Another Step Towards Equilibrium Offers in Unit Commitment Auctions with Nonconvex Costs: Multi-Firm Oligopolies

Joseph E. Duggan, Jr. * *and Ramteen Sioshansi* **

ABSTRACT

There are two uniform-price auction formats—centrally and self-committed—that are used commonly in wholesale electricity markets. Both formats are operated by an independent third-party market operator, which solicits supply offers from generators and determines how much energy they produce to serve customer demand. In centrally committed markets, generators submit complex offers that convey all of their non-convex operating costs and constraints. Conversely, generators submit simple offers in self-committed markets that only specify the price at which they are willing to supply energy. Thus, generators must internalize their non-convex costs and other operating constraints in submitting offers in a self-committed market. Centrally committed markets include also a provision that each generator is made whole on the basis of its submitted offers. No such guarantees exist in self-committed markets.

This paper builds on the work of Sioshansi and Nicholson (2011) and studies the energy-cost ranking and incentive properties of the two market designs in a multi-firm oligopoly setting. We derive Nash equilibria under both market designs. We find that equilibrium offer behavior across the two market designs is qualitatively similar to the duopoly model when demand is high. However, when demand is low, cost equivalence between the two market designs breaks down. This is because inframarginal generators are able to earn positive profits in certain states of low demand in self-committed markets, whereas all generators are constrained to earn zero profits in low-demand states in the centrally-committed market design.

Keywords: Electricity market, market design, unit commitment, Nash equilibrium, non-convex cost

JEL: C72, D43, D44, D47, L13, Q4

http://dx.doi.org/

1. INTRODUCTION

The organization of wholesale electricity markets is a question of immense importance. However, market designs of this nature remain an open area of inquiry. Two commonly utilized market designs are centrally and self-committed markets. In both market designs, generators offer their supply into a uniform-price auction that is operated by an independent third party, which we hereafter refer to as the market operator (MO). What distinguishes these market designs is the format of the offers and the way in which generator-operating decisions are made.

In a centrally committed market, each generator submits a complex offer, which contains the generator's complete cost and operating-constraint information, to the MO. The MO uses the complex offers that it receives to make financially binding decisions regarding the commitment (*i.e.*,

*Corresponding author. Department of Economics and Finance, University of Dayton, 300 College Park, Dayton, Ohio 45469-2251, United States of America. E-mail: jduggan1@udayton.edu.

**Department of Integrated Systems Engineering, The Ohio State University.

on/off status) of each generator and its dispatch (*i.e.*, production levels).[1] The MO makes these decisions by solving a unit commitment model, which is formulated normally as a mixed-integer optimization problem. Sheble and Fahd (1994); Baldick (1995); Hobbs et al. (2001); Padhy (2004) provide discussions and surveys of unit commitment and dispatch models.

Conversely, in a self-committed market generators make their own commitment decisions while dispatch decisions are left to the MO. A generator that decides to commit itself (*i.e.*, switch on) submits a simple offer, which specifies the minimum price at which it is willing to produce and sell energy. The MO does not decide which units to commit. However, the MO does determine the dispatch of each generating unit on the basis of the simple offers that are submitted. This is done by stacking the simple offers in merit order into a supply function, which is intersected with electricity demand to determine each generator's production level and the market-clearing price.

Centrally committed markets are commonplace in North America, whereas much of the rest of the world, including Australia and Western Europe, employ self-committed markets. Because of this regional divergence in designs, the debate regarding their relative merits is ongoing. Cramton (2017) notes that 'the exchange model [self commitment] operating in much of Europe is improving, but still falls far short in pricing transmission congestion both within and across countries.' Furthermore, while recognizing the benefits of the self-committed design in environments without non-convex costs, Cramton (2017) notes that a centrally committed market design 'better handles non-convexities and is simpler for participants.' Moreover, many European markets include now limited non-convex constraints, such as block bids, in their dispatch markets, which complicate price formation. These types of constraints are included to provide generators a mechanism with which to enforce some of their complicated operating constraints in dispatch solutions. Thus, some European markets are considering whether tighter co-ordination, for instance through a centrally committed design, may be beneficial. Imran and Kockar (2014) provide a comparison of market designs in North America and Europe.

The primary stated benefit of a centrally committed market is that the MO is in the best position to make commitment decisions. This is because the MO co-optimizes the commitment and dispatch of all units using all of the cost and constraint information that pertain to their operation. A self-committed market may be inefficient because generators making commitment decisions independently may not achieve the same level of co-ordination. On the basis of these observations, Ruff (1994); Hogan (1994) advocate centrally committed markets over self-committed designs.

However, in a centrally committed market the MO must rely on cost and constraint data that are provided by generators in their complex offers. Generators have weak incentives (at best) to provide true information in their offers. For example, Oren and Ross (2005) show that generators can misstate their ramping limits profitably. These misstated ramping limits reduce system efficiency because the MO mistakenly (*i.e.*, on the basis of incorrect constraint information) commits and dispatches more expensive units. Further to this point, Muñoz et al. (2018) examine the long-term efficiency of investments that are made within a market environment. They demonstrate that markets that rely on audited cost information for dispatch and pricing can reduce social welfare relative to bid-based market designs. This is because a cost-based market design can distort investment incentives. To illustrate this phenomenon, they discuss the example of opportunity costs diverging from direct fuel costs due to energy or start-up limits.

Another shortcoming of a centrally committed market design is price formation and generator cost recovery. Price formation in a self-committed market is relatively straightforward as the market-clearing price is given by the intersection of aggregate demand and supply. Generators must structure their offers to ensure that they recover all of their operating costs. In a centrally committed market, the wholesale energy price normally is set based on the value of the dual variables that are

[1]The commitment and dispatch decisions are financially binding in the sense that a generator that is committed can opt to not follow physically the commitment and dispatch instructions. In such an instance, it is financially liable for the cost of procuring replacement energy from another market participant.

Copyright © 2018 by the IAEE. All rights reserved.

associated with the load-balance constraints in the unit commitment model.[2]

The issue with pricing energy on the basis of these dual variables in a centrally committed market is that generators that are committed and dispatched may not recover all of their costs. This is a well known shortcoming of linear pricing in a setting with non-convexities and Scarf (1990, 1994) provides analyses and illustrative examples of this phenomenon. This economic confiscation can be overcome by some form of discriminatory pricing. O'Neill et al. (2005) propose addressing the confiscation by pricing the non-convexities explicitly. This is done by adding constraints to a linear relaxation of the unit commitment model that fix the commitment decisions to their optimal values. The dual variables that are associated with these added constraints are used to supplement energy payments that are made using the dual variables that are associated with the load-balance constraints. Sioshansi (2014) shows, however, that the dual variables that are associated with the commitment decisions may be negative. As such, inframarginal generators may 'lose' their inframarginal rents through the supplemental payments and be constrained to earn zero profits. Convex hull pricing is discussed as another approach to addressing the complications that non-convexities raise. Instead of starting with a convex dispatch problem and adding back non-convexities for pricing, convex hull pricing uses a 'convexified' unit commitment problem from the outset for operational planning and pricing. Schiro et al. (2016) provide an in-depth discussion of convex hull pricing.

In practice, revenue adequacy is ensured in centrally committed markets by giving generators linear payments for energy (based on the dual variables that are associated with the load-balance constraints) and make-whole payments. Make-whole payments provide any generator that would operate at a net loss on the basis of the costs that are specified in its complex offer (which may differ from its true costs) a supplemental payment that is equal exactly to its revenue shortfall. The rationale behind a make-whole payment is that absent the mechanism, generators have strong incentives to overstate their costs in their offers to ensure that they are not operated at a loss. Moreover, an MO economically confiscating a generating unit may run afoul of the owner's property rights. Make-whole payments are undesirable, however, because they are discriminatory and their costs must be recovered by the MO, which is done typically by uplifting to consumers.

Given these shortcomings of the centrally committed market design, there are advocates of self-committed markets. Elmaghraby and Oren (1999) argue that a self-committed market is more efficient than a centrally committed design, when all of the incentive problems and uplift payments are taken into account. Despite the tradeoffs between centrally and self-committed markets being an area of interest and discussion, there is very little formal and systematic examination of the two designs in the extant literature. Sioshansi and Nicholson (2011) compare offering incentives under the two market designs for a symmetric duopoly. They derive equilibrium offering strategies under the two market designs and show that if certain conditions are met, the two designs are expected-cost-equivalent (when accounting for strategic profit-maximizing offer behavior by the firms). Álvaro Riascos et al. (2016) conduct an econometric analysis of the Colombian electricity market, which transitioned from a self-committed to a centrally committed design in 2009. They find that while productive efficiency was increased after the transition, marginal cost markups and prices were higher after 2009 than they would have been absent the market-design change. They attribute this result to the possibility that firms are able to exercise market power more freely after the transition to a centrally committed design.

This paper adds to this literature by building on the work of Sioshansi and Nicholson (2011) and examining a general symmetric oligopoly. We characterize equilibrium offering strategies under the two market designs. Importantly, we find that the expected-cost equivalence between the two designs fails to hold in the oligopoly with more than two firms. The loss of expected-cost equivalence occurs because with more than two firms there are low-demand states in which generating firms earn

[2]Because the unit commitment model is formulated normally as a mixed-integer optimization problem, it does not, speaking strictly, have dual variables. As such, prices are in fact set by fixing the integer variables in the unit commitment model to their optimal values and solving the resulting continuous relaxation to obtain dual-variable values.

Copyright © 2018 by the IAEE. All rights reserved.

strictly positive profits under a self-committed market design. This cannot happen in low-demand states with a centrally committed market. Conversely, in a duopoly, firms are restricted to earning zero profits under both market designs in low-demand states. This is because the centrally committed market design allows for discriminatory make-whole payments that allow for lower settlement costs compared to the uniform-price nature of a self-committed design. This finding may have practical relevance in market-design debates.

An important aspect of market design and price formation that we ignore in our analysis is the recovery of investment costs. Sioshansi (2014); Muñoz et al. (2018) show that market-design choices can effect the incentives for capacity investment in the long-run. Although investment incentives are a vitally important consideration in market design, our focus is on how centrally and self-committed markets compare in terms of short-run operations and handling non-convex generation costs.

The remainder of this paper is organized as follows. Section 2 details the models of the market designs that are used in our analysis. Section 3 provides the derivations of equilibria in the two market designs. In Section 4 we use a numerical example to demonstrate our theoretical results and show the cost-comparison result between the two market designs. Section 5 concludes and provides some discussions on the regulatory and policy implications of our work. All of the technical proofs are relegated to the appendices.

2. SYSTEM AND MARKET MODELS

Our model follows largely that used by Sioshansi and Nicholson (2011). The important distinction between our model and theirs is that we examine general N-firm oligopolies, as opposed to restricting our attention to the duopoly case, as they do.

2.1 Power System and Firm Structure

The power system consists of N identical generating firms that are competing in a single-shot, single-period, uniform-price auction to serve exactly a deterministic and commonly known l-MW load. Each generator has the same commonly known generation capacities, $K > 0$. The total capacity is assumed sufficient to serve the load, meaning that $l < NK$.[3] For a given load, l, we define also N_l, such that:

$$(N_l - 1)K < l < N_l K. \tag{1}$$

N_l represents the minimum number of generators that must be committed to be able to serve the load. The strict inequalities in (1) rule out uninteresting knife-edge cases in which either $(N_l - 1)$ or N_l generators are needed to operate exactly at their capacities to serve the load. Such a case yields uninteresting perfectly competitive equilibria. For ease of exposition, we also define $l_r = l - (N_l - 1)K$ as the residual load that remains if $(N_l - 1)$ generators produce at their K-MW capacity.

Each generator incurs a fixed start-up cost, $S > 0$, if it is committed, which it must be to produce energy (otherwise its output must be zero). Each generator incurs also a constant marginal generating cost, $c > 0$, for each MW produced. Thus, the *actual* cost to each firm of producing q MW is:

$$C(q) = \begin{cases} 0, & \text{if } q = 0; \\ S + cq, & \text{if } 0 < q \leq K; \\ +\infty, & \text{if } q > K. \end{cases} \tag{2}$$

[3] The strict inequality rules out an uninteresting case in which all N generators must be dispatched at capacity. Such a case is uninteresting because each supplier is pivotal and the market clears at the offer caps (Sections 2.2 and 2.3 detail the offer caps).

Copyright © 2018 by the IAEE. All rights reserved.

This cost structure is consistent with a stylized model of thermal generation. The cost parameters, S and c, are commonly known to the generators but not to the MO. Instead, the generators submit offers to the MO in the two market designs that are meant to convey this cost information. These offers are the sole basis on which the MO clears the market.

2.2 Centrally Committed Market

In a centrally committed market, each generator submits a complex offer. Generator i's offer has the form $\omega_i = (\sigma_i, \epsilon_i)$, with σ_i and ϵ_i being its start-up and energy offers, respectively. We assume that there are offer caps, which we denote as $\bar{\sigma}$ and $\bar{\epsilon}$, on the two parts of the offer, respectively. These offer caps exist in all competitive wholesale electricity markets as a blunt means of mitigating the exercise of market power. Thus, we assume that $0 < \sigma_i \leq \bar{\sigma}$ and $0 \leq \epsilon_i \leq \bar{\epsilon}$ $\forall i = 1, \ldots, N$. σ_i is assumed strictly positive to ensure that the equilibria that are studied under the centrally committed market design differ from those under a self-committed market.

