On the Philosophical Work of Per Martin-Löf

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Per Martin-Löf began his work on logic and foundational issues in 1966 with a definition of the notion of a random sequence that has become classic. It was continued with the doctoral dissertation *Notes on Constructive Mathematics* that was written in 1968. Martin-Löf spent the academic year 1968–1969 in the United States, first at Chicago where, in December 1968, W. A. Howard gave him a copy of the handwritten manuscript *The formulae-as-types notion of construction* (that was subsequently published (1980) in the Curry Festschrift). In it an isomorphism à la Curry is established between axiomatic Hilbert-style systems for predicate logic and arithmetic and matching calculi in combinatory logic. The correspondence struck Martin-Löf as being of profound significance and he was determined to understand it fully. His first contribution to this end, as pointed out by Howard in a note added to the published paper, was to carry over the idea from Howard's Hilbert-style calculi to the framework of Gentzen's Natural Deduction that was known to him from Dag Prawitz's (1965) dissertation. In Martin-Löf's formulation, proof-theoretic reductions of natural-deduction derivations correspond to conversions of terms in an enriched lambda calculus. The now customary "Curry-Howard" isomorphism between natural-deduction derivations and the terms of a matching lambda-calculus (rather than the combinatory logic that was used by Howard) was then written up in a paper on *Infinite terms and a system of natural deduction* that was circulated in March 1969. Armed with the insight that natural-deduction derivations and lambda-calculus terms are essentially equivalent, Martin-Löf began a search for the optimal way of proving normalization results for systems of natural deduction. His investigation of iterated generalized inductive definitions, which was completed by March 1970, carried over the computability method of W. W. Tait (1967) to proofs of normalization for natural-deduction derivations, and the paper was presented at the *Second Scandinavian Logic Symposium* at Oslo in June 1970. Kreisel (1975, p. 177, footnote 6) reports that when travelling – with Prawitz and Martin-Löf – by train to Oslo, he mentioned J.-Y. Girard's very recent, but still unpublished, work on how to extend Gödel's *Dialectica* interpretation to second-order arithmetic ("Analysis") and gave Girard's manuscript to Martin-Löf, who already at the Oslo meeting convinced himself that Girard's insight would carry over into a normalization theorem also for second-order logic, giving a procedure of normalization, rather than completeness of the cut-free sequent-calculus rules for second-order logic (as in Prawitz's (1967) elegant demonstration of Takeuti's conjecture). After the Oslo meeting, Girard, Martin-Löf, and Prawitz all gave normalization proofs for second-order logic that were published in its *Proceedings* (Fenstad (1971)), and Girard (1971, p. 64) confirms that "les remarques de M. Martin-Löf sur la possibilité d'une démonstration de normalisation de l'Analyse ... on été déterminantes pour la suite de ce travail". Martin-Löf quickly extended the computability approach also to the intuitionistic simple theory of types (1970c). A characterization of the provable well-orderings of the theory of species also belongs to this period (1970d) of great creativity. Finally, in the autumn of 1970, the first system of Intuitionistic Type Theory was designed and presented in a seminar lecture at Stockholm as a deliberate attempt to extend the computability approach that had so amply demonstrated its worth at the Oslo conference to even stronger systems. Its main building blocks were Gentzen's natural-deduction style of formalization, with proof-theoretic reduction steps from Prawitz (1965), Gödel's *Dialectica* system T (1958), the Curry-Howard isomorphism, and Tait's (1967) computability method for normalization proofs. The written presentation, in a preprint dated February, revised October (1971a), was submitted for publication to *Acta Mathematica*. Martin-Löf's invited lecture at the Bucharest fourth LMPS conference in 1971b was also devoted to his novel intuitionistic type theory. However, both submissions were withdrawn from publication pursuant to Girard's discovery that the extreme impredicativity of the system allowed for the derivation of a version of Burali-Forti's paradox. Martin-Löf's revised version of his Intuitionistic Type Theory from 1972 was finally published in the proceedings of the conference at Venice that was held in 1995 to commemorate *Twenty-five Years of Constructive Type Theory*, and the proceedings of the 1973 Bristol Logic Colloquium contain the first published presentation of a – predicative – version of the type theory.

