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VENTURING FURTHER INTO EPISTEMIC TOPOLOGY

Johan van Benthem

joint work with Alexandru Baltag

28 January 2022

Formal Epistemology Seminar, CMU

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I

Background: Information in logic

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Knowledge and Information in Logic

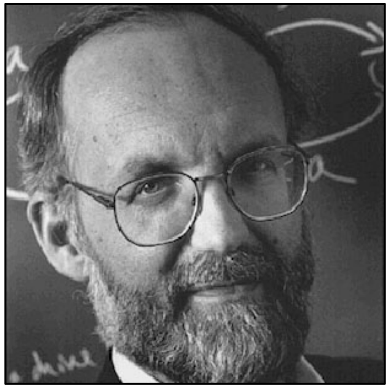
information as range

epistemic logic

Jaakko Hintikka

information as correlation

situation theory



Jon Barwise

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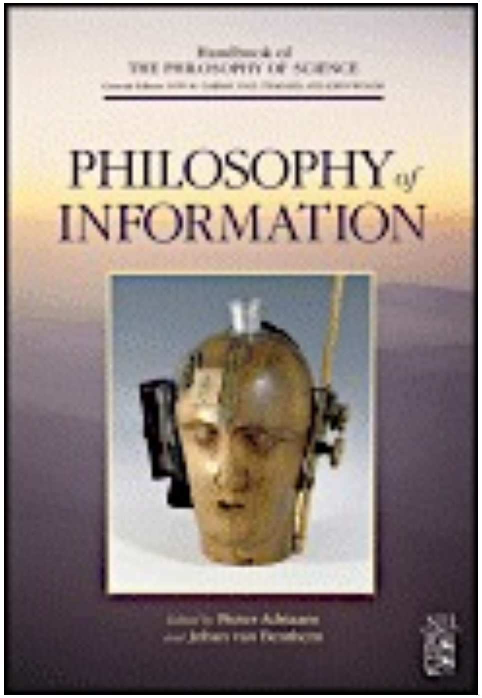
Further Varieties of Information in Logic

also **procedural information** (about process of inquiry)

information as code (deduction, computing)

and more ···



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II

Enter Dependence

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Dependence and Information

physical dependence of y on X in the world

determined, or correlated behavior

covers a family of notions/intuitions

epistemic dependence

if you know the values of X you know the value of y

related epistemic view: implication between questions

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A Dependence Table

Toy example: assignment of objects to variables

global dependence fixing value of *x* fixes that of *y*

x	*y*	*z*
0	**0**	**0**
0	**0**	**1**
1	**1**	**1**
2	**1**	**0**

$x \rightarrow y$: *y depends on x*

dependence graph

$x \rightarrow y$, not $y \rightarrow x$

not $z \rightarrow x$, $\{y, z\} \rightarrow x$

local dependence *x* depends on *y* ***at*** *(**0, 0, 0**)*

more basic! but not at *(**1, 1, 1**)*

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Dependence Models

$$\mathcal{M} = (V, O, S, P)$$

,

V	variables
O	objects (possible values of variables)
S	states: family of functions from V to O
	need not be full O^V: **gaps = dependence**
P	predicates of objects

$\mathbf{D^s_X y}$ $\quad \forall t \in S$: if $s =_X t$, then $s =_y t$ $\quad$ $\mathbf{D_X y}$ $\quad \forall s \in S: D^s_X y$

with $\quad s =_x t \ : s(x) = t(x), \quad s =_X t : \ \forall x \in X: s =_x t$

also $\quad \mathbf{D_X Y} : \forall y \in Y: D_X y$

Functional Definability

,

useful shift in perspective: **$x(s) := s(x)$**

variables as maps from states to objects

states might now be more than assignments

a simple factorization result

$D_X y$ holds globally in a dependence model

iff there exists a **(partial) function F** from

tuples of X-values to y-values s.t. for all s:

$$y(s) = \mathbf{F}(X(s)) \qquad \text{‘}y = F \circ X\text{’}$$

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III

Modal Dependence Logic LFD

Language and Semantics

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LFD Baltag & van Benthem 2021