Indeed, how we model the complex offers under a centrally committed market design effectively subsumes the self-committed design, insomuch as generators may submit offers with $\sigma_i \to 0^+$. This observation addresses a nuance of centrally committed markets, which is that typically they give generators the option to self commit their units, whereby a self-committed unit is dispatched on the basis of simple offers (*e.g.*, it participates in the market *à la* a self-committed design). As such, our analysis of a centrally committed market subsumes effectively a case in which generators can self commit (albeit imperfectly).

Because the MO does not know the values of S and c, it commits and dispatches generators based on the costs that are specified in their offers. Let $\omega = (\omega_1, \ldots, \omega_n)$ denote the entire vector of complex offers. We use the subscript '$-i$' (*e.g.*, ϵ_{-i}) to denote offers that are submitted by generator i's rivals.

Based on ω, the MO uses a unit commitment model to determine the generator commitments and dispatches. To formulate this model, we define the set of binary variables:

$$u_i = \begin{cases} 1, & \text{if generator } i \text{ is committed;} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

$\forall i = 1, \ldots, N$. We define also q_i as a continuous variable indicating how many MW are produced by generator i, for all $i = 1, \ldots, N$. The unit commitment problem is formulated then as:

$$\min_{u,q} \quad \sum_{i=1}^{N} (\sigma_i u_i + \epsilon_i q_i), \tag{4}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} q_i = l, \tag{5}$$

$$0 \leq q_i \leq K u_i, \qquad \forall i = 1, \ldots, N, \tag{6}$$

$$u_i \in \{0, 1\}, \qquad \forall i = 1, \ldots, N. \tag{7}$$

Objective function (4) minimizes total commitment and dispatch cost on the basis of the offers. Constraint (5) imposes load balance. Constraint set (6) imposes generator-capacity limits and requires a generator to be committed to produce a non-zero amount. Constraint set (7) requires the commitment variables to be binary.

The solution to the MO's unit commitment problem depends upon the demand level. If $N_l = N$, all N generators must be committed to serve the load. We refer to any load level such that $N_l = N$ as a *high-demand state*. Conversely, if $N_l < N$, which we refer to as being in a *low-demand state*, then at least one generator may be left inactive.

In high-demand states, the start-up offers are irrelevant for determining the optimal solution

Copyright © 2018 by the IAEE. All rights reserved.

to (4)–(7). This is because all of the generators must be committed and their start-up offers are sunk costs from the perspective of determining production levels. Thus, assuming random rationing, we can determine generator i's expected production level in a high-demand state, first by defining:

$$M_i^c = ||\{j|\epsilon_j = \epsilon_i\}||, \tag{8}$$

as the number of generators that submit the same energy offer that generator i submits (including generator i itself). Then, we define generator i's expected production level in a high-demand state as:

$$q_i^c(\omega; l) = \begin{cases} K, & \text{if } \exists j \text{ such that } \epsilon_j > \epsilon_i; \\ \frac{1}{M_i^c}(l - (N - M_i^c)K), & \text{if } \epsilon_j \leq \epsilon_i, \forall j. \end{cases} \tag{9}$$

There are two possible production levels for generator i in a high-demand state. First, generator i produces at capacity if another generator submits an energy offer that is strictly greater than generator i's offer. The second case in (9) arises if generator i submits the highest energy offer. In this case, all of the generators that submit energy offers that are strictly less than that of generator i, of which there are $(N - M_i^c)$, produce at their maximum capacity. All but one of the remaining M_i^c generators that submit the highest energy offer are assigned randomly to produce K MW while one serves the residual load.

In low-demand states, the load can be served with strictly less than N generators being committed. We show in the following lemma that because $\sigma_i > 0, \forall i = 1, \ldots, N$, it is optimal for only N_l generators to be committed and the remaining generators to be kept offline.

Lemma 1 *In low-demand states, it is suboptimal for the MO to commit more than N_l generators under a centrally committed market design.*

Proof. See Appendix 6.1. ■

Following from Lemma 1, once N_l generators are selected to be committed, $(N_l - 1)$ of those generators are dispatched (solely on the basis of their energy offers) to produce at their K-MW capacities. The remaining one generator with the highest marginal generating cost in its offer serves the residual load, l_r.

The energy price under a centrally committed market design is the highest of the accepted energy offers among generators that have a positive dispatch:

$$p^c = \max_{i=1,\ldots,N} \{\epsilon_i | q_i^c > 0\}, \tag{10}$$

where q_i^c denotes generator i's production level. In a high-demand state q_i^c is given explicitly by (9). We characterize q_i^c in low-demand states when deriving equilibria in Section 3.

Based on this energy price, generator i earns energy revenues that are equal to $p^c q_i^c$. These energy payments may be insufficient for each generator to recover its non-convex start-up cost. As such, the MO uses make-whole payments to ensure that no generator operates at a net loss, on the basis of the costs that are specified in its offer. If we let W_i and T_i denote the make-whole and total payments, respectively, that are given to generator i we have:

$$T_i = p^c q_i^c + W_i = p^c q_i^c + \max\{0, \sigma_i + (\epsilon_i - p^c)q_i^c\}, \tag{11}$$

meaning that:

$$W_i = \max\{0, \sigma_i + (\epsilon_i - p^c)q_i^c\}. \tag{12}$$

Equations (11) and (12) stem from the fact that generator i receives a non-zero make-whole payment if and only if its net operating profit from receiving energy payments alone (on the basis of its offer), which equals $(p^c - \epsilon_i)q_i^c - \sigma_i$, is strictly negative. In such a case, we have from (12) that generator i receives a make-whole payment of $W_i = \sigma_i + (\epsilon_i - p^c)q_i^c$, meaning that its net operating profit

Copyright © 2018 by the IAEE. All rights reserved.

from receiving the energy and make-whole payments is $(p^c - \epsilon_i)q_i^c - \sigma_i + W_i = 0$. Otherwise, if $(p^c - \epsilon_i)q_i^c - \sigma_i \geq 0$, generator i does not need a supplemental make-whole payment to recover the costs that are specified in its offer. In such a case, we have from (12) that $W_i = 0$, as desired.

2.3 Self-Committed Market

In a self-committed market each generator submits a simple offer consisting of a single parameter; δ_i specifies the minimum price at which generator i is willing to produce. We assume that this minimum energy price must be non-negative and that it is subject to an offer cap, which we denote as $\bar{\delta}$. All generators, including those that do not wish to commit, submit an offer. If generator i does not wish to commit its unit it may offer $\delta_i = \bar{\delta}$. Doing so withholds its capacity from the market economically. Many restructured wholesale electricity markets have such a requirement that generators offer their capacity at some price below the offer cap, to avoid unserved demand. We let $\delta = (\delta_1, \ldots, \delta_N)$ denote the offer vector.

As in the centrally committed market, because the MO does not know the values of S and c, in a self-committed market it dispatches the generators based solely on δ. This dispatch problem can be formulated as the following linear optimization problem:

$$\min_q \quad \sum_{i=1}^{N} \delta_i q_i, \tag{13}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} q_i = l, \tag{14}$$

$$0 \leq q_i \leq K, \quad \forall i = 1, \ldots, N, \tag{15}$$

where the variables, $q_i, \forall i = 1, \ldots, N$, retain the same definitions as in (4)–(7). The objective function and constraints of this dispatch problem are analogous to those in (4)–(7), except that start-up costs are not included in the objective function nor are there binary variables representing commitment decisions.

The solution to the dispatch problem depends upon the demand level and the rank ordering of the offers. The solution can be expressed in closed form by relabeling the offers in increasing order. Because the generators have identical *true* costs, without loss of generality we can relabel them such that $\delta_1 \leq \cdots \leq \delta_N$. We define:

$$M_i^s = ||\{j|\delta_j = \delta_i\}||, \tag{16}$$

as the number of generators that submit the same offer that generator i submits. Assuming random rationing, generator i's expected production level is:

$$q_i^s(\delta; l) = \begin{cases} K, & \text{if } \delta_i < \delta_{N_l}; \\ \frac{1}{M_i^s}(l - (N_l - M_i^s)K), & \text{if } \delta_i = \delta_{N_l}; \\ 0, & \text{if } \delta_i > \delta_{N_l}. \end{cases} \tag{17}$$

Generator i has three possible production levels in a self-committed market. If its offer is strictly lower than the N_lth offer (in ascending order), then generator i is dispatched to produce up to its K-MW capacity. If generator i's offer is strictly greater than the N_lth offer, then it is not dispatched and produces 0 MW. If its offer is equal to the N_lth offer, then it is assigned randomly either to produce K MW or to serve the residual load. This case is analogous to the treatment of ties in the highest energy offer that is given in (9).

Copyright © 2018 by the IAEE. All rights reserved.

The energy price under this market design is:

$$p^s = \max_{i=1,\ldots,N}\{\delta_i | q_i^s > 0\}, \tag{18}$$

which is the highest of the accepted offers among generators that are producing a positive amount. This yields generator i revenue that is equal to $p^s q_i^s$. Unlike in a centrally committed market, there are no make-whole payments under a self-committed design. Instead, generators that choose to commit themselves must ensure that they earn sufficient revenues from those sales to recover the start-up costs that they incur. As such, we assume that the offer cap, $\bar{\delta}$, is sufficiently large to ensure that any generator that produces a strictly positive amount of energy is able to recover its start-up cost. That is, we assume that $(\bar{\delta} - c)l_r \geq S$.

3. MARKET EQUILIBRIA

Both market designs have different types of equilibria, depending on the demand level. We have two types of *low-demand states*. We refer to the first type, in which only one generator is needed to serve the load, as *weak low-demand states*. We refer to the one generator that serves the load in a weak demand state as the *unique generator*. The unique generator's offer, production, profit, and payment are denoted by the superscript, 'U.' The other type of *regular low-demand states* require at least two generators to serve the load. Based on Lemma 1 and the discussion in Section 2.3, we know that the MO's problems in low-demand states (under both market designs) have optimal solutions in which $(N_l - 1)$ *inframarginal generators* operate at their K-MW capacity and one *marginal generator* serves the residual load. We refer to any generators that produce 0 MW as *inactive generators*. As needed, the offers, production, and profits of and payments to inframarginal, marginal, and inactive generators are denoted by the superscripts, 'I,' 'M,' and 'V,' respectively.

The other type of demand state, in which all N generators are pivotal, that we have is a *high-demand state*. We retain the same notion of inframarginal and marginal generators in high-demand states as we have in low-demand states. However, there are no inactive generators in a high-demand state.

We proceed by characterizing equilibria under the two market designs in different demand states.

3.1 Equilibria Under a Centrally Committed Market Design

Our analysis of the centrally committed market design begins with the following lemma, which characterizes payments to generators in different demand states.

Lemma 2 *Under a centrally committed market design the total payment to the unique generator is $T^U = \sigma^U + \epsilon^U l$ while all other generators receive zero payments in a weak low-demand state.*

In all other demand states the total payment to the marginal generator is $T^M = \sigma^M + \epsilon^M l_r$, the payment to inframarginal generator i is $T_i^I = \max\{\epsilon^M K, \sigma_i + \epsilon_i K\}$, and the payments to any inactive generators are zero under a centrally committed market design.

Proof. See Appendix 6.2.1. ■

Next, we proceed by characterizing equilibrium offering strategies in the two types of low-demand states and examining high-demand states.

3.1.1 Equilibria in Low-Demand States

We begin with the following lemma, which shows that in low-demand states the marginal generator earns zero profits.

Copyright © 2018 by the IAEE. All rights reserved.

Lemma 3 *In all low-demand states the marginal generator earns zero profits in a Nash equilibrium under a centrally committed market design.*

Proof. See Appendix 6.2.1. ■

Lemma 3 implies the following corollary, which dictates the behavior of inactive generators in a Nash equilibrium.

Corollary 1 *In all low-demand states all inactive generators submit offers,* $\omega^V = (\sigma^V, \epsilon^V)$*, such that:*

$$\sigma^V + \epsilon^V l_r \geq S + cl_r, \tag{19}$$

and this inequality is binding for at least one inactive generator in a Nash equilibrium under a centrally committed market design.

Proof. See Appendix 6.2.1. ■

Now we demonstrate that in weak low-demand states all generators earn zero profits.

Proposition 1 *In weak low-demand states the set of pure-strategy Nash equilibria under a centrally committed market design is characterized by at least two of the generators submitting offers of the form:*

$$\omega \in \Omega_W = \{(\sigma, \epsilon) | \sigma + \epsilon l = S + cl, \sigma \in (0, \bar{\sigma}], \epsilon \in [0, \bar{\epsilon}]\}, \tag{20}$$

and the remaining generators submitting offers of the form:

$$\omega \in \{(\sigma, \epsilon) | \sigma + \epsilon l \geq S + cl, \sigma \in (0, \bar{\sigma}], \epsilon \in [0, \bar{\epsilon}]\}. \tag{21}$$

All generators earn zero profits.

