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# Towards Equilibrium Offers in Unit Commitment Auctions with Nonconvex Costs

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**Abstract** We compare two types of uniform-price auction formats commonly used in wholesale electricity markets—centrally committed and self-committed markets. Auctions in both markets are conducted by an independent system operator that collects generator bids and determines which generators will operate and how much electricity each will produce. In centrally committed markets, generators submit two-part bids consisting of a startup cost and a variable energy cost. Self-committed markets force generators to incorporate their startup costs into a one-part energy bid. The system operator in a centrally committed system ensures that each generator recovers the startup and energy costs stated in its two-part bid, while no such guarantees are made in self-committed markets. The energy cost ranking and incentive properties of these market designs remains an open question. While the system operator can determine the most efficient dispatch with a centralized market, the auction mechanism used to solicit generator data compels generators to overstate costs. Self commitment might involve less efficient dispatch but have better incentive properties. We derive Nash equilibria for both market designs in a symmetric duopoly setting. We also derive simple conditions under which the two market designs will be expected cost-equivalent.

**Keywords** Market design · unit commitment · Nash equilibrium · non-convex costs

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## 1 Introduction

Wholesale electricity markets facilitate the trade of electricity across a system of transmission lines. Such markets often use uniform-price auctions to determine the price of electricity, and the generators that submit the lowest bids, or equivalently offer to produce electricity at the lowest price, are selected to produce electricity. The two key outcomes of the auction process are generator commitment (which generators startup), and generator dispatch (the amount of electricity each generator produces). Independent system operators (SOs) conduct the uniform-price auctions repeatedly throughout the day.

A debate exists as to which entity, the SO or the generators themselves, should make these decisions. In centrally committed markets, generators submit two-part bids, subject to offer caps, and the SO makes the commitment and dispatch decisions and guarantees that each generator recovers the startup costs stated in its energy offer. This guarantee is made through a make-whole payment, which is a supplemental payment given to a generator for any deficit between its as-bid cost and energy payments. In a self-committed market each generator makes its own commitment decision and submits a single-part bid for energy, also subject to an offer cap, and must incorporate its startup costs into this bid.[1]

An unresolved issue in wholesale electricity market design and regulation is what equilibrium bidding behavior, the total cost of electricity service, and system efficiency would be under central and self commitment. This design question is important, given the considerable size of the markets.[2] The revenues in these markets also have significant implications for investment in new generation capacity, which determines the future electricity costs. The debate over the two market designs centers on the tradeoff between efficient dispatch and commitment, and generator incentives to truthfully reveal startup and energy costs. [Ruff (1994), Hogan (1994), Hogan (1995), Hunt (2002)] support centrally committed markets because they give the SO, which has the best information about the electric system as a whole, the authority to make both commitment and dispatch decisions. However, [Oren and Ross (2005)] show that generators can have incentives to misstate their costs to increase profit if the SO collects multi-part bids. Moreover, [Johnson et al (1997), Sioshansi et al (2008)] claim that incentive compatibility issues in a centrally committed market can be further exacerbated if the SO must rely on suboptimal solutions to its unit commitment model. As such, [Wilson (1997), Elmaghraby and Oren (1999)] suggest that commitment decisions are ultimately more efficient in self-committed markets.

Despite the various claims about the two market designs, their incentive properties have not been directly compared. To this end, we develop a single-period symmetric duopoly model of two markets: a centrally committed market with two-part offers (energy and startup); and a self-committed market with one-part offers (energy only). By analyzing the market as a uniform-price auction with system-wide caps on

[1] Some electricity markets operate as a hybrid between the two designs highlighted here. For instance, the New York ISO incorporates some non-convex costs, such as startup costs, into the energy price.

[2] According to their 2007 Annual Reports, the sum of wholesale transactions in 2007 were: $30.5 billion in PJM Interconnection, $9.5 billion in New York ISO, $10 billion in ISO New England, and $1.9 billion in ERCOT.

each bid element, we are able to characterize Nash equilibria in each market. We further derive conditions on the offer caps in the two markets that will yield expected cost equivalence between the two market designs. We also use a numerical example to demonstrate and compare the nature of the equilibria of the two markets. The remainder of the paper is organized as follows: section 2 describes our model, section 3 derives our equilibrium and cost-equivalence results, section 4 presents our numerical example, and section 5 concludes and discusses some regulatory implications of our model and analysis. It is important to stress, and this is discussed further in section 5, that the model we use to analyze the unit commitment auction is a highly stylized representation of actual electricity markets. Thus this work should be viewed as an exploratory analysis of these types of markets.

## 2 Model

Two identical generators compete in a uniform-price auction to serve a deterministic load, $l$, that is commonly known and must be served exactly. The generators each have capacity constraints $K > 0$ on their generation. Each generator incurs a fixed startup cost $S > 0$ and a constant marginal generating cost, $c > 0$. The capacity of the two generators is assumed to always be sufficient to serve the load, or that $0 \le l < 2K$. Thus the cost to each firm of generating $q$ MWh is:

$$C(q) = \begin{cases} 0, & \text{for } q = 0; \\ cq + S, & \text{for } 0 < q \le K; \\ +\infty, & \text{for } q > K. \end{cases}$$

### 2.1 Centrally Committed Market

Under a centrally committed market, each generator submits a bid with two elements, an energy offer, $\varepsilon$, that specifies a marginal generating cost, and a fixed startup cost, $\sigma$, that is incurred if any positive quantity of electricity is produced. We let $\omega_i = (\varepsilon_i, \sigma_i)$ denote generator $i$'s bid. We assume that both bid components must be non-negative, and that there are caps, $\varepsilon^*$ and $\sigma^*$, on the two components.

The SO then uses a mixed-integer program (MIP) to determine the commitment and dispatch of each generator based on the two sets of bid. If we define $u_1$ and $u_2$ to be a binary variables indicating whether each generator is started up and $q_1$ and $q_2$ to be continuous variables indicating how much each energy generator produces, the SO's MIP is:

$$\begin{aligned} \min_{u,q} \quad & \sum_{i=1}^{2} (u_i \sigma_i + q_i \varepsilon_i) \\ \text{s.t.} \quad & \sum_{i=1}^{2} q_i = l \\ & 0 \le q_i \le u_i K \quad \forall\, i = 1,2 \\ & u_i \in \{0,1\} \quad \forall\, i = 1,2. \end{aligned}$$