SpringerLink

Open Access | Published: 24 March 2021

A Simple Logic of Functional Dependence

Alexandru Baltag & Johan van Benthem

Journal of Philosophical Logic **50**, 939–1005 (2021) | Cite this article

a modal logic of **functional dependence**

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Basic Modal-Style Dependence Logic

assignment of values to variables: state of the world

dependence because of gaps in full function space

dependence model: family of assignments

propositional language plus **$D_X\varphi$ | D_Xy**

D_Xy **local dependence** at state s: if $s =_X t$, then $s =_y t$

$D_X\varphi$ local modality/quantifier: for all t s.t. $s =_X t$, $t \models \varphi$

universal modality, global dependence definable

Expressive Power

• Changing *x* implies changing *y* $D_y x$

more complex versions with '(pre-causal) influence'

• $D_{\emptyset}\varphi$ is the **universal modality** $U\varphi$

• Global dependence defined from local: $UD_X y$

• Dependence as value restriction:

if *x* lies within some range, so does *y*: $U(Q_1 x \rightarrow Q_2 y)$

• **universal quantifier** $\forall X\varphi$: $\mathbf{D_{VAR-X}}\ \varphi$

Other dependence logics: CRS and DL

Extends **CRS**:

Generalized FOL semantics drops independence assumptions:

decidable sublogic remains

now add explicit dependence atoms

Different take from **DL**:

not second-order logic on teams

classical propositional base

(switching teams would require dynamic modalities)

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CRS First-Order Logic

Dependence models : ‘generalized assignment models’

$\boldsymbol{M} = (D, \mathcal{V}, I)$ with $\mathcal{V}$ set of ‘available’ assignments

$\boldsymbol{M}, s \models \exists x.\varphi$ iff there exists t in $\mathcal{V}$ with $s =^x t$ and $\boldsymbol{M}, t \models \varphi$

$s =^x t$: $s(y) = t(y)$ for all variables y distinct from x

Theorem The first-order validities on generalized assignment models are recursively axiomatizable and decidable.

Drops **independence** principles like $\exists x.\exists y.\varphi \rightarrow \exists y.\exists x.\varphi$:

these impose existential confluence properties on the set $\mathcal{V}$.

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Translation Into FOL on Standard Models

Thm There is a translation ***tr*** from the language of LFD into first-order logic making the following equivalent for modal formulas φ:

(a) φ is satisfiable in a dependence model,

(b) ***tr***(φ) is satisfiable in a *standard* first-order model.

Trick: finitely many variables ***x***, code that a tuple of values for ***x*** is an available assignment with new **dedicated predicate** *U****x***.

Corollary All logics that we will consider are RE (axiomatizable).

Caveat Not reduction of object-language, but meta-language.

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Axiom System for LFD

The proof system **LFD** consist of

(a) The principles of modal S5 for each separate $D_X\varphi$

(b) Monotonicity $D_X\varphi \rightarrow D_{X\cup Y}\varphi$

(c) Reflexivity, Transitivity, Monotonicity for atoms D_Xy

(d) **Transfer axiom** $\mathbf{(D_XY \wedge D_Y\varphi) \rightarrow D_X\varphi}$

(e) Invariance $(\neg)Q\mathbf{x} \rightarrow D_X(\neg)Q\mathbf{x}$, $(\neg)D_Xy \rightarrow D_X(\neg)D_Xy$

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IV

Independence and Correlation

Independence and Correlation

Independence is not the negation of dependence $\neg D_X y$.

Natural sense of **independence** of y from X:

fixing the values of *X* leaves *y* free to take on any value it can take in the model ('knowing *X* implies no knowledge about *y*').

Can be formalized as an **independence atom** $\mathbf{I_X y}$.

$\neg I_x y$ some local correlation, $D_x y$ total correlation

spectrum of (quantitative) **correlations in between**

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V

Completeness and Other System Results

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Some Results About the System

Thm **LFD** is sound, **complete**, **decidable**

Thm **LFD + I** axiomatizable, **undecidable**

Open **PL + just I** decidable?

some follow up results

LFD + terms + = undecidable Graedel & Puetzstueck

includes general analysis of localization and decidability

bisimulation analysis LFD Puetzstueck, Koudijs

LFD has Finite Model Property Koudijs

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Ongoing Related Projects

mutual interpretability LFD and **Guarded Fragment**

(solves computational complexity LFD)   Koudijs, ten Cate

dependence in **dynamical systems**

link up with temporal logic   Ba & vB with Dazhu Li

dependence & independence in Linear Algebra/Matroid Theory

modal logics of **vector spaces**   vB & Nick Bezhanishvili

Why mention all these technical LFD topics?

