Operational Modeling of the Nexus Between Water-Distribution and Electricity Systems

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Abstract

Purpose of Review: We survey operational models of water-distribution systems. Although such modeling is important in its own right, our focus is motivated by the growing desire to examine and manage the nexus between water-distribution and electricity systems. As such, our survey discusses computational challenges in modeling water-distribution systems, co-ordination dynamics between water-distribution and electricity systems, and gaps in the literature.
Recent Findings: Modeling water-distribution systems is made difficult by their highly non-linear and non-convex physical properties. Co-ordinating water-distribution and electricity systems, especially with the growing supply and

demand uncertainties of the latter, requires fast optimization techniques for real-time system management. Although many works suggest means of co-ordinating the two systems, practical applications are limited, due to the systems having separate and autonomous management and ownership. Nonetheless, recent works are navigating this challenge, by seeking methods to foster improved co-ordination of the two systems while respecting their autonomy. Additionally, with the backdrop of increased security threats, there is a growing need to bolster infrastructure protection, which is complicated by the intertwined nature of the two systems.
Summary: By providing a steady supply of potable water to satisfy residential, commercial, agricultural, and industrial demands, water-distribution systems are pivotal components of modern society and infrastructure. The extant literature presents many models and optimization strategies that are tailored for operating water-distribution systems. Yet, there remain unexplored problems, particularly related to simplifying model computation, capturing the flexibility of water-distribution systems, and capturing interdependencies between water-distribution and other systems and infrastructures. Future research that addresses these gaps will allow greater operational efficiency and resilience.

Keywords: Water-system operations, electricity-system operations, linearization, convexification

Introduction

By providing a reliable supply of clean and safe water to urban and rural areas, water-distribution systems are critical infrastructures that are necessities of modern life. These systems consist of numerous interconnected components, including pipes, pumps, valves, tanks, and reservoirs. Water-distribution systems are being expanded and developed continually, which reflects population and water-demand growth.

Modeling the operation of water-distribution systems is challenging, due to many non-linear and non-convex constraints being needed to represent the physical properties of the network. Moreover, operational modeling typically entails representing the operation of many system components (*e.g.*, pipes, pumps, tanks, and valves). The task becomes more complex if incorporating real-life considerations, such as uncertainty and construction staging. As such, improved mathematical techniques that simplify and speed these modeling exercises are needed.

Typically, electricity that is used to pump water represents the largest operational cost of a water-distribution system. As such, optimizing water-pump operations is a major concern in optimizing water-distribution systems, and this problem is a focus of the existing literature [1–7]. This concern is becoming more acute, as electricity prices continue to rise due to fuel cost and electricity decarbonization.

Optimizing electricity consumption can yield efficiencies and cost savings in operating a water-distribution system. In addition, there can be benefits to the electricity system that serves the water-distribution system. This benefit stems from using flexibility in the operation of the water-distribution system as a source of electricity-demand response [8]. In addition to economic-efficiency benefits, electricity-demand flexibility can help to integrate renewable-energy sources into the electricity-supply mix more efficiently [9–11]. Given this synergy, there are numerous works that examine co-optimized or co-operative operation of water-distribution and electricity systems to glean these benefits [12–22]. Such co-operation can be difficult to achieve in-practice, because normally water-distribution and electricity systems have independent owners, operators, and regulators. As such, there are works that examine different means of co-ordination between the two systems that respect their institutional autonomy [23–27]. As one example, Benders's decomposition can be employed as an algorithmic approach to co-ordinate the two systems while maintaining independent objectives and privacy of the two systems' operators [28, 29].

In addition to system efficiency, there is a growing need to bolster and protect infrastructures, which arises from increasing natural and human-caused attacks against them. These issues are complicated for water-distribution and electricity systems due to their interdependencies, which mean that an attack against one system can impact the other [27]. As such, there are works that examine interdependent infrastructures, including some that focus specifically upon the case of water-distribution and electricity systems [30, 31]. Many of these works examine approaches to protect system components or restore service following system disruptions.

These efficiency, reliability, and resilience benefits of improved water-system operation motivates the development of water-system models. Such tools allow researchers, policy-makers, and owners, operators, and regulators of water systems to understand the benefits and challenges from improved water-system operation. This understanding can include co-operative or co-ordinated operation of water-distribution and electricity systems. The aim of this paper is to provide a comprehensive survey of the literature concerning operational modeling of water-distribution systems. The survey includes a discussion of existing modeling and algorithmic approaches, and identification of challenges and gaps in the extant literature, which can help guide future research.

The remainder of this paper is organized as follows. We begin with an introduction to common components of water-distribution systems, which is followed by summaries of classes of operational models for water-distribution systems. This review includes models for water-distribution systems only and models that operate such systems in a co-operative or co-ordinated fashion with electricity systems. Next, we discuss challenges in modeling water-distribution systems, including complexities that arise from fluid dynamics and resultant non-linearities and non-convexities. The subsequent section provides a critical assessment of selected works, with a focus on their strengths, limitations, and the quality of the proposed models and optimization approaches. This section highlights also potential areas for future research. Finally, a concluding section summarizes the main challenges, identifies research gaps, and provides insights into future perspectives in modeling water-distribution systems. This section also summarizes the contribution of our survey towards advancing the modeling of water-distribution systems, especially within the context of their interactions with electricity systems.

Common Components of Water-Distribution Systems

We begin with an introduction to the common components, which include junctions, pipes, valves, reservoirs, and tanks, that constitute water-distribution systems. Understanding these components and their physical properties is an important step towards modeling water-distribution systems.

Pipes and Junctions

Two fundamental components of water-distribution systems are pipes and junctions. The former transport and distribute water, thereby ensuring its safe and efficient delivery from source to demand points. The choices of water-pipe material and dimension are critical for a water-distribution system being able to function effectively.

Junctions are points at which pipes intersect or branch from one another. As such, junctions allow the distribution of water from and to different points in the water-distribution system and are crucial in defining a system's structure and design. Water demands at junctions are modeled to enforce mass-conservation constraints at each junction. These constraints ensure that water flow into and out from each junction are equal to one another during all times. Water demands of junctions at which there is no water consumption are fixed equal to zero. It is common to model pressure head at junctions and to impose constraints on the minimum and maximum pressure head at each junction. The pressure-head difference between a pipe's inlet and outlet junctions provides the motive force to move water through the pipe.

Equality constraints are used to compute the pressure-head losses between the inlet and outlet junctions of pipes. These losses are caused primarily by friction within the pipe, which causes lower water pressure at the outlet junction of a pipe. The most common equations to compute pressure-head loss are Hazen-Williams and Darcy-Weisbach equations, both of which are discrete-time steady-state simplifications of fluid-dynamic models. The key parameters that govern these relationships are estimated empirically.

To specify these equations, we begin by defining $\mathcal{I}$ as the set of junctions in the water-distribution system. Next, we define $\mathcal{J}^P$ to be the set of pipes, whereby $(i,j) \in \mathcal{J}^P$ if there is a pipe in the water-distribution system that has i and j, respectively, as its inlet and outlet junctions. With these indices, $\forall (i,j) \in \mathcal{J}^P$, we let $F^D_{i,j}$ be the diameter (measured in m), $F^{F,H}_{i,j}$ be the unitless Hazen-Williams friction parameter, and $F^L_{i,j}$ be the length (measured in m) of pipe (i,j). These are fixed parameters that depend upon pipe material and dimension. Next, we let $\mathcal{T}$ denote the set of time steps in the model. With these definitions, $\forall t \in \mathcal{T}$ and $\forall (i,j) \in \mathcal{J}^P$, we let $L_{i,j,t}$ denote the time-t pressure-head loss (measured in m) within pipe (i,j). The time-t Hazen-Williams equation for pipe (i,j) is:

$$10.67 F^L_{i,j} \cdot (Q_{i,j,t})^{1.85} = \left(F^D_{i,j}\right)^{4.8655} \cdot \left(F^{F,H}_{i,j}\right)^{1.85} L_{i,j,t}; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^P; \quad (1)$$

where $Q_{i,j,t}$ is the time-t water flow within the pipe (measured in m^3/s) [1, 4, 7, 14, 17, 18, 20, 23, 27]. To specify the Darcy-Weisbach equation, $\forall (i,j) \in \mathcal{J}^P$ we define

$F^{F,D}_{i,j}$ as the unitless Darcy friction parameter of pipe (i, j). With this definition, the time-t Darcy-Weisbach equation for pipe (i, j) is:

$$8F^L_{i,j}F^{F,D}_{i,j} \cdot (Q_{i,j,t})^2 = \pi^2 g \cdot \left(F^D_{i,j}\right)^5 L_{i,j,t}; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^P; \tag{2}$$

where g is the gravitational constant and π has the standard definition as the ratio between the circumference and diameter of a circle [3, 6, 13, 15, 16, 19, 21, 22, 26, 28, 29]. Because (1) and (2) are highly non-linear and non-convex, some works use quadratic approximations to model pressure-head loss [2, 32].