If $l \leq K$ only one generator needs to be committed and dispatched to serve load, which will be the one with the bid that produces $l$ MWh at lowest total cost. The expected quantity sold by generator $i$ is thus given by:

$$q_i^c(\omega_i, \omega_j, l) = \begin{cases} \min\{l, K\}, & \text{if } \sigma_i + l\varepsilon_i < \sigma_j + l\varepsilon_j \text{ and } l \leq K; \\ \frac{1}{2}\min\{l, K\}, & \text{if } \sigma_i + l\varepsilon_i = \sigma_j + l\varepsilon_j \text{ and } l \leq K; \\ 0, & \text{if } \sigma_i + l\varepsilon_i > \sigma_j + l\varepsilon_j \text{ and } l \leq K; \end{cases}$$

and the uniform price of energy is set based on the $\varepsilon$ of the generator that is committed and dispatched. We assume that ties are broken with equal probability. Conversely if $l > K$, both generators must be committed and dispatched and the quantity sold by the generators will be based on energy cost only. Thus generator $i$'s expected production is:

$$q_i^c(\omega_i, \omega_j, l) = \begin{cases} K, & \text{if } \varepsilon_i < \varepsilon_j \text{ and } l > K; \\ \frac{1}{2}l, & \text{if } \varepsilon_i = \varepsilon_j \text{ and } l > K; \\ l - K, & \text{if } \varepsilon_i > \varepsilon_j \text{ and } l > K; \end{cases}$$

and the uniform energy price is $p = \max\{\varepsilon_i, \varepsilon_j\}$.

In both cases, the generators receive energy payments, $p \cdot q_i^c(\omega_i, \omega_j, l)$. However, the generators have non-convex costs due to their startup cost, so these energy payments alone may be confiscatory. The only information the SO has about the costs of the generators is their 'as-bid' costs in $\omega$, and the SO uses this information to ensure that no generator operates at a loss according to the bids. For example, suppose that $l \leq K$ and generator 1 wins the uniform-price auction. Hence $q_1^c = l$ and $p = \varepsilon_1$, however the net profit to generator 1, on the basis of its bids, is $(p - \varepsilon_1)q_1^c - \sigma_1 = -\sigma_1 \leq 0$.

Most centrally committed wholesale electricity markets overcome this problem by giving generators supplemental 'make-whole' payments, which cover any revenue shortfall based on the costs specified in each generator's bid. If the uniform price is $p$ and a generator is committed and dispatched to produce $q_i^c$ MWh, its total payment, $T_i$, from the SO is the sum of an energy payment and a make-whole payment, $W_i$:

$$\begin{aligned} T_i &= p \cdot q_i^c + W_i \\ &= p \cdot q_i^c + \max\{0, \sigma_i + q_i^c(\varepsilon_i - p)\}, \end{aligned}$$

which ensures that each generator recovers all of its costs, as-bid. We assume the centrally committed market includes such a make-whole payment provision.

## 2.2 Self-Committed Market

In a self-committed market each generator submits a single-part bid, $\delta$, which specifies the minimum price it is willing to accept to generate a MWh of energy. The generators decide independently whether to commit themselves and the SO does not provide make-whole payments,[3] however a dispatched generator is financially liable for serving its scheduled load. As before, we assume that the energy bids must be

[3] As we noted before, some markets, such as the New York ISO, do incorporate non-convex startup costs into the energy price.

non-negative and that there is a cap, $\delta^*$, below which the bids must be. Given the bids, the SO uses a uniform-price auction to dispatch the generators to serve the load at least cost and set the uniform energy price. Thus, generator $i$'s dispatch is given by:

$$q_i^s(\delta_i,\delta_j,l) = \begin{cases} \min\{l,K\}, & \text{if } \delta_i < \delta_j; \\ \frac{1}{2}[\min\{l,K\}+\max\{0,l-K\}] = \frac{1}{2}l, & \text{if } \delta_i = \delta_j; \\ \max\{l-K,0\}, & \text{if } \delta_i > \delta_j; \end{cases} \quad (1)$$

and the uniform price is:

$$p = \begin{cases} \min\{\delta_i,\delta_j\}, & \text{if } l \le K; \\ \max\{\delta_i,\delta_j\}, & \text{if } l > K. \end{cases}$$

## 3 Market Equilibria

In both markets, there will be different types of equilibria depending on whether the load can be served by a single generator or both generators are needed. If only one generator is needed, then under both market designs Bertrand-style competition will drive the market to perfectly competitive pure-strategy Nash equilibria with zero generator profits. If both generators are needed the generators' profits will be strictly positive in equilibrium. In this case the centrally committed market will only have mixed-strategy Nash equilibria, whereas the self-committed market will have both pure- and mixed-strategy equilibria. Moreover, the pure-strategy equilibria in the self-committed market will always be more costly to the SO than the mixed-strategy equilibria. We proceed by analyzing each market design under these two load scenarios separately.

In examining these scenarios, for cases in which $l \le K$ and only one generator is committed, the committed generator will be referred to as the *unique generator*, and its bid, payments, and profit will be denoted with the subscript, $U$. When $l > K$ and both generators are needed, we refer to the unit dispatched at full capacity as the *inframarginal generator* and the unit dispatched below its capacity as the *marginal generator*, and denote their bids, payments, and profits by the subscripts, $I$ and $M$, respectively.

### 3.1 Centrally Committed Market Equilibrium

We begin the analysis of the centrally committed market by characterizing the total payments to the generators under the two load scenarios.

**Lemma 1** *In a centrally committed market with $l \le K$, the total payment to the unique generator will be $T_U = \varepsilon_U l + \sigma_U$.*

*If $l > K$, both generators will produce a strictly positive amount, and the total payment to the marginal generator will be $T_M = \varepsilon_M(l-K) + \sigma_M$ and the total payment to the inframarginal generator will be $T_I = max\{\varepsilon_M K, \varepsilon_I K + \sigma_I\}$.*

*Proof* When $l \leq K$ the unique generator will be dispatched to serve the entire load, $l$, and the uniform price for energy is $p = \varepsilon_U$. Since the startup cost in its offer is non-negative, $\sigma_U \geq 0$, the unique generator's surplus from energy payments according to as-bid costs is $\varepsilon_U l - (\varepsilon_U l + \sigma_U) \leq 0$. Thus the make-whole payment will be $W_U = \max\{0, \sigma_U + l(\varepsilon_U - \varepsilon_U)\} = \sigma_U$. Hence, the unique generator's total payment is $T_U = \varepsilon_U l + \sigma_U$.

When $l > K$ the marginal generator will be dispatched to serve $(l - K)$ units of the load and the uniform price is $p = \varepsilon_M$. Again, since $\sigma_M \geq 0$, the marginal generator's as-bid surplus from energy payments will be non-positive, thus the total payments will be the sum of energy and make-whole payment, hence $T_M = \varepsilon_M(l - K) + \sigma_M$, where the make-whole payment is $W_M = \sigma_M$.