Proof. See Appendix 6.2.1. ■

Proposition 1 characterizes Nash equilibria in weak low-demand states. Characterizing equilibria in regular low-demand cases is more challenging. We begin with the following lemma, which shows that in a Nash equilibrium all inframarginal generators receive the same type of payment (*i.e.*, either they all receive make-whole or energy payments).

Lemma 4 *If at least one inframarginal generator receives energy payments (i.e., it does not receive make-whole payments) in a regular low-demand state under a centrally committed market design, then all inframarginal generators must receive energy payments in a Nash equilibrium.*

Proof. See Appendix 6.2.1. ■

Lemma 4 allows us to draw some inferences regarding equilibrium offers in a centrally committed market that allows generators to self commit. We can conclude that if some inframarginal generators are receiving make-whole payments, then there is not a Nash equilibrium in which other generators are receiving energy payments (which is akin to self committing). This is because when a generator self commits, it offers its generation to the market using simple offers, which precludes the possibility of receiving make-whole payments. However, Lemma 4 shows that such a set of offers cannot constitute a Nash equilibrium. This is because the self committing generator has an incentive to increase its offers to receive higher make-whole payments.

This finding is in line with private discussions with participants in centrally committed markets. Units in such markets are seldom self committed, as doing so raises revenue risk through the loss of potential make-whole payments. In some cases, a market participant may opt to self commit, if it foresees high loads beyond the one-day lookahead that is common in centrally committed markets. This is because in such instances, it may be more profitable in the long-run (*i.e.*, over several days) for a unit that would not be committed by the MO to be self committed. Indeed, baseload

Copyright © 2018 by the IAEE. All rights reserved.

generators (with low marginal generation but high start-up costs) may opt to self commit, depending on the volume of day-ahead market activity.

The following corollary to Lemma 4 shows that in a Nash equilibrium all inframarginal generators earn the same profits.

Corollary 2 *All inframarginal generators earn the same profits in a Nash equilibrium under a centrally committed market design in a regular low-demand state.*

Proof. See Appendix 6.2.1. ■

Now we proceed by examining three possible cases involving the offer of the marginal generator—those in which the marginal-cost portion of its offer, ϵ^M, is strictly less than, equal to, or strictly greater than its true marginal cost, c. We show in the following lemmata that equilibria are not possible in the first case, are possible in the second, and that all generators earn zero profits in the third.

Lemma 5 *There are no Nash equilibria in which the marginal generator submits an offer with $\epsilon^M < c$ in a regular low-demand state under a centrally committed market design.*

Proof. See Appendix 6.2.1. ■

Lemma 6 *There are pure-strategy Nash equilibria in a regular low-demand state under a centrally committed market design in which the marginal generator submits an offer with $\epsilon^M = c$. Such equilibria are characterized by the marginal generator submitting an offer with $\sigma^M = S$, all inframarginal generators submitting offers of the form:*

$$\omega \in \Omega_L = \{(\sigma, \epsilon) | \sigma + \epsilon K = S + cK, \sigma \in (0, \bar{\sigma}], \epsilon \in [0, \bar{\epsilon}]\}, \tag{22}$$

and the inactive generators submitting offers of the form:

$$\omega \in \{(\sigma, \epsilon) | \sigma + \epsilon l_r \geq S + c l_r, \sigma \in (0, \bar{\sigma}], \epsilon \in [0, \bar{\epsilon}]\}, \tag{23}$$

with this inequality being binding for at least one inactive generator. All generators earn zero profits.

Proof. See Appendix 6.2.1. ■

Lemma 7 *If the marginal generator submits an offer with $\epsilon^M > c$ in a regular low-demand state under a centrally committed market design, then all inframarginal generators earn zero profits in a pure-strategy Nash equilibrium.*

Proof. See Appendix 6.2.1. ■

3.1.2 Equilibria in High-Demand States

In high-demand states all generators must be active, meaning that the generators' start-up offers must all be incurred by the MO. Thus, dispatch decisions are made solely on the basis of the generators' energy offers. In the following proposition, we show that as a result of how the dispatch decisions are made, there are no pure-strategy Nash equilibria in high-demand states under a centrally committed market design.

Proposition 2 *There are no pure-strategy Nash equilibria in a high-demand state under a centrally committed market design.*

Copyright © 2018 by the IAEE. All rights reserved.

Proof. See 6.2.1. ■

Because there are no pure-strategy Nash equilibria in a high-demand state under a centrally committed market design, we characterise mixed-strategy Nash equilibria. Moreover, we derive an analytical expression for a symmetric mixed-strategy Nash equilibrium, should one exist. We let $F_i(\sigma_i, \epsilon_i)$ denote the cumulative distribution function of generator i's mixed strategy, Φ_i denote its support, and ϵ_i^- and ϵ_i^+ denote the infimum and supremum energy offers, respectively, in Φ_i. We let $\Phi = \cap_{i=1}^{N} \Phi_i$ denote the common support of the N cumulative distribution functions.

Lemma 8 *The infimum energy offers of the mixed strategies that are used by the generators in a high-demand state under a centrally committed market design are equal in a Nash equilibrium.*

Proof. See Appendix 6.2.1. ■

Lemma 9 *None of $F_1(\sigma_1, \epsilon_1), F_2(\sigma_2, \epsilon_2), \ldots, F_N(\sigma_N, \epsilon_N)$ have a mass point on Φ in a mixed-strategy Nash equilibrium in a high-demand state under a centrally committed market design.*

Proof. See Appendix 6.2.1. ■

Lemma 10 *Φ_i is a connected set $\forall i = 1, \ldots, N$, in a mixed-strategy Nash equilibrium in a high-demand state under a centrally committed market design.*

Proof. See Appendix 6.2.1. ■

Lemma 11 *The supremum energy offers of the mixed strategies that are used by the generators in a high-demand state under a centrally committed market design are equal in a Nash equilibrium.*

Proof. See Appendix 6.2.1. ■

Lemma 12 *The generators submit $\sigma_1 = \sigma_2 = \cdots = \sigma_N = \bar{\sigma}$ in a mixed-strategy Nash equilibrium in a high-demand state under a centrally committed market design.*

Proof. See Appendix 6.2.1. ■

As a result of Lemma 12, we can write the cumulative distribution function of generator i's mixed strategy as $F_i(\epsilon_i)$ (*i.e.*, it is not dependent on σ_i). The following proposition gives an analytical expression that characterizes a mixed strategy Nash equilibrium, if a symmetric equilibrium exists.

Proposition 3 *If a symmetric mixed-strategy Nash equilibrium exists in a high-demand state under a centrally committed market design, the cumulative distribution functions of the mixed strategies that are used by the generators satisfy the delay differential equation:*

$$f(\epsilon) = \frac{1}{N-1}\left[\frac{F(\epsilon)}{c-\epsilon} + \frac{F(\epsilon + \bar{\sigma}/K)^{N-1}K}{(l-NK)(c-\epsilon)F(\epsilon)^{N-2}}\right], \tag{24}$$

where $f(\epsilon)$ is the probability density function of the mixed strategies.

Proof. See Appendix 6.2.1. ■

Although we assume that the firms are symmetric, this on its own does not guarantee that a symmetric Nash equilibrium exists. Moreover, for (24) to give a valid equilibrium, it must have a solution that satisfies a number of properties of probability density and cumulative distribution functions (*i.e.*, $f(\epsilon) \geq 0$ for $\epsilon \in [\epsilon^-, \epsilon^+]$, $F(\epsilon) = 0$ for $\epsilon \leq \epsilon^-$, and $F(\epsilon) = 1$ for $\epsilon \geq \epsilon^+$). We do not have any theoretical guarantees that (24) yields a solution with such properties. However, our

Copyright © 2018 by the IAEE. All rights reserved.

numerical testing shows that we can find solutions to (24) that satisfy the requisite properties for many reasonable sets of cost, load, capacity, and offer-cap values. Equation (24) shows also that:

$$\lim_{\epsilon \to \bar{\epsilon}^-} f(\epsilon) = +\infty. \tag{25}$$

There are, however, examples of probability density functions (the Gamma distribution is one example) that exhibit such asymptotic behavior.

Lemma 13 *If a symmetric mixed-strategy Nash equilibrium exists in a high-demand state under a centrally committed market design, the equilibrium supremum energy offers of the mixed strategies that are used by the generators in a high-demand state under a centrally committed market design are equal to $\bar{\epsilon}$.*

Proof. See Appendix 6.2.1. ■

We conclude our analysis of Nash equilibria in a high-demand state under a centrally committed market design by computing the expected profits of the N generators.

Proposition 4 *If a symmetric mixed-strategy Nash equilibrium exists in a high-demand state under a centrally committed market design, each generator earns expected equilibrium profits that are equal to $\mathbb{E}[\pi^c] = \bar{\sigma} - S + (\bar{\epsilon} - c)l_r$.*

Proof. See Appendix 6.2.1. ■

3.2 Equilibria Under a Self-Committed Market Design

We proceed with the analysis of Nash equilibria under a self-committed market design by examining low-demand states first.

3.2.1 Equilibria in Low-Demand States

We show in the following proposition that in weak-demand states all generators earn zero profits in equilibrium.

Proposition 5 *In weak low-demand states the set of pure-strategy Nash equilibria under a self-committed market design is characterized by all of the generators submitting offers of the form:*

$$\delta \geq c + \frac{S}{l}, \tag{26}$$

with this inequality binding for at least two of them. All generators earn zero profits.

Proof. See Appendix 6.2.2. ■

Unlike weak low-demand states, regular low-demand states entail at least one inframarginal generator operating at full capacity while the marginal generator operates below its capacity. This distinction results in inframarginal generators earning strictly positive profits in regular low-demand states, as we demonstrate in the following proposition.

Proposition 6 *In regular low-demand states the set of pure-strategy Nash equilibria under a self-committed market design is characterized by $(N_l - 1)$ generators submitting offers of the form:*

$$\delta \leq c + \frac{S}{K}, \tag{27}$$

Copyright © 2018 by the IAEE. All rights reserved.

and the remaining generators submitting offers of the form:

$$\delta \geq c + \frac{S}{l_r}. \tag{28}$$

Of these remaining generators, at least two submit offers that make (28) binding. The marginal and inactive generators earn zero profits while the inframarginal generators earn strictly positive profits.

Proof. See Appendix 6.2.2. ■

The equilibria that are characterized in Proposition 6 are made possible by there being at least one inframarginal generator that is producing at capacity while the marginal generator is producing less than capacity. This does not occur in weak low-demand states. The marginal generator must produce below its full capacity but must recover its fixed and variable costs. Because its costs must be recovered through energy payments only (due to the lack of make-whole payments), the energy price must be sufficiently high to recover both cost components. This higher energy price allows the inframarginal generators to earn strictly positive profits. The inframarginal generators 'enforce' the equilibrium by submitting offers that are sufficiently low that neither the marginal nor the inactive generators have profitable deviations in which they undercut an inframarginal generator.

Before proceeding, we remark on one of the important distinctions between the multi-firm oligopoly setting that is studied here and the two-firm setting that Sioshansi and Nicholson (2011) examine. In the two-firm setting, low-demand states that do not require both of the generators to serve the load result in all generators earning zero profits under both the centrally and self-committed market designs. This equivalence no longer holds in the multi-firm setting. Indeed, we find that *ceteris paribus*, the self-committed market design is more expensive (in terms of remuneration to generators) in low-demand states than a centrally committed design is. This result shows also an important shortcoming of relying on linear energy payments only for recovering non-convex costs. The lack of a discriminatory payment mechanism (*i.e.*, make-whole payments) is what gives rise to the higher remuneration costs under a self-committed market design.

3.2.2 Equilibria in High-Demand States

Finally, we turn to the case of equilibria in high-demand states under a self-committed market design. Unlike the case of a centrally committed design, both pure- and mixed-strategy Nash equilibria exist in high-demand states under a self-committed design. We begin by characterizing the set of pure-strategy Nash equilibria in the following proposition.

Proposition 7 *In high-demand states the set of pure-strategy Nash equilibria under a self-committed market design is characterized by $(N-1)$ generators submitting offers of the form $\delta \leq (\bar{\delta}-c)l_r/K+c$ and the remaining generator submitting an offer of the form $\delta = \bar{\delta}$. All generators earn non-negative profits.*

Proof. See Appendix 6.2.2. ■

The intuition behind this set of pure-strategy Nash equilibria is straightforward. Because the generator submitting the highest offer is marginal and sets the energy price, this generator has an incentive to submit the highest offer possible. At the same time, the inframarginal generators must submit offers that are sufficiently small to ensure that the marginal generator has no incentive to undercut one of them. The threshold offer, $(\bar{\delta} - c)l_r/K + c$, is sufficiently small to ensure this. Because the market has a uniform price that is set by the marginal generator, the offers submitted by the inframarginal generators can be any value below this threshold (including offers that are below cost). Because $l_r < K$ we have that the threshold offer is valid per the market rules.

