CLARK GLYMOUR

Indistinguishable Space-Times and the Fundamental Group

It has recently been noted (Ellis, 1971; Dautcourt, 1971; Ellis and Sciama, 1972; Glymour, 1972; Trautman, 1965) that in some general relativistic cosmologies various global features of space-time may necessarily escape determination. In contrast to classical space-time theories, the fundamental group of space-time may itself be such a feature in a relativistic space-time. A precise account of what it means for two space-times to be "indistinguishable" will permit us to prove some elementary propositions concerning the classification of indistinguishable space-times which have distinct global topologies.

The equations of familiar space-time theories are local and therefore, even assuming a complete affine connection, do not of themselves determine a unique topology for space or for space-time. The equations of Newtonian theory (as given, for example, by Trautman, 1965) permit space-time to have any topology $V \times R$, where R is the reals and V is any three-dimensional manifold admitting a complete Riemannian connection of zero curvature. There are exactly eighteen such distinct space-forms. Many topologically different Newtonian models can be distinguished empirically either by making global journeys through space or by observing systems which have made such journeys. The possibility of such journeys results not solely from the fact that Newtonian theory allows arbitrarily fast causal signals, for even very slow signals can make transits of the universe, given enough time—and if the affine connection is complete, there is always enough time.

But let us look at the case of light. If space is not simply connected—if it is a 3-torus or, let us say, the topological product of a cylinder and the reals—then there will be points p, q and spatial paths α, β such that light can leave p and reach q either by α or by β, and the path $\alpha\beta^{-1}$ will not be

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homotopic to a constant map. Thus, as long as there are sources, "ghost images" of the sources will be observable in principle, and the pattern of such images will be determined by the fundamental group of space. For example, suppose space has topology $S \times R \times R$. If an observatory and a star are located on the same cylinder, then light from the star can reach the observatory by (1) going in either of two directions from the source to the observatory, but not spiraling completely around the cylinder, or (2) spiraling completely around the cylinder (in either of two directions) any finite number of times before reaching the observatory. Light that spirals around n times will be dimmer than light that spirals around m times, $m < n$, because it will have traveled farther. Moreover, if m is a large number, the images from m and $m + 1$ spiral paths will appear closer together than will the images of m and $m + 1$ spiral paths for small m. Thus the images of the star will appear to us roughly as they do in the accompanying picture.

Topologies with a different fundamental group will produce other distinctive patterns. Special relativity, too, admits alternative topologies, but in this case we do not have a classification theorem. If, however, we consider only those product topologies $V \times R$, where R is timelike, then V must be a three-dimensional Euclidean space form. Now in special relativity there are not arbitrarily fast causal chains, but still we can, in principle, always determine something about the global topology of space-time. Let us write $X << Y$ if there is a future-directed timelike path from X to Y. Let $I^-(Y)$ be $\{X \in M: X << Y\}$ (and similarly, $I^+(Y) = \{X \in M: Y << X\}$). Consider the projection map $p: V \times R \rightarrow V$ given by $p(v, r) = v$. Then for any $Y \in V \times R$, $p(I(Y)) = V$. This means that at any time an observer in a special relativistic cosmology could in principle determine his topology in exactly the same way that an observer in a Newtonian cosmology with the same topology might do.

In Newtonian and special relativistic cosmologies we are not presented with causally inaccessible regions. In general relativity we often are, even in the simplest of cases. The Schwarzchild solution already contains a region of space-time from which, once inside it, nothing can escape. Early writers on cosmology, Weyl (1922) and Tolman (1934) for example, recognized that the determination of the curvature of space would not of itself determine the global topology of space. They assumed, however, that global topology could be determined by the appearance of ghost images and other phenomena and they did not entertain the possibility that causal inaccessibility might prevent us from making the global discriminations possible in classical cosmologies. To investigate the question we need a precise notion of what it might mean for two general relativistic cosmologies to be indistinguishable. Clearly they must be locally isometric; but as Marder (1962) has shown, that is not sufficient. Now the events of which an observer can have knowledge are exhausted by what happens at points connected to his world-line by future-directed timelike or null curves. So an intuitive requirement for indistinguishability is that there be local isometries which extend over the whole causal past of any world line. This idea can readily be made more precise.

By a space-time we shall mean a four-dimensional differentiable manifold with a smooth pseudo-Riemannian metric form of Lorentz signature. Where convenient, we identify a curve with its image on a manifold.