Moreover, because of the make-whole provision, the SO will ensure the inframarginal generator's as-bid surplus is $\max\{(\varepsilon_M - \varepsilon_I)K - \sigma_I, 0\}$. If $\max\{(\varepsilon_M - \varepsilon_I)K - \sigma_I, 0\} = (\varepsilon_M - \varepsilon_I)K - \sigma_I$, then $\varepsilon_M K \geq \varepsilon_I K + \sigma_I$ and the total payment to the inframarginal generator is simply the energy payment, $\varepsilon_M K$, because the energy payment alone is sufficient to cover the inframarginal generator's (as-bid) startup and variable operating costs. Otherwise, if $\max\{(\varepsilon_M - \varepsilon_I)K - \sigma_I, 0\} = 0$ then $\varepsilon_M K < \varepsilon_I K + \sigma_I$, and the total payment to the inframarginal generator is:

$$
\begin{aligned}
T_I &= pK + W_I \\
&= \varepsilon_M K + \max\{0, \sigma_I + K(\varepsilon_I - \varepsilon_M)\} \\
&= \varepsilon_I K + \sigma_I,
\end{aligned}
$$

which is the desired expression.

Having characterized generator payments under the centrally committed market, we now prove the following result, which gives the set of Nash equilibria when only one of the generators is needed to serve the load.

**Proposition 1** *If $l \leq K$, the unique set of pure-strategy Nash equilibria of the centrally committed market consists of offers such that $\omega_i \in B$ for $i = 1, 2$, where $B$ is the set:*

$$
B = \left\{(\varepsilon, \sigma) \in \mathbb{R}^2 \mid \varepsilon l + \sigma = cl + S,\ \varepsilon \in [0, \varepsilon^*],\ \text{and } \sigma \in [0, \sigma^*]\right\},
$$

*and each generator has an expected profit of zero.*

*Proof* Given that $l \leq K$, the SO only needs to commit and dispatch one generator and the SO does so in the least-costly way. Thus, the SO selects the generator with the lowest total cost. The dispatch is determined by the ranking of these costs, which for simplicity we refer to as $b_i = \varepsilon_i l + \sigma_i$ for $i = 1, 2$. This game is thus isomorphic to a simple Bertrand game, but in this case, each generator submits a total cost $b_i = \varepsilon_i l + \sigma_i$. The total cost of each generator, $b_i$ is such that $b_i = cl + S$ for $i = 1, 2$ and generators earn zero profit in equilibrium. Clearly, there are many $\omega$ that belong to the set $B$ but all vectors are payoff-equivalent because they result in the same expected commitment, dispatch, and profits. Moreover, since the total cost of the offers equal actual costs, expected profits are zero in equilibrium.

We now turn to the case in which $l > K$ and both generators must be committed and dispatched to serve the load. Since both generators must be committed, their startup costs must be borne, thus the optimal commitment and dispatch decisions will be made purely on the basis of each generator's energy offer, $\varepsilon$. As we show in the following lemmas and propositions, this characteristic of an optimum, coupled with the generators' binding capacity constraints, eliminates the possibility of a pure-strategy Nash equilibrium in the bidding game. As such, we assume that the generators follow mixed-strategy equilibria. This, in turn, implies that each generator has a strictly positive probability of receiving make-whole payments, and as such each generators' expected profit function is a non-decreasing function of its startup bid. Thus, each generator will submit an offer with a startup cost equal to the startup offer cap, $\sigma^*$.

**Proposition 2** *If $l > K$, no pure-strategy Nash equilibria exist in the centrally committed market.*

*Proof* Suppose $(\tilde{\varepsilon}_i, \tilde{\sigma}_i)$, for $i = 1, 2$, constitute a pure-strategy Nash equilibrium, and assume without loss of generality that the generators have been labeled such that $\tilde{\varepsilon}_1 \leq \tilde{\varepsilon}_2$.

Suppose first that $\tilde{\varepsilon}_1 < \tilde{\varepsilon}_2$. Then generator 1 is the inframarginal generator and its profit is:

$$\tilde{\Pi}_1 = \max\{\tilde{\varepsilon}_2 K, \tilde{\varepsilon}_1 K + \tilde{\sigma}_1\} - cK - S.$$

If $\max\{\tilde{\varepsilon}_2 K, \tilde{\varepsilon}_1 K + \tilde{\sigma}_1\} = \tilde{\varepsilon}_1 K + \tilde{\sigma}_1$ then generator 1 can profitably deviate by changing the energy portion of its offer to $\hat{\varepsilon}_1 = \tilde{\varepsilon}_2 - \eta$, with $\eta > 0$ and small, since its profits are increasing in $\varepsilon_1$. If, instead, $\max\{\tilde{\varepsilon}_2 K, \tilde{\varepsilon}_1 K + \tilde{\sigma}_1\} = \tilde{\varepsilon}_2 K$ then generator 1 can profitably deviate by changing its offer to $(\hat{\varepsilon}_1, \hat{\sigma}_1)$ such that $\hat{\varepsilon}_1 = \tilde{\varepsilon}_2 - \eta$, with $\eta > 0$ and small, and $\hat{\sigma}_1 > 0$ and sufficiently large, so that $\max\{\tilde{\varepsilon}_2 K, \hat{\varepsilon}_1 K + \hat{\sigma}_1\} = \hat{\varepsilon}_1 K + \hat{\sigma}_1 > \tilde{\varepsilon}_2 K$.

Suppose instead that $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2 = e$. Then both generators' expected profits are given by:

$$\mathbb{E}[\tilde{\Pi}_i] = \frac{1}{2} l (e - c) + \sigma_i - S.$$

Suppose $e \leq c$, then either generator can profitably deviate by submitting an offer with a higher $\varepsilon_i$, since this will guarantee it a strictly positive margin on energy sold whereas an offer of $e$ gives it a non-positive margin. Otherwise, if $e > c$, either generator can profitably deviate by submitting an energy offer of $\hat{\varepsilon}_i = e - \eta$, with $\eta > 0$ and small. This gives generator $i$ an expected profit of:

$$\mathbb{E}[\hat{\Pi}_i] = (e - c)K + \sigma_i - S,$$

which is greater than $\mathbb{E}[\tilde{\Pi}_i]$ for $\eta$ sufficiently small, since $K > \frac{1}{2} l$.

Having ruled-out pure-strategy Nash equilibria, we will let $F_i(\varepsilon_i, \sigma_i)$ denote the cumulative distribution function (CDF) of generator $i$'s mixed-strategy Nash equilibrium, let $\Phi_i$ denote the support of $F_i$, and let $\underline{\varepsilon}_i$ and $\overline{\varepsilon}_i$ denote the infimum and supremum energy offers, respectively, of $\Phi_i$. We also define $\Phi = \Phi_1 \bigcap \Phi_2$ as the common support of the two CDFs. We show in the following lemmas that the range of energy offers in $\Phi_1$ and $\Phi_2$, must intersect, and as such generators will always submit the highest possible startup cost.

