A Stringy Mechanism for a Small Cosmological Constant

Henry Tye
with Yoske Sumitomo

Institute for Advanced Study, Hong Kong University of Science and Technology
Cornell University

August 26, 2012

Workshop on Cosmic Acceleration, Carnegie Mellon University, Pittsburgh, PA

This talk is based on work with **Yoske Sumitomo** :
arXiv:1204.5177 and a forthcoming paper

Applied to :
Large Volume Flux Compactification Scenario in Type IIB String Theory

Balasubramanian and Berglund, hep-th/0408054
Balasubramanian, Berglund, Conlon and Quevedo, hep-th/0502058
Westphal, hep-th/0611332
de Alwis and Given, arXiv:1106.0759
Rummel and Westphal, arXiv:1107.2115

Background

- There is very strong evidence that we are living in a de-Sitter vacuum with a very small positive cosmological constant Λ,

$$\Lambda \sim +10^{-122} M_P^4$$

- Why dark energy contributes 70% of the content of our universe ?

Background

- There is very strong evidence that we are living in a de-Sitter vacuum with a very small positive cosmological constant Λ,

$$\Lambda \sim +10^{-122} M_P^4$$

- Why dark energy contributes 70% of the content of our universe ? Why not 99.999999....999999% ?

Background

- There is very strong evidence that we are living in a de-Sitter vacuum with a very small positive cosmological constant Λ,

$$\Lambda \sim +10^{-122} M_P^4$$

- Why dark energy contributes 70% of the content of our universe ? Why not 99.999999....999999% ?
- There is strong evidence that our universe has gone through an inflationary period, when the vacuum energy is below the Planck scale but much higher than the TeV scale.

What is considered to be a natural explanation for the observed dark energy ?

- Given the scale of the underlying theory, how the observed value emerges ?
  E.g., In gravity, we have M_P, so we have to explain why

$$\Lambda \sim +10^{-122} M_P^4$$

What is considered to be a natural explanation for the observed dark energy ?

- Given the scale of the underlying theory, how the observed value emerges ?
  E.g., In gravity, we have M_P, so we have to explain why

$$\Lambda \sim +10^{-122} M_P^4$$

- In another theory with a different scale, it must generate both M_P and Λ.

What is considered to be a natural explanation for the observed dark energy ?

- Given the scale of the underlying theory, how the observed value emerges ?
  E.g., In gravity, we have M_P, so we have to explain why

$$\Lambda \sim +10^{-122} M_P^4$$

- In another theory with a different scale, it must generate both M_P and Λ.
  E.g., Quintessence: To start, one has to put $\Lambda = 0$. That is a fine-tuning.

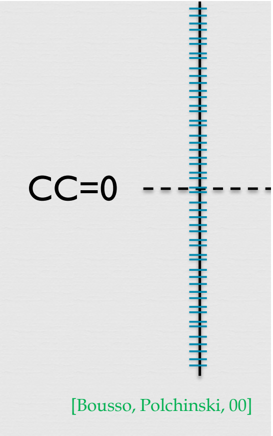
What is considered to be a natural explanation for the observed dark energy ?

- Given the scale of the underlying theory, how the observed value emerges ?
  E.g., In gravity, we have M_P, so we have to explain why

$$\Lambda \sim +10^{-122} M_P^4$$

- In another theory with a different scale, it must generate both M_P and Λ.
  E.g., Quintessence: To start, one has to put $\Lambda = 0$. That is a fine-tuning.
  E.g., String theory has string scale M_S, so it must generate both M_P and Λ from M_S.

The situation in string theory : J types of 4-form fluxes $F^i_{\mu\nu\rho\sigma}$



$$\lambda = \lambda_{\text{bare}} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2 \ .$$

Pressing Question

String theory may have 10^{500} possible solutions. Surely many will have Λ at about the right value.

Pressing Question

String theory may have 10^{500} possible solutions. Surely many will have Λ at about the right value.

Why nature picks such a very small positive Λ ?