Using the pressure-head loss, as computed by (1), (2), or an approximation, the pressure-head levels at junctions can be computed. To give this relationship, $\forall i \in \mathcal{I}$, we let $\bar{H}_i$ be the elevation of junction i and $\forall t \in \mathcal{T}$, we let $H_{i,t}$ be the time-t pressure head of junction i (all measured in m). Then, we have that:

$$H_{i,t} - H_{j,t} + \bar{H}_i - \bar{H}_j = L_{i,j,t}; \forall t \in \mathcal{T}; \forall (i,j) \in \mathcal{J}^P.$$

Most works assume that pipes have fixed water-flow directions. Recent advances relax this assumption and allow for bi-directional flows that can be determined endogenously within an operational model [6, 13, 16, 21, 26]. Another recent advance is capturing dynamics, *e.g.*, transfer delays [22]. Water-transfer delays arise from continuous spatio-temporal variation of flow—when an input flow is introduced at the inlet junction of a pipe, a non-trivial amount of time may be needed for output flow at the outlet junction to match the input flow.

Pumps

With the exception of gravity-fed water-distribution systems (*i.e.*, systems that rely upon elevation differences between the inlet and outlet junctions of pipes), pumps are needed to pressurize water and provide motive force to move it from source to demand junctions. Pump operation is captured by computing the pressure-head gain or water-pressure increase that the pump induces. Pump-operation models distinguish between fixed- and variable-speed pumps.

The operation of a fixed-speed pump is modeled typically using a quadratic relationship. To specify this relationship, we reuse the notation that is used in (1). In addition, we define $\mathcal{J}^M$ to be the set of pumps, whereby $(i, j) \in \mathcal{J}^M$ if there is a pump in the water-distribution system that has i and j, respectively, as its inlet and outlet junctions. Finally, $\forall (i, j) \in \mathcal{J}^M$ and $\forall t \in \mathcal{T}$, we let $P_{i,j,t}$ represent the time-t pressure-head gain (measured in m) that is induced by pump (i, j). With these definitions we have the equalities:

$$P_{i,j,t} = P^a_{i,j} \cdot (Q_{i,j,t})^2 + P^b_{i,j} Q_{i,j,t} + P^c_{i,j}; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^M; \tag{3}$$

where $P^a_{i,j}$, $P^b_{i,j}$, and $P^c_{i,j}$ are fixed parameters that represent the physical capabilities of the pump and are measured empirically [1, 2, 13–16, 19, 22, 26, 33, 34]. Some works use cubic polynomials to compute pressure-head gain [35], whereas others use linear relationships [21, 29]. Another approach to modeling pressure-head gain is to ignore completely its relationship with water flow [6]. With such an approach, the pressure-head gain is constant when the pump is operating and zero otherwise. Some real-world water-distribution systems have a bypass valve that is connected in parallel to a pump. As such, if the pump is not operating, water can flow freely in either direction through the bypass valve. Some works that model such bypass valves assume no pressure-head loss when it is in-use, whereas others model pressure-head loss in the same manner as for a standard pipe.

For variable-speed pumps, the common equation to represent the time-t head gain that is induced by pump (i,j) is:

$$P^a_{i,j} \cdot (Q_{i,j,t})^2 + P^b_{i,j} Q_{i,j,t} W_{i,j,t} + P^c_{i,j} \cdot (W_{i,j,t})^2 = P_{i,j,t} \qquad (4)$$
$$0 \leq W_{i,j,t} \leq \bar{W}_{i,j};$$

$\forall t \in \mathcal{T}$ and $\forall (i,j) \in \mathcal{J}^M$, where $W_{i,j,t}$ measures the time-t p.u. (relative to its maximum) operating speed of pump (i,j) and $\bar{W}_{i,j}$ is the pump's maximum p.u. operating speed [3–5, 7, 14, 17, 18, 20, 23, 27, 28].

Based on the pressure-head gain, whether computed using (3), (4), or another relationship, the pressure-head difference between the inlet and outlet junction of a pump can be computed as:

$$H_{j,t} - H_{i,t} + \bar{H}_j - \bar{H}_i = P_{i,j,t}; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^M.$$

To compute the electricity consumption of water pumps, $\forall (i,j) \in \mathcal{J}^M$ and $\forall t \in \mathcal{T}$, we define $E_{i,j,t}$ as time-t electricity consumption (measured in MWh) of pump (i,j). $E_{i,j,t}$ for fixed- and variable-speed pumps is calculated normally using the relationship:

$$\eta^p_i E_{i,j,t} = \rho g P_{i,j,t} Q_{i,j,t}; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^M; \qquad (5)$$

where η^p_i is a p.u. measure of pump (i,j)'s efficiency, ρ is the density of water (measured in kg/m^3), and g is the standard gravitational constant (measured in m/s^2)[3]. Most works assume that pumps are operational during all times, whereas others model the on/off status of pumps [2, 3, 5, 6, 13, 14, 16, 18, 22, 24, 26, 29]. Optimizing the on/off status of pumps endogenously within a model requires the use of binary variables.

Valves

Valves play an essential role in helping to maintain suitable water pressure within a water-distribution network, by allowing pressure relief when and where it is necessary. Depending upon their designs, valves can be controlled remotely (based on observed water-network conditions) or may operate on a predefined schedule. Works that model valves account for their pressure-reducing effect by representing the head loss that is induced by the valve between its inlet and outlet junctions [3, 15, 18, 22, 26, 28]. Some works account for the on/off status of valves and minimum and maximum water flows through valves while they are operational [3, 22].

A standard relationship to give the pressure-head loss that is induced by a valve is:

$$H_{i,t} - H_{j,t} + \bar{H}_i - \bar{H}_j = V_{i,j,t}; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^V;$$

where $\mathcal{J}^V$ is the set of valves, whereby $(i,j) \in \mathcal{J}^V$ if there is a valve in the water-distribution system that has i and j, respectively, as its inlet and outlet junctions, and $V_{i,j,t}$ is the time-t pressure-head loss (measured in m) that is induced by valve (i,j). An advanced design could allow a valve to regenerate energy through the pressure reduction that is provides. As an example, a valve could be replaced with a so-called pump as turbine [18, 36].

Tanks

Tanks are used to store water, which can be used to meet subsequent demand. Water storage can be economically beneficial (*e.g.*, to manage the cost of operating the water-distribution system) or can serve as a safety stock to manage emergency conditions. Many operational models of water-distribution networks include tanks as a key system component [1–7, 14–24, 26–29]. The change in the volume of stored water that is in a tank is determined by the difference in inflow and outflow water volume. The outlet pressure is related to the water level of the tank. Most works assume that the inlets and outlets of water tanks are completely open, while others model explicitly the opening and closing of tank inlets and outlets [6, 14, 18].

The key relationship that is needed to represent tanks is that between pressure-head levels of its inlet and outlet junctions and water flow through the tank's inlet and outlet junctions. We let $\mathcal{Z}$ be the set of tanks. For all $z \in \mathcal{Z}$, tank z has inlet and outlet junctions, which we denote as $i(z)^+$ and $i(z)^-$, respectively. In addition, we define $\mathcal{J}^{z,+}$ and $\mathcal{J}^{z,-}$ to be, respectively, the set junctions from which water flows directly to junction $i(z)^+$ and to which water flows directly from junction $i(z)^-$. For all $z \in \mathcal{Z}$, we define K_z^A as the cross-sectional area (measured in m^2) and $\bar{K}_z$ as the capacity (measured in m^3) of tank z. The standard relationship between pressure-head levels and water flows is:

$$K_z^A \cdot \left(H_{i(z)^-,t} - H_{i(z)^-,t-1}\right) = \Delta_{\mathcal{T}} \cdot \left(\sum_{j \in \mathcal{J}^{z,+}} Q_{j,i(z)^+,t} - \sum_{j \in \mathcal{J}^{z,-}} Q_{i(z)^-,j,t} \right); \quad \forall t \in \mathcal{T}; z \in \mathcal{Z};$$

where $\Delta_{\mathcal{T}}$ is a conversion factor to account for the duration of a time step in defining $\mathcal{T}$. Many models have hourly time steps whereas water flow is measured in m^3/s, which gives $\Delta_{\mathcal{T}} = 3600$ as the appropriate conversion factor.