We turn now to characterizing and deriving a closed-form expression for a mixed-strategy Nash equilibrium in high-demand states under a self-committed market design. Unlike the case of

Copyright © 2018 by the IAEE. All rights reserved.

a centrally committed market design, existence of a symmetric mixed-strategy Nash equilibrium is guaranteed under the self-committed design. This is because the equilibrium can be derived explicitly as the solution of an ordinary differential equation, which exhibits all of the requisite properties of probability density and cumulative distribution functions. We let $G_i(\delta_i)$ denote the cumulative distribution function of generator i's mixed strategy, Ψ_i denote its support, and δ_i^- and δ_i^+ denote the infimum and supremum offers, respectively, in Ψ_i. We let $\Psi = \cup_{i=1}^N \Psi_i$ denote the common support of the N cumulative distribution functions.

Lemma 14 *The infimum offers of the mixed strategies that are used by the generators in a high-demand state under a self-committed market design are equal in a Nash equilibrium.*

Proof. See Appendix 6.2.2. ■

Lemma 15 *None of $G_1(\delta_1), G_2(\delta_2), \ldots, G_N(\delta_N)$ have a mass point on Ψ in a mixed-strategy Nash equilibrium in a high-demand state under a self-committed market design.*

Proof. See Appendix 6.2.2. ■

Lemma 16 *Ψ_i is a connected set $\forall i = 1, \ldots, N$, in a mixed-strategy Nash equilibrium in a high-demand state under a self-committed market design.*

Proof. See Appendix 6.2.2. ■

Lemma 17 *The supremum offers of the mixed strategies that are used by the generators in a high-demand state under a self-committed market design are all equal to $\bar{\delta}$ in a Nash equilibrium.*

Proof. See Appendix 6.2.2. ■

We derive now a closed-form expression of each generator's equilibrium mixed strategy in high-demand states under a self-committed market design.

Proposition 8 *There is a symmetric mixed-strategy Nash equilibrium in a high-demand state under a self-committed market design. The cumulative distribution functions of the mixed strategies that are used by the generators are given by:*

$$G(\delta) = \left(\frac{\delta - c}{\bar{\delta} - c}\right)^{\lambda}, \tag{29}$$

where:

$$\lambda = \frac{l_r}{(K - l_r)(N - 1)}. \tag{30}$$

Proof. See Appendix 6.2.2. ■

We conclude our analysis of the mixed-strategy Nash equilibria in a high-demand state under a self-committed market design by determining the expected profits of the generators.

Proposition 9 *In a symmetric mixed-strategy Nash equilibrium in a high-demand state under a self-committed market design each generator earns expected profits that are equal to $\mathbb{E}[\pi^s] = -S + (\bar{\delta} - c)l_r$.*

Proof. See Appendix 6.2.2. ■

4. EXPECTED-COST COMPARISON OF MARKET DESIGNS

We begin our cost comparison of the two market designs by showing analytical results, which rely on the properties of the equilibria that are derived in Section 3. Then we demonstrate the cost comparisons using a numerical example.

Copyright © 2018 by the IAEE. All rights reserved.

4.1 Analysis of Market Equilibria

We show first conditions under which the two market designs are expected-cost-equivalent and cases in which cost equivalence fails to hold. Importantly, we find cases in which cost equivalence holds in the duopoly case but fails to hold in the multi-firm case.

Proposition 10 *Centrally and self-committed market designs have the same remuneration costs in weak low-demand states.*

Proof. See Appendix 6.3. ■

Proposition 11 *Centrally and self-committed market designs do not have the same remuneration costs in regular low-demand states.*

Proof. See Appendix 6.3. ■

Cost comparison in high-demand states depends upon the offer caps that are set in the two markets. As such, we compare the expected remuneration cost of the two market designs under a range of random high-demand states. We add the following assumption that the offer caps under the two market designs are 'equivalent' to one another. In essence, we assume that the market (regardless of which design is adopted) clears repeatedly (*e.g.*, hourly) at different load levels. The offer caps under the two market designs are assumed to be set such that generators are not disadvantaged under one market design relative to another (*i.e.*, by one market design having an unduly low cap compared to the other).

Assumption 1 *The offer caps of the two market designs are set to satisfy:*

$$\bar{\sigma} - S + (\bar{\epsilon} - c)\mathbb{E}[l_r|l > (N-1)K] = -S + (\bar{\delta} - c)\mathbb{E}[l_r|l > (N-1)K]. \tag{31}$$

The way that Assumption 1 compares the offer caps under the two market designs is based on the expected profits of a generator submitting offers equal to the caps in high-demand states. A generator submitting an offer at that cap is marginal with probability 1. Thus, the two terms in the equality defining Assumption 1 are the expected profits of a generator offering at the cap. Assumption 1 can be interpreted as the MO having a forecast of expected demand in high-demand states. Using this forecast, the MO determines the expected residual load of the marginal generator. On the basis of this expected residual load, the MO can set offer caps to guarantee that a generator that submits an offer that is equal to the cap earns the same profit under either market design. This is critical for a meaningful comparison between the two designs. What is of interest is the fundamental properties of the two market designs, rather than artificially generating differences between them by manipulating the offer caps. Assumption 1 allows us to make an 'apples-to-apples' comparison of cost between the two designs.

The 'equivalent' offer cap under one market design can be determined from the offer cap of the other market design without knowing the true generator costs. This is because the equality in Assumption 1 can be simplified to $\bar{\sigma} + \bar{\epsilon}\mathbb{E}[l_r|l > (N-1)K] = \bar{\delta}\mathbb{E}[l_r|l > (N-1)K]$. This is desirable, as policymakers may not know true generator costs when designing markets and setting offer caps. Indeed, if a policymaker knows true generator costs, one would not need to solicit offers in the market as is examined in this paper.

We show now the cost-comparison properties of the two market designs in high-demand states when Assumption 1 holds.

Proposition 12 *Centrally and self-committed market designs have the same expected remuneration costs in high-demand states if Assumption 1 holds and the generators follow mixed-strategy Nash equilibria.*

Copyright © 2018 by the IAEE. All rights reserved.

Proof. See Appendix 6.3. ■

Corollary 3 *Centrally and self-committed market designs do not have the same expected remuneration costs in high-demand states if Assumption 1 holds and the generators follow a pure-strategy Nash equilibrium under a self-committed market design.*

Proof. See Appendix 6.3. ■

These two results regarding cost comparisons in high-demand states have analogues in the duopoly case. Sioshansi and Nicholson (2011) show that the two market designs are expected-cost-equivalent in the duopoly if Assumption 1 holds and generators follow mixed-strategy Nash equilibria. Otherwise, if duopolists follow a pure-strategy Nash equilibrium under a self-committed market design, the self-committed market is more costly.

4.2 Numerical Example

We conclude this section with a simple numerical example that illustrates equilibrium prices and remuneration costs under the two market designs. Table 1 lists the parameter values that are assumed in the example. The offer caps that are in Table 1 satisfy Assumption 1 if l_r is distributed uniformly in high-demand states. We examine cases with different values for N. Increasing the number of firms has two impacts on market equilibria. First, higher values of N mean that the load must be higher for the market to be in a high-demand state. Second, having more firms in the market tends to result in less aggressive offers (and lower expected energy prices) in high-demand states.

Table 1: Data for Numerical Example

Parameter	Value
c	\$30/MWh
S	\$10000
K	500 MW
$\mathbb{E}[l_r \vert l > (N-1)K]$	250 MW
$\bar{\epsilon}$	\$1000/MWh
$\bar{\sigma}$	\$25000
$\bar{\delta}$	\$1100

Although the primary distinction between the two market designs arises in regular low-demand states or if firms follow pure-strategy Nash equilibria in high-demand states under the self-committed market, our analysis of the example focuses on high-demand states with the firms following mixed strategy equilibria. This is because it is easier to conceptualize the prices and market outcomes that arise in low-demand states or from firms following pure strategies. The numerical example provides a visual feel for the relationship between equilibrium offering behavior and the residual load and number of firms if the firms follow mixed strategies.

Figure 1 shows the cumulative distribution functions that are derived from (24) with different numbers of firms and residual-load levels. The figure shows, as expected, that:

$$\lim_{\epsilon \to \epsilon^-} f(\epsilon) = +\infty,$$

which is seen by the functions being nearly vertical as they approach the infimum energy offer for each parameter set. The infimum energy offer depends on the number of firms and residual load level. This is because as the residual load increases, the opportunity cost of having a higher offer (which increases the likelihood of a generator being marginal as opposed to inframarginal) decreases. The figure shows also that having more firms results in the mixed strategies that are employed being more competitive, as the cumulative distribution functions with $N = 5$ are first-order stochastically dominated by the corresponding ones with $N = 3$.

Copyright © 2018 by the IAEE. All rights reserved.

Figure 1: Cumulative distribution functions of mixed strategies that are derived from (24) with different numbers of firms and residual-load levels.

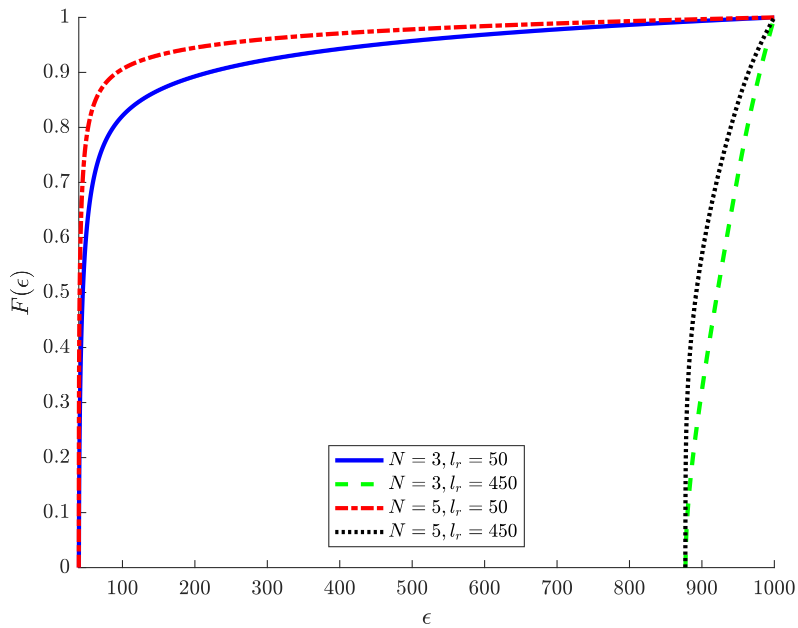

Figure 2 shows expected energy prices under the two market designs as a function of the residual load with different values of N, if the firms follow mixed-strategy Nash equilibria. Expected energy prices under a self-committed market design are computed analytically, using the closed-form expression for the equilibrium cumulative distribution function. Expected energy prices under a centrally committed market design are approximated numerically by solving the delay differential equation using finite differences to obtain the equilibrium cumulative distribution function. Then, the cumulative distribution function is integrated using numerical quadrature to approximate the expected energy price.

Figure 2 shows that with a greater number of firms, the expected energy price decreases under both market designs. The figure shows also that for low residual-load levels the expected energy price under a self-committed market design is lower than that under a centrally committed design. This is because lower residual-load levels imply greater profit losses from being the marginal as opposed to an inframarginal generator. Thus, generators submit more aggressive offers with lower residual loads, resulting in lower expected prices. With higher residual loads expected energy prices under the two market designs reverse.

Table 2 summarizes the total expected remuneration cost to the MO (which is likely borne by customers) in the high-demand states that are shown in Figure 2, assuming that the residual loads in the high-demand states are distributed uniformly between 50 MW and 450 MW and that the firms follow mixed-strategy equilibria. As expected from Proposition 12, the expected settlement costs are the same under the two market designs. The table shows that the make-whole payments represent a non-trivial portion of the total cost, constituting about 5% of costs with $N = 2$ and decreasing to 2% of costs with $N = 10$.

5. DISCUSSION AND CONCLUDING REMARKS

This paper studies equilibrium offering behavior under centrally and self-committed wholesale electricity market designs with an N-generator symmetric oligopoly. We show that the equilibrium results concerning high-demand states are qualitatively unchanged between the duopoly and oligopoly

Copyright © 2018 by the IAEE. All rights reserved.

Figure 2: Expected energy price as a function of residual load for different values of N if firms follow mixed-strategy Nash equilibria under centrally and self-committed market designs.