Pressing Question

String theory may have 10^{500} possible solutions. Surely many will have Λ at about the right value.

Why nature picks such a very small positive Λ ?

- We present a Stringy Mechanism why a very small Λ may be preferred.

Pressing Question

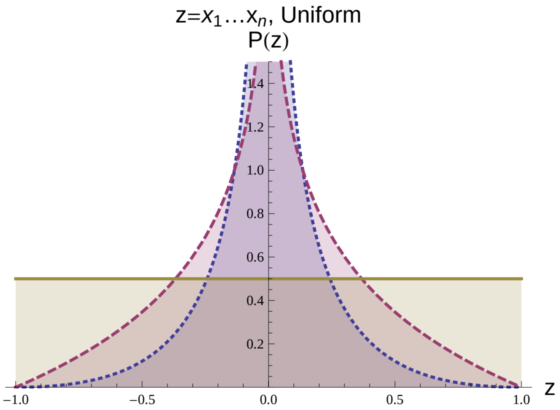
String theory may have 10^{500} possible solutions. Surely many will have Λ at about the right value.

Why nature picks such a very small positive Λ ?

- We present a Stringy Mechanism why a very small Λ may be preferred.

The Basic Idea is very simple :
Consider a set of random variables x_i ($i = 1, 2, ..., n$). Let the probability distribution of each x_i be uniform in the range $[-1, +1]$. What is the probability distribution of their product ?

Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$



$$P(z) = \frac{1}{2(n-1)!}\left(\ln\frac{1}{|z|}\right)^{n-1}$$

Basic Idea

Let x_j to have a uniform distribution $f(x_j) = 1$ between 0 and 1. What is the probability distribution $P(z)$ of the product $z = x_1 x_2$?

$$P(z) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x_1 x_2 - z) = \int_z^1 dx_1 \frac{1}{x_1} = \ln \frac{1}{z}$$

for $0 \leq z \leq 1$.

Basic Idea

Let x_j to have a uniform distribution $f(x_j) = 1$ between 0 and 1. What is the probability distribution $P(z)$ of the product $z = x_1 x_2$?

$$P(z) = \int_0^1 dx_1 \int_0^1 dx_2\, \delta(x_1 x_2 - z) = \int_z^1 dx_1 \frac{1}{x_1} = \ln \frac{1}{z}$$

for $0 \leq z \leq 1$.

For $z = x_1 x_2 ... x_n$, we have

$$< z^N > = < x_1^N > < x_2^N > \cdots < x_n^N >$$

Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$

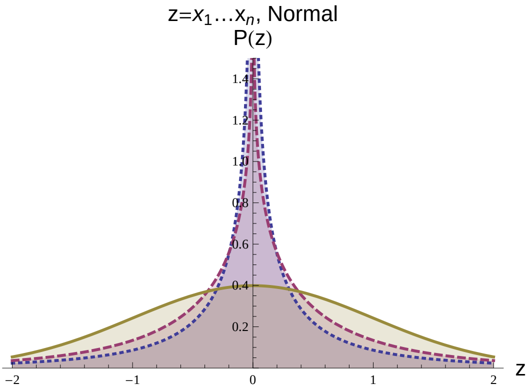
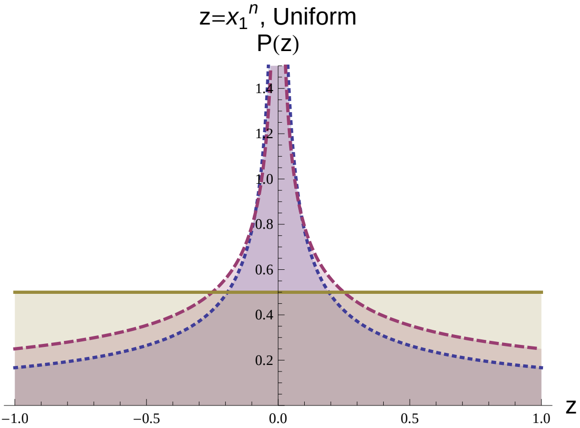


Figure: The product distribution $P(z)$ is for $z = x_1$ (solid brown curve for normal distribution), $z = x_1 x_2$ (red dashed curve), and $z = x_1 x_2 x_3$ (blue dotted curve), respectively. In general, the curves are given by the Meijer-G function.