The task that faced a mathematical logician from Frege's days until 1930 was a challenging one: to graft the Fregean notion of a formal system onto the Aristotelian conception of demonstrative science. That is, one should design a sizeable formal system with clearly delimited axioms and rules of inference. The system should be adequate for the needs of analysis after the then novel fashion of Weierstraß and Dedekind. In particular, it should admit of classical logic (as well as impredicative quantification). The axioms and (primitive) rules of inference should be rendered immediately evident from the meaning explanations for the (primitive signs of the) formal language of the formal system in question. Frege, Whitehead-Russell, Lesniewski, and the early logical works of Curry, Church, and Quine, are good examples here; as is well known, the early logicist attempts were not successful, owing to the use of axioms, such as Reducibility, that were not rendered evident by the relevant meaning-explanations. Carnap's contribution to the famous conference on the philosophy and foundations of mathematics at Königsberg in November 1930 can be seen as the last stand of Logicism. Thereafter, it seemed clear that Logicism was no longer a live option. This meant that the foundations of mathematics

were faced with the two horns of a dilemma: either we retain classical logic in a mathematical object-language, but give up hope for meaning explanations, or we insist on retaining a contentual language with meaning explanations, but have to jettison classical logic. Hilbert in his programme chose the first option, whereas the second one was preferred by Brouwer and elaborated by Heyting in his articles on the formalization and interpretation of intuitionist logic.

Martin-Löf's first logical writings belong to the metamathematical paradigm. The formal languages and systems dealt with are principally objects of metamathematical study, and in that spirit normalization *theorems* are established for the early versions of type theory and other systems. However, in 1974, stimulated by reading Wittgenstein, as well as by listening to Michael Dummett's lecture *The Philosophical Basis of Intuitionistic Logic* at the Bristol Logic Colloquium 1973, Martin-Löf turned to the theory of meaning, and thereby brought type theory within the contentual approach in logic. Peter Hancock's words in the lectures that marked the meaning-theoretical turn (1975b, p. 13) speak with quiet confidence and could be used unchanged also today:

> Especially in its later stages, Martin-Löf's work has been developed with the principal aim that it should admit a detailed and coherent semantics. We are not concerned here, except by implication, to subject other languages to a destructive criticism. We are ignorant of a comparable project that has been carried through for a different language. So we do not have to say at this stage why our account is to be preferred to another. You will just have to make up your mind about that if and when there *is* another account.

He has devoted himself ever since to the realization of the second horn of the above dilemma in the foundations of mathematics with a project that is comparable to that of Frege's *Grundgesetze*, but without classical logic and impredicativity: design a full-scale formal language, with explicit meaning-explanations, that is adequate for the needs of mathematical analysis in the style of Errett Bishop's (1967) revolutionary constructive presentation. In the first two papers after his meaning-theoretical turn, Martin-Löf made an experimental attempt to view the natural-deduction *elimination* rules as basic, or meaning giving. A trace of this approach can be found in the sole reference (1991, p. 280) to Martin-Löf in Michael Dummett's William James lectures (that were given at Harvard in 1976, shortly after Martin-Löf's course at All Souls in Michaelmas Term 1975, and was later published as Dummett (1975)). The paper (1975a), written jointly with Peter Hancock, on primitive recursive arithmetic, has attracted a measure of attention also as a contribution to the proper interpretation of mathematics in Wittgenstein's *Tractatus*, whereas the lecture notes (1975b) gave meaning explanations for the language of type theory. The experiment with the elimination rules as basic was soon abandoned, though, and the lecture notes left uncompleted, after it became clear that the approach could not be carried through. In Martin-Löf's (1979) lecture at LMPS 6 in Hannover, the pattern of meaning explanations based on the introduction-rule constructors that yield canonical proof-objects is introduced. These explanations have essentially remained constant ever since, particularly in the Padova lecture notes from 1980 by Giovanni Sambin that were published by Bibliopolis. (The only notable change from the expositions in 1979 and 1980 lies in Martin-Löf's current use of *intensional* identity rather than the previous extensional one.) In this mature form, Martin-Löf's constructive type theory, from the point of view of the foundations of mathematics, constitutes an impressive, mathematically precise rendering of the BHK meaning-explanations that were first given by Arend Heyting in 1930.