Definition: Two isochronous space-times, M and N, are indistinguishable if and only if for every maximal curve, σ, on M, whose tangent vector field is everywhere timelike, there is a maximal curve, τ, on N whose tangent vector field is everywhere timelike and $I^-(\sigma) = U_{x\epsilon\sigma} I^-(x)$ is isometric to $I^-(\tau) = U_{y\epsilon\tau} I^-(y)$, and likewise with M and N interchanged.

Remark: $I^-(\tau)$ is an open set. As long as there are no maximal timelike curves with a future-most point, we need not explicitly consider points z connected to a point x by a future-directed null curve because for any isochronous space-time, if z is connected to x by a future-directed null curve, and x is connected to y by a future-directed timelike curve, then z is connected to y by a future-directed timelike curve (see Geroch and Penrose, 1972). It should be clear that indistinguishability is an equivalence relation.

With a little strengthening, the usual cosmological assumptions amount to the requirement that space-time have a product structure, $M = V \mathbf{X} R$, where V is a complete three-dimensional Riemannian space of constant curvature, R the reals, and M carries the pseudo-Riemannian metric form $dt \otimes dt - R^2(t)\, dv \otimes dv$ where dt is the obvious 1-form on the reals, $dv \otimes dv$ is the Riemannian form on V, and $R(t)$ is a smooth function of the real variable t. Such a cosmological model will be called *standard*. We note that all standard space-times are assumed complete. The propositions given subsequently are stated for standard models, but all of them, save the first, apply as well if we require only that the models be products $V \mathbf{X} I$ (where I is an interval of the reals) and hence not necessarily complete. Many of the most popular cosmological models, e.g., the Friedman models, satisfy this weaker condition.

The following simplification is elementary:

Proposition 1: Two standard space-times, M and N, are indistinguishable if and only if for every $x \epsilon M$ $(y \epsilon N)$ there is a $y \epsilon N$ $(x \epsilon M)$ such that $I^-(x)$ is isometric to $I^-(y)$.

It is easy to give conditions sufficient for a standard model to have an indistinguishable but nonhomeomorphic counterpart.[1] If $M = V \mathbf{X} R$ is standard and (1) Δ is a nontrivial group of isometries acting freely and properly discontinuously on M; (2) for every $\delta \epsilon \Delta$ there is an isometry g on V such that $\delta(v, t) = (g(v), t)$; and (3) for every $\delta \epsilon \Delta$ and for every $x \epsilon M$, if $\delta \neq 1$ implies that $I^-(x) \cap I^-(\delta x)$ is empty, then M is indistinguishable from the quotient space-time M/Δ. Such conditions are not, however, sufficiently informative; we should like, in addition, quasi-local topological conditions sufficient to guarantee that a standard model has a covering from which it is indistinguishable. Ideally such conditions should represent the kind of information given by ghost images. It is evident that in general no purely topological conditions together with local isometry will be necessary and sufficient for two standard space-times to be indistinguishable.

Consider a point, x, in a standard model $M = V \mathbf{X} R$. Let $\alpha: [0, 1] \rightarrow M$, $\beta: [0, 1] \rightarrow M$ be curves such that (1) $\alpha(0) = \beta(0) = x$; (2) $\alpha(1) = \beta(1) \epsilon I^-(x)$; and (3) the tangent vector fields to α and β are timelike. The product curve $\alpha\beta^{-1}$ is then a closed loop through x. Let C be the set of all closed loops formed from all pairs of curves, α, β, meeting the above conditions, together with the constant map $e: [0, 1] \rightarrow \{x\}$. C is understood to contain

$\beta\alpha^{-1}$ if it contains $\alpha\beta^{-1}$. Now form the class, S, of all products of loops in C. Just as in homotopy theory, we proceed to define an equivalence relation on the curves in S and turn the set of equivalence classes into a group. We take two curves in S to be equivalent if they are homotopic by a homotopy every curve of which is itself in S. That is, curves $\omega, \tau \in S$ are equivalent if and only if there is a continuous map F: $[0, 1] \mathbf{X} [0, 1] \to M$ such that

$$F(0, t) = \omega(t)$$
$$F(1, t) = \tau(t)$$
$$F(t, 0) = x$$
$$F(t, 1) = x$$

and for every $u \in [0, 1]$ the curve $\sigma_u(t) = F(u, t)$ is in S. Denote the equivalence class of $\sigma \in S$ by $[\sigma]$ and define the product $[\sigma] \cdot [\tau]$ of $[\sigma]$ and $[\tau]$ to be $[\sigma\tau]$, the equivalence class of the product curve $\sigma\tau$, and similarly define $[\sigma]^{-1}$ to be $[\sigma^{-1}]$. The standard treatments of the fundamental group readily show that the relation on S given above is an equivalence relation, that the product and inverse operations on the equivalence classes are well defined, and that the equivalence classes form a group under the operations. We denote the group thus defined by "$r(x)$."

