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Benefits of Stochastic Optimization for Scheduling Energy Storage in Wholesale Electricity Markets

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Abstract We propose a two-stage stochastic model for optimizing the operation of energy storage. The model captures two important features: (i) uncertain real-time prices when day-ahead operational commitments are made and (ii) the price impact of charging and discharging energy storage. We demonstrate that if energy storage has full flexibility to make real-time adjustments to its day-ahead commitment and market prices do not respond to charging and discharging decisions, there is no value in using a stochastic modeling framework (*i.e.*, the value of stochastic solution is always zero). This is because in such a case the energy storage behaves purely as a financial arbitrageur day ahead, which can be captured using a deterministic model. We show also that prices responding to its operation can make it profitable for energy storage to 'waste' energy, for instance by charging and discharging simultaneously, which is sub-optimal normally. We demonstrate our model and how to calibrate the price-response functions from historical data with a real-world case study.

Keywords Energy storage · stochastic optimization · value of stochastic solution

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1 Introduction

Energy storage is experiencing a renaissance, which is driven by a number of developments. These include the advent of markets that provide price signals for many of the services that energy storage can provide [1] and the role that energy storage can play in accommodating the variable and uncertain real-time availability of many renewables [2, 3]. As such, there is a diversity of energy storage technologies that are at varying stages of development and commercialization [4].

Energy storage can provide many services, such as generation shifting, transmission and distribution relief and deferral, and ancillary services [5, 6]. Generation shifting is among the most studied of these and generates value by arbitraging wholesale energy prices. Many works study generation shifting using a deterministic price-taking approach, wherein prices are assumed to be fixed and known with certainty *a priori* [7]. Other works extend these models. Some consider a stochastic-optimization framework, wherein market prices are important sources of uncertainty [8, 9]. Other works relax the assumption of fixed prices and model price-making energy storage that can influence the price through its charging and discharging [10–12].

The operation of energy storage can impact wholesale prices numerous ways. One is a direct merit-order effect—charging or discharging energy storage can result in the market clearing further up or lower down the merit order of the generation-dispatch stack [13]. Depending upon where it is located within the transmission network, energy storage also can alleviate or exacerbate congestion, which can increase or decrease locational marginal prices. This latter effect can be pronounced for bulk energy-storage technologies, such as pumped hydroelectric storage (PHS). This is because many PHS plants have high power capacities (*e.g.*, GW-scale) and can be in remote areas of the transmission network (due to geological requirements). Thus, operating a PHS plant can congest or decongest a radial transmission line that connects it to the power system.

Given these operational properties, we propose and examine the use of a two-stage stochastic model for optimizing the operation of energy storage. The model accounts for price impacts of operating energy storage in a relatively simple manner that captures merit-order effects and transmission congestion. Other works in the literature [14–16] represent the market-price impact of energy storage in more sophisticated ways. Although we could employ such techniques, we believe that a linear relationship between wholesale energy prices and the operation of energy storage provides a reasonable balance between model fidelity and tractability. The two model stages correspond to scheduling day-ahead and real-time charging and discharging. Under our stylized assumptions of a price-making storage operator and a linear relationship between prices and net loads, we find that the benefits of using a two-stage stochastic model are related critically to two important market-design assumptions. Under a case in which prices are insensitive to the operation of energy storage and in which energy storage has full flexibility to adjust its real-time operations relative to its day-ahead schedule, there is no value to using stochastic optimization. Otherwise, if *either* of these assumptions are relaxed, there is value in employing a stochastic model. We find also that if wholesale energy prices are sufficiently responsive to its operation, there can be cases in which it is beneficial for energy storage to waste energy (*e.g.*, by charging and discharging simultaneously). This finding is contrary to other analyses, which observe simultaneous charging and discharging of energy storage only in the presence of negative prices or transmission congestion.

The remainder of this paper is organized as follows. Section 2 details our model formulations. Sections 3 and 4 illustrate the models using a simple example and a comprehensive case study, respectively. Section 5 concludes. Appendix A details how historical data can be used to calibrate the function that is used to represent the impact of energy storage on market prices.

2 Model Formulation

This section provides detailed model formulations. Section 2.1 overviews the assumed market and model structures. Model notation is defined in Section 2.2. Sections 2.3 and 2.4 provide two-stage stochastic and deterministic model formulations, respectively. Section 2.5 details the computation of the value of stochastic solution (VSS).