Around this time, that is, the late 1970s, Martin-Löf also began a study of the philosophy of Edmund Husserl and from then on the phenomenological perspective has been a rich source of inspiration. In conversation Martin-Löf has indicated that he regards his syntactic-semantic method of logical exploration as a version of phenomenology. To this time belongs also a second period of experimentation, in which Martin-Löf attempted to avoid the use of type-theoretical abstractions, and instead explored the alternative of working within predicate logic, eschewing the use of proof-objects and the form of judgement a: proof(A), working instead with the predicate logic form A true as principal form of judgement. This work culminated in a set of influential lectures that were given in 1983 at Siena, on which basis a compact course was published, and where considerable systematic effort was spent on relating the work to issues in the philosophy of logic. With the creation of the higher type structure, Martin-Löf returned to a type-theoretical formulation of his ideas. The full language of type theory, now using both sets and types, has been in place since 1986, when it was first presented in a lecture at Edinburgh. A lecture series given at Florence in 1987 gave meaning explanations also for the higher type theory. It was further extended in 1992 with a calculus of explicit substitutions in order to make the treatment fully formal. In 1993, Martin-Löf gave a semester-long series of lectures on the *Philosophical Aspects of Type Theory* at Leiden, in which he presented the meaning explanations in full detail and dealt with a number of topics from the philosophy of logic and language.

Martin-Löf's mature philosophical outlook is characterized by three main tenets that make it unique among contemporary philosophical positions within the foundations of mathematics. First, and most significantly, constructive type theory is an *interpreted formal language*. The importance of this cannot be stressed firmly enough. Today, as a rule, the metamathematical "expressions" employed in mathematical logic are mere objects of study, but do not express. On the contrary, they are objects that may serve as referents of real expressions. In constructive type theory, on the other hand, the expressions used are real expressions that carry meaning. In a nutshell, the language is endowed with meaning by turning the proof-theoretic reductions into steps of meaning explanation. Just like the formulae of Frege's ideography, or of the language of *Principia Mathematica*, the type-theoretic formulae are actually intended to say something. They do not essentially serve as objects of metamathematical study. This, of course, in no way precludes Martin-Löf's contentual system from being studied metamathematically.

Secondly, Martin-Löf has restored the notion of *judgement* to pride of place in logic. In the metamathematical tradition, the same well-formed formulae serve in different roles: wff's are built up from wff's that have been generated earlier, for instance, when φ and χ are wff's, then so is $(\varphi \supset \chi)$. Furthermore, a wff φ may be a derived theorem, that is, the end-formula of a closed derivation (with no open assumptions). From Martin-Löf's contentual point of view, a proposition A is a set

of (canonical) proof-objects. Such propositions serve as building blocks for more complex propositions. The contentual role of a derivational end-formula, however, is not propositional; here we do not have just an occurrence of the proposition A, but an *assertion* that

proposition A is true.

Martin-Löf's propositions are given via *proof conditions*, that is, to each proposition A there is associated a type proof(A) that is explained in terms of how canonical proof-objects for A may be formed and when two such proof-objects are the same. Truth of propositions is explained by the "truth-maker" analysis