**Lemma 2** *If $l > K$, then the infimum energy offers in a Nash equilibrium are equal.*

*Proof* Suppose that the infimum energy bids of the two generators differ in equilibrium and that the generators are labeled such that $\underline{\varepsilon}_1 < \underline{\varepsilon}_2$. Generator 1 has a profitable deviation because it can move all of the density in the interval $[\underline{\varepsilon}_1, \underline{\varepsilon}_2)$, to $\underline{\varepsilon}_2 - \eta$ for $\eta > 0$ small, as doing so increases generator 1's expected profit and does not change the probability that it is the inframarginal generator.

We further characterize equilibrium CDFs by showing that they cannot have mass points on their common support and that $\Phi_1$ and $\Phi_2$ are connected and have a common supremum.

**Lemma 3** *If $l > K$, then neither $F_1$ nor $F_2$ can have a mass point on $\Phi$.*

*Proof* Suppose for contradiction that there is a $\tilde{\varepsilon} \in \Phi$ which is a mass point of $F_i$. Then there exist $\eta > 0$ and $\rho > 0$ such that generator $j$ would have a profitable deviation by moving the density assigned to the interval $[\tilde{\varepsilon}, \tilde{\varepsilon} + \eta)$ to $\tilde{\varepsilon} - \rho$, since the profit from offers in the interval $[\tilde{\varepsilon}, \tilde{\varepsilon} + \eta)$ is at most:

$$(\tilde{\varepsilon} + \eta - c)(l - K) + \sigma_j - S,$$

and the profit from an offer of $\tilde{\varepsilon} - \rho$ is:

$$(\tilde{\varepsilon} - c)K + \sigma_j - S,$$

which is greater for $\eta$ sufficiently small, contradicting the assumption of a mass point in an equilibrium.

**Lemma 4** *If $l > K$, then $\Phi_i$ is a connected set (interval) for both generators.*

*Proof* Suppose for contradiction that there is an interval $[\tilde{\varepsilon}, \tilde{\varepsilon} + \eta]$, with $\eta > 0$ on which generator $i$ places zero density. Consider a deviation by generator $j$ wherein it moves the density assigned to the interval $(\tilde{\varepsilon} - \rho, \tilde{\varepsilon})$ to an energy offer of $\tilde{\varepsilon} + \eta - \xi$, with $\rho > 0$ and $\eta > \xi > 0$. We can bound the change in generator $j$'s expected profits depending on whether it would be the marginal or inframarginal generator with the original strategy and deviation:

– If generator $j$ is the inframarginal generator and would have been the inframarginal generator without deviating, its expected profits will either increase by at least $(\eta - \xi)K$ if it receives make-whole payments or not change if it does not receive make-whole payments.
– If generator $j$ is the marginal generator and would have been the marginal generator without deviating, the deviation will increase the price of energy and generator $j$'s expected profits will increase by at least $(\eta - \xi)(l - K)$.
– If generator $j$ is the marginal generator but would have been the inframarginal generator without deviating, its expected profits will change by at most $(\tilde{\varepsilon} + \eta - \xi)(l - K) - \tilde{\varepsilon}K$.

Thus, the only cost to generator $j$ involves situations where it would have been the inframarginal generator without deviating but becomes the marginal generator as a result of deviating. However $\rho$ can be chosen to make the probability of this event arbitrarily close to zero.

**Lemma 5** *If $l > K$, then the supremum energy offers in a Nash equilibrium are equal.*

*Proof* Suppose the suprema are different such that $\overline{\varepsilon}_i < \overline{\varepsilon}_j$. Generator $i$ has a profitable deviation, which is to move some density from the interval $(\overline{\varepsilon}_i - \eta, \overline{\varepsilon}_i]$ for some small $\eta > 0$, just below $\overline{\varepsilon}_j$. Doing so increases generator $i$'s expected profit without decreasing the probability that generator $i$ will be the inframarginal generator, as there is no density in the interval $(\overline{\varepsilon}_i, \overline{\varepsilon}_j)$.

**Lemma 6** *If $l > K$, then in equilibrium each generator submits the maximum possible startup cost in its offer. That is, $\sigma_1 = \sigma_2 = \sigma^*$, almost surely.*

*Proof* Because there are only mixed-strategy Nash equilibria and $\Phi_1 = \Phi_2$, each generator has a strictly positive probability of being the marginal generator. Since the payoff to the marginal generator is strictly increasing in $\sigma$, the payoff to the inframarginal generator is non-decreasing in $\sigma$, and the value of $\sigma$ does not impact the dispatch of the generators, it is optimal to submit an offer with $\sigma_i = \sigma^*$.

The essence of Lemma 6 is that because the SO's dispatch depends solely on the energy portion of the generators' offers, the two-dimensional offer problem (energy and startup costs) collapses into a one-dimensional offer problem with only an energy cost. Therefore, we will hereafter denote the equilibrium CDFs as $F_i(\varepsilon_i)$. The next step is to determine $F_i(\varepsilon_i)$ by optimizing the profit function of the generators, which are symmetric.

In order to find an equilibrium CDF, we first express generator $i$'s expected profit as a function of its energy offer, $\varepsilon_i$, assuming generator $j$ follows the CDF, $F(\varepsilon_j)$. This expected profit is:

$$\begin{aligned}\mathbb{E}[\pi_i^C(\varepsilon_i)] = F(\varepsilon_i)&[(l-K)(\varepsilon_i - c) + \sigma^* - S] \\ &+ [F(\varepsilon_i + \sigma^*/K) - F(\varepsilon_i)][(\varepsilon_i - c)K + \sigma^* - S] \\ &+ \int_{\varepsilon_i+\sigma^*/K}^{\varepsilon^*} [(\varepsilon_j - c)K - S] dF(\varepsilon_j).\end{aligned} \tag{2}$$

The first term in equation (2) gives generator $i$'s expected profit conditional on being the marginal generator, whereas the other two give the expected profit conditional on being the inframarginal generator. The first-order necessary condition (FONC) for an expected profit-maximizing choice of $\varepsilon_i$ is:

$$f(\varepsilon_i)(l-2K)(\varepsilon_i - c) + F(\varepsilon_i)(l-2K) + F(\varepsilon_i + \sigma^*/K)K = 0,$$

which can be re-written as:

$$f(\varepsilon_i) = \frac{F(\varepsilon_i)}{c - \varepsilon_i} + \frac{F(\varepsilon_i + \sigma^*/K)K}{(l-2K)(c-\varepsilon_i)},$$

or as:

$$f(\varepsilon) = \frac{F(\varepsilon)}{c-\varepsilon} + \frac{F(\varepsilon + \sigma^*/K)K}{(l-2K)(c-\varepsilon)}, \tag{3}$$

since the equilibrium is symmetric.