2.1 Market and Model Structures

We model energy storage that participates in day-ahead and real-time markets. Day-ahead charging and discharging are scheduled first knowing day-ahead prices but with incomplete knowledge of real-time prices. Adjustments to energy purchases and sales (which determine the device's actual net operating profile) can be made thereafter in reaction to real-time prices.

The market is assumed to employ a two-settlement system wherein day-ahead and real-time transactions are settled at the corresponding day-ahead and real-time prices, respectively. One effect of uncertain real-time prices that we explore is flexibility in making adjustments to day-ahead transactions. We do this by imposing constraints that relate real-time adjustments to the operating schedule and the day-ahead schedule.

Another aspect of energy storage participating in the energy market that we examine is its impact on prices. We capture price impacts by assuming that day-ahead and real-time prices react to day-ahead and real-time energy transactions that are scheduled by the energy storage.

2.2 Model Notation

This section defines sets, indices, model parameters, and decision variables.

2.2.1 Sets, Indices, and Parameters

We define T as the set of operating periods, which in our case study are assumed to be hour-long time steps. $t \in T$ is defined as the corresponding time index. We define Ω as a set of second-stage scenarios in the two-stage model formulation. We define $\omega \in \Omega$ as the corresponding scenario index. We let ϕ^{ω} denote the probability with which scenario ω occurs.

We characterize energy storage through four technical parameters. $C^{\max}$ and $D^{\max}$ represent the device's charging and discharging power capacities, respectively, which are measured in MW. $S^{\max}$ represents the device's maximum state of charge (SOC) in MWh. $\eta \in (0, 1)$ is a unitless measure of the device's round-trip efficiency. It represents the MWh of energy that can be discharged from the device per MWh that is charged. $\eta < 1$ implies that there are net energy losses from cycling energy through the device. S_0 denotes the device's SOC at the beginning of the optimization horizon.

To capture the price effects of energy storage, we assume that day-ahead and real-time prices have a linear relationship with the amount of energy that is transacted by the device [17, 18]. Specifically, we have that the hour-t day-ahead energy price, which is measured in \$/MWh, is given by $\alpha_t + \beta_t Z_t$, where Z_t represents the net MWh of energy that the energy storage purchases from the day-ahead market in hour t. Uncertainties in real-time prices are captured using second-stage scenarios. Thus, we assume that the hour-t real-time price in scenario ω, which also is measured in \$/MWh, is given by $\alpha_t^\omega + \beta_t^\omega Z_t^\omega$, where Z_t^ω represents the net amount of energy (measured in MWh) that the energy storage purchases from the real-time market in hour t of scenario ω. Our assumption of a linear relationship between prices and the operation of energy storage represents a balance between model tractability and fidelity. Some of our results stem from this assumption.

The parameter, γ, specifies (on a per-unit basis relative to $C^{\max}$ and $D^{\max}$) the extent to which the device's operation can be adjusted in real time relative to its day-ahead schedule.

2.2.2 Decision Variables

We define two sets of variables, which correspond to transactions that are scheduled in the day-ahead and real-time markets. c_t and d_t denote, respectively, hour-t MW that are scheduled to be charged into and discharged from the device in the day-ahead market. We define Δc_t^ω and Δd_t^ω, respectively, as the hour-t incremental changes in MW that are scheduled to be charged into and discharged from the device in the real-time market under scenario ω.

We define two sets of SOC-related variables. s_t denotes the MWh of energy that is held in the device at the end of hour t from following the day-ahead schedule. We define s_t^ω as the ending hour-t SOC of the device, measured in MWh, in scenario ω from following the real-time schedule.