In addition, it is common to have the outlet-junction pressure head no greater than the inlet-junction pressure head:

$$H_{i(z)^-,t} \leq H_{i(z)^+,t}; \forall t \in \mathcal{T}; z \in \mathcal{Z};$$

and a capacity constraint on the water levels of tanks:

$$0 \leq K_z^A H_{i(z)^-,t} \leq \bar{K}_z; \forall t \in \mathcal{T}; z \in \mathcal{Z}.$$

Reservoirs

In most cases, reservoirs are sources of treated water that meets the standards for direct use. Typically, reservoirs are intended to provide sufficient storage to accommodate major water-demand fluctuations, firefighting needs, and emergencies, such as breakdowns or repairs of water-distribution networks. Reservoirs help also to stabilize pressures within the water-distribution system. Consequently, most models treat reservoirs as infinite water sources with a constant pressure head of zero.

Water-Treatment and -Desalination Plants

Some works include other water sources—in addition to or *in lieu* of reservoirs—such as water-treatment and -desalination plants [7, 17]. Typically, both types of plants employ high-pressure pumps and reverse-osmosis membrane filtration systems. Reverse osmosis is a widely adopted technology for medium- to large-scale water-treatment and -desalination plants. During reverse osmosis, pressurized wastewater or seawater that is fed into the plant is separated into a freshwater permeate stream and a concentrated brine retentate stream.

In terms of modeling, water-treatment and -desalination plants typically are represented similarly. This is because both facilities utilize reverse-osmosis technology to process wastewater and seawater. A key distinction between the types of plants is different salinity coefficients for wastewater and seawater. These salinity coefficients impact directly the amount of energy that is required to purify the water, making it essential to account for them in a model that represents energy use [17].

Classes of Operational Models for Water-Distribution Systems

This section surveys four common classes of operational models for water-distribution systems. The first considers a water-distribution system in isolation. The following two consider water-distribution and electricity systems, with different levels of coordination between the two systems. The final model class considers resilience of water-distribution systems to natural or human-caused attacks.

Water-Distribution-System Models

A number of works propose optimization approaches for scheduling water-system operations. Typically, these models are formulated to achieve cost-minimal operation of the system's components while respecting hydraulic constraints. A major optimization challenge is computational issues that arise from the complex fluid dynamics and the large physical scale that is typical of water-distribution systems.

Giacomello *et al.* [1] propose a hybrid optimization method using linear optimization and a greedy algorithm. This method is intended to provide high-fidelity pump-scheduling solutions in a computationally efficient manner. The method starts by solving a linear relaxation of the original nonlinear optimization problem. The solution that is obtained from the linearized model is used as a starting point for the greedy algorithm, with the goal of finding a near-globally optimal solution.

Naoum-Sawaya *et al.* [2] model the operation of a water-distribution system and replacement of pipelines. The model is solved using the EPANET simulation-optimization software package [37].

Oikonomou *et al.* [4] study the operation of a water-distribution system to provide electricity-demand response. They consider the flexibility that water tanks and variable-speed pumps provide. Their proposed approach consists of two optimization models—the first minimizes the operation cost of the water-distribution system. The second model maximizes the profit of the water-distribution system, with consideration of electricity-demand response and frequency regulation. In both cases, exogenous electricity-market prices are used as model inputs. This modeling approach is extended to study the flexibility of water-desalination plants [7].

Fooladivanda and Taylor [3] propose a model to optimize the operation of a water-distribution system, with consideration of variable-speed pumps and explicit modeling of their on/off status. They solve the resultant mixed-integer non-linear optimization problem by using a mixed-integer convex relaxation. Singh and Kekatos [6] extend the representation of non-convexities in operating water-distribution systems by considering the on/off status of fixed-speed pumps, valves, tanks, and water-flow directions on pipes that allow bi-directional flows. They formulate the problem as a mixed-integer second-order cone problem, which has feasibility and optimality guarantees under certain sufficient conditions.

Mkireb *et al.* [5] introduce a chance-constrained problem that determines optimal electricity-demand-response offers into an electricity market from a flexible water-distribution system with uncertain water demands. They examine flexibility that stems from fluctuating water demand and focus upon demand-response mechanisms that are employed in the French electricity market. The model seeks to minimize electricity costs that are related to water pumping and to maximize revenue from providing demand response.

Water-Distribution-System and Electricity-System Co-Optimization and Co-Ordination Models

A limitation of the aforementioned works is that they model a water-distribution system in isolation of the system that supplies it with electricity or they consider only a static representation of the electricity system (*e.g.*, through exogenous prices). To exploit fully the inherent flexibility of a water-distribution system to provide electricity-demand response, some form of co-optimization or co-ordination is beneficial. This co-optimization or co-ordination can be achieved by having a single decision-maker that operates the two systems. An alternative approach is to have autonomous decision-makers, with sufficient behavioral assumptions and information exchange to achieve such co-ordination. To this end, numerous works study such co-optimization or co-ordination [13–22, 25, 28].

Li *et al.* [13] investigate the use of 'hidden' controllable water loads, such as that of an irrigation system, as virtual energy storage. This use of such loads can increase the electricity-demand flexibility of a water-distribution system.

Oikonomou and Parvania [17] co-optimize the operation of both an electricity and water-distribution system. Their focus is flexible energy-intensive components, such as water-treatment and -desalination plants, of the latter system.

Moazeni *et al.* [14] optimize the daily electricity cost of islanded water-distribution and electricity systems. They consider energy-storage and electricity-generation units, including combined-heat-and-power, diesel- and natural-gas-fired, wind, and solar-photovoltaic technologies. Their model minimizes day-ahead operation costs with an optimal set of operation schedules for the two systems. Subsequent work expands this model by planning the number and locations of pump-as-turbine units, which regenerate electricity by relieving pipeline pressure [18]. They demonstrate that adding pump-as-turbine units yield operational-cost improvements to the joint electricity and water-distribution systems.

Yao *et al.* [22] model transfer delays in water-distribution systems (*e.g.*, delays in a pumping or another control action changing downstream water flows and pipeline pressures). They introduce a method which is based on an electricity/water analogy to model these delays. Their model aids the co-operative functioning of electricity and water-distribution systems, potentially increasing electricity-demand flexibility of the water-distribution system and reducing total operating costs.

Some works employ mathematical decomposition to solve the so-called optimal water-and-power flow problem [28, 29]. These decomposition techniques are distributed, in the sense that the two systems are operated autonomously. However, with proper privacy-preserving information exchange, these techniques achieve operating decisions that are optimal for the joint system.

Other works delve into ensuring stability under extreme conditions through co-optimization or co-ordination between electricity and water-distribution systems. One approach uses two-stage distributionally robust optimization [16, 19]. These works examine an integrated system, with consideration of electricity, water-distribution, and natural-gas infrastructures and electricity-supply uncertainty that is caused by renewable generating units. Zhao *et al.* [19] tackle the computational complexity of the resultant problem through the use of Benders's decomposition. Wang *et al.* [16] devise a two-step solution procedure that embeds a penalty convex-concave procedure into a column-and-constraint-generation algorithm. Numerical results demonstrate the economic value of co-optimizing the three systems [16, 19].

Stuhlmacher and Mathieu [15] employ chance-constrained optimization to design pumping schedules and real-time control mechanisms that respond to unpredictable water and electricity demands. They extend this work by examining a robust variant of a water-pumping problem [21]. This model includes electricity- and water-distribution-system constraints and aim to support electricity-system voltage levels and current frequency. They demonstrate the ability of the water-distribution system to provide multiple services simultaneously to the electricity system.

Alhazmi *et al.* [20] formulate an optimization model with the specific purpose of operating water-distribution and electricity systems under emergency conditions.

