Brane Localization and Stabilization via Regional Physics

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Outline

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Why Extra Dimensions and Braneworlds?

They are an interesting possibility and may offer an explanation of some outstanding issues in fundamental physics, e.g.

- Weak Hierarchy ($M_{\text{Weak}} \ll M_{pl}$): "Large" Extra Dimensions (Arkani-Hamed, et al. '98, Randall & Sundrum '99,...)
- Dark Energy ($\rho_{\text{DE}} \ll M_{Weak}^4$): Modifying Gravity in the IR with infinite extra d's (DGP '00, de Rham, et al. '07,...)

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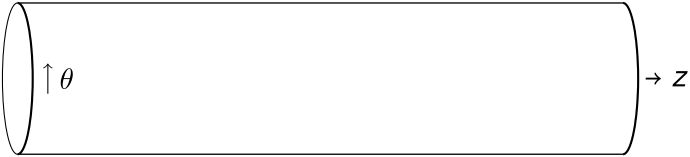
A Simple Example
The Cylinder

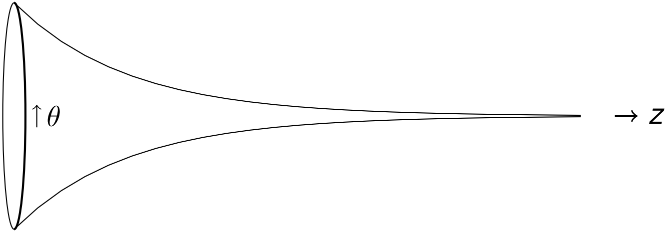
- A cylinder – one dimension of E^2 compactified



- Locally homogeneous and isotropic, however the interactions with fields tell you that isotropy is broken

A More Interesting Example
The 2d Horn

- Contrast with the 2d horn ($\mathcal{H}^2/\Gamma$), obtained by compactifying one dimension of the hyperbolic space ($\mathcal{H}^2$)

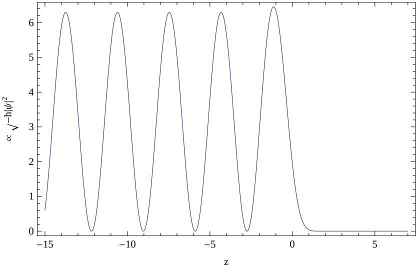


- Again, the geometry is homogeneous and isotropic, but now fields can tell you that *both* symmetries are broken
- Result: “Things look different" depending on *where* you are

The 2d Horn

An Example Wavefunction

- Typical probability density for low-energy bulk wavefunctions:

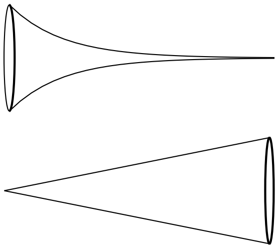


- Suppressed when wavelengths $>$ horn circumference, affecting e.g. interaction between brane and bulk fields

In general...

- **Generic Point:** Even if the geometry is locally homogeneous and isotropic, the physics may very well not be because it is sensitive to global conditions
- In particular, position (of e.g. a brane) can be physically relevant, and valuable for model building!

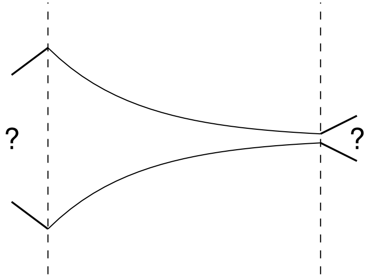
The 2d Horn and Cone



Nice models for more generic spaces because

- one is curved, the other is flat
- one is infinite in extent, one ends at a vertex
- both break translation invariance (important later)
- both have “large" and “small" regions
- Field modes can be solved for analytically

The Horn/Cone as a Regional Approximation



- On more general manifolds some *regions* may be well-approximated by a horn or cone
- *Regional* physics – depends on more than the local geometry, but not necessarily on the full manifold structure