Probability distribution $P(z)$ for $z = x_1^n$



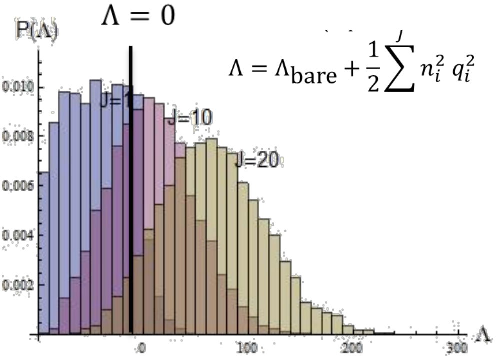
$$P(z) \sim z^{-1+1/n}$$

Probability distribution $P(z)$

z	Asymptote of $P(z)$ at $z=0$
$x_1 \cdots x_n$	$(\ln(1/\|z\|))^{n-1}$
x_1^n	$z^{-1+1/n}$
$x_1^n \cdots x_m^n$	$z^{-1+1/n}(\ln(1/\|z\|))^{m-1}$
$x_1^m x_2^n$	$(z^{-1+1/m} - z^{-1+1/n})/(m-n)$
$x_1 \cdots x_m \;/\; y_1 \cdots y_n$	$(\ln(1/\|z\|))^{m-1}$
x_1^m / y_1^n	$z^{-1+1/m}$
$x_1^{n_1} + \cdots + x_m^{n_m}$	$z^{-1+1/n_1+\cdots 1/n_m}$
$x_1 x_2$, $0 < c = x_1/x_2 < \infty$	smooth
$x_1 x_2$, $0 \leq c = x_1/x_2$ or $c \leq \infty$	$\ln(1/\|z\|)$

Non-interacting case: e.g., Sum of terms

No peaking behavior for $P(\Lambda)$ if Λ is a sum of terms.



Toy Model

$$V(\phi) = a\phi - \frac{b}{2}\phi^2 + \frac{c}{3!}\phi^3$$

If ϕ is arbitrary $\rightarrow P(V = \Lambda)$ is smooth at $\Lambda = 0$.

Toy Model

$$V(\phi) = a\phi - \frac{b}{2}\phi^2 + \frac{c}{3!}\phi^3$$

If ϕ is arbitrary $\rightarrow P(V = \Lambda)$ is smooth at $\Lambda = 0$.
Bring in dynamics : stabilize at meta-stable vacuum

$$\Delta \equiv \sqrt{b^2 - 2ac} > 0, \quad \phi_{\min} = \frac{b + \Delta}{c}$$

$$\Lambda \equiv V_{\min} = \frac{(b + \Delta)^2 (b - 2\Delta)}{6c^2}$$

$\rightarrow P(\Lambda)$ peaks at $\Lambda = 0$.

Toy Model

$$V(\phi) = a\phi - \frac{b}{2}\phi^2 + \frac{c}{3!}\phi^3$$

If ϕ is arbitrary $\rightarrow P(V = \Lambda)$ is smooth at $\Lambda = 0$.
Bring in dynamics : stabilize at meta-stable vacuum

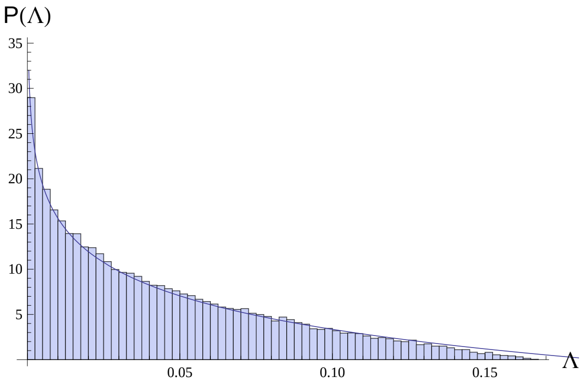
$$\Delta \equiv \sqrt{b^2 - 2ac} > 0, \quad \phi_{\rm min} = \frac{b + \Delta}{c}$$

$$\Lambda \equiv V_{\rm min} = \frac{(b+\Delta)^2(b - 2\Delta)}{6c^2}$$

$\rightarrow P(\Lambda)$ peaks at $\Lambda = 0$.