Equation (3) is a differential difference equation (DDE) characterizing a symmetric Nash equilibrium energy offer density function. We can find a particular solution of the DDE if we specify an interval of boundary conditions of width $(\sigma^*/K)$. We do this by showing that the common supremum of the Nash equilibrium CDFs must be the offer cap, $\varepsilon^*$, which implies that $F(\varepsilon) = 1$ for all $\varepsilon \geq \varepsilon^*$.

**Lemma 7** *If $l > K$, then a Nash equilibrium energy offer density function must have $\overline{\varepsilon} = \varepsilon^*$:*

*Proof* Suppose that $\overline{\varepsilon} < \varepsilon^*$ in an equilibrium. Then generator $j$ has a profitable deviation whereby it moves the density assigned to the interval $(\overline{\varepsilon} - \eta, \overline{\varepsilon})$ to an energy offer of $\varepsilon^*$, with $\eta > 0$. We can bound the change in generator $j$'s expected profits depending on whether it would be the marginal or inframarginal generator with the original strategy and deviation:

– If generator $j$ is the inframarginal generator and would have been the inframarginal generator without deviating, its expected profits will either increase by at least $(\varepsilon^* - \overline{\varepsilon})(l-K)$ if it receives make-whole payments or will not change if it does not receive make-whole payments.
– If generator $j$ is the marginal generator and would have been the marginal generator without deviating, the deviation will increase the price of energy and generator $j$'s expected profits will increase by at least $(\varepsilon^* - \overline{\varepsilon})(l-K)$.
– If generator $j$ is the marginal generator but would have been the inframarginal generator without deviating, its expected profits will change by at most $\varepsilon^*(l-K) - \overline{\varepsilon}K$.

Thus, the only cost to generator $j$ involves situations where it would have been the inframarginal generator without deviating but becomes the marginal generator as a result of deviating. However $\eta$ can be chosen to make the probability of this event arbitrarily close to zero.

Because this is a mixed-strategy Nash equilibrium, all energy offers in the support of the equilibrium CDF must yield the two generators the same expected profit. Because the equilibrium CDF has no mass point, we know that a generator that submits the energy offer cap will necessarily be the marginal generator, and as such will yield an expected profit of:

$$\mathbb{E}[\pi^C(\varepsilon^*)] = (l-K)(\varepsilon^* - c) + \sigma^* - S.$$

We further know that any energy offer will yield the same expected profit, or that:

$$\mathbb{E}[\pi^C(\varepsilon)] = (l-K)(\varepsilon^* - c) + \sigma^* - S, \ \forall\, \varepsilon \in \Phi. \tag{4}$$

## 3.2 Self-Committed Market Equilibrium

With a self-committed market design, generators independently decide whether to commit themselves, and submit single-part energy offers, $\delta \in [0, \delta^*]$ to the uniform-price auction conducted by the SO. The only revenue available to generators is the energy payment, $p \cdot q_i^s$, where $p$ is the uniform electricity price and $q_i^s$ is the quantity sold in the self-committed market, as defined in equation (1). We assume the offer cap is sufficiently high so that the generators can always recover their startup cost if they bid $\delta^*$. Thus, if $l \leq K$ we assume that $(\delta^* - c)l \geq S$ and if $l > K$ that $(\delta^* - c)(l - K) \geq S$. Otherwise, the market would not clear because one or both of the generators would choose not to participate.

We again proceed by analyzing equilibrium behavior depending on whether one generator or both are needed to serve the load. We first consider the case in which only one generator must be dispatched, the energy offer of which will set the uniform energy price. We can easily characterize this game as having a Bertrand-type Nash equilibrium in which the generators' expected profits are both zero.

**Proposition 3** *If $l \leq K$, then the unique pure-strategy Nash equilibrium of the self-committed market is for each generator to offer $\delta_1 = \delta_2 = c + S/l$, with each generator having an expected profit of zero.*

*Proof* The proof of this proposition follows that of Proposition 1—since $l \leq K$ the SO will only dispatch one generator, which the SO will select on the basis of the energy bids. Because the generators are competing on the basis of price without any binding capacity constraints, the game is isomorphic to a Bertrand-type game and will have a Bertrand equilibrium.

We now turn to the case in which $l > K$ and both generators must be dispatched to serve the load. The dispatch of the two generators will be determined by their energy offers. Again, the generator with the lower energy offer will be dispatched to its capacity, $K$, while the other generator will serve the residual load, $(l - K)$. Once again, it is trivial to show that the uniform energy price will equal the energy offer of the marginal generator. We now rely on a result of [Fabra et al (2006)] which shows that this game can have either pure- or mixed-strategy equilibria, and also gives some properties of the mixed-strategy equilibrium.

**Proposition 4** *If $l > K$, then the self-committed market will have both pure- and mixed-strategy Nash equilibria. The pure-strategy Nash equilibria will be of the form $\delta_M = \delta^*, \delta_I \leq \hat{\delta}$, for some $\hat{\delta} \in (c, \delta^*)$. The mixed-strategy Nash equilibrium will be unique and symmetric, and have a differentiable distribution function and an atomless density function.*

*Proof* The generators in this market are submitting energy bids into a uniform-price auction with two capacity-constrained generators. [Fabra et al (2006)] show the existence of pure-strategy Nash equilibria (Proposition 1) and the properties of the mixed-strategy Nash equilibrium (Lemma 3). As such we do not repeat the proofs.

The pure-strategy Nash equilibria of this auction are relatively straightforward—the marginal generator bids at the offer cap, $\delta^*$, and the inframarginal generator submits a bid sufficiently low so that the marginal generator has no incentive to undercut it. Moreover, the threshold value, $\hat{\delta}$, below which the inframarginal generator must bid is easy to derive. To see this, note that the marginal generator's profit in the pure-strategy equilibrium is:

$$(\delta^* - c)(l - K) - S,$$

whereas the maximum profit it can earn from deviating (by undercutting the inframarginal generator) is:

$$(\delta_I - c)K - S.$$

The Nash equilibrium requires:

$$(\delta^* - c)(l - K) - S \geq (\delta_I - c)K - S,$$

or

$$\delta_I \leq (\delta^* - c)\frac{l - K}{K} + c = \hat{\delta}.$$

Moreover, because $(l - K)/K \in (0,1)$ if $l > K$, we have that $\hat{\delta} \in (c, \delta^*)$. Finally, it is important to note that under these pure-strategy Nash equilibria the energy price will always be at the offer cap, $\delta^*$, regardless of the load (so long as $l > K$). Thus the profit of the inframarginal generator will be:

$$\pi_I^{S,P} = (\delta^* - c)K - S, \tag{5}$$

while the profit of the marginal generator will be:

$$\pi_M^{S,P} = (\delta^* - c)(l - K) - S. \tag{6}$$

To find the mixed-strategy Nash equilibrium of the self-committed market we express the expected profit of generator $i$ as a function of its energy bid, assuming generator $j$ bids according to the CDF $G(\delta_j)$, and analyze the FONC. Generator $i$'s expected profit is given as:

$$\mathbb{E}[\pi_i^{S,M}(\delta_i)]^4 = G(\delta_i)[(l - K)(\delta_i - c) - S] + \int_{\delta_i}^{\delta^*} [K(\tau - c) - S]dG(\tau). \tag{7}$$