Proposition 2: Let $M = V \mathbf{X} R$ be a standard model, $x \in M$, p: $M \to V$ the projection map. Then p induces a surjective group homomorphism $p_*: r(x) \to \pi_1(pI^-(x), px)$.

Proof: Since for $\alpha, \beta \in S$, the projection map p generates a homotopy of $p\alpha$ and $p\beta$ from a homotopy of α and β, it is obvious that the map p_* taking the homotopy class of α to the homotopy class of $p\alpha$ is well defined and a homomorphism. To show that p_* is surjective, we argue that any generator of $\pi_1(pI^-(x), px)$ contains a curve that is the composition of two other curves which are the projections of suitable timelike future-directed curves in $I^-(x)$.

Since the space-times in question are standard, whether or not a differentiable curve α in $pI^-(x)$ ending in $p(x)$ is the projection of a timelike future-directed curve in $I^-(x)$ depends only on the length of α. If $pI^-(x)$ is bounded, then there will be a number $r > 0$ such if α has length $< r$, then α is the projection of a timelike curve. In fact, if t_x is the time coordinate of x, we may set $r = \displaystyle\int_{-\infty}^{t_x} \frac{dt}{R(t)}$. (If $I^-(x)$ is not bounded, i.e., $pI^-(x) = V$, then p_* is obviously surjective.) If U is the universal Riemannian covering of V, then any arc-component of $pI^-(x)$ will be an open r-ball in U. Let q: $U \to V$ be the covering map, and let $\tilde{X} \subset U$ be an arc-component of $q^{-1}pI^-(x)$ and denote by $\tilde{x}$ the pre-image of px in $\tilde{X}$. If g is a generator in $\pi_1(pI^-(x), px)$, then there is a curve $\alpha \in g$ such that α is the composition of $q(\tilde{\alpha}_1)$, $q(\tilde{\alpha}_2)$ where $\tilde{\alpha}_1$, $\tilde{\alpha}_2$ are geodesic segments in $\tilde{X}$. $\tilde{\alpha}_1$ is an arc beginning at $\tilde{x}$ and ending at a point $\tilde{a} \in X$; $\tilde{\alpha}_2$ is an arc beginning at $\tilde{b} \in \tilde{X}$ and ending at $\tilde{x}$; and $q(\tilde{a}) = q(\tilde{b})$. The length of $\tilde{\alpha}_1$ and of $\tilde{\alpha}_2$ is less than r in each case, and the same must be true of $q(\tilde{\alpha}_1)$ and $q(\tilde{\alpha}_2)$. Thus there are future-directed timelike curves s_1, s_2 in $I^-(x)$ that project onto $q(\tilde{\alpha}_1)$ and $q(\tilde{\alpha}_2)$ respectively, and the element of $r(x)$ containing $s_1 \cdot s_2^{-1}$ is mapped to g by p_*. It follows that p_* is surjective.

The group $r(x)$ could, ideally, be calculated from the information available from ghost images. In Newtonian cosmology $r(x)$ is necessarily isomorphic to the fundamental group of space, but in general relativistic cosmology it need not be. When it is not, the next result gives us a partial classification of indistinguishable counterparts.

Proposition 3: Let $M = V \mathbf{X} R$ be a standard space-time with expansion function $R(t)$. Suppose that for all $x \in M$, $i_*p_*r(x)$ is a proper subgroup of $\pi_1(V, px)$, where i: $pI^-(x) \to V$ is the inclusion map. Let G be any normal subgroup of $\pi_1(V, v)$ such that for all $z \in M$ with $pz = v$, G contains a conjugate of $i_*p_*r(z)$. Then there is a standard space-time $N = \tilde{V} \mathbf{X} R$ indistinguishable from M, and $\pi_1(\tilde{V})$ is isomorphic to G.