proposition A is true = proof(A) exists,

where the existence in question is the constructive Brouwer-Weyl notion that is explained in terms of possession of an instance. It should be noted that these are proofs of *propositions*. Previously in the history of logic, all proving (better: all *demonstration*) took place at the level of assertions, and not at the level of propositions (that are traditionally seen as unasserted contents of theorems). Such proofs of propositions were first considered in Heyting's seminal writings from the early 1930s. Martin-Löf's crucial notion of a judgement is explained in terms of an *assertion condition* that lays down what one has to know in order to have the right to make the judgement in question. Thus for instance, in order to have the right to make the judgement that the proposition A&B is true, one has to have exhibited a proof-object c that either is of, or evaluates to, the form <a, b>, where a is a proof-object for the proposition A and b is a proof-object for the proposition B. This reintroduction of judgements into logic, with the concomitant distinction between judgements and propositions, also leads to a crucial distinction between (epistemic) *demonstrations* of judgements, and *proofs*, in the sense of proof-objects, of propositions. This distinction between propositions and judgements, and the ensuing distinction between proofs of propositions and demonstrations of judgements, also brings about a corresponding distinction between (epistemic) *inference* from premise to conclusion judgements and relations of *consequence* between antecedent and consequent propositions.

Finally, the emphasis on judgement and the acquaintance with the works of Husserl has led Martin-Löf to an uncommon epistemological perspective. In contemporary analytical philosophy, epistemology takes a very "factual", almost ontological stance. Knowledge is invariably seen from a third person perspective as an ontological state that obtains in the world and makes true propositions such that *agent A knows proposition p*. Here the main concern is not what it is to know something oneself, but rather what it is for *someone else* to know something. Thus, on the linguistic level, one is concerned with the meaning of the locution "A knows p" rather than with "I know p". Martin-Löf's approach to meaning, on the other hand, is squarely *first person*. To him every assertion by means of an utterance of a declarative sentence contains an *implicit* first-person knowledge claim. Martin-Löf explains a declarative by means of an "assertion condition" that lays down *what one has to know* in order to have the right to make the assertion by means of an utterance of the declarative in question. Accordingly, the *legitimacy* of the counter-question *How do you know that?* serves as a criterion by which assertoric uses of declarative sentences can be recognized.

The effect of this can be seen, for instance, in Moore's paradox concerning an assertion by means of:

> "It is raining, but I do not believe it."

As is well known, the use of the present tense and the first person is essential here. No paradox results in the imperfect: "It was raining, but I did not believe it." Similarly, non-assertoric occurrences of the crucial sentence are not problematic: "If it is raining, but I do not believe it, then I will get wet when I go outside." Also any third-person assertion by me, of "It is raining, but X does not believe it," where X is placeholder for the name of a person, is not paradoxical, even though we may choose X = Göran Sundholm here. Only the first person poses problems, owing to the implicit first-person "I know" that is contained in every assertion and that contradicts the second clause of the Moorean assertion. Martin-Löf's insistence on this kind of first-person knowledge comes out time and again in his philosophy, for instance, in the quotation above from 1975b, but perhaps most clearly in his pertinent request to the reader in the first full contentual exposition of constructive type theory (1979, p. 166):

> For each of the rules of inference, the reader is asked to try to make the conclusion evident on the presupposition that he knows the premises. This does not mean that further verbal explanations are of no help in bringing about an understanding of the rules, only that this is not the place for such detailed explanations. But there are also certain limits to what verbal explanations can do when it comes to justifying axioms and rules of inference. In the end, everybody must understand for himself.

In a series of published philosophical lectures from 1985 to 2004, Martin-Löf has explored central notions within the philosophy of logic, such as judgement, evidence, rightness, and knowledge, as well as in the philosophy of mathematics, for instance concerning the Axiom of Choice, and spelled out consequences for his views on metaphysics and epistemology. He has, however, been a frequent invited speaker at conferences and the list of topics covered in unpublished material is long: it comprises Frege's distinction between *Sinn* and *Bedeutung*, intensionality of objects, Tarski's truth definition and the notion of a model for type theory, predicativity in mathematics, propositions versus contents, categories, and general methodology in the philosophy of logic. In recent years, since his Gödel lecture *The two layers of logic* at the annual meeting of the Association for Symbolic Logic at Montréal in 2006, Martin-Löf's philosophical work has been directed to the question, Logic, epistemological or ontological? that also gave the title for his lecture at the Uppsala meeting.