Their model aims to help system operators understand the static security of these integrated networks.

Models of Independent Water-Distribution and Electricity Systems

Co-optimization or perfect co-ordination between electricity and water-distribution systems is an ideal case, because it exploits fully the flexibilities of the two systems. However, the assumptions that underlie the analysis of such cases may be unrealistic, because in most cases electricity and water-distribution systems have different owners and operators. Moreover, the owners and operators of the two systems may have limited incentives (or strong disincentives) to share pertinent information that would be needed for the operations of the two systems to be co-ordinated. To address this reality, some works explore methods for co-ordinating the operations of these systems, while allowing for independent management and limited information exchange between the two. Collectively, these studies underline the potential benefits of adopting various co-ordination strategies between electricity and water-distribution systems, while preserving the independent ownership, operation, and management of each.

Oikonomou and Parvania [23] propose a flexibility model for a water-distribution system. The premise of their work is that the water-distribution-system operator offers to the electricity-system operator a flexible-capacity range of energy-demand levels that are feasible for operating the water-distribution system. In turn, the electricity-system operator provides to the water-distribution-system operator the lowest-cost water-pumping schedule that satisfies the flexible-capacity range.

Zuloaga *et al.* [24] and Zuloaga and Vittal [25] study co-ordination between an electricity and water-distribution system under constrained conditions, including water shortages and generator outages. In particular, they consider operational constraints that can arise due to limited cooling water for electricity generators and electricity for water pumps. They develop an integrated simulation engine, with which to examine long-term simulations for these interdependent infrastructures. Specifically, they combine a genetic algorithm with EPANET [37]. The separate operational optimization of the two systems are linked *via* limited information sharing between the two. Their work demonstrates that their optimization methodology is effective for long-term contingency planning for both electricity-generator outages and droughts. Rodriguez-Garcia *et al.* [27] propose an analytic model to study the operation of water-distribution and electricity systems under significant electricity-supply disruptions. In particular, they quantify the extent to which resilience of the two systems are interdependent.

Yao *et al.* [26] propose an optimization model to determine optimal tank sizes for water-distribution systems. The model is formulated to maximize electricity-demand flexibility that can be offered to the electricity system that supplies energy to the water-distribution system. Their work studies the effect of tank size on water quality, with the goal of ensuring that the quality of supplied water is not compromised by the provision of electricity-demand flexibility.

Resilience Models

Resilience is defined as the capacity of a system to withstand disruptions and to recover quickly from such disruptions. Thus, enhancing the resilience of an infrastructure system depends upon reducing its vulnerability or improving its recoverability. Critical infrastructures are crucial for the economy and social well being. As the number of natural and human-caused attacks and incidents against infrastructures increase, research on bolstering infrastructure protection has become imperative. The resilience of water-distribution systems is complicated by their interdependence on electricity systems. As such, contemporary works that examine the resilience of water-distribution systems examine the impacts of interdependent systems.

Ghorbani-Renani *et al.* [30] propose a tri-level model that examines resilience of interdependent water-distribution and electricity systems. Their model examines protection, interdiction, and restoration of the systems and aims to achieve an optimal balance between vulnerability and recoverability of the systems before and after disruptions. The tri-level model structure represents decisions made at the top level by a defender to minimize network vulnerability. The middle level represents an attacker, which aims to maximize system disruption. The bottom level of the model represents a defender, which aims to maximize post-disruption recoverability.

In practice, it is common for human attackers to possess fairly complete information about the network's structure and protection strategies. On the other hand, the defender can lack knowledge of the attacker's disruption strategies. This information asymmetry can yield an incomplete-information game between the attacker and the system defender. Li *et al.* [31] examine the resilience of interdependent water-distribution and electricity systems in a community context. They propose a joint optimization method, which is based on an incomplete-information game under attacks. Utilizing an index that is based on maximal flow as a performance metric, they develop a multi-objective planning approach for optimizing the interdependency structure and joint protection of the two systems.

Challenges with Operational Modeling of Water-Distribution Systems

Their inherent complexity and physical scale creates computational challenges in operational modeling of water-distribution systems. Water-distribution systems consist of numerous interconnected and interrelated components, including pipes, water pumps, water tanks, and valves. Thus, operational modeling of a water-distribution system entails representing the status of these components. In addition, equations, some of which are highlighted above, introduce non-linear and non-convex relationships in the status of different water-distribution-system components. Taken together, operational models of water-distribution systems can involve non-linear and non-convex relationships and may entail discrete (*e.g.*, binary) variables.

As the size of a water-distribution system increases and more realistic aspects (*e.g.*, uncertainty and construction staging) are incorporated, the complexity of the optimization problem grows significantly. In some cases, models of water-distribution systems must be solved quickly with a strict time limit, *e.g.*, if water-pumping actions react to real-time electricity-supply conditions and associated uncertainties. These realities and challenges demand the development of efficient and effective mathematical methods and algorithms to simplify and speed the process of solving optimization problems that involve water-distribution systems.

This section surveys three sources of modeling complexity: non-linear and non-convex relationships, discrete decisions, and complex dynamics and multi-scale phenomena.

Non-Linear and Non-Convex Relationships

There are numerous relationships between different components in water-distribution systems that are non-linear and non-convex. Among the most common causes of these non-linearities and non-convexities are relationships, such as (1) and (2), which give pressure-head losses on pipes; pump-operation constraints, such as (3) and (4); and relationships, such as (5), which give electricity consumption by pumps. Due to inherent challenges in finding global optima of non-linear and non-convex optimization problems, these relationships can make solutions difficult to find or optimization models intractable [38]. Given these difficulties, the literature proposes approaches, including convexifications, to simplify these models.

Piecewise Linearization of Relationships

One standard approach to simplify a non-linear or non-convex relationship is to approximate it as being piecewise-linear [39]. In doing so, the relationship can be approximated as being linear or mixed-integer linear. A number of works take this approach to simplify the aforementioned types of non-linear or non-convex relationships [4, 7, 14, 17, 20, 23]. These works apply this technique and show that the piecewise-linear approximation yields fairly accurate solutions but entails lower computation times.

For a given $t \in \mathcal{T}$ and a given $(i,j) \in \mathcal{J}^P$, Hazen-Williams equations (1) and Darcy-Weisbach equations (2) include non-linear relationships of the generic form:

$$L_{i,j,t} = \zeta_{i,j} \cdot (Q_{i,j,t})^n; \tag{6}$$

where $\zeta_{i,j}$ and n are constants. To derive a piecewise-linear approximation of (6), we divide the range of values that $Q_{i,j,t}$ can take into $K-1$ intervals, with the breakpoints:

$$q^1_{i,j,t}, q^2_{i,j,t}, \dots, q^K_{i,j,t}.$$

Next, we define two sets of auxiliary variables. For all $k = 1, \dots, K-1$, we let $\alpha^k_{i,j,t}$ be a binary variable that equals 1 if:

$$Q_{i,j,t} \in \left[q^k_{i,j,t}, q^{k+1}_{i,j,t}\right];$$

and equals 0 otherwise. Next, $\forall k = 1, \dots, K$, we let $\lambda^k_{i,j,t}$ denote the weight that is placed on $q^k_{i,j,t}$ in defining $Q_{i,j,t}$.