The first term in equation (7) is generator $i$'s expected profit conditional on it being the marginal generator, while the integral term is generator $i$'s expected profit conditional on it being the inframarginal generator. The FONC for an expected profit-maximizing choice of $\delta_i$, which must hold for all $\delta_i$ in the support of $G$, is:

$$g(\delta_i)[(l - K)(\delta_i - c) - S] + G(\delta_i)(l - K) - g(\delta_i)[K(\delta_i - c) - S] = 0,$$

which can be re-written as:

$$g(\delta_i) - \frac{G(\delta_i)(l - K)}{(2K - l)(\delta_i - c)} = 0,$$

[4] The superscript, $M$, on $\pi_i^{S,M}$ is meant to denote the fact that this expected profit function is for the mixed-strategy Nash equilibrium.

or

$$g(\delta)-\lambda\frac{G(\delta)}{\delta-c}=0, \tag{8}$$

where we have dropped the subscripts, due to the symmetry of the equilibrium, and defined $\lambda=(l-K)/(2K-l)$. The differential equation (8) can be solved by defining the integrating factor:

$$\begin{aligned}\mu(\delta) &= \exp\left\{-\int_a^\delta \frac{\lambda}{\tau-c}d\tau\right\}\\ &= \left(\frac{\delta-c}{a-c}\right)^{-\lambda},\end{aligned}$$

where $a$ is an arbitrary constant. Multiplying both sides of equation (8) by $\mu(\delta)$ and integrating with respect to $\delta$ yields:

$$\begin{aligned}G(\delta) &= b\exp\left\{\int_a^\delta \frac{\lambda}{\tau-c}d\tau\right\}\\ &= b\left(\frac{\delta-c}{a-c}\right)^{\lambda},\end{aligned}$$

where $b$ is a constant of integration. In order to specify an exact solution to the differential equation we use the boundary condition that neither generator has a mass point at the supremum offer, $\delta^*$, hence $G(\delta^*)=1$ which gives:

$$b\left(\frac{\delta^*-c}{a-c}\right)^{\lambda}=1\Longrightarrow b=\left(\frac{a-c}{\delta^*-c}\right)^{\lambda}\Longrightarrow G(\delta)=\left(\frac{\delta-c}{\delta^*-c}\right)^{\lambda},$$

which is the CDF of the mixed-strategy Nash equilibrium.

We can also derive expressions for the expected energy offer and expected profit of each generator under this mixed-strategy Nash equilibrium. The expected energy offer is derived by first determining the infimum energy offer and then using the property of mixed-strategy Nash equilibria that all offers in the support must yield the same expected profit. The infimum energy offer, $\underline{\delta}$, is determined by solving the equation $G(\underline{\delta})=0$, which implies that $\underline{\delta}=c$.

Next, the expected profit of each generator as a function of its bid is given by:

$$\mathbb{E}[\pi^{S,M}(\delta)]=(l-K)(\delta-c)G(\delta)+K\int_\delta^{\delta^*}(\eta-c)dG(\eta)-S.$$

More specifically, we have that:

$$\mathbb{E}[\pi^{S,M}(\delta^*)]=(\delta^*-c)(l-K)-S,$$

and:

$$\begin{aligned}\mathbb{E}[\pi^{S,M}(c)] &= K\int_\delta^{\delta^*}(\eta-c)dG(\eta)-S\\ &= K(\mathbb{E}[\delta]-c)-S,\end{aligned}$$

Equating these two terms gives:

$$(\delta^* - c)(l - K) - S = K(\mathbb{E}[\delta] - c) - S,$$

which can be rewritten as:

$$\mathbb{E}[\delta] = \frac{(\delta^* - c)(l - K)}{K} + c, \tag{9}$$

and gives an expression for the expected energy bid in the Nash equilibrium.

Equation (9) also shows a number of properties of the mixed-strategy equilibrium in the self-committed market. One is that the expected energy offer is increasing in $l$, which is in keeping with economic theory. The higher the load is, the lower the cost of being the marginal generator. As the load decreases the generators become more aggressive and submit lower energy offers:

$$\lim_{l \to K^+} \mathbb{E}[\delta] = c,$$

and the reverse is also true:

$$\lim_{l \to 2K^-} \mathbb{E}[\delta] = \delta^* \tag{10}$$

Moreover:

$$\frac{\partial}{\partial l}\mathbb{E}[\delta] = \frac{\delta^* - c}{K} > 0, \tag{11}$$

showing that the expected energy price is strictly monotonically increasing in $l$ and will always be less than $\delta^*$ if $l < 2K$. This implies that the expected energy price will always be less under the mixed-strategy Nash equilibrium than with the pure-strategy one. Additionally, $\mathbb{E}[\delta]$ is increasing in the energy bid cap, $\delta^*$, and the marginal cost, $c$. Finally, the expected energy bid is only indirectly related to the fixed startup cost, $S$, insomuch as we imposed the assumption $(\delta^* - c)(l - K) \geq S$ on $\delta^*$.

We can also derive an expression for the expected profit of each generator, since in a mixed-strategy Nash equilibrium all energy offers in the support of $G$ must yield the same expected profit. Since we know that bidding the offer cap yields an expected profit of $(\delta^* - c)(l - K) - S$, we know that:

$$\mathbb{E}[\pi^{S,M}(\varepsilon)] = (l - K)(\delta^* - c) - S, \ \forall\ \delta \in [c, \delta^*]. \tag{12}$$

### 3.3 Expected Cost Equivalence of Central- and Self-Committed Market Designs

Equations (4)–(6) and (12) give expressions for the expected profit of each generator under the two market designs. These equations show that generator capacities and costs, the total load, and the offer caps set in the two markets will determine generator profits and settlement costs. Thus, we can derive conditions under which the two markets will be expected cost-equivalent. In deriving these conditions, we will assume that the markets will operate over multiple settlement periods, which will have different loads, and that $\mathbb{E}[l]$ is the expected load over the settlement periods.

We now prove the following proposition which states that if the offer caps of the two markets are set such that expected revenues to a generator from bidding the cap in both markets are equivalent and the generators follow the mixed-strategy Nash equilibria when $l > K$, then the markets will have the same expected settlement cost.