Add a constant v_0 to $V(\phi) \rightarrow P(\Lambda)$ is smooth at $\Lambda = 0$.

Preference for Small Λ



Applied to String Theory

- Treat all parameters that enter into the model as random variables with some probability distributions, say uniform within a certain range. Almost all can take zero values.

Applied to String Theory

- Treat all parameters that enter into the model as random variables with some probability distributions, say uniform within a certain range. Almost all can take zero values.
- Solve for the meta-stable vacuum, so all moduli are determined in terms of the parameters.

Applied to String Theory

- Treat all parameters that enter into the model as random variables with some probability distributions, say uniform within a certain range. Almost all can take zero values.
- Solve for the meta-stable vacuum, so all moduli are determined in terms of the parameters.
- Determine Λ in terms of the parameters (i.e., random variables) and find the probability distribution $P(\Lambda)$ for Λ.

Applied to String Theory

- Treat all parameters that enter into the model as random variables with some probability distributions, say uniform within a certain range. Almost all can take zero values.
- Solve for the meta-stable vacuum, so all moduli are determined in terms of the parameters.
- Determine Λ in terms of the parameters (i.e., random variables) and find the probability distribution $P(\Lambda)$ for Λ.
- If the functional form of Λ is a single product (or something equivalent), then we have a peaked probability distribution $P(\Lambda)$. (This is necessary but not sufficient for a small Λ.)

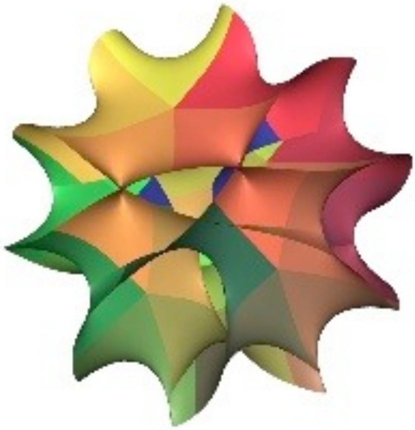
Applied to String Theory

- Treat all parameters that enter into the model as random variables with some probability distributions, say uniform within a certain range. Almost all can take zero values.
- Solve for the meta-stable vacuum, so all moduli are determined in terms of the parameters.
- Determine Λ in terms of the parameters (i.e., random variables) and find the probability distribution $P(\Lambda)$ for Λ.
- If the functional form of Λ is a single product (or something equivalent), then we have a peaked probability distribution $P(\Lambda)$. (This is necessary but not sufficient for a small Λ.)
- The expected value/magnitude of Λ can be exponentially small as the number of such moduli (and parameters) increases.

The Large Volume Scenario in Type IIB String Theory

$$
\begin{aligned}
V =& e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right), \\
K =& -2\ln(\mathcal{V} + \alpha'^3 \hat{\xi}/2) \\
\mathcal{V} =& \gamma_1 (T_1 + \bar{T}_1)^{3/2} - \sum_{i=2} \gamma_i (T_i + \bar{T}_i)^{3/2}, \\
\alpha'^3 \hat{\xi} =& -\frac{\zeta(3)\chi(M)}{4\sqrt{2}(2\pi)^3} (\frac{S+\bar{S}}{2})^{3/2}, \\
W =& W_0(U_i, S) + \sum_{i=1}^{N_K} A_i e^{-a_i T_i}, \\
W_0(U_i, S) =& c_1 + \sum_j b_j U_j - s(c_2 + \sum_j d_j U_j)
\end{aligned} \tag{1}
$$

Calabi-Yau 3-fold



Executive summary

- Consider the Large Volume Scenario (LVS) in Type II B string theory.