Interesting to see later which ones make sense

(and if so, how) in a topological setting

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VI

Empirical Inquiry, Measurement, Topology

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From Sharp Values to Empirical Measurement

LFD semantics assumes sharp values and

suggests that we can know these

this works for many epistemic puzzles and scenarios

but it is highly idealized in many settings

empirical inquiry yields only **approximate measurements**

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Epistemic Topology

open sets

outcomes of possible **observations**

approximation of values now essential

many versions of this idea:

Vickers, **Parikh & Moss**

also **Intuitionistic Logic** since 1930s

Topology of information states, or:

stages in temporal history of inquiry

Felix Hausdorff

Marshall Stone

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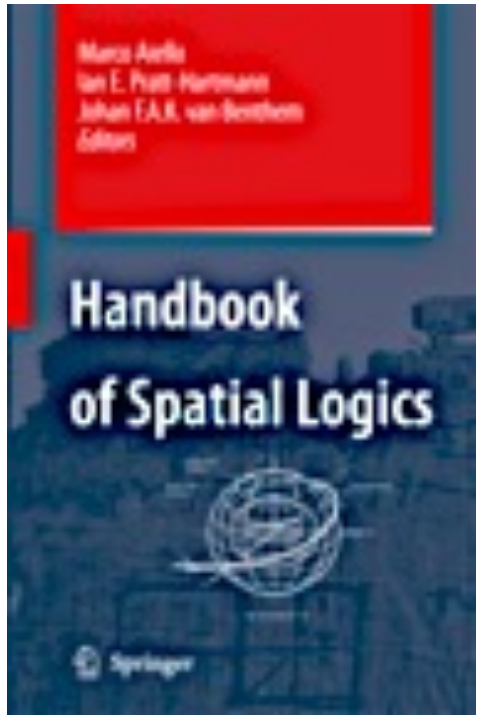
Background: Logics of Space



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VII

Topological Dependence Logic

Topo-Dependence Models

M = $(S, O, \{\mathbf{O}_x\}_{x \text{ in } VAR}, V)$

set of admissible states **S**

variables x map states to objects x(s) in their ranges O_x

these ranges carry topologies $\mathbf{O}_x$

measurements of x yield opens in $\mathbf{O}_x$

* separation axioms make epistemic sense: T_0

* lift to finite sets of variables using **product topology**

topologies on values can be retracted to $\mathbf{O}_x$ on states:

the smallest topology that makes the map x continuous

often makes for more intuitive formulations

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Knowledge and Knowability

local modal semantic format **$M, s \models \varphi$**

$U\varphi$ what is true in all states is **hard knowledge**

[can be made more sophisticated in various standard ways]

$K_X\varphi$ local **knowability** under **new information**

φ true in some open X-neighborhood of s

the standard topological interpretation of modal logic

S4 instead of **S5** lots of theory in epistemic logic

(also on modal logics for product topologies)

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Important Special Topologies

discrete topology contains our earlier **LFD** approach

Alexandrov topology each point has smallest open NBD

~ standard relational models: **specialization pre-orders**

Proposition *In an Alexandroff topo-dependence model* $\mathbf{M}$*, the above semantical clauses are equivalent to the following relational versions:*

$$s \models_{\mathbf{M}} K_X\varphi \quad \textit{iff} \quad \forall t \in S(s \leq_X t \Rightarrow t \models \varphi),$$

also for later notions

$$s \models_{\mathbf{M}} K_X Y \quad \textit{iff} \quad \forall t, w \in S(s \leq_X t \leq_X w \Rightarrow t \leq_Y w),$$

where $\leq_X$ *is the specialization preorder for the* X*-topology* $\tau^S_{\mathbf{X}}$ *on states.*

technical uses our completeness proofs all pass via

representation theorems for such relational models

Alternative/Richer Frameworks

topo-logic (Parikh & Moss)

dynamic-epistemic update logics like PAL and DEL for explicit changes of topological models