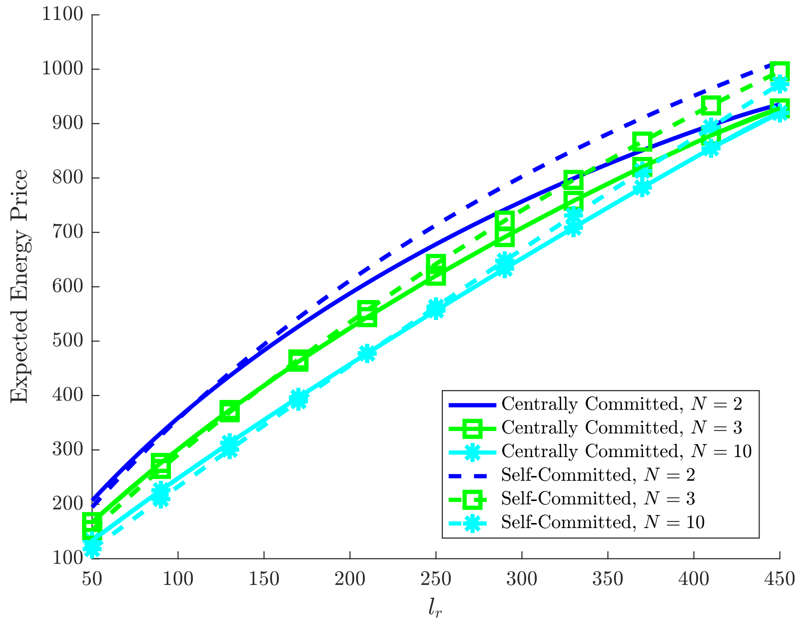

Table 2: Total Expected Remuneration Cost to MO Under Centrally and Self-Committed Market Designs for Different Values of N if Firms Follow Mixed-Strategy Nash Equilibria

	Centrally Committed			Self-Committed		
	$N = 2$	$N = 3$	$N = 10$	$N = 2$	$N = 3$	$N = 10$
Expected Energy Price [$/MWh]	645	598	548	678	620	557
Expected Make-Whole Payments [$ thousand]	28	30	46	n/a	n/a	n/a
Expected Settlement Cost [$ thousand]	535	803	2675	535	803	2675

cases. There is, however, a stark point of departure between the two market designs in low-demand states. As with a duopoly, all generators in a multi-firm oligopoly are constrained to earn zero profits under a centrally committed market design in low-demand states. However, inframarginal generators in a multi-firm oligopoly can earn strictly positive profits in low-demand states under a self-committed market design. This difference is driven by low-demand states that require the marginal generator to produce a positive amount of energy that is strictly less than its capacity. For the marginal generator to recover all of operating its costs, it must submit an energy offer (which sets the uniform energy price) that results in the inframarginal generators earning strictly positive profits. This outcome is ruled out in the duopoly case because a low-demand state in that case results in only one generator having to produce energy. Indeed, the equilibrium outcome in low-demand states in a duopoly are analogous to weak low-demand states in the multi-firm oligopoly that we study here. The upshot of this finding is that the uniform nature of payments under a self-committed market design guarantees that inframarginal generators earn positive profits. Conversely, the discriminatory nature of payments under a centrally-committed market design can result in lower prices and generator profits.

Our results showing strictly positive profits under a self-committed market design in low-demand states hold important policy and market-design implications. The analysis of a duopoly by Sioshansi and Nicholson (2011) suggests that centrally and self-committed market designs are equiv-

Copyright © 2018 by the IAEE. All rights reserved.

alent from the perspective of expected energy costs. Our analysis of multi-firm oligopolies shows this conclusion to be invalid. Importantly, the lack of expected-cost equivalence is not due to one market design being more prone to the exercise of market power. Indeed, expected-cost equivalence still holds in high-demand states, wherein firms have the most potential to exercise market power. Rather, expected-cost equivalence fails in low-demand states due to a deficiency of the payment mechanism under a self-committed market design. The discriminatory nature of the make-whole payment that is employed under a centrally committed market design allows for the non-convex costs of the marginal generator to be recovered without impacting the profits of inframarginal generators. Because a self-committed market design is restricted to relying on a uniform energy payment, cost recovery by the marginal generator results in positive profits for inframarginal generators. Thus, our results suggest that self-committed market designs suffer a flaw that arises from the simple offers and payment mechanism that they employ.

In a broader policy context, our results suggest that market designs that rely on simple offers (such as those that are employed in much of Australia and Western Europe) may be overly costly to consumers compared to the centrally committed designs that are now the archetype in the United States. There is an important *caveat* of our analysis, in that we do not consider the recovery of long-run investment and maintenance costs. The additional profits that generators earn in low-demand states under a self-committed market design may embiggen the recovery of such costs. An important nuance of these additional profits is that they are not earned due to scarcity. Rather, these profits arise from inframarginal generators exploiting a defect in the remuneration mechanism that is employed with a self-committed market to increase their profits. Thus, understanding the incentive properties of centrally and self-committed market designs within the context of long-run investment is an open question for further inquiry.

While we generate insights that go beyond the duopoly case, our model is highly stylized. We assume symmetric firms and fixed deterministic demand. The cost structure that we assume is consistent with thermal generation. The penetration of non-thermal (especially renewable) generation is growing in many parts of the world. Nevertheless, we focus on modeling thermal generation because its operation entails non-convexities that complicate market design, pricing, and remuneration. Given that renewable generation does not have start-up costs, it does not raise the same types of market-design issues on which we focus. There is a growing body of work, including those of Gugler et al. (2018); Grubb and Newbery (2018), that explores the integration of renewables into spot energy markets and necessary market reforms that they raise. As such, future work might consider a mixed model with characteristics of both thermal and renewable generation.

Promising work also remains to be done by relaxing the symmetric-firm and deterministic-demand assumptions and exploring more general market designs. Such work may generate further regulatory and market-design insights. That being said, we expect that our findings regarding the lack of cost equivalence between the two market designs in low-demand states to be robust to these assumptions. This is because the lack of cost equivalence arises from a limitation in relying on linear uniform prices for the recovery of non-convex costs. We do not anticipate that load uncertainty or firm asymmetry would overcome this fundamental limitation. However, relaxing some of our assumptions may provide firms with added avenues to exercise market power. Along these lines, LaCasse (1995) studies how the possibility of prosecution alters incentives for firms to collude in government procurements. Fabra (2003) considers the susceptibility of discriminatory versus uniform auctions to collusion in a repeated game.

Another important consideration that we neglect in our work is incentive-compatibility. Chao and Wilson (2002) consider an optimal auction mechanism for the procurement of electricity reserves. A promising direction, in which to take future work, is examining centrally committed and self-committed markets from a theoretical mechanism design perspective, taking into account explicit incentive-compatibility constraints. We neglect also distributional and market-efficiency impacts of a centrally committed market that arise from the MO's market model not being solved

Copyright © 2018 by the IAEE. All rights reserved.

to complete optimality. Johnson et al. (1997); Sioshansi et al. (2008); Sioshansi and Tignor (2012) show that this limitation of market modeling software can impact wholesale market prices and the profits of individual generators. This introduces randomness in commitment, dispatch, and pricing decisions. Finally, while the study of electricity auctions is growing within the theoretical literature, it also remains a promising arena for future experimental work. Denton et al. (2001) conduct an experimental analysis of spot-market design for electricity, while Rassenti et al. (2003) study price volatility in discriminatory- and uniform-price auctions for electricity in an experimental setting. An experimental analysis of centrally and self-committed market designs may generate useful information regarding the behavior of participants in these markets as well as potential mechanisms for their improvement.

ACKNOWLEDGEMENTS

Thank you to R. O'Neill, S. Oren, and A. Sorooshian for helpful discussions and suggestions. This work was supported by National Science Foundation grant 1548015. Any opinions, conclusions, and errors expressed in this paper are solely those of the authors.

REFERENCES

Álvaro Riascos, Bernal, M., de Castro, L., Oren, S. S., 2016. Transition to Centralized Unit Commitment: An Econometric Analysis of Colombia's Experience. The Energy Journal 37, 271–291.

Baldick, R., February 1995. The generalized unit commitment problem. IEEE Transactions on Power Systems 10, 465–475.

Chao, H.-P., Wilson, R. B., 2002. Multi-Dimensional Procurement Auctions for Power Reserves: Robust Incentive-Compatible Scoring and Settlement Rules. Journal of Regulatory Economics 22, 161–183.

Cramton, P., 2 November 2017. Electricity market design. Oxford Review of Economic Policy 33, 589–612.

Denton, M. J., Rassenti, S. J., Smith, V. L., April 2001. Spot market mechanism design and competitivity issues in electric power. Journal of Economic Behavior & Organization 44, 435–453.

Elmaghraby, W., Oren, S. S., October 1999. Efficiency of Multi-Unit Electricity Auctions. The Energy Journal 20, 89–115.

Fabra, N., September 2003. Tacit Collusion in Repeated Auctions: Uniform Versus Discriminatory. The Journal of Industrial Economics 51, 271–293.

Grubb, M., Newbery, D., June 2018. UK Electricity Market Reform and the Energy Transition: Emerging Lessons. EPRG Working Paper 1817, Energy Policy Research Group, University of Cambridge.

Gugler, K., Haxhimusa, A., Liebensteiner, M., 2018. Integration of European Electricity Markets: Evidence from Spot Prices. The Energy Journal 39, 41–66.

Hobbs, B. F., Rothkopf, M. H., O'Neill, R. P., Chao, H.-P. (Eds.), 2001. The Next Generation of Electric Power Unit Commitment Models. Kluwer, Norwell, Massachusetts.

Hogan, W. W., 1994. An Efficient Bilateral Market Needs a Pool. Testimony Before the California Public Utilities Commission.

Imran, K., Kockar, I., March 2014. A technical comparison of wholesale electricity markets in North America and Europe. Electric Power Systems Research 108, 59–67.

Johnson, R. B., Oren, S. S., Svoboda, A. J., March 1997. Equity and Efficiency of Unit Commitment in Competitive Electricity Markets. Utilities Policy 6, 9–19.

LaCasse, C., Autumn 1995. Bid Rigging and the Threat of Government Prosecution. The RAND Journal of Economics 26, 398–417.

Muñoz, F. D., Wogrin, S., Oren, S. S., Hobbs, B. F., 2018. Economic Inefficiencies of Cost-based Electricity Market Designs. The Energy Journal 39, 51–68.

O'Neill, R. P., Sotkiewicz, P. M., Hobbs, B. F., Rothkopf, M. H., Stewart, W. R., 1 July 2005. Efficient market-clearing prices in markets with nonconvexities. European Journal of Operational Research 164, 269–285.

Oren, S. S., Ross, A. M., October 2005. Can we prevent the gaming of ramp constraints? Decision Support Systems 40, 461–471.

Padhy, N. P., May 2004. Unit Commitment—A Bibliographical Survey. IEEE Transactions on Power Systems 19, 1196–1205.

Copyright © 2018 by the IAEE. All rights reserved.

Rassenti, S. J., Smith, V. L., Wilson, B. J., March 2003. Discriminatory Price Auctions in Electricity Markets: Low Volatility at the Expense of High Price Levels. Journal of Regulatory Economics 23, 109–123.

Ruff, L. E., June 1994. Stop Wheeling and Start Dealing: Resolving the Transmission Dilemma. The Electricity Journal 7, 24–43.

Scarf, H. E., May-June 1990. Mathematical Programming and Economic Theory. Operations Research 38, 377–385.

Scarf, H. E., Autumn 1994. The Allocation of Resources in the Presence of Indivisibilities. The Journal of Economic Perspectives 8, 111–128.

Schiro, D. A., Zheng, T., Zhao, F., Litvinov, E., September 2016. Convex Hull Pricing in Electricity Markets: Formulation, Analysis, and Implementation Challenges. IEEE Transactions on Power Systems 31, 4068–4075.

Sheble, G. S., Fahd, G. N., February 1994. Unit commitment literature synopsis. IEEE Transactions on Power Systems 9, 128–135.

Sioshansi, R., December 2014. Pricing in Centrally Committed Markets. Utilities Policy 31, 143–145.

Sioshansi, R., Nicholson, E., August 2011. Towards Equilibrium Offers in Unit Commitment Auctions with Nonconvex Costs. Journal of Regulatory Economics 40, 41–61.

Sioshansi, R., O'Neill, R. P., Oren, S. S., May 2008. Economic Consequences of Alternative Solution Methods for Centralized Unit Commitment in Day-Ahead Electricity Markets. IEEE Transactions on Power Systems 23, 344–352.

Sioshansi, R., Tignor, A., 2012. Do Centrally Committed Electricity Markets Provide Useful Price Signals? The Energy Journal 33, 97–118.

Copyright © 2018 by the IAEE. All rights reserved.

6. APPENDIX

6.1 Proof of Lemma from Section 2

Proof of Lemma 1. Suppose for contradiction that there is a solution to the MO's problem in which $\tilde{N}$ generators are committed, where $\tilde{N} > N_l$. Without loss of generality, suppose that the generators are labeled so that $\epsilon_1 \leq \epsilon_2 \leq \ldots \leq \epsilon_{\tilde{N}}$ are the generators that are committed and the remaining generators are not committed.

Consider an alternate solution in which generators 1 through $(N_l - 1)$ are dispatched to produce K MW, generator N_l is dispatched to produce l_r MW, and the remaining generators are dispatched to produced nothing and switched off. Such a solution is less costly. ■

6.2 Proof of Lemmata, Corollaries, and Propositions from Section 3

6.2.1 Proof of Lemmata, Corollaries, and Propositions from Section 3.1

Proof of Lemma 2. In a weak low-demand state only one generator is required to serve demand. As such, this generator sets the energy price, $p^c = \epsilon^U$. Moreover, because $\sigma^U > 0$, this generator must receive make-whole payments. Thus, from (11) we have that total payments to the unique generator are $T^U = \epsilon^U l + \max\{0, \sigma^U + (\epsilon^U - \epsilon^U)l\} = \sigma^U + \epsilon^U l$. Because inactive generators produce nothing and incur zero cost, they receive neither energy nor make-whole payments.