With these definitions, the piecewise-linear approximation of (6) is defined as:

$$\sum_{k=1}^{K} \zeta_{i,j} \cdot \left(q^k_{i,j,t}\right)^n \lambda^k_{i,j,t}; \tag{7}$$

where we add the auxiliary constraints:

$$\sum_{k=1}^{K-1} \alpha^k_{i,j,t} = 1 \tag{8}$$

$$\sum_{k=1}^{K} \lambda^k_{i,j,t} = 1 \tag{9}$$

$$\sum_{k=1}^{K} q^k_{i,j,t}\lambda^k_{i,j,t} = Q_{i,j,t} \tag{10}$$

$$0 \leq \lambda^k_{i,j,t} \leq \alpha^k_{i,j,t}; \forall k = 1, \ldots, K-1 \tag{11}$$

$$0 \leq \lambda^k_{i,j,t} \leq \alpha^{k-1}_{i,j,t}; \forall k = 2, \ldots, K \tag{12}$$

$$\alpha^k_{i,j,t} \in \{0, 1\}; \forall k = 1, \ldots, K-1. \tag{13}$$

Equation (7) approximates (6) as being a convex combination of:

$$\zeta_{i,j} \cdot \left(q^k_{i,j,t}\right)^n;$$

and:

$$\zeta_{i,j} \cdot \left(q^{k+1}_{i,j,t}\right)^n;$$

where the choice of the indices, k and $k+1$, corresponds to the value of $Q_{i,j,t}$. Because $q^k_{i,j,t}$ and $q^{k+1}_{i,j,t}$ are constant, (7) is linear in the decision variables, $\lambda^1_{i,j,t}, \ldots, \lambda^K_{i,j,t}$, which appear in (7). Constraint (8) ensures that exactly one of $\alpha^1_{i,j,t}, \ldots, \alpha^{K-1}_{i,j,t}$ is equal to 1. Constraint (9) ensures that the weights, $\lambda^1_{i,j,t}, \ldots, \lambda^K_{i,j,t}$, sum to 1, which ensures that (7) computes the piecewise-linear approximation as a convex combination of:

$$\zeta_{i,j} \cdot \left(q^1_{i,j,t}\right)^n, \ldots, \zeta_{i,j} \cdot \left(q^K_{i,j,t}\right)^n.$$

Constraint (10) ensures that the weights, $\lambda^k_{i,j,t}$ and $\lambda^{k+1}_{i,j,t}$, are chosen appropriately so that:

$$\lambda^k_{i,j,t}q^k_{i,j,t} + \lambda^{k+1}_{i,j,t}q^{k+1}_{i,j,t} = Q_{i,j,t}.$$

Constraint sets (11) and (12) enforce the needed relationships between $\lambda^1_{i,j,t}, \ldots, \lambda^K_{i,j,t}$ and $\alpha^1_{i,j,t}, \ldots, \alpha^{K-1}_{i,j,t}$. Specifically, $\lambda^k_{i,j,t}$ and $\lambda^{k+1}_{i,j,t}$ can be non-zero if and only if $\alpha^k_{i,j,t}$ is non-zero, which fits the definition of $\alpha^k_{i,j,t}, \forall k = 1, \ldots, K-1$. Finally, (13) enforces the binary restriction on $\alpha^k_{i,j,t}, \forall k = 1, \ldots, K-1$. If (6) is a non-linear but convex relationship, then (13) can be relaxed.

Piecewise-linearization of the relationships that govern variable-speed pumps is complicated by (4) having bilinear terms that involve the product of $Q_{i,j,t}$ and $W_{i,j,t}$, for a given $t \in \mathcal{T}$ and a given $(i,j) \in \mathcal{J}^P$. Moreover, substituting (4) into (5) yields additional terms that interact $Q_{i,j,t}$ and $W_{i,j,t}$. The piecewise linearization that is outlined by (7)–(13) can be extended to this case by approximating (4) using triangles, as compared to piecewise-linearization of (6), which uses line segments for the approximation [4, 39].

To derive the piecewise-linear approximation of (4), we break the range of values that $Q_{i,j,t}$ can take into $\mathcal{V}-1$ segments, with breakpoints:

$$q^1_{i,j,t}, \dots, q^{\mathcal{V}}_{i,j,t}.$$

Similarly, we break the range of values that $W_{i,j,t}$ can take into $\mathcal{M}-1$ segments, with breakpoints:

$$w^1_{i,j,t}, \dots, w^{\mathcal{M}}_{i,j,t}.$$

Next, we define three sets of auxiliary variables. First, $\forall v = 1, \dots, \mathcal{V}-1$ and $\forall m = 2, \dots, \mathcal{M}$, we define $h_+^{v,m}$ as a binary variable that equals 1 if $(Q_{i,j,t}, W_{i,j,t})$ is contained in the 'upper-left' triangle that has:

$$\left(q^v_{i,j,t}, w^m_{i,j,t}\right);$$

$$\left(q^v_{i,j,t}, w^{m+1}_{i,j,t}\right);$$

and:

$$\left(q^{v+1}_{i,j,t}, w^{m+1}_{i,j,t}\right);$$

as vertices and equals 0 otherwise. Analogously, $\forall v = 2, \dots, \mathcal{V}$ and $\forall m = 1, \dots, \mathcal{M}-1$, we define $h_-^{v,m}$ as a binary variable that equals 1 if $(Q_{i,j,t}, W_{i,j,t})$ is contained in the 'lower-right' triangle that has:

$$\left(q^v_{i,j,t}, w^m_{i,j,t}\right);$$

$$\left(q^{v+1}_{i,j,t}, w^m_{i,j,t}\right);$$

and:

$$\left(q^{v+1}_{i,j,t}, w^{m+1}_{i,j,t}\right);$$

as vertices and equals 0 otherwise. Finally, $\forall v = 1, \dots, \mathcal{V}$ and $\forall m = 1, \dots, \mathcal{M}$, we let $\gamma^{v,m}_{i,j,t}$ denote the weight that is placed on $q^v_{i,j,t}$ and $w^m_{i,j,t}$ in defining $Q_{i,j,t}$ and $W_{i,j,t}$.

With these definitions, the piecewise-linear approximation of the bilinear term that is in (4) is given by:

$$P^b_{i,j} Q_{i,j,t} W_{i,j,t} \approx \sum_{v=1}^{\mathcal{V}} \sum_{m=1}^{\mathcal{M}} P^b_{i,j} q^v_{i,j,t} w^m_{i,j,t} \gamma^{v,m}_{i,j,t}; \tag{14}$$

and we add the auxiliary constraints:

$$\sum_{v=1}^{\mathcal{V}-1} \sum_{m=2}^{\mathcal{M}} h_+^{v,m} + \sum_{v=2}^{\mathcal{V}} \sum_{m=1}^{\mathcal{M}-1} h_-^{v,m} = 1 \tag{15}$$

$$\sum_{v=1}^{\mathcal{V}} \sum_{m=1}^{\mathcal{M}} \gamma_{i,j,t}^{v,m} = 1 \tag{16}$$

$$\sum_{v=1}^{\mathcal{V}} \sum_{m=1}^{\mathcal{M}} q_{i,j,t}^{v} \gamma_{i,j,t}^{v,m} = Q_{i,j,t} \tag{17}$$

$$\sum_{v=1}^{\mathcal{V}} \sum_{m=1}^{\mathcal{M}} w_{i,j,t}^{m} \gamma_{i,j,t}^{v,m} = W_{i,j,t} \tag{18}$$

$$0 \le \gamma_{i,j,t}^{v,m} \le h_{+}^{v-1,m}; \forall v = 2, \ldots, \mathcal{V}; m = 2, \ldots, \mathcal{M} \tag{19}$$

$$0 \le \gamma_{i,j,t}^{v,m} \le h_{+}^{v,m+1}; \forall v = 1, \ldots, \mathcal{V}-1; m = 1, \ldots, \mathcal{M}-1 \tag{20}$$

$$0 \le \gamma_{i,j,t}^{v,m} \le h_{+}^{v,m}; \forall v = 1, \ldots, \mathcal{V}-1; m = 2, \ldots, \mathcal{M} \tag{21}$$

$$0 \le \gamma_{i,j,t}^{v,m} \le h_{-}^{v,m-1}; \forall v = 2, \ldots, \mathcal{V}; m = 2, \ldots, \mathcal{M} \tag{22}$$

$$0 \le \gamma_{i,j,t}^{v,m} \le h_{-}^{v+1,m}; \forall v = 1, \ldots, \mathcal{V}-1; m = 1, \ldots, \mathcal{M}-1 \tag{23}$$

$$0 \le \gamma_{i,j,t}^{v,m} \le h_{-}^{v,m}; \forall v = 2, \ldots, \mathcal{V}; m = 1, \ldots, \mathcal{M}-1 \tag{24}$$

$$h_{+}^{v,m} \in \{0,1\}; \forall v = 1, \ldots, \mathcal{V}-1; m = 2, \ldots, \mathcal{M} \tag{25}$$

$$h_{-}^{v,m} \in \{0,1\}; \forall v = 2, \ldots, \mathcal{V}; m = 1, \ldots, \mathcal{M}-1; \tag{26}$$

which are analogous to (8)–(13). Specifically, (15) is analogous to (8), in that it allows exactly one triangle to be selected (depending upon the values of $Q_{i,j,t}$ and $W_{i,j,t}$). Constraint (16) is analogous to (9). Constraints (17) and (18) are analogous to (10) and ensure that the weights are selected appropriately, depending upon the values of $Q_{i,j,t}$ and $W_{i,j,t}$. Constraints (19)–(24) are analogous to (11) and (12), and enforce the needed relationships between the variables, $\gamma_{i,j,t}^{v,m}$, $\forall v = 1, \ldots, \mathcal{V}$ and $\forall m = 1, \ldots, \mathcal{M}$, $h_{+}^{v,m}$, $\forall v = 1, \ldots, \mathcal{V}-1$, $\forall m = 2, \ldots, \mathcal{M}$, and $h_{-}^{v,m}$, $\forall v = 2, \ldots, \mathcal{V}$, $\forall m = 1, \ldots, \mathcal{M}-1$. Finally, (25) and (26) enforce integrality restrictions.

