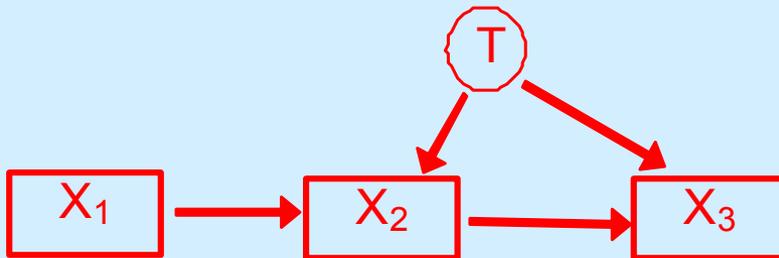
$X$  d-sep  $Y$  relative to  $Z = \{W, Z_2\}$  ?      *Yes*

# D-separation



$X_3$  and  $X_1$  d-sep by  $X_2$ ?

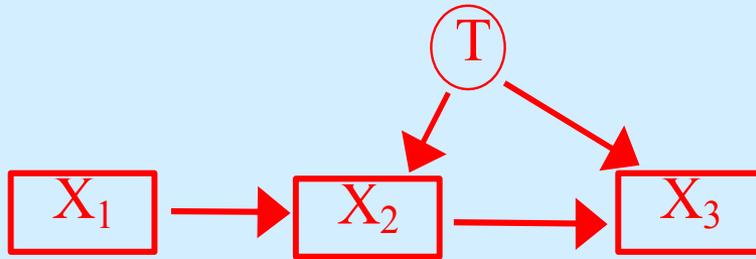
Yes:  $X_3 \perp\!\!\!\perp X_1 \mid X_2$



$X_3$  and  $X_1$  d-sep by  $X_2$ ?

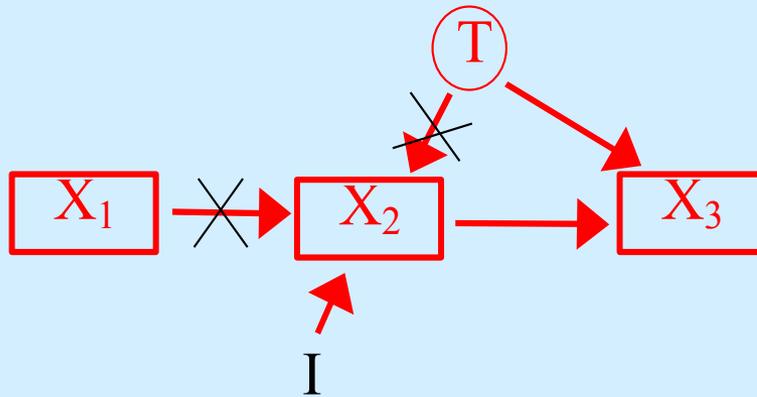
No:  $X_3 \not\perp\!\!\!\perp X_1 \mid X_2$