Works by Per Martin-Löf

Martin-Löf has often been reticent in publishing his papers and sometimes there is a considerable delay between the distribution of the preprint version and its ultimate publication, if any. I therefore cite his works after the year of distribution rather than of publication.

Martin-Löf, Per. 1966. The definition of random sequences. *Information and Control* 9(6): 602–619.

Martin-Löf, Per. 1968. *Notes on constructive mathematics* (doctoral dissertation). Stockholm: Almqvist & Wiksell, 1970.

Martin-Löf, Per. 1969. Infinite terms and a system of natural deduction. *Compositio Mathematica* 24 (1972): 93–103.

Martin-Löf, Per. 1970a. Hauptsatz for the intuitionistic theory of iterated inductive definitions. In *Proceedings of the 2nd Scandinavian logic symposium*, ed. J.E. Fenstad, 179–216. Amsterdam: North-Holland Publishing Company, 1971.

Martin-Löf, Per. 1970b. Hauptsatz for the intuitionistic theory of species. In *Proceedings of the 2nd Scandinavian logic symposium*, ed. J.E. Fenstad, 217–233. Amsterdam: North-Holland Publishing Company, 1971.

Martin-Löf, Per. 1970c. Hauptsatz for intuitionistic simple type theory. In *Logic, methodology and philosophy of science IV, Proceedings of the fourth international congress for logic, methodology and philosophy of science, Bucharest, 1971*, ed. P. Suppes, L. Henkin and A. Joja, 279–290. Amsterdam: North-Holland Publishing Company, 1973.

Martin-Löf, Per. 1970d. A construction of the provable wellorderings of the theory of species. In *Logic, meaning and computation*, ed. A. Anthony Anderson and M. Zeleny, 343–352. Dodrecht: Kluwer, 1997.

Martin-Löf, Per. 1971a. An intuitionistic theory of types, unpublished preprint.

Martin-Löf, Per. 1971b. On the strength of intuitionistic reasoning, preprint of a contribution to the Symposium on Perspectives in the Philosophy of Mathematics, *LMPS* IV, Bucharest, September 1971.

Martin-Löf, Per. 1972. An intuitionistic theory of types. In *Twenty-five years of constructive type theory*, ed. G. Sambin and Jan J. Smith, 127–172. Oxford: Clarendon Press, 1998.

Martin-Löf, Per. 1973. An intuitionistic theory of types: Predicative part. In *Logic colloquium'73*, ed. H.E. Rose and J. Shepherdson, 73–118. Amsterdam: North-Holland Publishing Company, 1975.

Martin-Löf, Per. 1975a. Syntax and semantics of the language of primitive recursive functions (written jointly with Peter Hancock), unpublished preprint.

Martin-Löf, Per. 1975b. Syntax and semantics of mathematical language (Notes by Peter Hancock on a lecture series given at All Souls College, Oxford, Michaelmas term, 1975), unpublished preprint.

Martin-Löf, Per. 1979. Constructive mathematics and computer programming. In *Logic, methodology and philosophy of science VI, Proceedings of the 1979 international congress at Hannover, Germany*, ed. L.J. Cohen, J. Los, H. Pfeiffer and K.-P. Podewski, 153–175. Amsterdam: North-Holland Publishing Company, 1982.

Martin-Löf, Per. 1980. Intuitionistic type theory: Notes by Giovanni Sambin of a series of lectures given in Padua, June 1980. Napoli: Bibliopolis 1984.

Martin-Löf, Per. 1983. On the meanings of the logical constants and the justifications of the logical laws. *Nordic Journal of Philosophical Logic* 1(1): 11–60, 1996 ("Siena lectures").

Martin-Löf, Per. 1985. Truth of a proposition, evidence of a judgement, validity of a proof. *Synthese* 73: 407–420.

Martin-Löf, Per. 1988. Mathematics of infinity. In *Colog 88*, ed. P. Martin-Lof and G. Mints, 146–197. Berlin: Springer, 1990.