Executive summary

- Consider the Large Volume Scenario (LVS) in Type II B string theory.
- Besides the dilation S, introduce $N_K = h^{1,1}$ number of Kähler moduli T_i and $n = h^{2,1}$ number of complex structure moduli U_i. (Note $\chi(M) = 2(h^{1,1} - h^{2,1})$.)

Executive summary

- Consider the Large Volume Scenario (LVS) in Type II B string theory.
- Besides the dilation S, introduce $N_K = h^{1,1}$ number of Kähler moduli T_i and $n = h^{2,1}$ number of complex structure moduli U_i. (Note $\chi(M) = 2(h^{1,1} - h^{2,1})$.)
- All parameters introduced are treated as random variables with smooth probability distributions that include zero. Determine Λ of meta-stable vacuum in terms of them.

Executive summary

- Consider the Large Volume Scenario (LVS) in Type II B string theory.
- Besides the dilation S, introduce $N_K = h^{1,1}$ number of Kähler moduli T_i and $n = h^{2,1}$ number of complex structure moduli U_i. (Note $\chi(M) = 2(h^{1,1} - h^{2,1})$.)
- All parameters introduced are treated as random variables with smooth probability distributions that include zero. Determine Λ of meta-stable vacuum in terms of them.
- The functional form of Λ on the parameters are more complicated than a simple product. However, some of them will still lead to the peaking property in the probability distribution $P(\Lambda)$ for Λ.

Executive summary

- Consider the Large Volume Scenario (LVS) in Type II B string theory.
- Besides the dilation S, introduce $N_K = h^{1,1}$ number of Kähler moduli T_i and $n = h^{2,1}$ number of complex structure moduli U_i. (Note $\chi(M) = 2(h^{1,1} - h^{2,1})$.)
- All parameters introduced are treated as random variables with smooth probability distributions that include zero. Determine Λ of meta-stable vacuum in terms of them.
- The functional form of Λ on the parameters are more complicated than a simple product. However, some of them will still lead to the peaking property in the probability distribution $P(\Lambda)$ for Λ.
- The expected value/magnitude of Λ becomes exponentially small as the number of complex structure moduli increases.

Typical Manifolds Studied

Manifold	$N_K = h^{1,1}$	$n = h^{2,1}$	χ
$\mathcal{P}^4_{[1,1,1,6,9]}$	2	272	-540
$\mathcal{F}_{11}$	3	111	-216
$\mathcal{F}_{18}$	5	89	-168
$\mathcal{CP}^4_{[1,1,1,1,1]}$	1	$\mathcal{O}(100)$	$\mathcal{O}(-200)$

Single Kähler Modulus Model

- Consider the superpotential

$$W = W_0 - Ae^{-x}$$

where W_0 and A are parameters and x is a Kähler modulus.

- A stable vacuum can exist at $x = x_m$

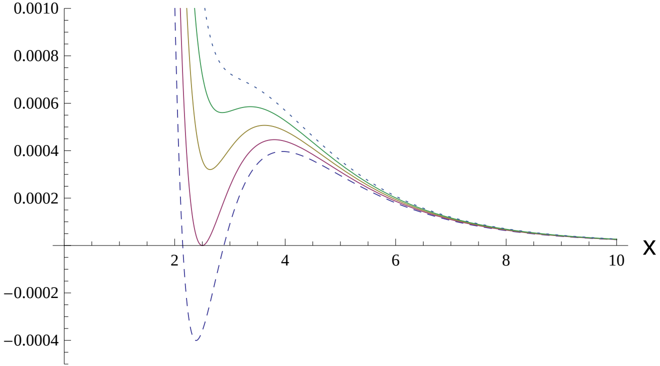
$$\Lambda = V_{min} = BW_0A\hat{\xi}(x_m - 2.5)$$

where B is a constant.