Convex-Hull-Based Relaxation of Relationships

An alternative to piecewise-linearization is to convexify non-convex relationships based on their convex hulls. To illustrate this approach, consider (3), which relates water flow to pressure-head gain that is induced by a fixed-speed pump. In many cases, $P_{i,j}^{a}$ is small in magnitude compared to the other terms and $P_{i,j}^{a}$ is negative. Thus, neglecting the term that includes $P_{i,j}^{a}$ and simplifying (3) to:

$$P_{i,j,t} = P_{i,j}^{b} Q_{i,j,t} + P_{i,j}^{c}; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^{M}; \tag{27}$$

yields a reasonable linear approximation [13, 15, 16].

Next, substituting (27) into (5) yields:

$$\eta_{i}^{p} E_{i,j,t} = \rho g \cdot \left[P_{i,j}^{b} \cdot (Q_{i,j,t})^{2} + P_{i,j}^{c} Q_{i,j,t} \right]; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^{M}; \tag{28}$$

which can be written equivalently as:

$$\eta_i^p E_{i,j,t} \geq \rho g \cdot \left[P_{i,j}^b \cdot (Q_{i,j,t})^2 + P_{i,j}^c Q_{i,j,t} \right] \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^M \tag{29}$$

$$\eta_i^p E_{i,j,t} \leq \rho g \cdot \left[P_{i,j}^b \cdot (Q_{i,j,t})^2 + P_{i,j}^c Q_{i,j,t} \right]; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^M. \tag{30}$$

Similarly, (2) can be written equivalently as:

$$\pi^2 g \cdot \left(F_{i,j}^D\right)^5 L_{i,j,t} \geq 8 F_{i,j}^L F_{i,j}^{F,D} \cdot (Q_{i,j,t})^2; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^P \tag{31}$$

$$\pi^2 g \cdot \left(F_{i,j}^D\right)^5 L_{i,j,t} \leq 8 F_{i,j}^L F_{i,j}^{F,D} \cdot (Q_{i,j,t})^2; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^P. \tag{32}$$

Both (29) and (31) are convex second-order cone constraints whereas (30) and (32) are non-convex. These two non-convex constraint sets can be replaced with convex-hull-based relaxations, which are:

$$\eta_i^p E_{i,j,t} \leq \rho g \cdot \left(b_{i,j}^0 + b_{i,j}^1 Q_{i,j,t} \right); \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^M \tag{33}$$

$$\pi^2 g \cdot \left(F_{i,j}^D\right)^5 L_{i,j,t} \leq 8 F_{i,j}^L F_{i,j}^{F,D} \cdot (f_{i,j}^0 + f_{i,j}^1 Q_{i,j,t}); \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^P; \tag{34}$$

respectively, where $\forall (i,j) \in \mathcal{J}^M$, $b_{i,j}^0$ and $b_{i,j}^1$, and $\forall (i,j) \in \mathcal{J}^P$, $f_{i,j}^0$ and $f_{i,j}^1$ are constants that define the upper bounds for the convex hulls of (30) and (32) [15].

Figure 1 provides a graphical illustration of the convex-hull-based relaxation of (28). The relaxation of (2) has the same geometric property. Figure 1 shows two curves. The green-colored solid curve is the original equality, (28), which is a non-convex relationship between $Q_{i,j,t}$ and $E_{i,j,t}$. The first step of the convex-hull-based relaxation, which is given by (29), is to replace this non-convex equality with a convex inequality, which gives the feasible region that is above the green-colored solid curve. The second step, which is given by (33), adds an inequality that defines the upper bound of the convex hull of (28). This inequality is represented in Fig. 1 by the blue-colored dashed line segment. Thus, the convex-hull-based relaxation requires $Q_{i,j,t}$ and $E_{i,j,t}$ to be in the space that is between the two curves. This relaxation tends to be 'tight', because, for a given value of $Q_{i,j,t}$, it allows energy consumption of the pump, which is given by $E_{i,j,t}$, to be greater than that which (28) requires. Because optimization models tend to minimize energy consumption by the water-distribution system, these relaxations tend to yield solutions that obey (28).

Other Approaches

In addition to piecewise-linear approximation and convex-hull-based relaxation of non-convex relationships, there are other techniques that are applied to models of water-distribution systems. One approach uses EPANET for simulation-optimization [24, 37]. Another approach employs successive linear approximations of the relationships that govern pipe and pump dynamics in an iterative fashion [29]. These successive linear approximations are intended to yield a near-optimal solution that obeys the original non-linear and non-convex relationships. Some works simplify (3) by assuming a fixed pressure-head gain when a pump is operated and zero pressure-head gain when it is

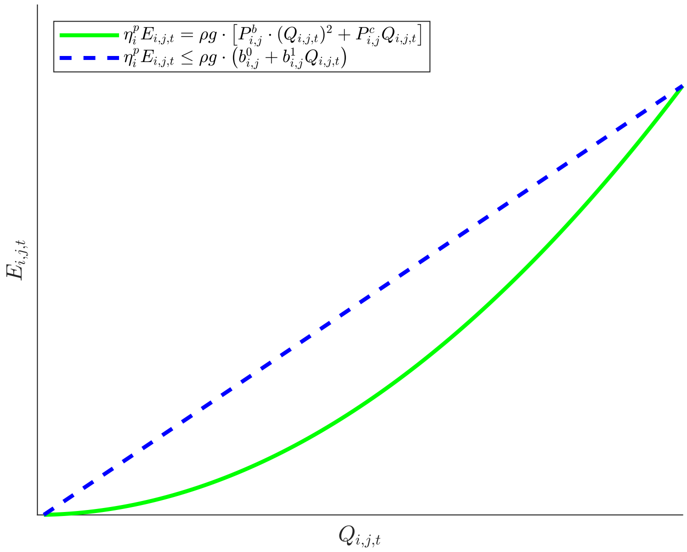

Fig. 1 Illustration of convex-hull-based relaxation of (28).

not [6]. Singh and Kekatos [6] simplify (2) by replacing it is with convex inequality (31) and adding a term to the objective function that penalizes violations of (2).

Giacomello *et al.* [1] use a linearized model in a two-step greedy algorithm. During the first step, the original non-linear and non-convex model is linearized by fixing the values of variables. The fixed values are obtained from a simulation model, which assumes that pumps follow a fixed operating schedule. The linearized model is used to find a near-optimal pump-scheduling solution. During the second step, the best solution from the linearized model is used as a starting point to solve the original non-linear and non-convex model with a greedy algorithm.

Convexification of models that include variable-speed pumps is more complicated and no widely used method is proposed currently. One possible approach is to replace (4) with the convex inequality set:

$$P_{i,j,t} \leq P^a_{i,j} \cdot (Q_{i,j,t})^2 + P^b_{i,j} Q_{i,j,t} \bar{W}_{i,j,t} + P^c_{i,j} \cdot (\bar{W}_{i,j,t})^2; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^M; \quad (35)$$

where $\bar{W}_{i,j,t}$ is fixed and convexity follows because $P^a_{i,j} \leq 0$ [3, 28]. A nice property of this relaxation is that for any $P_{i,j,t}$ and $Q_{i,j,t}$ that are obtained from (35), (4) gives a unique value of $W_{i,j,t}$ that is feasible in (4) [3, 28].