2.3 Two-Stage Stochastic Model

Our two-stage stochastic model is formulated as:

$$\max \sum_{t\in T} \Bigg\{ [\alpha_t + \beta_t \cdot (c_t - d_t)](d_t - c_t) + \sum_{\omega\in\Omega} \phi^\omega \cdot [\alpha_t^\omega + \beta_t^\omega \cdot (c_t + \Delta c_t^\omega - d_t - \Delta d_t^\omega)] \cdot (\Delta d_t^\omega - \Delta c_t^\omega) \Bigg\}; \quad (1)$$

$$\text{s.t. } s_t = s_{t-1} + \eta c_t - d_t, \quad \forall t \in T; \quad (2)$$
$$0 \le c_t \le C^{\max}, \quad \forall t \in T; \quad (3)$$
$$0 \le d_t \le D^{\max}, \quad \forall t \in T; \quad (4)$$
$$0 \le s_t \le S^{\max}, \quad \forall t \in T; \quad (5)$$
$$s_t^\omega = s_{t-1}^\omega + \eta \cdot (c_t + \Delta c_t^\omega) - d_t - \Delta d_t^\omega, \quad \forall t \in T, \omega \in \Omega; \quad (6)$$
$$0 \le c_t + \Delta c_t^\omega \le C^{\max}, \quad \forall t \in T, \omega \in \Omega; \quad (7)$$
$$0 \le d_t + \Delta d_t^\omega \le D^{\max}, \quad \forall t \in T, \omega \in \Omega; \quad (8)$$
$$0 \le s_t^\omega \le S^{\max}, \quad \forall t \in T, \omega \in \Omega; \quad (9)$$
$$-\gamma C^{\max} \le \Delta c_t^\omega \le \gamma C^{\max}, \quad \forall t \in T, \omega \in \Omega; \quad (10)$$
$$-\gamma D^{\max} \le \Delta d_t^\omega \le \gamma D^{\max}, \quad \forall t \in T, \omega \in \Omega. \quad (11)$$

Objective function (1) gives total expected profit from energy transactions. There are two terms in (1), which give profits that are earned in the day-ahead and real-time markets, respectively. The hour-t energy price is computed as $\alpha_t + \beta_t \cdot (c_t - d_t)$, thereby taking into account that $(c_t - d_t)$ represents the net amount of hour-t energy that the device purchases from the day-ahead market. The hour-t real-time price in scenario ω is computed as $\alpha_t^\omega + \beta_t^\omega \cdot (c_t + \Delta c_t^\omega - d_t - \Delta d_t^\omega)$, which accounts for *net* real-time purchases by the device. The device's net real-time purchase is equal to purchases that are scheduled in the day-ahead market, in addition to any incremental real-time changes. Because of the assumed two-settlement system, the quantity, $(d_t - c_t)$, which is sold day-ahead, is settled financially at the day-ahead price. Only the incremental net purchases, which are defined by $(\Delta d_t^\omega - \Delta c_t^\omega)$, are settled at the real-time price.

There are two sets of model constraints. The first, (2)–(5), pertain to the first stage whereas the remaining pertain to the second stage. Energy-balance equalities (2) define the device's SOC at the end of each hour if it follows the day-ahead schedule. Constraints (3) and (4) impose non-negativity and capacity limits on charging and discharging, respectively, that are scheduled in the day-ahead market. Constraints (5) impose non-negativity and energy-capacity limits on the device's SOC if it follows the day-ahead schedule.

Constraints (6) define the ending SOC of energy storage in each hour of each scenario from following the net real-time schedule. Constraints (7)–(9) are analogous to (3)–(5) and impose non-negativity and upper bounds on net real-time charging and discharging of the device and the device's SOC, respectively.

Constraints (10) and (11) impose limits on real-time charging and discharging deviating from the day-ahead schedule. If $\gamma = 0$, there is no flexibility to adjust real-time charging and discharging, because Δc_t^ω and Δd_t^ω are fixed equal to zero, $\forall t \in T, \omega \in \Omega$. $\gamma = 1$ yields the opposite case of complete flexibility to adjust real-time charging and discharging.

Some energy-storage models have constraints that bar simultaneous charging and discharging. We do not include such constraints for two reasons. First, simultaneous charging and discharging of energy storage is often sub-optimal, because energy is wasted [15, 19]. Second, many energy storage technologies have the physical capability to charge and discharge simultaneously. For instance, a PHS plant can operate its pump and turbine simultaneously. In some instances in which wasting energy is beneficial (*e.g.*, if energy prices are negative) our model allows such operation. Our model can be modified to restrict simultaneous charging and discharging by introducing binary variables that represent whether the device is in charging or discharging mode in each hour [20–23]. Doing so would impose added computational costs on the model.