In all other demand states we have $N_l \geq 2$. The marginal generator is dispatched to serve the residual load, l_r, and it sets the energy price. This is because it is optimal for the MO to have the generator with the highest value of ϵ_i in its offer (among the N_l generators that it commits) serve the residual load. Thus, we have $p^c = \epsilon^M$. Following the same logic as in the weak low-demand state, because $\sigma^M > 0$ the marginal generator must receive make-whole payments. Thus, the total payments to the marginal generator are $T^M = \epsilon^M l_r + \max\{0, \sigma^M + (\epsilon^M - \epsilon^M)l_r\} = \sigma^M + \epsilon^M l_r$.

The MO ensures that inframarginal generators earn revenues that cover their incurred costs (as calculated on the basis of their offers). We know that inframarginal generators are dispatched to produce K MW. Thus, from (11) we have that total payments to inframarginal generator i are $T_i^I = \epsilon^M K + \max\{0, \sigma_i + (\epsilon_i - \epsilon^M)K\}$. If $\max\{0, \sigma_i + (\epsilon_i - \epsilon^M)K\} = \sigma_i + (\epsilon_i - \epsilon^M)K$ this means that $\sigma_i + \epsilon_i K \geq \epsilon^M K$ and that energy payments alone are insufficient to recover the offer-based cost that is incurred by generator i. Thus, generator i is given a make-whole payment that is equal to $\sigma_i + \epsilon_i K$. Otherwise, if $\max\{0, \sigma_i + (\epsilon_i - \epsilon^M)K\} = 0$ we have that $\sigma_i + \epsilon_i K \leq \epsilon^M K$ meaning that energy payments are sufficient to recover the offer-based cost that is incurred by generator i. In such a case, generator i is given an energy payment that is equal to $\epsilon^M K$. Thus, in the end, an inframarginal generator receives the larger of $\sigma_i + \epsilon_i K$ and $\epsilon^M K$ in total payments, meaning that $T_i^I = \max\{\epsilon^M K, \sigma_i + \epsilon_i K\}$. Finally, we have (as with weak low-demand states) that any inactive generators receive neither energy nor make-whole payments. ■

Proof of Lemma 3. Assume for contradiction that the marginal generator earns nonzero profits. If its profits are negative, the marginal generator prefers to be inactive and earn zero profits. If the marginal generator is earning positive profits, its offer can be profitably undercut by an inactive generator. Thus, the profits of the marginal generator must be zero. ■

Proof of Corollary 1. Based on Lemma 2 we know that the marginal generator receives $T^M = \sigma^M + \epsilon^M l_r$, in total payments. We know from Lemma 3 that the marginal generator earns zero profits. Thus, we have that $\sigma^M + \epsilon^M l_r = S + cl_r$.

Now, assume for contradiction that no inactive generator submits an offer that satisfies (19). This means that at least one inactive generator, generator j, submits an offer $\omega_j = (\sigma_j, \epsilon_j)$, such that $\sigma_j + \epsilon_j l_r < S + cl_r$. However, in this case it would be optimal for the MO to commit and dispatch generator j in place of the marginal generator, giving a contradiction.

Copyright © 2018 by the IAEE. All rights reserved.

To prove the second part of the corollary, assume for contradiction that no inactive generator submits an offer that makes (19) binding. Let generator i be the inactive generator with the offer that gives the lowest value of $\sigma_i + \epsilon_i l_r$. Then the marginal generator has a profitable deviation in which it increases its offer to undercut $\sigma_i + \epsilon_i l_r$, which increases the make-whole payments that the marginal generator receives. ■

Proof of Proposition 1. In a weak low-demand state the load is served by a single unique generator, which is also, by definition, the marginal generator. From Lemma 3, we know that the unique generator must earn zero profit. Combining this with Lemma 2 tells us that $\omega^U \in \Omega_W$. Corollary 1 shows that at least one inactive generator must submit an offer from the set Ω_W and the property regarding offers of the remaining inactive generators. From Lemma 2 we know that all generators earn zero profits. ■

Proof of Lemma 4. Assume for contradiction that there exists at least one inframarginal generator that receives energy payments and another inframarginal generator, generator j, that receives make-whole payments. This means that generator j receives total payments of $\sigma_j + \epsilon_j K$ which is greater than $\epsilon^M K$, the amount earned by the inframarginal generator receiving energy payments. Then, the inframarginal generator receiving energy payments has a profitable deviation, in which it matches generator j's offer. ■

Proof of Corollary 2. From Lemma 4 we know that all inframarginal generators receive the same type of payments. If they all receive energy payments, then this result is true trivially.

If, on the other hand, they receive make-whole payments assume for contradiction that inframarginal generator i receives a greater make-whole payment than inframarginal generator j. This implies that $\sigma_i + \epsilon_i K > \sigma_j + \epsilon_j K$. In such a case, however, generator j can deviate profitably and match generator i's offer. ■

Proof of Lemma 5. Assume for contradiction that the marginal generator submits an offer with $\epsilon^M < c$. By Lemma 3 we must have $\sigma^M > S$ and the marginal generator receives make-whole payments to recover its actual incurred costs (and earn zero profits). This means that $\sigma^M + \epsilon^M l_r = S + c l_r$. Given that $l_r < K$ and the assumption that $\epsilon^M < c$, it must be true that $\sigma^M + \epsilon^M K < S + cK$, which in turn implies that $\epsilon^M K < S + cK$ (because $\sigma^M > 0$). Thus, inframarginal generators must receive make-whole payments, otherwise they earn negative profits.

For inframarginal generators to earn non-negative profits from make-whole payments, they must submit offers such that $\sigma^I + \epsilon^I K \geq S + cK$. This inequality implies, however, that $\sigma^M + \epsilon^M K < S + cK \leq \sigma^I + \epsilon^I K$, meaning that it would be less costly (from the perspective of the MO) for the marginal generator to be dispatched as an inframarginal generator. This gives a contradiction showing that there are no equilibria in which $\epsilon^M < c$. ■

Proof of Lemma 6. We prove that a set of offers satisfying these conditions constitute a Nash equilibrium by examining, in turn, the offers of the marginal, inactive, and inframarginal generators. We know from Lemma 3 that the marginal generator must earn zero profits in a Nash equilibrium. Combining this with Lemma 2 tells us that if $\epsilon^M = c$ then $\sigma^M = S$. Corollary 1 gives us the properties of the offers of the inactive generators.

Finally, consider the offers of inframarginal generators. If an inframarginal generator submits an offer from the set, Ω_L, it must receive make-whole payments (because the marginal generator sets the energy price equal to c, which is insufficient to recover the inframarginal generator's start-up cost) and earns zero profits. This inframarginal generator has no incentive to submit an offer, $\tilde{\omega} = (\tilde{\sigma}, \tilde{\epsilon})$, such that $\tilde{\sigma} + \tilde{\epsilon} K < S + cK$, because doing so results in negative profits. Moreover, consider a deviation in which this inframarginal generator submits an offer, $\tilde{\omega}$, such that $\tilde{\sigma} + \tilde{\epsilon} K > S + cK$. The marginal generator is submitting an offer with true costs. Thus, we have $\tilde{\sigma} + \tilde{\epsilon} K > S + cK = \sigma^M + \epsilon^M K$, meaning that the inframarginal generator becomes a marginal or inactive generator (resulting in it earning zero profits) if it deviates and offers $\tilde{\omega}$. Thus, we conclude that the inframarginal generator has no profitable deviation from submitting an offer in the set Ω_L.

Copyright © 2018 by the IAEE. All rights reserved.

We can conclude also that there are no Nash equilibria in which any inframarginal generator submits an offer that is not in the set Ω_L. To see this, consider inframarginal generator i submitting an offer, $\omega_i = (\sigma_i, \epsilon_i)$, such that $\sigma_i + \epsilon_i K < S + cK$. In this case, generator i earns negative profits and has a profitable deviation in which it submits a sufficiently high offer that it becomes inactive. Conversely, if inframarginal generator i submits an offer such that $\sigma_i + \epsilon_i K > S + cK$, then because the marginal generator is submitting an offer with its actual costs, we know that it is not possible for generator i to be inframarginal (*i.e.*, there is a lower-cost solution to the MO's problem in which generator i is not an inframarginal generator). ■

Proof of Lemma 7. Assume for contradiction that there is a Nash equilibrium in which $\epsilon^M > c$ and in which the inframarginal generators earn strictly positive profits. We examine two cases, those in which the inframarginal generators receive energy payments and those in which they receive make-whole payments (we know from Lemma 4 that all inframarginal generators receive the same type of payments in equilibrium).

First, consider the case in which inframarginal generators receive energy payments. For this to be true, it must be the case that $\epsilon^M K > \sigma_i + \epsilon_i K$, where generator i is an inframarginal generator. For generator i's profits to be strictly positive, it must also be the case that $\epsilon^M K > S + cK$. Moreover, because $\sigma^M > 0$ we must have that $\sigma^M + \epsilon^M K > \epsilon^M K > S + cK$. Thus, generator i has a profitable deviation in which it submits the offer, $\tilde{\omega}_i = (\tilde{\sigma}_i, \tilde{\epsilon}_i)$, such that $\sigma^M + \epsilon^M K > \tilde{\sigma}_i + \tilde{\epsilon}_i K > \epsilon^M K$. Doing so results in generator i receiving make-whole payments (as opposed to energy payments), which are greater than the energy payments it otherwise receives. Thus, we have a contradiction showing that if $\epsilon^M > c$ there are no pure-strategy Nash equilibria in which inframarginal generators receive energy payments and earn strictly positive profits.

Next, consider the case in which inframarginal generator i receives make-whole payments. For generator i to be inframarginal, we need $\sigma^M + \epsilon^M K \geq \sigma_i + \epsilon_i K$. We consider now cases in which this inequality is binding and not. If $\sigma^M + \epsilon^M K > \sigma_i + \epsilon_i K$, generator i has a profitable deviation in which it submits the offer, $\tilde{\omega}_i = (\tilde{\sigma}_i, \tilde{\epsilon}_i)$, such that $\sigma^M + \epsilon^M K > \tilde{\sigma}_i + \tilde{\epsilon}_i K > \sigma_i + \epsilon_i K$, as doing so increases the make-whole payments that it receives. In the other case that:

$$\sigma^M + \epsilon^M K = \sigma_i + \epsilon_i K, \tag{32}$$

we note that for generator i to earn strictly positive profits we need that $\sigma_i + \epsilon_i K > S + cK$. Combining this inequality with (32) gives $\sigma^M + \epsilon^M K = \sigma_i + \epsilon_i K > S + cK$. This inequality gives the marginal generator a profitable deviation, in which it undercuts generator i's offer slightly, earning it strictly positive profits. Thus, we again have a contradiction showing that if $\epsilon^M > c$ there are no pure-strategy Nash equilibria in which inframarginal generators receive make-whole payments and earn strictly positive profits.

Finally, we can conclude that the inframarginal generators cannot earn strictly negative profits in an equilibrium, as each would have a profitable deviation in which it increases its offer to become inactive. Thus, the inframarginal generators must earn exactly zero profits in equilibrium. ■

Proof of Proposition 2. Assume for contradiction that the offers, $(\sigma_1, \epsilon_1), (\sigma_2, \epsilon_2), \ldots, (\sigma_N, \epsilon_N)$, constitute a pure-strategy Nash equilibrium. If any generator is operating at a net profit loss, it can deviate and submit the offer, $(\bar{\sigma}, \bar{\epsilon})$. By assumption, such offers are high enough to guarantee cost recovery. Thus, we assume henceforth that no generator is operating at a net profit loss under the assumed equilibrium.

Without loss of generality, suppose that the generators are labeled so that $\epsilon_1 \leq \epsilon_2 \leq \cdots \leq \epsilon_N$. Because all N generators must be committed, we know that they are dispatched based solely on the merit order of the marginal costs that are specified in their offers. As such, generators 1 through $(N-1)$ are inframarginal generators, generator N is the marginal generator, and the energy price is ϵ_N. We consider now two cases, depending on whether there is a tie between the marginal costs specified in the offers of generators $(N-1)$ and N.

Consider, first, the case in which $\epsilon_{N-1} < \epsilon_N$. If generator $(N-1)$ receives make-whole

Copyright © 2018 by the IAEE. All rights reserved.

payments, then generator $(N-1)$ has a profitable deviation in which it submits an offer, $\tilde{\omega}_{N-1} = (\sigma_{N-1}, \epsilon_N - \eta)$, with $\eta > 0$ sufficiently small. Offering $\tilde{\omega}_{N-1}$ results in generator $(N-1)$ remaining an inframarginal generator but earning higher make-whole payments. Conversely, if generator $(N-1)$ receives energy payments, this means that $\epsilon_N K > \sigma_{N-1} + \epsilon_{N-1} K$. Generator $(N-1)$ has also a profitable deviation in this case, wherein it submits the offer, $\tilde{\omega}_{N-1} = (\sigma_N, \epsilon_N - \eta)$, with $\eta > 0$ sufficiently small so that $\sigma_N + (\epsilon_N - \eta)K > \epsilon_N K$. Offering $\tilde{\omega}_{N-1}$ results in generator $(N-1)$ remaining an inframarginal generator but receiving make-whole payments, which are greater than the energy payments, $\epsilon_N K$. The profitable deviations for generator $(N-1)$ in both of these cases show that there are no pure-strategy Nash equilibria in which there is a unique marginal generator with the highest marginal cost in its offer.