Proof: Given a subgroup G of $\pi_1(V, v)$, there is a covering $(\tilde{V}, q)$, q: $\tilde{V} \to V$, of V such that for $q(\bar{v}) = v$, $q_* \pi_1(\tilde{V}, \bar{v}) = G$[2] where q_* is the injective homomorphism of the fundamental group induced by q. We claim, first, that if G is a subgroup of $\pi_1(V, v)$ satisfying the hypothesis of the theorem and $(\tilde{V}, q)$ a covering of the kind just mentioned, then for any $z \in M$ such that $pz = v$, $pI^-(z)$ is an admissible set. To show this, it suffices to prove that q, when restricted to any arc-component of $pI^-(z)$, has a continuous inverse.

$pI^-(z)$ is a connected, locally arc-wise connected, open set. Let $\tilde{A}$ be an arc-component of $q^{-1}pI^-(z)$ and let $\tilde{v} \in \tilde{A}$ be such that $q(\tilde{v}) = v = pz$. Let $\tilde{b}$, $\tilde{c} \in \tilde{A}$ and suppose that $q(\tilde{b}) = q(\tilde{c}) = b \in V$. Choose arcs $\tilde{\beta}$, $\tilde{\gamma}$, in $\tilde{A}$ from $\tilde{v}$ to $\tilde{b}$ and to $\tilde{c}$ respectively. Then letting $\beta = q(\tilde{\beta})$ and $\gamma = q(\tilde{\gamma})$, $\beta\gamma^{-1}$ is a loop in $pI^-(z)$ through $pz = v$. The collection of all subgroups $q_*\pi_1(\tilde{V}, \tilde{x})$ for $\tilde{x} \in q^{-1}(v)$ is exactly a conjugacy class of subgroups of $\pi_1(V, v)$.[3] By the hypothesis of the theorem, $i_* p_* r(z)$ is conjugate to a subgroup of $G = q_* \pi_1(\tilde{V}, \tilde{v})$, so by proposition 2, $i_* \pi_1(pI^-(z), pz)$ is conjugate to a subgroup of G. Now the homotopy class $[\beta\gamma^{-1}]$ of $\beta\gamma^{-1}$ is an element of $i_* \pi_1(pI^-(z), pz)$, and therefore there must be some $\tilde{x} \in \tilde{V}$ such that $[\beta\gamma^{-1}] \in q_* \pi_1(\tilde{V}, \tilde{x})$. Since q_* is injective, the corresponding lift of $\beta\gamma^{-1}$ must be closed in $\tilde{V}$. But G is normal, and hence either every lift of $\beta\gamma^{-1}$ is closed or no lift is closed; therefore every lift of $\beta\gamma^{-1}$ is closed. This proves that $\tilde{b} = \tilde{c}$ and hence q restricted to any arc-component of $q^{-1}pI^-(z)$ is injective. Since q is an open continuous map, the restriction of q to any arc-component of $q^{-1}pI^-(z)$ is a homeomorphism.

V is a differentiable manifold, and there is a unique differentiable structure on $\tilde{V}$ for which q is a differentiable map of maximal rank.[4] We take $\tilde{V}$ to be endowed with this structure and, letting u be the metric form on V, define a Riemannian metric $\tilde{u}$ on $\tilde{V}$ by $\tilde{u}(X, Y) = u(q_*X, q_*Y)$ for all vectors, X, Y, in the tangent space of any point in $\tilde{V}$. Then every admissible set is isometric to any of its arc-components, and $\tilde{V}$ is a Riemannian space of the same constant curvature as V. The group D of deck transformations of $(\tilde{V}, q)$ are isometries acting freely and properly discontinuously on $\tilde{V}$; since $pI^-(z)$ is admissible for any $z \in M$, for any arc-component $\tilde{A}$ of $pI^-(z)$, and any $d \in D$, $d \neq I$, $d\tilde{A} \cap \tilde{A}$ is empty.

Now consider the space-time $N = \tilde{V} \mathbf{X} R$ with the same expansion function, $R(t)$, as obtains on M. We claim that N is indistinguishable from M. The group Δ of isometries of N of the form $\delta(\tilde{v}, t) = (d\tilde{v}, t)$ for $d \in D$ acts freely and properly discontinuously on N. Thus if ϕ: $N \rightarrow M$ is the map defined by $\phi(\tilde{v}, t) = (q(\tilde{v}), t)$, (N, ϕ) is a pseudo-Riemannian covering of M and $M = N | \Delta$, the quotient of N by Δ. It follows from the property of the group D established in the preceding paragraph that for every $y \in N$ and $\delta = (d, I) \in \Delta$, if δ is not the identity, then $I^-(y) \cap I^-(\delta y)$ is empty and $I^-(y)$ is an arc-component of an admissible set $I^-(\phi(y))$ of M. Thus for each $y \in N$ there is an $x \in M$ such that $I^-(y)$ is isometric to $I^-(y)$, and, conversely, so by proposition 1 M and N are indistinguishable.