Martin-Löf, Per. 1991. A path from logic to metaphysics. In *Atti del Congresso Nuovi Problemi della Logica e della Filosofia della Scienza, Viareggio, 8–13 gennaio, 1990*, ed. G. Corsi and G. Sambin, 141–149. Bologna: CLUEB, 1991.
Martin-Löf, Per. 1992. Analytic and synthetic judgements in type theory. In *Kant and contemporary epistemology*, ed. P. Parrini, 87–99. Dordrecht: Kluwer, 1994.
Martin-Löf, Per. 1994. Verificationism then and now. In *The foundational debate: Complexity and constructivity in mathematics and physics*, ed. W. DePauli-Schimanovich et al, 187–196. Dordrecht: Kluwer, 1995.
Martin-Löf, Per. 1995. Truth and knowability: On the principles C and K of Michael Dummett. In *Truth in mathematics*, ed. H.G. Dales and G. Oliveri, 105–114. Oxford: Clarendon Press, 1998.
Martin-Löf, Per. 2004. 100 years of Zermelo's Axiom of Choice: What was the problem with it? In *Logicism, intuitionism, and formalism – What has become of them?*, ed. S. Lindström, E. Palmgren, K. Segerberg and V. Stoltenberg-Hansen, 209–219. Dordrecht: Springer, 2009.
Martin-Löf, Per. 2008. The Hilbert-Brouwer controversy resolved? In *One hundred years of intuitionism (1907–2007). The Cerisy conference*. Publications of the Henri Poincaré Archives. Science around 1900, ed. M. van Atten, P. Boldini, M. Bourdeau and G Heinzmann, 243–256. Basel: Birkhäuser, 2008.

Works by Others

Bishop, Errett. 1967. *Foundations of constructive analysis*. New York: McGraw-Hill.
Dummett, Michael. 1975. The philosophical basis of intuitionistic logic. In *Logic colloquium'73*, ed. H.E. Rose and J. Shepherdson, 5–40. Amsterdam: North-Holland Publishing Company.
Dummett, Michael. 1991. *The logical basis of metaphysics*. London: Duckworth.
Fenstad, J.E. (ed.). 1971. *Proceedings of the second Scandinavian logic symposium*. Amsterdam: North-Holland Publishing Company.
Girard, J.-Y. 1971. Une extension de l'interprétation de Gödel à l'analyse, et son application à l'élimination des coupures dans l'analyse et la théorie des types. In *Proceedings of the 2nd Scandinavian logic symposium*, ed. J.E. Fenstad, 63–92. Amsterdam: North-Holland Publishing Company, 1971.
Gödel, Kurt. 1958. Über eine bisher noch nocht benützte Erweiterung des finiten Standpunktes. *Dilalectica* 12: 280–287.
Howard, W.A. 1980. The formulae-as-types notion of construction. In *To H. B. Curry: Essays on combinatory logic, lambda calculus and formalism*, ed. J.P. Seldin and J.R. Hindley, 479–490. London: Academic.
Kreisel, Georg. 1975. Observations on a recent generalization of completeness theorems due to Schütte. In *ISILC Proof Theory Symposion, Dedicated to Kurt Schütte on the occasion of his 65th birthday. Proceedings of the International Summer Institute and Logic Colloquium, Kiel 1974. Springer Lecture Notes in Mathematics*, vol. 500, ed. J. Diller and G.H. Müller, 164–181. Berlin: Springer.
Prawitz, Dag. 1965. *Natural deduction* (doctoral dissertation). Stockholm: Almqvist & Wiksell. (Second edition at Dover Publishing Company, 2006).
Prawitz, Dag. 1967. Completeness and Hauptsatz for second order logic. *Theoria* 33: 246–258.
Prawitz, Dag. 1971. Ideas and results in proof theory. In *Proceedings of the 2nd Scandinavian logic symposium*, ed. J.E. Fenstad, 235–307. Amsterdam: North-Holland Publishing Company, 1971.
Tait, W. W. 1967. Intensional interpretations of functionals of finite type. *Journal of Symbolic Logic* 32: 198–212.

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