Now, consider the second case, in which $\epsilon_{N-1} = \epsilon_N$. We know from (9) that the expected profit of generator $(N-1)$ is $\sigma_{N-1} - S + (\epsilon_N - c)\frac{1}{M_N^c}(l - (N - M_N^c)K)$, where M_N^c is defined by (8). Consider a deviation, in which generator $(N-1)$ submits the offer, $\tilde{\omega}_{N-1} = (\sigma_{N-1}, \epsilon_N - \eta)$, with $\eta > 0$ sufficiently small. Offering $\tilde{\omega}_{N-1}$ causes generator $(N-1)$ to become an inframarginal generator that receives make-whole payments. As such, its profit changes to $\sigma_{N-1} - S + (\epsilon_N - \eta - c)K$. We have that $\sigma_{N-1} - S + (\epsilon_N - \eta - c)K > \sigma_{N-1} - S + (\epsilon_N - c)(l - (N - M_N^c)K)/M_N^c$, because $K > (l - (N - M_N^c)K)/M_N^c$, whereas η can be made arbitrarily small. Thus, we have a contradiction showing that there are no pure-strategy Nash equilibria in which there are multiple generators tied with the highest energy offer, proving the desired result. ■

Proof of Lemma 8. Assume for contradiction that $\epsilon_i^- < \epsilon_j^-$. In such a case, generator i has a profitable deviation wherein it moves the density that is assigned to the interval, $[\epsilon_i^-, \epsilon_j^-)$, to $\epsilon_j^- - \eta$, with $\eta > 0$ sufficiently small. By doing so, generator i increases its expected profit without decreasing the probability that it is an inframarginal generator. Thus, we have a contradiction showing that the infimum energy offers in a mixed-strategy Nash equilibrium must be equal. ■

Proof of Lemma 9. Assume for contradiction that $F_i(\sigma_i, \epsilon_i)$ has a mass point, which we denote as $\hat{\epsilon}_i$. There exist $\eta > 0$ and $\rho > 0$ such that some generator $j \neq i$ can deviate profitably by moving the density that is assigned to the interval, $[\hat{\epsilon}_i, \hat{\epsilon}_i + \eta)$, to the offer $\hat{\epsilon}_i - \rho$, contradicting the assumption of an equilibrium mass point.

To see that this is a profitable deviation, we consider the following three possible cases of whether generator j is marginal or inframarginal before and after the deviation. First, if generator j is inframarginal before and after the deviation, there is no profit loss if it receives energy payments. Otherwise, if it receives make-whole payments its profit loss is at most $(\eta + \rho)K$. Second, if generator j is marginal before and after the deviation, there is a profit loss of at most $(\eta + \rho)K$. Finally, if generator j is marginal before the deviation and inframarginal after the deviation, its profit increases by at least $(\hat{\epsilon}_i - c)(K - l_r) - \rho K - \eta l_r$. For η and ρ sufficiently small, the profit increase in the third case outweighs the profit losses in the first two. ■

Proof of Lemma 10. Assume for contradiction that there exists an interval, $[\hat{\epsilon}_i, \hat{\epsilon}_i + \eta]$ with $\eta > 0$, on which generator i places zero density in a mixed-strategy Nash equilibrium. Generator $j \neq i$ has a profitable deviation in which the density that is assigned to the interval, $(\hat{\epsilon}_i - \rho, \hat{\epsilon}_i)$, is assigned to the offer, $\hat{\epsilon}_i + \eta - \xi$, where $\rho > 0$ and $\xi \in (0, \eta)$, contradicting the assumption that Φ_i is not connected.

To see that this is a profitable deviation, we consider the following three possible cases of whether generator j is marginal or inframarginal before and after the deviation. First, if generator j is inframarginal before and after the deviation, there is no profit change if it receives energy payments. Otherwise, if it receives make-whole payments its profits increase by at least $(\eta - \xi)K$. Second, if generator j is marginal before and after the deviation, its profits increase by at least $(\eta - \xi)l_r$. Finally, if generator j is inframarginal before the deviation and marginal after the deviation, its profits change by at most $(\hat{\epsilon}_i - c)(l_r - K) + (\eta - \xi)l_r + \rho K$. Thus, generator j only stands to have a profit loss in the third case. However, ρ can be chosen to make the probability of this event arbitrarily close to zero. ■

Copyright © 2018 by the IAEE. All rights reserved.

Proof of Lemma 11. Assume for contradiction that $\epsilon_i^+ < \epsilon_j^+$. Generator i could deviate profitably by moving the density that is assigned to the interval, $(\epsilon_i^+ - \eta, \epsilon_i^+]$, with $\eta > 0$ sufficiently small, to $\epsilon_j^+ - \rho$, with $\epsilon_j^+ - \rho > \epsilon_i^+$. By doing so, generator i increases its expected profits without decreasing the probability that it is an inframarginal generator. This contradiction shows that the supremum energy offers must be equal. ■

Proof of Lemma 12. There are only mixed-strategy Nash equilibria in a high-demand state under a centrally committed market design. Moreover, we have that $\Phi_1 = \Phi_2 = \cdots = \Phi_N$. Thus, each generator has a strictly positive probability of being the marginal generator. From Lemma 2 we know that the marginal generator's profit is strictly increasing in σ^M. Moreover, the profits of the inframarginal generators are non-decreasing in σ^I. Finally, we know that the assignment of the N generators to being marginal or inframarginal does not depend on the values of $\sigma_1 = \sigma_2 = \cdots = \sigma_N$ in a high-demand state. Thus, it is expected-profit-maximizing for each generator to submit the highest possible value for the start-up cost, $\bar{\sigma}$, in its offer. ■

Proof of Proposition 3. We begin by assuming that all of generator i's rivals follow the symmetric equilibrium that is given by the probability density function, $f(\epsilon)$, and the associated cumulative distribution function, $F(\epsilon)$. We can write generator i's expected profit, as a function of its own offer, ϵ_i, as:

$$\begin{aligned}
\mathbb{E}[\pi_i^c(\epsilon_i)] &= F(\epsilon_i)^{N-1}[\bar{\sigma} - S + (\epsilon_i - c)l_r] \\
&\quad + \left[F(\epsilon_i + \bar{\sigma}/K)^{N-1} - F(\epsilon_i)^{N-1}\right][\bar{\sigma} - S + (\epsilon_i - c)K] \\
&\quad + \int_{\epsilon_i + \bar{\sigma}/K}^{\bar{\epsilon}} [-S + (\epsilon_{-i}^{\max} - c)K]\, d\left(F(\epsilon_{-i}^{\max})^{N-1}\right) \\
&= F(\epsilon_i)^{N-1}[\bar{\sigma} - S + (\epsilon_i - c)l_r] \\
&\quad + \left[F(\epsilon_i + \bar{\sigma}/K)^{N-1} - F(\epsilon_i)^{N-1}\right][\bar{\sigma} - S + (\epsilon_i - c)K] \\
&\quad + \int_{\epsilon_i + \bar{\sigma}/K}^{\bar{\epsilon}} [-S + (\epsilon_{-i}^{\max} - c)K](N-1)F(\epsilon_{-i}^{\max})^{N-2} f(\epsilon_{-i}^{\max})\, d\epsilon_{-i}^{\max},
\end{aligned} \tag{33}$$

where $\epsilon_{-i}^{\max}$ represents the maximum energy offer of generator i's rivals.

The first-order necessary condition (FONC) for maximizing (33) with respect to ϵ_i can be written as:

$$f(\epsilon_i) = \frac{1}{N-1}\left[\frac{F(\epsilon_i)}{c - \epsilon_i} + \frac{F(\epsilon_i + \bar{\sigma}/K)^{N-1}K}{(l - NK)(c - \epsilon_i)F(\epsilon_i)^{N-2}}\right]. \tag{34}$$

Finally, because we are assuming a symmetric equilibrium, we drop the subscript, i, which gives the desired delay differential equation. ■

Proof of Lemma 13. Assume for contradiction that $\epsilon_1^+ = \epsilon_2^+ = \cdots = \epsilon_N^+ = \epsilon^+ < \bar{\epsilon}$ in a Nash equilibrium. Then generator i has a profitable deviation in which it moves the density that is assigned to the interval, $(\epsilon^+ - \eta, \epsilon^+]$, where $\eta > 0$, to the energy offer, $\bar{\epsilon}$, contradicting the assumption of a Nash equilibrium in which $\epsilon^+ < \bar{\epsilon}$.

To see that this is a profitable deviation, we consider the following three possible cases of whether generator i is marginal or inframarginal before and after the deviation. First, if generator i is inframarginal before and after the deviation, there is no change in its profits if it receives energy payments. Otherwise, if it receives make-whole payments its profits increase by at least $(\bar{\epsilon} - \epsilon^+)K$. Second, if generator i is marginal before and after the deviation, its profits increase by at least $(\bar{\epsilon} - \epsilon^+)l_r$. Finally, if generator i is inframarginal before the deviation and marginal after the deviation, its profits change by at most $(\bar{\epsilon} - c)l_r - (\epsilon^+ - c)K$. Thus, generator i only stands to have a profit loss in the third case. However, η can be chosen to make the probability of this event arbitrarily close to zero. ■

Copyright © 2018 by the IAEE. All rights reserved.

Proof of Proposition 4. From (33) we have that generator i earns an expected profit of:

$$\begin{aligned}\mathbb{E}[\pi_i^c(\bar{\epsilon})] &= F(\bar{\epsilon})^{N-1}[\bar{\sigma} - S + (\bar{\epsilon} - c)l_r] && (35)\\ &\quad + \left[F(\bar{\epsilon} + \bar{\sigma}/K)^{N-1} - F(\bar{\epsilon})^{N-1}\right][\bar{\sigma} - S + (\bar{\epsilon} - c)K]\\ &\quad + \int_{\bar{\epsilon}+\bar{\sigma}/K}^{\bar{\epsilon}} [-S + (\epsilon_{-i}^{\max} - c)K](N-1)F(\epsilon_{-i}^{\max})^{N-2} f(\epsilon_{-i}^{\max}) d\epsilon_{-i}^{\max} && (36)\\ &= \bar{\sigma} - S + (\bar{\epsilon} - c)l_r, && (37)\end{aligned}$$

if it submits an energy offer that is equal to $\bar{\epsilon}$. Because the mixed strategies constitute a Nash equilibrium, all energy offers in Φ must yield the same expected profit for each generator. Thus, we have that $\mathbb{E}[\pi_i^c(\epsilon)] = \bar{\sigma} - S + (\bar{\epsilon} - c)l_r, \forall \epsilon \in \Phi, i = 1, \ldots, N$, showing the desired result. ■

6.2.2 Proof of Propositions and Lemmata from Section 3.2

Proof of Proposition 5. Without loss of generality, assume that the generators are labeled so that $\delta_1 \leq \delta_2 \leq \cdots \leq \delta_N$. Because we are in a weak low-demand state, we know that generator 1 is the unique generator, which is dispatched to produce l MW, and its profits are $-S + (\delta_1 - c)l$.

First, assume for contradiction that $\delta_1 < c + S/l$. In this case, generator 1's profits are negative and it has a profitable deviation in which it submits an offer that is strictly greater than δ_2. Doing so makes generator 1 inactive and its profits zero. Thus, we have a contradiction showing that in an equilibrium, all generators submit offers satisfying (26).

Next, assume for contradiction that $\delta_1 > c + S/l$. In this case, generator 1's profits are strictly positive and all of generators 2 through N have profitable deviations in which they undercut slightly δ_1. Doing so results in the deviating generator becoming the unique generator (in place of generator 1) and earning strictly positive profits. This contradiction shows that in an equilibrium, inequality (26) must be binding for at least one generator.

To show that (26) must be binding for at least two generators, assume for contradiction that $\delta_1 = c + S/l$ while $\delta_2 > c + S/l$. In this case generator 1 earns zero profits. However, generator 1 has a profitable deviation in which it increases it offer to undercut generator 2's offer slightly. Doing so allows generator 1 to remain the unique generator but earn strictly positive profits. This contradiction shows that in an equilibrium at least two generators must submit offers that make (26) binding.

Finally, we have that the unique generator exactly breaks even and earns zero profits. Moreover, because the remaining generators are inactive, they earn zero profits as well. ■

Proof of Proposition 6. Without loss of generality, assume that the generators are labeled so that $\delta_1 \leq \delta_2 \leq \cdots \leq \delta_N$. Because we are in a regular low-demand state, we know that generators 1 through $(N_l - 1)$ are inframarginal and dispatched to produce K MW, while generator N_l is marginal and produces $l_r < K$.

First, assume for contradiction that $\delta_{N_l} < c + S/l_r$. In this case, generator N_l's profits are negative and it has a profitable deviation in which it submits an offer that is strictly greater than δ_{N_l+1}. Doing so makes generator N_l inactive and its profits zero. Thus, we have a contradiction showing that in an equilibrium $\delta_{N_l}, \delta_{N_l+1}, \ldots, \delta_N$ must satisfy (28).