temporal logics with topological structure (dynamic topological logic **DTL**)

here we will stick with the simplest base setting

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VIII

Continuity and Learnability

Continuity as Information-Carrying Dependence

we may know that a dependence exists

but it may not be useful for approximation

$k_x y$ approximable dependence of y on X:

get arbitrarily close to $y(s)$ by measuring $X(s)$

locally$_1$ continuous function at state s

may still not yield knowledge:

we do not know which state s we are at

local continuity can be fragile

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$Local_2$ $Local_1$ Continuity

maybe the more natural topological sense is **$locality_2$**: truth **throughout** some open neighborhood of s

knowable dependence K_Xy y depends on X continuously over some open neighborhood U of s

I.e. dependence function y(X) on states factors over continuous map from X-values to y-values on the set U

similar factoring views for our other notions to come

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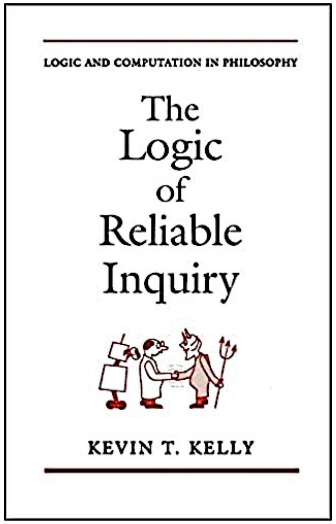
Connections to Formal Learning Theory

if the spaces are **first-countable**

(countable basis for open NBHs)

continuous y is **gradually learnable**

from X under some X-data stream



Proposition *Let* $X : S \to (\mathbb{D}_X, \tau_X)$ *and* $Y : S \to (\mathbb{D}_Y, \tau_Y)$ *be observational variables The following are equivalent:*

1. *Y is gradually learnable from X-observations;*
2. *there exists a continuous function* $F : (\mathbb{D}_X, \tau_X) \to (\mathbb{D}_Y, \tau_Y)$ *s.t.* $F \circ X = Y$*;*
3. *there is a* known epistemic dependence *between the variables* $X : S \to (\mathbb{D}_X, \tau_X)$ *and* $Y : S \to (\mathbb{D}_Y, \tau_Y)$*, i.e. we have* $S \models K(X; Y)$.

here $K(X ; Y) = U\ k_x y$

IX

The System LCD

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LCD Documents since 2020

The Logic of Continuous Dependence

Alexandru Baltag & Johan van Benthem

Knowability as Continuous Dependence

Alexandru Baltag, ILLC, Amsterdam

Based on joint work with **Johan van Benthem**.

conceptual/mathematical analysis of epistemic notions

design of simple **modal base logics**

sequence of **completeness** and **decidability** results

uniform approach via **representation theorems**

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LCD Language and Semantics

$\mathbf{M}, s \models K_X \varphi$ iff $\exists U \in O_X$: $X(s) \in U$ & $\forall t$ with $t(X) \in U$: $\mathbf{M}, t \models \varphi$

equivalent in our earlier terms: some open NBH

in the retracted O_X-topology at s is contained in $[[\varphi]]$

the usual topological interpretation of modal logic

$\mathbf{M}, s \models k_X y$ iff $\forall U \in O_y$ s.t. $s(y) \in U$ $\exists V$: $\forall t$ with $t(X) \in V$: $t(y) \in U$

or: every y-open NBD of s contains an X-open NBD of s

again too fragile? replace by local_2 version

$\mathbf{M}, s \models K_X y$ iff $\mathbf{M}, s \models \mathbf{K}_X k_X y$

Digression: First-Order Perspective

what can we expect a priori?