2.4 Deterministic Model

To formulate the deterministic model, we define first the expected values of the α_t^ω's and β_t^ω's as:

$$\bar{\alpha}_t = \sum_{\omega \in \Omega} \phi^\omega \alpha_t^\omega, \forall t \in T;$$

and:

$$\bar{\beta}_t = \sum_{\omega \in \Omega} \phi^\omega \beta_t^\omega, \forall t \in T.$$

We define three additional sets of decision variables. $\Delta c'_t$ and $\Delta d'_t$, respectively, denote hour-t incremental changes in MW that are scheduled to be charged into and discharged from the device in the real-time market. s'_t denotes the device's ending hour-t SOC, measured in MWh, from following the real-time schedule.

The deterministic model is formulated as:

$$\max \sum_{t \in T} \{[\alpha_t + \beta_t \cdot (c_t - d_t)](d_t - c_t) + \left[\bar{\alpha}_t + \bar{\beta}_t \cdot (c_t + \Delta c'_t - d_t - \Delta d'_t)\right](\Delta d'_t - \Delta c'_t)\} \tag{12}$$

$$\text{s.t. } s_t = s_{t-1} + \eta c_t - d_t, \quad \forall t \in T; \tag{13}$$

$$0 \le c_t \le C^{\max}, \quad \forall t \in T; \tag{14}$$

$$0 \le d_t \le D^{\max}, \quad \forall t \in T; \tag{15}$$

$$0 \le s_t \le S^{\max}, \quad \forall t \in T; \tag{16}$$

$$s'_t = s'_{t-1} + \eta \cdot (c_t + \Delta c'_t) - d_t - \Delta d'_t, \quad \forall t \in T; \tag{17}$$

$$0 \le c_t + \Delta c'_t \le C^{\max}, \quad \forall t \in T; \tag{18}$$

$$0 \le d_t + \Delta d'_t \le D^{\max}, \quad \forall t \in T; \tag{19}$$

$$0 \le s'_t \le S^{\max}, \quad \forall t \in T; \tag{20}$$

$$-\gamma C^{\max} \le \Delta c'_t \le \gamma C^{\max}, \quad \forall t \in T; \tag{21}$$

$$-\gamma D^{\max} \le \Delta d'_t \le \gamma D^{\max}, \quad \forall t \in T. \tag{22}$$

Objective function (12) computes total profit that is earned from scheduling energy storage in the day-ahead and real-time markets. Because this model is deterministic, the profit term that corresponds to the real-time market in (12) does not compute expected profits under multiple second-stage scenarios. Rather, profits that are earned in the real-time market are computed under the expected real-time price.

The constraints are analogous to those that are in the two-stage stochastic model. The notable difference is that the constraints in the deterministic model that correspond to scheduling the energy storage in the real-time market are not imposed on a per-scenario basis. Specifically, Constraints (13)–(16) are analogous to (2)–(5) and (17)–(22) are analogous to (6)–(11).

2.5 Calculation of Value of Stochastic Solution

VSS measures the benefit of using a stochastic model for decision making [24]. We compute VSS by using the deterministic model to determine how the energy storage is scheduled in the day-ahead market without stochastic optimization. $\tilde{c}_t$ and $\tilde{d}_t$ denote the optimal values of c_t and d_t, respectively, that are obtained from solving (12)–(22). Then, we solve (1)–(11), but fix $c_t = \tilde{c}_t$ and $d_t = \tilde{d}_t$, $\forall t \in T$. Model (1)–(11) is guaranteed to be feasible with c_t and d_t fixed in this way, because Constraints (2)–(5) and (13)–(16) are the same in the two models and $\Delta c_t^\omega = \Delta d_t^\omega = 0$, $\forall t \in T, \omega \in \Omega$ is feasible in (6)–(11). Solving (1)–(11) gives an objective-function value, which we denote as z_D^*.

We solve also (1)–(11), without fixing any variables. This gives an objective-function value, which we denote as z_S^*. The normalized difference between these two objective-function values, $(z_S^* - z_D^*)/z_S^*$, gives the VSS. The VSS measures the amount by which the expected objective-function value increases (*i.e.*, decisions are made sub-optimally) if a deterministic as opposed to stochastic model is used for day-ahead scheduling.

3 Example

In this section we use a simple example to explore the behavior of the two-stage model.