Next, assume for contradiction that $\delta_{N_l} > c + S/l_r$. In this case, generator N_l's profits are strictly positive and all of generators $(N_l + 1)$ through N have profitable deviations in which generator $i > N_l$ undercuts slightly generator N_l's offer. Doing so results in generator i becoming marginal (in place of generator N_l) and earning strictly positive profits. This contradiction shows that in an equilibrium, inequality (28) must be binding for at least one generator.

To show that (28) must be binding for at least two generators, assume for contradiction that $\delta_{N_l} = c + S/l_r$ while $\delta_{N_l+1} > c + S/l_r$. In this case, generator N_l earns zero profits. However, this generator has a profitable deviation in which it increases its offer to undercut slightly generator $(N_l + 1)$'s offer. Doing so allows generator N_l to remain marginal but earn strictly positive profits. This

Copyright © 2018 by the IAEE. All rights reserved.

contradiction shows that in an equilibrium at least two generators must submit offers that make (28) binding.

To derive (27), assume for contradiction that $c+S/K < \delta_{N_l-1}$. In this case, generator N_l has a profitable deviation in which it undercuts generator $(N_l - 1)$'s offer. Doing so makes generator N_l inframarginal and generator $(N_l - 1)$ marginal. Moreover, the energy price under this deviation is $p^s = \delta_{N_l-1} > c + S/K$. This means that generator N_l's profits are $-S + (\delta_{N_l-1} - c)K > -S + (c + S/K - c)K > 0$, after the deviation. Thus, we have a contradiction showing that in an equilibrium, $\delta_1, \delta_2, \ldots, \delta_{N_l-1}$ must satisfy (27).

Finally, we have that the inactive generators earn zero profits, because they produce nothing. The marginal generator also earns zero profit, because the energy price is exactly high enough for it to recover its costs. The profits of each inframarginal generator are equal to $-S + (\delta_{N_l} - c)K = -S + (c + S/l_r - c)K = S \cdot (K/l_r - 1) > 0$, because $l_r < K$. ■

Proof of Proposition 7. Without loss of generality, assume that the generators are labeled so that $\delta_1 \leq \delta_2 \leq \cdots \leq \delta_N$. Because we are examining a high-demand state, we know that generators 1 through $(N - 1)$ are inframarginal and dispatched to produce K MW, while generator N is marginal and produces $l_r < K$, and that the energy price is δ_N.

First, we show that generator N does not have a profitable deviation. If generator N submits an offer less than δ_N but greater than δ_{N-1}, it remains the marginal generator and its profits decrease strictly because the energy price decreases. Next, consider a deviation in which generator N undercuts one of the other generators. In such an instance, it becomes an inframarginal generator and the energy price is at most $(\bar{\delta} - c)l_r/K + c$. Thus, its profits after this deviation are at most $-S + [(\bar{\delta} - c)l_r/K + c - c]K = -S + (\bar{\delta} - c)l_r$. However, its profits before the deviation are $-S + (\bar{\delta} - c)l_r$, which is non-negative by assumption. This means that the marginal generator does not have a profitable deviation in which it undercuts the offer of an inframarginal generator.

Thus, this shows that the marginal generator must submit an offer equal to $\bar{\delta}$ in a pure-strategy Nash equilibrium. Moreover, we can conclude that the inframarginal generators must submit offers no greater than $(\bar{\delta} - c)l_r/K + c$ for the marginal generator not to have a profitable deviation. We have finally that the profits of inframarginal generators are $-S + (\bar{\delta} - c)K > 0$. ■

Proof of Lemma 14. Assume for contradiction that $\delta_i^- < \delta_j^-$. In such a case, generator i has a profitable deviation wherein it moves the density that is assigned to the interval, $[\delta_i^-, \delta_j^-)$, to $\delta_j^- - \eta$ with $\eta > 0$ sufficiently small. By doing so, generator i increases its expected profit without decreasing the probability that it is an inframarginal generator. Thus, we have a contradiction showing that the infimum energy offers must be equal in a mixed-strategy Nash equilibrium. ■

Proof of Lemma 15. Assume for contradiction that $G_i(\delta_i)$ has a mass point, which we denote as $\hat{\delta}_i$. There exist $\eta > 0$ and $\rho > 0$ such that some generator $j \neq i$ can deviate profitably by moving the density that is assigned to the interval, $[\hat{\delta}_i, \hat{\delta}_i + \eta)$, to the offer, $\hat{\delta}_i - \rho$, contradicting the assumption of an equilibrium mass point.

To see that this is a profitable deviation, we consider the following three possible cases of whether generator j is marginal or inframarginal before and after the deviation. First, if generator j is inframarginal before and after the deviation, there is no profit loss. Second, if generator j is marginal before and after the deviation, there is a profit loss of at most $(\eta + \rho)K$. Finally, if generator j is marginal before the deviation and inframarginal after the deviation, its profit increases by at least $(\hat{\delta}_i - c)(K - l_r) - \rho K - \eta l_r$. For η and ρ sufficiently small, the profit increase in the third case outweighs the profit losses in the second. ■

Proof of Lemma 16. Assume for contradiction that there exists an interval, $[\hat{\delta}_i, \hat{\delta}_i + \eta]$, with $\eta > 0$, on which generator i places zero density in a mixed-strategy Nash equilibrium. Generator $j \neq i$ has a profitable deviation in which the density that it assigns to the interval, $(\hat{\delta}_i - \rho, \hat{\delta}_i)$, is assigned to the offer, $\hat{\delta}_i + \eta - \xi$, where $\rho > 0$ and $\xi \in (0, \eta)$, contradicting the assumption that Ψ_i is not connected.

To see that this is a profitable deviation, we consider the following three possible cases of whether generator j is marginal or inframarginal before and after the deviation. First, if generator j

Copyright © 2018 by the IAEE. All rights reserved.

is inframarginal before and after the deviation, there is no profit change. Second, if generator j is marginal before and after the deviation, its profits increase by at least $(\eta-\xi)l_r$. Finally, if generator j is inframarginal before the deviation and marginal after the deviation, its profits change by at most $(\hat{\delta}_i - c)(l_r - K) + (\eta - \xi)l_r + \rho K$. Thus, generator j only stands to have a profit loss in the third case. However, ρ can be chosen to make the probability of this event arbitrarily close to zero. ■

Proof of Lemma 17. To first show that the supremum energy offers are the same, we assume, for contradiction, that $\delta_i^+ < \delta_j^+$. Generator i could deviate profitably by moving the density that it assigns to the interval, $(\delta_i^+ - \eta, \delta_i^+]$, with $\eta > 0$ sufficiently small, to $\delta_j^+ - \rho$ with $\delta_j^+ - \rho > \delta_i^+$. By doing so, generator i increases its expected profits without decreasing the probability that it is an inframarginal generator. This contradiction demonstrates that the supremum energy offers must be equal.

To show that the supremum energy offers equal $\bar{\delta}$, we assume, for contradiction, that $\delta_1^+ = \delta_2^+ = \ldots = \delta_N^+ = \delta^+ < \bar{\delta}$ in a Nash equilibrium. Generator i has a profitable deviation in which it moves the density that is assigned to the interval, $(\delta^+ - \eta, \delta^+]$, where $\eta > 0$, to the energy offer, $\bar{\delta}$, contradicting the assumption of a Nash equilibrium in which $\delta^+ < \bar{\delta}$.

To see that this is a profitable deviation, we consider the following three possible cases of whether generator i is marginal or inframarginal before and after the deviation. First, if generator i is inframarginal before and after the deviation, there is no change in its profits. Second, if generator i is marginal before and after the deviation, its profits increase by at least $(\bar{\delta} - \delta^+)l_r$. Finally, if generator i is inframarginal before the deviation and marginal after the deviation, its profits change by at most $(\bar{\delta} - c)l_r - (\delta^+ - c)K$. Thus, generator i only stands to have a profit loss in the third case. However, η can be chosen to make the probability of this event arbitrarily close to zero. ■

Proof of Proposition 8. We assume that all of generator i's rivals follow the symmetric equilibrium that is given by the probability density function, $g(\delta)$, and the cumulative distribution function, $G(\delta)$. Thus, generator i's expected profit, as a function of its offer, δ_i, is given by:

$$\mathbb{E}[\pi_i^s(\delta_i)] = G(\delta_i)^{N-1}[-S + (\delta_i - c)l_r] + \int_{\delta_i}^{\bar{\delta}} [-S + (\delta_{-i}^{\max} - c)K] d\left(G(\delta_{-i}^{\max})^{N-1}\right), \tag{38}$$

where $\delta_{-i}^{\max}$ represents the maximum offer of generator i's rivals. Differentiating (38) with respect to δ_i gives the FONC for generator i's profit-maximizing offer, which can be written as $g(\delta_i) - \lambda G(\delta_i)/(\delta_i - c) = 0$, where λ is given by (30). Because we assume a symmetric equilibrium, we can eliminate the subscripts, giving:

$$g(\delta) - \lambda \frac{G(\delta)}{\delta - c} = 0. \tag{39}$$

Differential equation (39) can be solved explicitly by defining the integrating factor:

$$\mu(\delta) = \exp\left(-\int_a^\delta \frac{\lambda}{x - c} dx\right) = \left(\frac{\delta - c}{a - c}\right)^{-\lambda}, \tag{40}$$

where a is an arbitrary constant. Multiplying both sides of (39) by $\mu(\delta)$ gives:

$$\left(\frac{\delta - c}{a - c}\right)^{-\lambda} g(\delta) - \left(\frac{\delta - c}{a - c}\right)^{-\lambda} \lambda \frac{G(\delta)}{\delta - c} = 0. \tag{41}$$

Integrating the left-hand side of this equation with respect to δ gives:

$$\left(\frac{\delta - c}{a - c}\right)^{-\lambda} G(\delta) - b = 0, \tag{42}$$

Copyright © 2018 by the IAEE. All rights reserved.

where b is the constant of integration. Thus, we have that:

$$G(\delta) = b \cdot \left(\frac{\delta - c}{a - c}\right)^{\lambda}. \tag{43}$$

Because we know that $G(\bar{\delta}) = 1$, we have that:

$$b = \left(\frac{a - c}{\bar{\delta} - c}\right)^{\lambda}, \tag{44}$$

which gives the desired result when substituted into (43). ■

Proof of Proposition 9. From (38) we have that generator i earns an expected profit of:

$$\mathbb{E}[\pi_i^s(\bar{\delta})] = G(\bar{\delta})^{N-1}[-S+(\bar{\delta}-c)l_r]+\int_{\underline{\delta}}^{\bar{\delta}}[-S+(\delta_{-i}^{\max}-c)K]d\left(G(\delta_{-i}^{\max})^{N-1}\right) = -S+(\bar{\delta}-c)l_r, \tag{45}$$

if it submits an offer equal to $\bar{\delta}$. Because the mixed strategies constitute a Nash equilibrium, all offers in Ψ must yield the same expected profit for all generators. Thus, we have that $\mathbb{E}[\pi_i^s(\delta)] = -S + (\bar{\delta} - c)l_r, \forall \delta \in \Psi, i = 1, \ldots, N$, which is the desired result. ■

6.3 Proof of Propositions and Corollary from Section 4

Proof of Proposition 10. This result follows immediately from Propositions 1 and 5, which show that all generators earn zero profits under both market designs in weak low-demand states. ■

Proof of Proposition 11. From Lemma 6 we know that in regular low-demand states there are pure-strategy Nash equilibria under a centrally committed market design in which all generators earn zero profits. On the other hand, we know from Proposition 6 that in a regular low-demand state the set of pure-strategy Nash equilibria under a self-committed market design results in inframarginal generators earning strictly positive profits. Thus, cost equivalence fails to hold. ■

Proof of Proposition 12. From Propositions 4 and 9, respectively, we have that expected generator profits in high-demand states are $\mathbb{E}[\pi^c] = \bar{\sigma} - S + (\bar{\epsilon} - c)l_r$, under a centrally committed market design and $\mathbb{E}[\pi^s] = -S + (\bar{\delta} - c)l_r$, if generators follow mixed-strategy Nash equilibria under a self-committed market design. Expected-cost-equivalence would require the expectation (with respect to l_r) of these two terms to be equal, or:

$$\mathbb{E}_l\left[\mathbb{E}[\pi^c]|l > (N-1)K\right] = \mathbb{E}_l\left[\mathbb{E}[\pi^s]|l > (N-1)K\right] \tag{46}$$

$$\bar{\sigma} - S + (\bar{\epsilon} - c)\mathbb{E}[l_r|l > (N-1)K] = -S + (\bar{\delta} - c)\mathbb{E}[l_r|l > (N-1)K], \tag{47}$$

which is the equality in Assumption 1. ■

Proof of Corollary 3. From Proposition 7 we know that if generators follow a pure-strategy Nash equilibrium under a self-committed market design the energy price is equal to $\bar{\delta}$. We know also from Proposition 8 that if the generators follow a mixed-strategy Nash equilibrium under a self-committed market design there is a non-zero probability that they submit energy offers that are strictly less than $\bar{\delta}$ (meaning that the energy price is less than $\bar{\delta}$). Because the mixed-strategy Nash equilibrium under a self-committed market design is expected-cost-equivalent to the mixed-strategy Nash equilibrium under a centrally committed market design, it must be the case that the pure-strategy Nash equilibrium under a self-committed market design is more costly. ■

Copyright © 2018 by the IAEE. All rights reserved.