LFD and its related systems were all

translatable effectively into first-order logic

explains the axiomatizability of these logics

definition of topological space is second-order, but:

if we think of the outcomes of measurements as a base

attractive **three-sorted FOL-translatable** versions exist

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LCD Proof System

(I)	**Axioms and rules of LFD**
(II)	***S*4 Axioms for knowability:**
(K-Necessitation)	From φ, infer $K_X\varphi$
(K-Distribution)	$K_X(\varphi \to \psi) \to (K_X\varphi \to K_X\psi)$
(Veracity)	$K_X\varphi \to \varphi$
(Positive Introspection)	$K_X\varphi \to K_X K_X\varphi$
(Knowable Determination)	$K_X\varphi \Rightarrow D_X\varphi$
(Knowledge of Necessity)	$A\varphi \Rightarrow K\varphi$
(IV)	**Axioms for knowable dependence:**
(Inclusion)	K_XY, provided that $Y \subseteq X$
(Additivity)	$(K_XY \wedge K_XZ) \to K_X(Y \cup Z)$
(Transitivity)	$(K_XY \wedge K_YZ) \to K_XZ$
(Knowable Dependence)	$K_XY \to (D_XY \wedge K_XK_XY)$
(Knowledge of Constants)	$C(Y) \Rightarrow KY$
(Knowability Transfer)	$K_XY \to (K_Y\varphi \to K_X\varphi)$

{t | s $=_X$ t} contained in every X-open around s, but need not itself be open.

KT expresses continuity of dependence, even modal corrrespondence-style.

Table 1: The proof system **LCD**, where we used the notations $A\varphi := D_\emptyset\varphi$, $C(Y) := D_\emptyset Y$, and $K := K_\emptyset$ (for both formulas φ and sets $Y \subseteq V$).

explanation: look like **LFD** axioms but different content

LCD Results

LCD modal logic of knowability plus knowable continuity

Thm LCD is complete for validity

Open Axiomatize full language with local continuity

Thm LCD is decidable

Key proof steps Standard modal completeness for 'general relational models'. Representation theorem general as standard relational models: yields Alexandrov topologies.

Decidability via finite general relational model property.

mathematical logic Generalize Tarski-McKinsey Thm for **S4**

now not just produce the reals, but also continuous maps over them

X

Computability and Domain Theory

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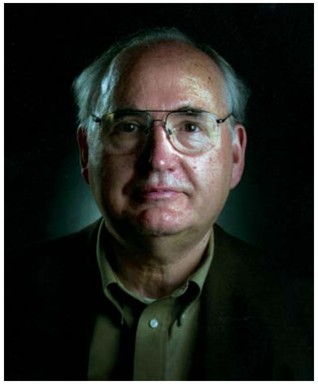
Toward Computable Dependence

Specialization Computability in **Domain Theory**

complete lattices of 'information pieces'

Scott topology with base of finite information pieces (not the upset topology for inclusion)

Scott continuity ~ abstract **computability**



Dana Scott

Thm Complete and decidable logic is **LCD +** one extra axiom for bottom elements in domain structure

$$K(K_X\varphi \vee K_X\psi) \to (K\varphi \vee K\psi).$$

K is universal modality, look at bottom element for X-value domain

Representation proof more complex so as to produce domains!

XI

Independence in Topological Models

Independence in Epistemic Topology

topological **independence**

$$s \models Ig_X Y \quad \text{iff} \quad Y(U) = \mathbb{D}_Y \text{ for all } U \in \tau_X^S(s).$$

subtly different weaker than LFD-style independence

which wants to see all y-values on the X-equal states

also symmetric(!) **global versions**

$$I(X;Y) \Leftrightarrow I(Y;X), \qquad Ig(X;Y) \Leftrightarrow Ig(Y;X).$$

scope for modal logic: some connections

$$I_X Y \Rightarrow Ig_X Y, \qquad I(X;Y) \Rightarrow Ig(X;Y)$$

$$I(X;Y) \Rightarrow I_X Y, \qquad Ig(X;Y) \Rightarrow Ig_X Y$$

open problem Topological I-logic axiomatizable, decidable?

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Everywhere Surjective Functions

Y as function of X is **everywhere surjective** Lebesque 1904

sort of opposite to continuous functions

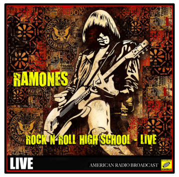
Thm The following are equivalent:

1. $D(X;Y) \wedge Ig(X;Y)$ *holds in the model* $\mathbf{M}$ *(at any/all states);*
2. *there exists some everywhere-surjective map* $F : \mathbb{D}_X \to \mathbb{D}_Y$ *s.t.* $F \circ X = Y$.

interesting recent mathematical theory

Bernardi and Rainaldi 2018

Anti-Learning Theory?