- Let us treat W_0 and A as random variables where $W_0/A \sim C$ is constrained: $3.65 \lesssim C \lesssim 3.89, \quad 2.50 \leq x_m \lesssim 3.11$
- What is the probability distribution $P(\Lambda)$ of Λ ?

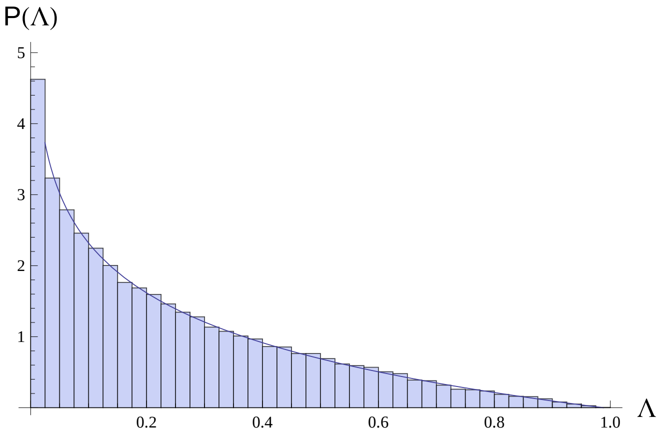
The form of $V(x)$ for the Single Kähler Modulus Case

$$\frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}$$



$$W_0 A_1 \leq 0$$

$P(\Lambda)$ for the Single Kähler Modulus Case



Multi-Complex Structure Moduli case

Now consider the case with n complex structure moduli U_i + the dilaton S + 1 Kähler modulus x. We solve for U_i and S at the supersymmetric point and then insert the resulting $W_{0,min}$ into the Kähler uplift case to solve for x. There are $2n + 6$ parameters.

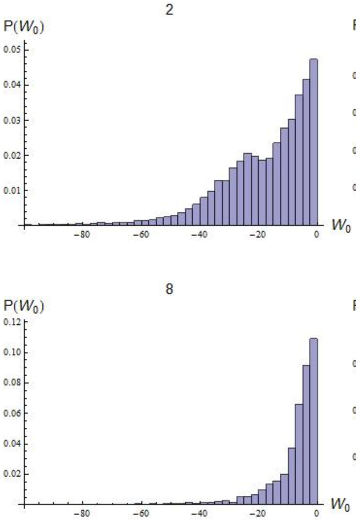
Multi-Complex Structure Moduli case

Now consider the case with n complex structure moduli U_i + the dilaton S + 1 Kähler modulus x. We solve for U_i and S at the supersymmetric point and then insert the resulting $W_{0,min}$ into the Kähler uplift case to solve for x. There are $2n+6$ parameters.

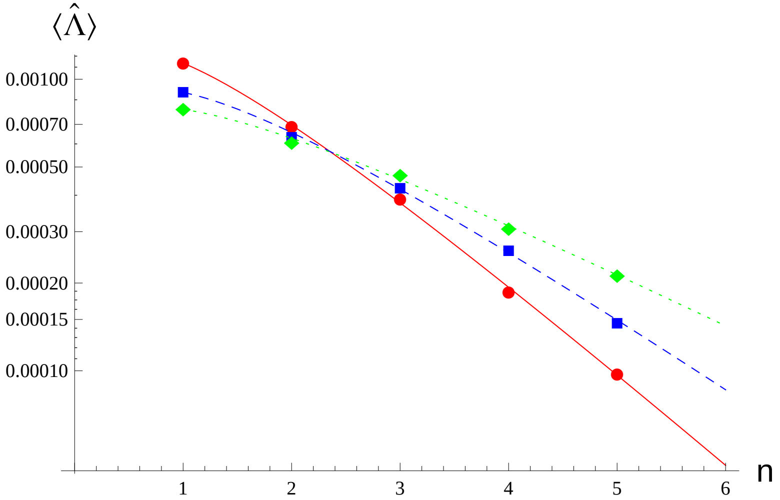
$$W_0|_{\min} = \frac{2(c_1 + sc_2)\Pi_1^n(b_i - sd_i)}{\sum_i (b_i + sd_i)\Pi_{j\neq i}(b_j - sd_j)}$$

$$\Lambda = \frac{e^{-5/2}}{9}\left(\frac{2}{5}\right)^2 \frac{-W_0|_{\min} a_1^3 A_1}{\gamma_1^2}\left(x_m - \frac{5}{2}\right)$$