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Motivations (Braneworld Scenarios)

- Regional features of the bulk manifold will affect, e.g.
  - probability for the brane fields to interact with bulk fields
  - the apparent dimensionality of the spacetime
- So there are two questions:
  - So, where is the brane? (localization)
  - What keeps it there? (stabilization)
    Constraint: no massless scalars

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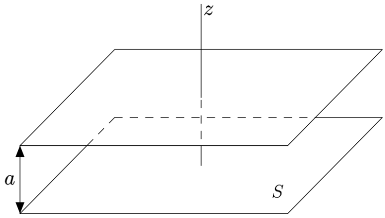
The Casimir Effect
Canonical Example: Parallel Conducting Plates

Bordag, et al. (2009)



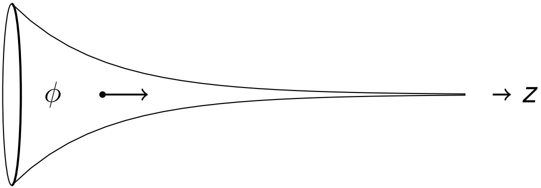
Energy/area = $-\frac{\pi^2}{720}\frac{\hbar c}{a^3}$ Pressure = $-\frac{\pi^2}{240}\frac{\hbar c}{a^4}$

- Force on plates because seperation, $a \Rightarrow \{k\}$ and $E_0 = {}^1\!/_2 \Sigma \hbar\omega(k)$
- Since $E(a)$, the force, $F \sim -\frac{d}{da}E(a)$

Casimir Energy

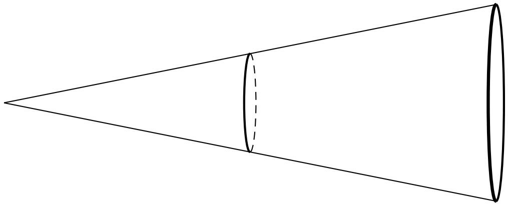
Forces on Branes from Bulk Fields

- Bulk fields satisfying boundary conditions on the brane can provide a position-dependent Casimir energy/force



Energy from Brane Geometry

- If the brane geometry is non-trivial, e.g. when it wraps



- Energy from brane tension acts to shrink brane
- Energy from its extrinsic curvature acts to flatten the brane
- These can be position-dependent

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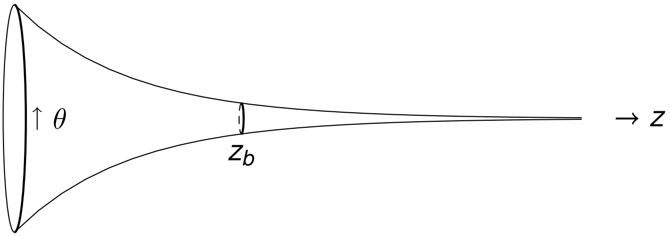
The Models

$\mathcal{M}^4 \times \mathcal{H}^2/\Gamma$ (Hyperbolic Horn)



- The full spacetime manifold is $\mathcal{M}^4 \times \mathcal{H}^2/\Gamma$

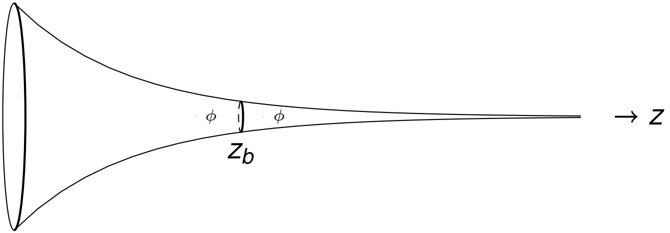
$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2z/z_\star} z_\star^2 d\theta^2 + dz^2$$

- SM fields can propagate in θ (it's "universal"), but in z they're confined to a codimension-1 brane at z_b