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Numerical Correlations

How to logicize **more general accuracy correlations**

in between independence and functional dependence?

And also: **inverse accuracy correlations**

like that between position and momentum in

Heisenberg's Uncertainty Principle?

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XII

Epistemic Know-How: Uniform Continuity

From Continuity to Uniform Continuity

epistemic know-how in empirical inquiry

knowing **an approximation** that works: **uniform continuity**

$$\forall \varepsilon > 0 \forall s \in S \exists \delta > 0 \forall t \in S \ (d_X(X(s), X(t)) \leq \delta \Rightarrow d_Y(Y(s), Y(t)) \leq \varepsilon).$$

$$\forall \varepsilon > 0 \exists \delta > 0 \forall s \in S \forall t \in S \ (d_X(X(s), X(t)) \leq \delta \Rightarrow d_Y(Y(s), Y(t)) \leq \varepsilon).$$

LCD-style modal logic on **metric dependence models**

U(X ; Y) uniformly continuous dependence map Y(X)

$U_X Y$ locally uniform continuous dependence

Proposition *Given empirical variables $X : S \to (\mathbb{D}_X, d_X)$ and $Y : S \to (\mathbb{D}_Y, d_Y)$, the following are equivalent:*

1. *$U(X;Y)$ holds;*
2. *there exists a uniformly continuous map $F_{X;Y} : (\mathbb{D}_X, d_X) \to (\mathbb{D}_Y, d_Y)$ s.t. $F \circ X = Y$ holds on S.*

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The Complete Logic LUD

(I)	**All axioms and rules of the system LCD**
(II)	**Axioms for uniform dependence:**
(U-Inclusion)	$U(X;Y)$, provided that $Y \subseteq X$
(U-Additivity)	$(U(X;Y) \wedge U(X;Z)) \to U(X;Y \cup Z)$
(U-Transitivity)	$(U(X;Y) \wedge U(Y;Z)) \to U(X;Z)$
(Uniform Dependence is Known)	$U(X;Y) \to KU(X;Y)$
(Uniformity implies Continuity)	$U(X;Y) \to K(X;Y)$

some explanations

Thm LUAD is complete and decidable.

proof runs via ‘pseudo metric models’:

requires yet more complex representation argument

open problem how to deal with $U_X Y$

Uniform Spaces

uniform spaces with **entourages** Weil 1937

Family $\mathcal{U}$ of ref-sym relations ~ plus refinement closure:

for every $R \in \mathcal{U}$ there is $S \in \mathcal{U}$ with $S \circ S \subseteq R$

Sets $\{y \mid y \sim x\}$ generate a topology

qualitative **uniform continuity**

$$\forall U \in \mathcal{U} \exists V \in \mathcal{V} \forall x \in D \forall y \in D \, ((x, y) \in V \Rightarrow (F(x), F(y)) \in U)$$

Analogy with margin-of-error relations in epistemology

How to best bring into our kind of logic?

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One Option: Modal Logic of Accuracy Dynamics

u assigns binary relations $\sim_x$ to variables x

$r \leq_X r'$ more refined relations on all X-variables

$r \models S_X\varphi$ φ true in all $U(X)$-close points

$U_X y$ y uniformly continuous in X

dynamic modalities $[\downarrow_X]\varphi$

φ true for all X-refinements of current r

logic now includes reasoning about accuracy refinements

~ modal logics of relation change, e.g.:

The Modal Logic of Stepwise Removal

Johan van Benthem, Krzysztof Mierzewski & Francesca Zaffora Blando

Review of Symbolic Logic

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XIII

Discussion and Conclusions

Limitations 1

Our topological models consider global objects 'known'

a global dependence function is known in this sense

But what if we want to **learn a dependence function**?

need to lift our semantics to **families** of dependence models

LFD paper: 'dependence universes'

optimal logic still to be developed

analogies with learning the content of a set:

suggests lifting of public announcement logic

plus richer static base language

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Limitations 2

our topological models assume sharply defined functions

but is not this **at odds** with the imprecise measurement setting?

possible alternative: **point-free topology**

primitive **approximation maps** run backwards

'as if' representation results

create points out of primitive opens,

produce continuous function out of approximations

General Enterprise: Epistemize Analysis

develop parallel mathematical notions ~ epistemic notions

various directions:

- common sense analogues of math notions

continuity/uniform continuity: move to knowing how?