Proposition 4: Let $M = V \mathbf{X} R$ be a spatially compact standard space-time; suppose compact V has zero curvature and for all $x \in M$, $r(x)$ is trivial. Then for any (topological) space-form S admitting a metric of zero curvature, there is a space-time $N = S \mathbf{X} R$ indistinguishable from M.

Sketch of Proof: By proposition 3 we know that because $r(x)$ is trivial for all $x \in M$, so that $i_* p_* r(x)$ is the identity element and thus a normal subgroup

of $i_* \pi_1(pI^-(x), px)$, M is indistinguishable from its universal covering $U = R^3 \mathbf{X} R$. The idea is that for every Euclidean space-form S there is a space-time that is topologically $S \mathbf{X} R$ and indistinguishable from U and hence from M. This will follow if the projection on R^3 of the chronological past of every world-line in U is suitably bounded.

Let q: $R^3 \rightarrow V$ be a Riemannian covering map. From the proof of proposition 3 we know that the map ϕ: $U \rightarrow M$ given by $\phi(a, t) = (q(a), t)$ is a covering taking the chronological past of every world-line on U isometrically onto the chronological past of some world-line in M. For every γ on M, then, $I^-(\gamma)$ must be an admissible set for the covering (U, ϕ); it follows that $pI^-(\gamma)$ must be an admissible set for the covering (R^3, q) of V, so $pI^-(\gamma)$ must be simply connected.

Let $B(b, r)$ be an open ball in R^3 with radius r containing an arc-component of $pI^-(\gamma)$. Then $B(b, r)$ must also contain the projection $pI^-(\sigma)$ for some world-line σ on U. Moreover, since the volume of $pI^-(x)$ for $x \in M$ is a function only of the time coordinate and does not depend on the location of px in space, the same value of r may be chosen for every world-line σ on M. In fact, r may be taken to be the length of any curve in a generator of the fundamental group of V. It follows that for every world-line σ on U, $pI^-(\sigma)$ is contained in an open ball of radius r.

Now let S' be a (topological) Euclidean space-form. S' is homeomorphic to a space S that is the quotient of R^3 by some group G of isometries of R^3. Every such group is described by a finite set of generators and their relations so that if two sets of generators satisfying these relations generate groups G, G', then R^3/G is diffeomorphic to R^3/G'. For each group, the generators consist of translations and possibly compositions of translations and rotations, and there are no restrictions on how large the translations may be; that is, for any positive n, we can choose translations t in the generators so that the distance from x to $t(x)$ is at least n. Thus we may choose generators so that the shortest distance a point is moved by any generator (and hence by any element of G other than the identity)[5] is at least n. In particular, for any space-form S' we may choose G so that $S = R^3/G$, with S homeomorphic to S', and the shortest distance any point in R^3 is moved by any element of G other than the identity is $2r$.

The quotient space $S = R^3/G$ admits a differentiable and metric structure such that the covering map q: $R^3 \rightarrow S$ is differentiable, of maximal rank, and a local isometry. Consider the space-time $N = S \mathbf{X} R$ with the same expansion function as U. Exactly as in the proof of proposition 3, we may define a covering of N by U in terms of the covering of S by R^3, show that for all $y \in N$, $I^-(y)$ is admissible, and thus prove that N and U are indistinguishable. It follows that N and M are indistinguishable, since U is indistinguishable from M and indistinguishability is an equivalence relation.

Proposition 5: Let $M = S^3 \mathbf{X} R$ be standard, and suppose that the length (in radians) of the projection on S^3 of every null geodesic on M is less than or equal to π/n, $n > 0$. Then M is indistinguishable from a space-time $N = S^3/Z_m \mathbf{X} R$, where Z_m is a cyclic group of isometries of order $m \leq n$. If $M = S^3/Z_n \mathbf{X} R$ and $r(x)$ is trivial everywhere, then M is indistinguishable from $N = S^3/Z_m \mathbf{X} R$, $m \leq n$.