Fooladivanda and Taylor [3] address the bilinear term that is in (5) by employing alternating direction method of multipliers, which is an iterative technique with convergence guarantees to a point that satisfies Karush-Kuhn-Tucker conditions [38, 40]. Zamzam *et al.* [28] employ a feasible point pursuit-successive convex approximation algorithm to solve an optimal water-and-power flow problem. The algorithm has two major steps. First, a feasibility step aims to find a feasible solution to the problem by solving approximations of the original problem iteratively. Next, a refinement step refines the feasible set that is obtained from the first step to identify an optimal or near-optimal solution.

Discrete Decisions

Most works assume that water-flow directions within a water-distribution system are fixed. Another common simplification is to assume that all pumps and valves are operational during all times. These assumptions are reasonable if a water-distribution system has a network structure with a clear upstream-to-downstream water flow and if the pumps are all variable-speed. Some works consider bi-directional water flow in pipes and model endogenously the on/off status of pumps [3, 13, 16, 21]. These considerations can yield a model that is more accurate and applicable to a wider set of water-distribution systems, but require integer variables, which represent water-flow directions and operating status of pumps. Integer variables increase computational complexity, by yielding a mixed-integer non-linear optimization problem, which calls for model linearizations and convexifications.

Big-M Method

Big-M Method is used widely to simplify disjunctive constraints that are related to the working status of pumps or water-flow directions in a water-distribution system. To illustrate this, consider equalities (3), which specify the pressure-head gain that is induced by a fixed-speed pump. For all $t \in \mathcal{T}$ and $\forall (i,j) \in \mathcal{J}^M$, we define $\beta_{i,j,t}$ as a binary variable that equals 1 if pump (i,j) is operating during time t and equals 0 otherwise. With this definition, a disjunctive constraint set that could replace (3) is:

$$\begin{cases} P_{i,j,t} = P^a_{i,j} \cdot (Q_{i,j,t})^2 + P^b_{i,j} Q_{i,j,t} + P^c_{i,j}; & \text{if } \beta_{i,j,t} = 1; \\ P_{i,j,t} = 0; & \text{otherwise;} \end{cases} \tag{36}$$

$\forall t \in \mathcal{T}$ and $\forall (i,j) \in \mathcal{J}^M$. Equalities (36) are non-linear, due to the 'if' statement, but can be linearized using Big-M Method and replaced by:

$$-M \cdot (1 - \beta_{i,j,t}) \leq P_{i,j,t} - P^a_{i,j} \cdot (Q_{i,j,t})^2 - P^b_{i,j} Q_{i,j,t} - P^c_{i,j} \leq M \cdot (1 - \beta_{i,j,t}) \tag{37}$$

$$0 \leq P_{i,j,t} \leq M \beta_{i,j,t}; \tag{38}$$

$\forall t \in \mathcal{T}$ and $\forall (i,j) \in \mathcal{J}^M$, where M is a sufficiently large constant.

For a given $t \in \mathcal{T}$ and $(i,j) \in \mathcal{J}^M$, if $\beta_{i,j,t}$ is held constant, then the right-hand side of (37) yields a convex feasible region whereas the left-hand side does not. However, the aforementioned convex-hull-based relaxation techniques can be applied to the left-hand side of (37), which simplifies the mathematical structure of (37) further. Endogenous representation of the on/off status of variable-speed pumps, valves, and tanks and bi-directional water flows on pipes can be modeled using Big-M Method, following the approach that is outlined in (36)–(38) [3, 6, 14, 21].

Quasi Convex-Hull-Based Relaxation

Head-loss equations (1) and (2) have the generic form that is given by (6). Considering bi-directional water flows changes the generic equation set (6) into an equation set of the form:

$$L_{i,j,t} = \zeta_{i,j} \cdot \mathrm{sgn}(Q_{i,j,t})(Q_{i,j,t})^n; \forall t \in \mathcal{T}; (i,j) \in \mathcal{J}^P; \tag{39}$$

where sgn(·) equals 1 if its argument is non-negative and equals −1 otherwise. The green solid curve that is in Fig. 2 shows the shape of the relationship between $L_{i,j,t}$ and $Q_{i,j,t}$ that is given by (39) for a given $t \in \mathcal{T}$ and $(i,j) \in \mathcal{J}^P$. The red dashed line segments that are in Fig. 2 show the shape of a quasi convex-hull-based relaxation of (39), which is an extension of the relaxation of (30) and (32) that is given by (33) and (34) [13, 16]. This is called a quasi convex-hull-based relaxation, because the red dashed line segments that are in Fig. 2 do not yield exactly the convex hull of the feasible region that is given by (39).

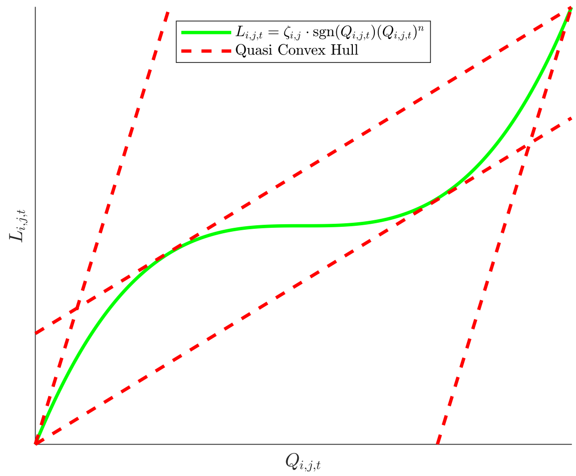

Fig. 2 Illustration of quasi convex-hull-based relaxation of (39).

Other Approaches

Yao *et al.* [22] use an iterative technique to capture discrete decisions. The first step is to determine the operational status of pumps and valves, which are fixed in the subsequent optimization model. This relaxation technique reduces computation time, but there are no general guarantees regarding the optimality of solutions that are obtained.

Complex Dynamics and Multi-Scale Phenomena

Yao *et al.* [22] consider transfer delays in a water-distribution system, *i.e.*, delays between when control actions are taken and when water flows are impacted. They demonstrate that because balancing of water supply and demand is not instantaneous, the ability of a water-distribution system to control its electricity-consumption profile may be limited. They model the dynamics of water flows within a pipe as:

$$\frac{1}{4}\pi g \cdot \left(F_{i,j}^{D}\right)^2 \frac{\partial H}{\partial x} + \frac{\partial Q}{\partial t} + \frac{2F_{i,j}^{F,D} Q|Q|}{\pi \cdot \left(F_{i,j}^{D}\right)^3} = 0; \tag{40}$$

where:

$$\frac{\partial H}{\partial x};$$

is the rate of change of pressure head along the length of the pipe and:

$$\frac{\partial Q}{\partial t};$$

is the time rate of change of water flow along the pipe. If we assume steady-state water flow, then we have:

$$\frac{\partial Q}{\partial t} = 0;$$

in which case (40) simplifies to:

$$L = \frac{8F_{i,j}^{F,D} Q|Q| F_{i,j}^{L}}{\pi^2 g \left(F_{i,j}^{D}\right)^5};$$

from which we obtain Darcy-Weisbach relationships (2) because, by definition, we have that head loss is:

$$L = -\frac{\partial H}{\partial x}.$$

Yao *et al.* [22] propose an equivalent circuit model to represent transfer delay using a simplified set of constraints to represent the dynamics of the pipes in continuous time and space domains. Furthermore, they develop a graph-based model of the water-transmission process through the water-distribution system which has an analogy to electricity flows through a transmission network.

Literature Gaps and Future Research Directions

Strengths and Limitations of the Literature

From a mathematical perspective, piecewise linear and convex-hull-based relaxations are useful approaches to simplify the computational complexity of water-distribution-system models while retaining model fidelity. Piecewise linearization is used in many works [4, 7, 17, 20, 23], but introduces integer variables. As such, piecewise linearization may be more suitable for models that include integer variables already, as it would result in converting a mixed-integer non-linear optimization to a mixed-integer linear optimization. Conversely, convex-hull-based relaxation does not linearize but only convexifies relationships [13, 15]. Thus, this technique may be more suitable for non-linear models that do not include integer variables, because it would result in converting a non-convex non-linear optimization to a convex non-linear optimization.