- common sense epistemic principles as

high-level expressions of mathematical notions

e.g., $K_X y \to U_x y$ (epistemic de dicto to de re)/local compactness?

Digression: Where is the Dynamics?

epistemic actions

model change, LFD + PAL

dependence universes

Conference Paper | PDF Available

The Logic of Public Announcements and Common Knowledge and Private Suspicions.

January 1998

Source · DBLP

Conference: Proceedings of the 7th Conference on Theoretical Aspects of Rationality and Knowledge (TARK-98), Evanston, IL, USA, July 22-24, 1998

Authors:

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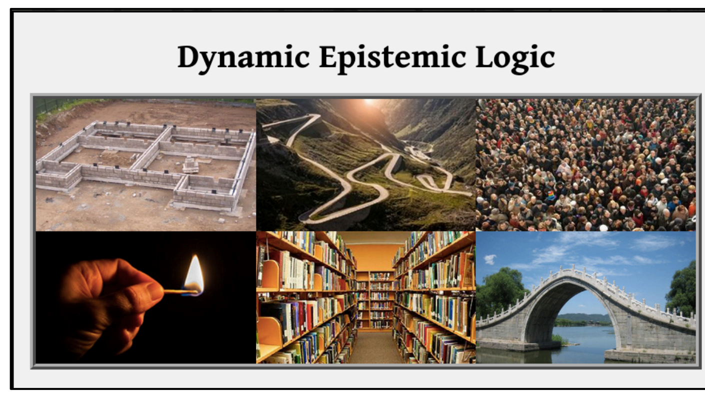
BMS

in the topology/metrics:

DEL **accuracy** dynamics



causality: dependence

plus **interventions** that

change causal models

Amsterdam Dynamics Portal

Summary: Richer Epistemic Topology

Exploit more topology explicitly in epistemic logic

Unify topological and computational aspects

Venture beyond topology into Analysis

not discussed today:

Add further structure beyond dependence: causality

Step up abstraction: category theoretic-framework for LCD style logics (current work by Ye Lingyuan)

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Questions and Answers: Just a Few Points

$K_X y$ Why is 'knowable' $k_x y$ only based on more **X**-info?

Yes, could be based on measuring other variables Z. Simple case: when X depends on Z. In general, we would need $K_Z\, k_X y$ which is not yet in our language⋯

What are we learning?

Just the actual state s of the system. Like the actual world in epistemic scenarios. Learning the state space itself, or a dependence function, requires richer models. We can see this as LCD over more structured states, but we might want a richer language describing the structural content of states.

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Questions and Answers: Just a Few Points

Could not we also learn about y by other means than measurement of other variables? [Hope I got this right]

Even in LCD, measuring X gives information about y in combination with another source of information: the structure of the state space and its assignment gaps. And yes of course, other informational events could take place, PAL- or DEL-style, not based on measurement.

How to capture more general intuitions about approximate knowledge in terms of ‘getting enough information about X produces enough information about y’?

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Questions and Answers: Continued

Spelling out the 'enough' is interesting as a typical qualitative device in stating the gist of mathematical notions. In which direction does it run here: from y to X, or vice versa? Could entourages offer a good way of studying this?

Is 'knowability' a felicitous term? Better 'will be known' or some other term from current learning-theoretic topology?

Our framework is more about information than knowledge, and yes, there may be better names for what we study.

Connections with knowability in intuitionistic logic?

May be helpful to bring out intuitionistic structures inside LCD.

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Questions and Answers: Continued

Note: intuitionistic mathematics make heavy use of (uniform) continuity in its logical analysis of the informational surplus in constructive statements and proofs.

Why is LFD independence complex qua logic?
What about topological independence?

In LFD: independence atoms can enforce a full Cartesian product for three variables, and FO(3) is undecidable. Topo-independence is weaker, and no such reduction may exist, leaving room for a decidable D + I logic.

*Aside: independence makes computational **practice** easier, but this regular structure is precisely what can make for a more complex logical **theory**.*

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Questions and Answers: Aftermath

for posing further questions or getting better answers:

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