The proof of proposition 5 is omitted, since it involves no new ideas and is immediate from the structure of the groups of isometries (given in Wolf, 1967, p. 224). It should be noted that certain global assumptions will reduce or eliminate the variety of indistinguishable space-times. If it is required that space-times be standard and satisfy the global cosmological principle—that is, that the group of global isometries of space act transitively—then any two indistinguishable space-times of constant negative space curvature are isometric. The possible topologies for standard space-times of zero curvature are reduced to $R^m \mathbf{X} T^{3-m} \mathbf{X} R$, where $m < 3$ and T^{3-m} is the $(3-m)$-dimensional torus. Calabi and Marcus (1962) have shown that if the global perfect cosmological principle—that the group of space-time isometries act transitively—is introduced, then the only complete standard space-times of constant positive curvature are the De Sitter space-times.

We note some examples. The De Sitter model is a hyper-hyperboloid in five-dimensional Minkowski space, with the metric induced therefrom. The metric can be given the standard form

$$ds^2 = dt^2 - \cosh^2(t)\,(dX^2 + \sin^2 X(d\Omega^2)),\ 0 \leq X \leq \pi.$$

Consider the class of models, (D, n), of this kind given by the family of expansion functions

$$\cosh^2(nt)$$

where n is a positive integer. The maximum spatial coordinate distance traveled by a light beam leaving a point on the equatorial sphere is

$$\int_0^\infty \frac{dt}{\cosh(nt)} = \frac{\pi}{2n}.$$

Thus the maximum spatial distance traveled in all of time is just π/n. By proposition 5 we have that the models (D, n) are indistinguishable from models with topology $S^3/Z_m \mathbf{X} R$, $m \leq n$.

Consider the family of spatially open, Euclidean models, M, with cosmological constant $\Lambda \geq -8\pi\rho_{00}$. The metric form can be written

$$ds^2 = dt^2 - e^{1/2g(t)}(dx_1^2 + dx_2^2 + dx_3^2)$$

and we have (see Tolman, 1934, p. 403) the differential equation

$$\frac{de^{1/2g(t)2}}{dt} = \frac{8\,\pi\rho_{00}}{3}\,e^{g(t)} + \frac{\Lambda}{3}\,e^{g(t)}$$

which integrates to

$$g = 1/2kt + d$$

d a constant, $k = \frac{8\pi\rho_{00}}{3} + \frac{\Lambda^{1/2}}{3}$. The radial velocity of a light ray is therefore

$$dr/dt = \pm e^{-1/4(kt+d)}$$

and hence the coordinate distance traveled in all of time by a light ray leaving its source at an arbitrary time is finite. So by the argument of proposition 4, for every time t, and for every Euclidean space-form V, there is a space-time N that is topologically $V \mathbf{X} R$ and is indistinguishable from the space-time M_t obtained by deleting from M all points occurring at time t or earlier. All of these space-times are incomplete, but since they are strongly causal it follows from the work of Clarke (1970) that they can be made null complete by a conformal change in the metric.

Notes

1. I shall assume that the reader is familiar with the standard terminology and facts about covering spaces. See, for example, Wolf (1967), section 1.8.
2. See Wolf (1967), p. 39.
3. See Wallace (1967), p. 155.
4. Wolf (1967), p. 41.
5. I omit the argument (for seventeen different cases) that the generators can be so chosen that if n is the shortest distance a point is moved by any generator in G, then every element of G other than the identity moves every point at least a distance d.

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DAVID MALAMENT

Observationally Indistinguishable Space-times

In his paper "Indistinguishable Space-times and the Fundamental Group"[1] Clark Glymour poses a criterion for the observational indistinguishability of space-time models and presents two sets of examples from the subclass of Robertson-Walker models. The underlying idea is quite intuitive.

In some space-time models studied in relativity theory any particular observer can receive signals from, and hence directly acquire information about, only a limited region of space-time. This happens, for instance, in a rapidly expanding universe in which galaxies that might try to signal one another are actually receding from one another at velocities approaching that of light. It may turn out in these cases that the information from that limited region of space-time which any one observer can have access to is compatible with quite different overall space-time structures. Two space-times are observationally indistinguishable under Glymour's criterion if, for precisely these reasons, no observer in either space-time would have grounds for deciding which of the two, if either, was his. No observer would be able to discriminate observationally between the two even if he did nothing but sit and record signals beamed at him from all directions all day long, even if the signals themselves coded all the spatio-temporal information that the sender had to offer, and even if the observer lived eternally.

Glymour is proposing a reason why the spatio-temporal structure of the universe might be underdetermined by all observational data that we could ever, even just in principle, obtain. Some claims of underdetermination in science are of a very general sort, to the effect that no body of evidence will ever force a particular scientific hypothesis upon us

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