**Proposition 5** *If the offer caps of the two markets are set such that:*

$$\delta^*(\mathbb{E}[l \mid l > K] - K) = \varepsilon^*(\mathbb{E}[l \mid l > K] - K) + \sigma^*, \tag{13}$$

*and the generators follow the mixed-strategy Nash equilibrium under each market design whenever $l > K$, then the two market designs will be expected cost-equivalent.*

*Proof* If $l \leq K$ then both markets are cost equivalent, since generators will reveal their true total cost and generator profits will be zero. Thus, we only need to consider cases in which $l > K$. If $l > K$ then the expected profit to a generator in the centrally committed market as a function of the load is:

$$\pi^C(l) = (l - K)(\varepsilon^* - c) + \sigma^* - S,$$

while the expected profit in the self-committed market is:

$$\pi^{S,M}(l) = (l - K)(\delta^* - c) - S.$$

If the two markets have the same expected settlement costs, then the expected profit to the generators will be equivalent under the two market designs, or:

$$\begin{aligned} \mathbb{E}[\pi^C] &= \mathbb{E}[\pi^{S,M}] \\ \mathbb{E}[\pi^C(l) \mid l > K] &= \mathbb{E}[\pi^{S,M}(l) \mid l > k] \\ (\mathbb{E}[l \mid l > K] - K)(\varepsilon^* - c) + \sigma^* - S &= (\mathbb{E}[l \mid l > K] - K)(\delta^* - c) - S \\ \varepsilon^*(\mathbb{E}[l \mid l > K] - K) + \sigma^* &= \delta^*(\mathbb{E}[l \mid l > K] - K), \end{aligned}$$

which is the desired condition.

It bears mentioning that the condition yielding cost-equivalence is a property that one should naturally expect of the market designs: the caps should be set such that generators are not disadvantaged in one market due to constraints on available bidding strategies. Moreover, setting the cost-equivalent offer cap in one market based on the offer caps used in the other does not require the SO or regulator to have any information about the generators' costs. Rather, only the expected load and generating capacity of the generators is needed. Generator cost information would obviously be needed, however, in order to set reasonable offer caps (based on actual generator costs).

As a corollary, we now show that cost equivalence will not hold if generators follow the pure-strategy Nash equilibria in the self-committed market and the offer caps are set to satisfy equation (13). Rather, the self-committed market will yield higher expected settlement costs.

**Corollary 1** *If the offer caps of the two markets are set such that:*

$$\delta^*(\mathbb{E}[l] - K) = \varepsilon^*(\mathbb{E}[l] - K) + \sigma^*,$$

*and the generators follow a pure-strategy Nash equilibrium in the self-committed market whenever $l > K$, then the self-commmitted market will have a higher expected settlement cost than the centrally committed one.*

*Proof* The result follows immediately from equations (10) and (11). Since we know that the energy price will always be $\delta^*$ when $l > K$ if the generators follow a pure-strategy Nash equilibrium in the self-committed market, the expected energy price when generators follow the mixed-strategy Nash equilibrium will always be (weakly) less than the price when they follow the pure-strategy equilibrium.

## 4 Numerical Example

We use a numerical example to compare equilibrium behavior, energy prices, and settlement costs of the centralized and self-committed market designs. Because both markets result in the same perfectly competitive outcome when $l \le K$, we assume in our computations that $l > K$. We further consider markets with offer caps that satisfy the cost-equivalence condition in equation (13).

Table 1 summarizes the parameter assumptions underlying our example. The load profile is computed by assuming that the loads have a minimum of 550 MW and a maximum of 950 MW.[5] We then compute 8760 loads in such a way to fit the 2006 load duration curve of the California ISO. The pure- and mixed-strategy equilibria and associated expected costs for the self-committed market are computed directly in closed form. The DDE for the centrally committed market is approximated numerically using the `dde23` function in Matlab, which is based on a Runge-Kutta algorithm.

**Table 1** Generator characteristics, expected load, and offer caps used for numerical example

| Parameter | Value |
|---|---|
| $c$ | \$30/MWh |
| $S$ | \$10,000 |
| $K$ | 500 MW |
| $\mathbb{E}[l]$ | 703 MW |
| $\varepsilon^*$ | \$1,000/MWh |
| $\sigma^*$ | \$25,000 |
| $\delta^*$ | \$1,123/MWh |

Table 2 shows the expected cost to the SO of the two market designs, considering both the pure- and mixed-strategy Nash equilibria in the self-committed market. The costs given are per-settlement period, meaning that this is the SO's expected cost each time it conducts the auction and determines the dispatch and payments of the generators. The expected settlement cost of the centrally committed market, which is given in the last row of the table, include make-whole payments. The table shows an approximately \$5 difference in expected settlement costs between the two market designs when the mixed-strategy Nash equilibrium is used in the self-committed market, which amounts to less than 0.001% of the total settlement cost. We attribute this difference to approximation errors in solving and integrating the DDE characterizing the centrally committed equilibrium. Otherwise, the two markets have the same

[5] The DDE characterizing the equilibrium of the centrally committed market becomes difficult to solve if $l$ is too close to either $K$ or $2K$, which is why we choose these particular upper and lower bounds on $l$.

expected settlement costs. Although the two markets are expected cost-equivalent, energy prices will not generally be the same in the two markets. This is shown both in table 2 and in figure 1, which shows the expected energy price as a function of the load under the two market designs. Instead, the centrally committed market tends to have lower prices than the self-committed, but make-whole payments account for a nontrivial portion of settlement costs (about $38/MWh or 6% of total settlement costs in this example), which must be paid by the SO and borne by ratepayers. Moreover, the expected settlement costs are not equal under all load scenarios. Rather, at low load levels settlement costs are higher in the centrally committed market than under the self-committed design, due to the make-whole payments. Table 2 also shows that when generators follow a pure-strategy Nash equilibrium in the self-committed market, the expected energy price and settlement costs rise significantly compared to the mixed-strategy equilibrium and the centrally committed market.

**Table 2** Expected per-settlement period energy price and cost to SO of centrally and self-committed markets

| | Centrally Committed | Self-Committed Mixed-Strategy | Self-Committed Pure-Strategy |
|---|---|---|---|
| Expected Energy Price | 615 | 652 | 1123 |
| Expected Make-Whole Payments | 26,791 | n/a | n/a |
| Expected Settlement Costs | 465,648 | 465,653 | 789,893 |

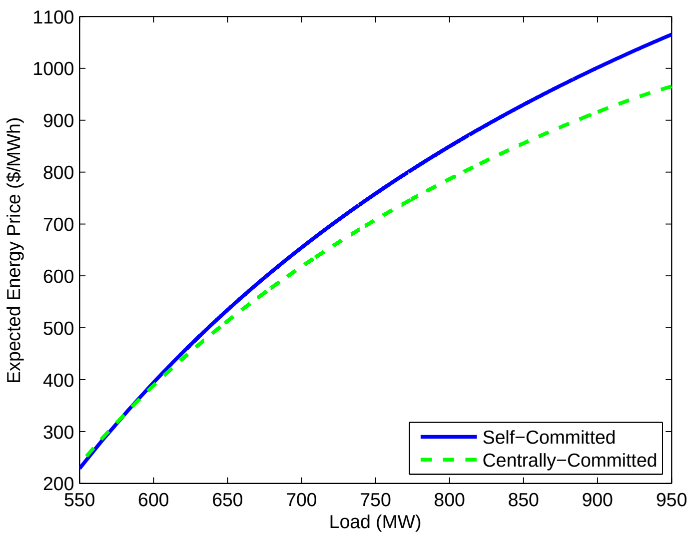


**Fig. 1** Expected energy prices in centrally and self-committed markets when generators use mixed-strategy Nash equilibrium in self-committed market