Probability Distribution $P(W_{0,min}) \rightarrow P(\Lambda)$



Expectation value of Λ



Expectation value of Λ

When the parameters in the model are treated as random variables with uniform distributions in the range $[-1, 1]$:

$$\begin{aligned} \langle|\Lambda|\rangle_{N_K=1} =& 0.00251 n^{0.436} e^{-0.791 n}, \\ \langle|\Lambda|\rangle_{N_K=2} =& 0.00170 n^{0.457} e^{-0.633 n}, \\ \langle|\Lambda|\rangle_{N_K=3} =& 0.00125 n^{0.342} e^{-0.464 n}. \end{aligned}$$

Expectation value of Λ

When the parameters in the model are treated as random variables with uniform distributions in the range $[-1,1]$:

$$\begin{aligned} \langle|\Lambda|\rangle_{N_K=1} &= 0.00251 n^{0.436} e^{-0.791 n}, \\ \langle|\Lambda|\rangle_{N_K=2} &= 0.00170 n^{0.457} e^{-0.633 n}, \\ \langle|\Lambda|\rangle_{N_K=3} &= 0.00125 n^{0.342} e^{-0.464 n}. \end{aligned}$$

We need about 350 complex structure moduli to reach $\Lambda \sim 10^{-122} M_P^4$.

Expectation value of Λ

When the parameters in the model are treated as random variables with uniform distributions in the range $[-1, 1]$:

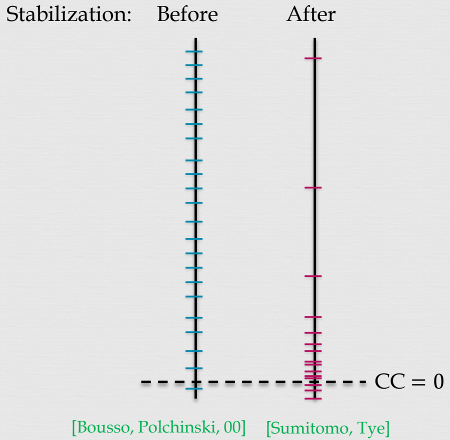
$$\begin{aligned}\langle|\Lambda|\rangle_{N_K=1} =& 0.00251 n^{0.436} e^{-0.791 n}, \\ \langle|\Lambda|\rangle_{N_K=2} =& 0.00170 n^{0.457} e^{-0.633 n}, \\ \langle|\Lambda|\rangle_{N_K=3} =& 0.00125 n^{0.342} e^{-0.464 n}.\end{aligned}$$

We need about 350 complex structure moduli to reach $\Lambda \sim 10^{-122} M_P^4$.
If the parameters A_i for the Kähler moduli are also peaked, as expected, then we may need a lot less than 350.

Summary and Remarks

- At high vacuum energies, no stable vacua
- At lower vacuum energies, heavy stabilized moduli may be frozen
- At very low vacuum energies, meta-stable vacua begin to appear

New Picture


Stabilization:
Before
After
CC = 0
[Bousso, Polchinski, 00]
[Sumitomo, Tye]

Summary and Remarks

- At high vacuum energies, no stable vacua
- At lower vacuum energies, heavy stabilized moduli may be frozen
- At very low vacuum energies, meta-stable vacua begin to appear

The picture is very encouraging. Many directions to explore :

- What is the back-reaction due to SUSY breaking ?
- What about higher (α' and loop) corrections ?
- How about the light moduli problem ?
- What about other compactification (e.g., KKLT) scenarios ?