# Statistical Control $\neq$ Experimental Control



Statistically control for  $X_2$

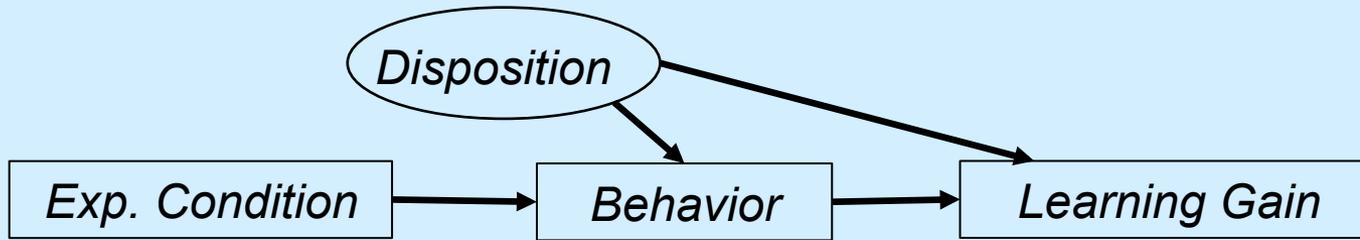
$$X_3 \perp\!\!\!\perp X_1 \mid X_2$$



Experimentally control for  $X_2$

$$X_3 \perp\!\!\!\perp X_1 \mid X_2(\text{set})$$

# Statistical Control $\neq$ Experimental Control



Exp. Cond  $\not\parallel$  Learning Gain

Exp  $\rightarrow$  Learning

Exp. Cond  $\parallel$  Learning Gain | Behavior, Disposition

Exp  $\rightarrow$  Learning is Mediated by Behavior

Exp. Cond  $\parallel$  Learning Gain | Behavior *set*

Exp  $\rightarrow$  Learning is Mediated by Behavior

Exp. Cond  $\not\parallel$  Learning Gain | Behavior *observed*

Exp  $\rightarrow$  Learning is *not* Mediated by Behavior

*or*

Unmeasured Confounder

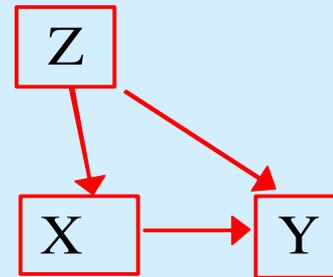
# Regression & Causal Inference

# Regression & Causal Inference

Typical (non-experimental) strategy:

1. Establish a prima facie case (X associated with Y)

But, omitted variable bias



2. So, identify and measure potential confounders **Z**:
  - a) prior to X,
  - b) associated with X,
  - c) associated with Y
3. Statistically adjust for **Z** (multiple regression)

# Regression & Causal Inference

Strategy threatened by measurement error – ignore this for now

Multiple regression is provably unreliable

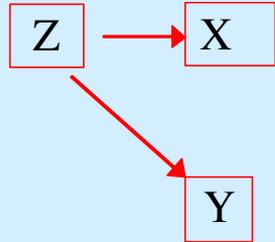
for causal inference unless:

- X prior to Y
- X, **Z**, and Y are causally sufficient (no confounding)

Regression  
Y: outcome  
X, Z, Explanatory

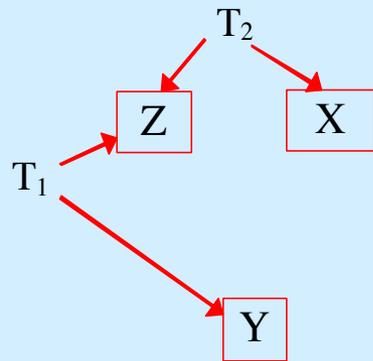
Alternative?

Truth



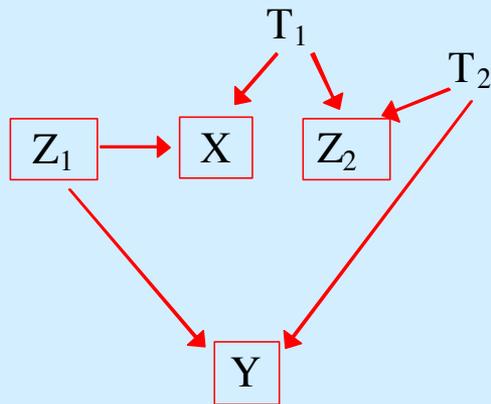
$$\beta_X = 0$$

$$\beta_Z \neq 0$$



$$\beta_X \neq 0$$

$$\beta_Z \neq 0$$



$$\beta_X \neq 0$$

$$\beta_{Z_1} \neq 0$$

$$\beta_{Z_2} \neq 0$$

# Better Methods Exist

Causal Model Search (since 1988):

- Provably Reliable
- Provably Rumsfeld

Tetrad Demo