There are important limitations regarding the level of detail with which water-distribution systems can be modeled. A common assumption, which is shown in (27), is to ignore the quadratic term that is in (3) [13, 15]. This can be a reasonable assumption in many cases, because the quadratic term tends to be small in magnitude compared to the others. However, if the water-flow rate is sufficiently high, the quadratic term can become non-trivial, which can give inaccurate representations of the pressure-head gain that is induced by a pump. A further simplification relaxes (3) completely and assumes a constant pressure-head gain when the pump is operating [6]. Giacomello *et al.* [1] propose a two-step method to speed the computation of pump-scheduling models. However, their method has no guarantees regarding global optimality of solutions.

Some works consider a wider range of water-distribution-system components (other than pumps and tanks) that can provide electricity-demand response and other forms of flexibility. Oikonomou and Parvania [17] investigate the potential for incorporating water-treatment and -desalination plants into the system. Li *et al.* [13] explore the use of hidden controllable water loads, such as irrigation systems. Yao *et al.* [26] optimize the size of water-storage tanks to improve the flexibility of a water-distribution system. On the other hand, some works focus only on certain water-distribution-system components. Li *et al.* [13] investigate the electricity-demand-response potential of water-distribution systems, but do not consider the role of water tanks. Similarly, Giacomello *et al.* [1] consider neither tanks nor valves in their analysis.

Another body of work considers water-distribution systems within the nexus of electricity and natural-gas networks [16, 19]. Rodriguez-Garcia *et al.* [27] study the interdependence between electricity and water systems, due to the use of water for hydroelectric generation and thermal-generator cooling. Future work could build upon this by examining pumped hydroelectric energy storage, which can enhance the resilience of both systems. Other works consider water consumption for electricity-to-gas conversion processes and for combined-heat-and-power systems [16, 19]. However, these works do not consider domestic water demand.

There is a variety approaches to designing optimization models, based on the research questions that they are meant to address. Oikonomou *et al.* [4] develop a two-step modeling approach to maximize profit of a water-distribution system that provides electricity-demand response and frequency regulation to the electricity system that serves it. The second model step maximizes demand response and frequency regulation that is offered by the water-distribution system, subject to an operational schedule that is determined by the first step. However, the first step considers only water-procurement cost, without consideration of energy consumption by the water-distribution system. Thus, the model structure could yield sub-optimal solutions. Oikonomou and Parvania [17, 23] model electricity consumption of a water-distribution system that provides electricity-demand response. Electricity consumption is optimized using an electricity-price forecast. However, the provision of electricity-demand response may alter prices. As such, the electricity-consumption profile may be suboptimal, due to its not capturing such effects endogenously. Moreover, Oikonomou and Parvania [23] assume that total daily electricity consumption of the water pumps is unchanged, reflecting only the available range of hourly demand flexibility. If the water-distribution system uses tanks for water storage, total energy use for water pumping would differ, even if aggregate supplied water demand remains unchanged. This model does not capture these relationships between electricity-consumption and water-flow profiles.

Stuhlmacher and Mathieu [15, 21] model the coupling between electricity and water-distribution systems, with the objective of minimizing energy-procurement costs of the latter. The model takes the perspective of an independent water-distribution-system operator that provides flexibility to help the electricity system maintain power and voltage balance. The model allows the water-distribution system to make real-time adjustments to water-pumping actions to decrease the probability of loss of electricity or water-pumping demand. However, such an action may increase costs to the water-distribution system. It is unclear if the water-distribution-system operator would be willing to help maintain power and voltage balance of the electricity system, given potential cost increases. Zamzam *et al.* [28] formalize the optimal water-and-power flow problem, which aims to minimize the total operation cost of water-distribution and electricity systems. However, they calculate the cost of operating the water-distribution system using fixed electricity prices, which do not reflect real-time marginal costs of electricity production.

Another set of works model the derating of thermal electricity generation during periods of drought, by adjusting the cooling-water demands of the electricity-generation fleet [24, 25]. These works model the water-distribution system from the perspective of supplying cooling water to electricity generators, without considering the provision of domestic water supply. Rodriguez-Garcia *et al.* [27] analyze the resilience of water-distribution and electricity systems to electricity-supply outages. However, their work minimizes curtailed demands, without consideration of operation and recovery costs.

Finally, one common feature of these works is that they do not provide complete and accurate units for parameters and variables. Modeling water-distribution systems involves complex fluid dynamics. A lack of clear units can cause formulation and computation errors by researchers who aim to employ or develop further the models that are proposed by these works.

Areas for Further Research

Based on the aforementioned strengths and limitations of the reviewed literature, we identify several areas for further research. First, there remains a need for improved model formulations and solution algorithms that allow for computationally efficient and high-fidelity study of water-distribution systems. As an example, the complex model structures that are needed to study system dynamics, such as tank-design and transfer delays, can yield computationally complex tri-level optimization problems or models with partial differential equations or integer variables [22, 26].

Second, future research can develop means with which to model a wider range of water-distribution-system components, beyond pipes, pumps, valves, and tanks. One such example is exploring the role of water-treatment or -desalination demands in providing flexibility to the electricity system to which it is coupled. Another example is investigating the potential for pumped hydroelectric energy storage to enhance the resilience of both water-distribution and electricity systems.

Third, most works consider energy consumption of water pumps as the sole point of coupling between water-distribution and electricity systems. The coupling between the two systems can be extended to consider cooling-water requirements of the latter. Coupling can be extended further to consider the impacts of extreme conditions, for instance as they pertain to component protection and optimizing recovery from disruptive events. Another extension is to consider the range of options for co-ordinating or co-optimizing the operation and planning of the two systems.

Finally, models that examine the design of water-distribution systems do not consider fully the role and impact of such systems within electricity systems. This lack of focus hinders the potential for water-distribution systems to provide flexibility to electricity systems. As an example, Yao *et al.* [26] model the design of water tanks. This type of work could be extended to examine all aspects of water-distribution-system design, considering points of coupling with the electricity system that serves it.

Conclusions

Water-distribution systems are crucial infrastructure assets that are vital for delivering clean and safe water to urban and rural areas. The complexity of modeling the operation of water-distribution systems arises from the need to solve numerous simultaneous nonlinear equations and to capture the operational status of system components. This survey provides a thorough examination of current modeling and computational techniques that are employed in representing water-distribution systems. Our focus is identifying gaps in existing research. Despite the strides in mathematical modeling to simplify and speed system optimization, our review reveals persistent limitations in applying these methods to large-scale systems or in integrating complex components, (*e.g.*, variable-speed pumps) and multi-level and equilibrium modeling. We stress the need for further advances that can address the realities of modern water-distribution systems and address the computational complexity of independencies with other infrastructure systems.

Our survey is focused on problems that arise from tapping potential synergies between water-distribution and electricity systems. Such synergies are increasingly important with increasing penetrations of renewable-electricity sources, which introduce electricity-supply variability and uncertainty. We advocate a two-prong approach to studying these synergies. One is to examine co-optimization of the two systems. The other examines co-ordination strategies that respect the operational independence of these infrastructure systems.

Additionally, our review suggests value in modeling non-traditional components and dynamics of water-distribution systems, including transfer delays, pumped hydroelectric storage, and water-treatment and -desalination facilities. These elements are perfectly cromulent for enhancing the flexibility of electricity systems but often are overlooked in conventional studies. Another emerging linkage between water-distribution and electricity systems is the need for water to cool thermal generating units. Co-ordinating this linkage can have efficiency and resilience impacts. Another consideration is expanding the scope of models to include longer-term decision support, *e.g.*, planning and design of water-distribution, electricity, or both systems.

In conclusion, our paper provides an extensive review of existing approaches that are used to model and optimize water-distribution systems. Our survey aids researchers to develop models of water-distribution systems and offers directions for future research that will enhance modeling capabilities further. These efforts are crucial for enhancing the resilience and adaptability of infrastructure systems that are faced with evolving global demographics and the challenges that are posed by climate change.

Acknowledgments. The authors thank A. Sorooshian for helpful discussions, comments, and suggestions. Any errors, opinions, and conclusions that are expressed in this paper are solely those of the authors.

Declarations

- Funding: This work was supported by The Ohio State University, National Science Foundation grants 1463492, 1808169, and 1922666, Electric Power Research Institute grant DKT230520, Carnegie Mellon Electricity Industry Center, and Wilton E. Scott Institute for Energy Innovation.
- Conflict of interest/Competing interests: The authors declare that they have no competing interests
- Authors' contributions: W.G. wrote the manuscript text and R.S. edited the manuscript text.

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