## 5 Discussion and Conclusion

This paper uses a symmetric duopoly model to examine equilibrium behavior, pricing, and settlement costs in centrally and self-committed electricity markets. Given the size of wholesale electricity markets and the potential for allocative or productive efficiency losses, regulators need to understand the implications of different market designs. We show that if the load can be served by a single generator, then a Bertrand-style equilibrium, in which costs are truthfully revealed and generator profits are zero, will occur. When both generators are needed, however, the equilibria will generally be non-competitive with bids above marginal costs and strictly positive generator profits. In this case, the centrally committed market has only a mixed-strategy Nash equilibrium whereas both pure- and mixed-strategy Nash equilibria exist in the self-committed market. The nature of the equilibria show that when both generators are needed to serve the load, the generators are not more likely to bid their true costs under one market design than the other. In the centrally committed market generators always bid their startup costs at the offer cap and mix over the full range of energy offers. With self-commitment the generators will either mix over the full range of energy offers, or follow strategies which will necessarily yield an energy price equal to the energy offer cap.

We also show that if the offer caps in the two markets are set so that a generator bidding the caps receives the same expected revenue in both markets, then the outcomes of the two auction formats will be expected cost-equivalent if the generators follow the mixed-strategy Nash equilibria. Otherwise, if they follow pure-strategy Nash equilibria in the self-committed market, then costs will be higher with self commitment. Thus, if one expects the generators to follow the mixed-strategy equilibrium, the two market designs can be made cost equivalent on an expected basis by setting the offer caps in a very natural manner. Importantly, computation of the cost-equivalent offer caps does not require the SO or regulator to know the generators' cost parameters, but only the expected load and their generating capacities. Otherwise, if pure-strategy Nash equilibria are expected, the cap in the self-committed market must be lowered. These findings suggest that regulators should be especially cautious in implementing self-committed markets, due to potentially higher settlement and consumer energy costs.

Our results also show that self-committed markets will tend to have higher energy prices than under centralized commitment. This is because self-committed markets typically do not include a make-whole provision and as such energy payments must be sufficiently high for generators to recover both their fixed and marginal costs. These higher energy prices can result in allocative efficiency losses if demand is price-responsive, as demonstrated by [Sioshansi et al (2010)]. Depending on how make-whole payments are allocated to ratepayers, these efficiency losses may be mitigated in a centrally committed market. If the make-whole payments are incorporated into a linear retail electricity tariff, then similar efficiency losses will persist. If, however, the regulator imposes a two-part tariff, in which the make-whole payments are included in the lump-sum payment, then marginal energy prices and consumption decisions will not be affected. This would be difficult to implement in practice because such a tariff involves treating heterogeneous customers with different load profiles symmetrically, despite the fact that they impose different costs on the SO. Directly assigning joint costs to customers with drastically different load profiles, such as residential and industrial customers, is a complex process that takes place in hearings before State Public Utility Commissions rather than being decided by the SO.

Despite these and other findings, the issue of how electricity markets should be organized and how much authority the SO should have to make binding market decisions has and will continue to be contentious. Although centrally committed markets can, in theory, maximize welfare, they have computational issues. These can introduce incentive problems, in addition to those that we have analyzed here. Self-committed markets can overcome these problems, however the non-convex nature of the generators' commitment decisions and costs can lead to some coordination and productive efficiency losses. [Sioshansi et al (2008),Sioshansi et al (2010)] demonstrate this using a numerical example based on the ISO New England system as well as through some stylized examples. They show that even under perfect competition, the system can experience non-trivial productive efficiency losses under self commitment. This is due to an inherent limitation of using linear prices in a system with non-convex costs—the prices cannot properly signal the benefits or costs of activities to market participants—and has been examined in the context of unit commitment pricing by [O'Neill et al (2005)]. With strategic generator behavior, the lack of coordination and associated efficiency losses may be even greater. These incentive and efficiency issues can be resolved by implementing a different auction format, such as a Vickrey-Clarke-Groves (VCG) mechanism. [Chao and Wilson (2002)], for example, derive an optimal auction mechanism for procurement of electricity reserves. Similarly a VCG auction would, in theory, address both the incentive and efficiency issues of the unit commitment auction. The VCG mechanism raises other challenges of its own, however. One is the computational burden of determining the transfer payments—the VCG will require the centralized unit commitment model to be solved to complete optimality $N+1$ times, where $N$ is the number of bidders in the auction. Since the centralized unit commitment cannot currently be solved to complete optimality once (within the time window during which the SO must determine generators' day-ahead schedules), this is presently impossible. A second issue with the VCG mechanism is that it is generally not budget balanced, and would require the SO or another entity to subsidize the transfer payments. We do not directly address this type of a mechanism design issue in this paper, rather we derive equilibrium behavior and resulting pricing and settlement costs in two popular electricity market designs. Our results are nevertheless informative, as they suggest that a centrally committed market may be preferred if the regulator or SO believes that generators may follow a pure-strategy Nash equilibrium in the self-committed design.

As noted before, it is important to qualify the findings of this paper. This analysis is based on a stylized model of an electricity market with two symmetric generators competing in a static single-shot auction without transmission constraints or demand uncertainty. This model is rather divorced from actual electricity systems, which typically have multiple generators, multiblock bids, intertemporal constraints, and other factors complicating the structure of the bids and market settlements. Moreover, SOs and regulators often limit the extent to which generators can adjust their bid elements. For instance, the California ISO only allows generators to adjust their startup bids every six months. Furthermore, regulators often empower SOs to conduct market mitigation, whereby they can scrutinize bids that seem excessively high or uncompetitive. These types of factors are not included in our analysis either, which is reflected in the nature of the equilibria that we derive. For instance, the pure-strategy Nash equilibria that we find in the self-committed market would likely lead to scrutiny, and perhaps more drastic regulatory action. In light of these limitations, this work should be viewed as an exploratory step in comparing the incentive, price, and cost properties of the two market designs. Future work that examines the impacts of asymmetric costs, demand uncertainty, and multiple generators will be needed to more fully understand these properties. We nevertheless feel that regulators, SOs, and others can use these results to better inform their market design decisions.

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