The Models

$\mathcal{M}^4 \times \mathcal{H}^2/\Gamma$ (Hyperbolic Horn)

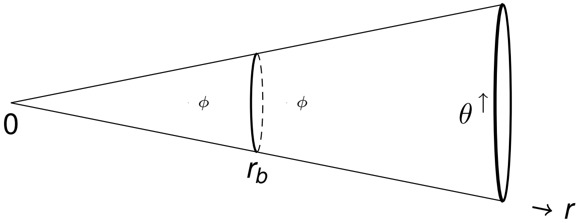
- We assume a bulk scalar field, ϕ



- Brane boundary conditions (e.g. $\phi(z_b) = 0$) determine the spectrum, hence affect the energy in quantum fluctuations

The Models
$\mathcal{M}^4 \times$ Cone

Same setup as horn



- Flat space, but $\theta \leftrightarrow \theta + 2\pi\,(1-\delta)$

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + r^2 d\theta^2$$

Local (Geometric) Energies of the Brane

$$E_{\text{ten}} = \int d^4x \sqrt{|\gamma|}\sigma$$

$$E_{\text{curv}} = \int d^4x \sqrt{|\gamma|} \left(h_1 K^2 + h_2 K_{ab} K_{ab} + \ldots \right)$$

Casimir Energy

- Vacuum Energy in bulk field, ϕ

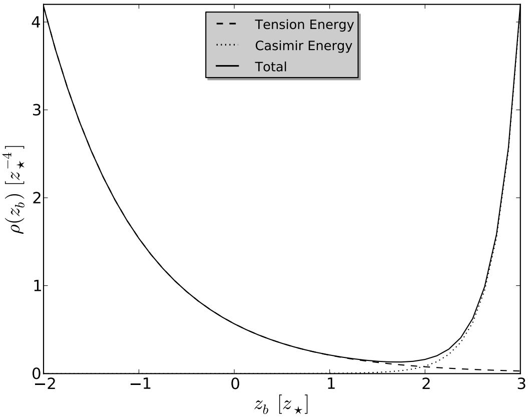
$$E_0 = \frac{1}{2}\sum_{\mathbf{i}} \omega_{\mathbf{i}}$$

- Zeta fn. regularization used to tame divergences

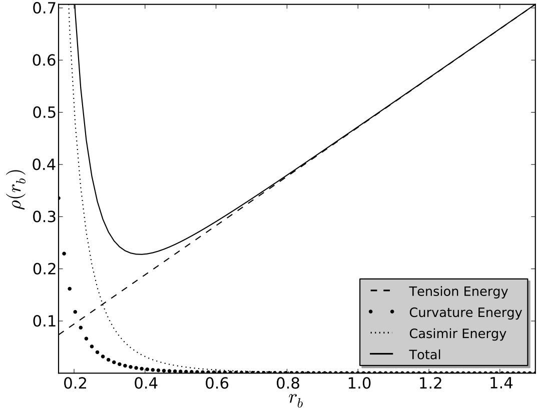
$$E_0 = \lim_{s\to 0} E_0(s) \equiv \lim_{s\to 0} \frac{\mu^{2s}}{2}\sum_{\mathbf{i}} \omega_{\mathbf{i}}^{1-2s}$$
$$= E_0^{\text{div}}(s) + E_0^{\text{finite}}$$

$E_0^{\text{div}}(s) \sim \frac{1}{s}$, but is actually irrelevant for the Casimir force.

Brane Potential (Horn, Dirichlet)



Brane Potential (Cone, Dirichlet)



Summary

- In a general braneworld scenario, it is very relevant where the brane resides – can use non-trivial bulk manifolds for interesting model building
- Different contributions to the brane position effective potential shown to provide stability by pitting *at least two* effects against each other
- Casimir is particularly important, as it might be the only mechanism available for a codimension-d brane (with d extra dimensions), i.e. point-like in the bulk