3.1 Example Data

Our example assumes a twenty-hour optimization horizon and ten second-stage scenarios. Figure 1 summarizes the hourly day-ahead prices and the range of real-time prices in the second-stage scenarios. Specifically, the figure shows the value of α_t in each hour as well as the maximum and minimum values (across the scenarios) of α_t^ω. We examine cases with different values of β_t and β_t^ω, which are assumed to be uniform across all of the hours and scenarios in each case that we examine. The scenarios all have equal probabilities (*i.e.*, $\phi^\omega = 1/10$). We assume that $S^{\max} = 1500$, $C^{\max} = D^{\max} = 100$, $\eta = 0.75$, and $S_0 = 200$.

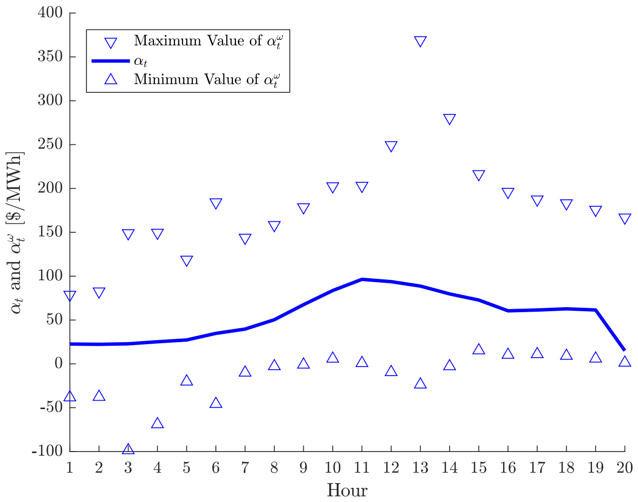

Fig. 1: Values of α_t and Range of Values (Across Scenarios) of α_t^ω in the Example in Section 3

3.2 Example Results

Figure 2 summarizes the scheduled operation of the device in a base case with $\beta_t = \beta_t^\omega = 0, \forall t \in T, \omega \in \Omega$ and $\gamma = 1$. The figure shows hourly day-ahead, expected real-time, and realized scenario-8 real-time prices. Because all of the β_t's and β_t^ω's are zero, the day-ahead and real-time prices in the figure do not respond to the operation of energy storage. Moreover, because $\gamma = 1$, the device is fully flexible to adjust its real-time operation.

Under these two conditions, the energy storage behaves as a purely financial arbitrageur in the day-ahead market, in the sense that its day-ahead schedule is divorced from its eventual real-time schedule (*cf.* hours 1, 3, 13, 14, and 17–20). The day-ahead schedule is determined solely to arbitrage differences between day-ahead and expected real-time prices. The device discharges (charges) day ahead during hours in which the

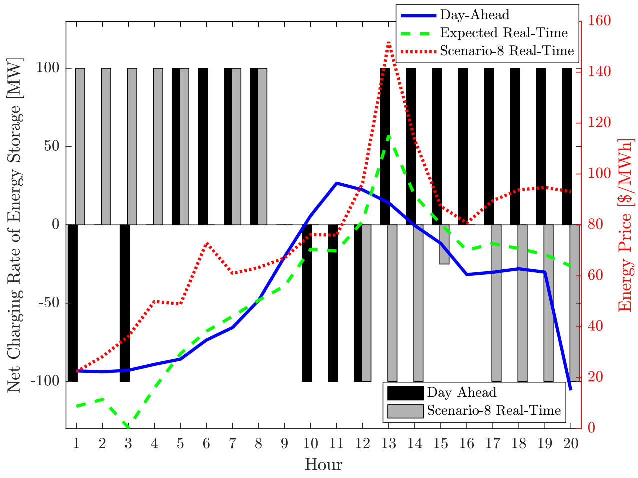

Fig. 2: Day-Ahead and Real-Time Scenario-8 Dispatch of Energy Storage in Base Case of the Example in Section 3 With $\beta_t = \beta_t^\omega = 0, \forall t \in T, \omega \in \Omega$ and $\gamma = 1$

day-ahead price is greater (less) than the expected real-time price. This behavior of the energy storage as a financial arbitrageur stems from our assumptions that it has full flexibility in adjusting its real-time operation (*i.e.*, $\gamma = 1$) and that the prices are insensitive to the operation of energy storage (*i.e.*, $\beta_t = \beta_t^\omega = 0$, $\forall t \in T, \omega \in \Omega$). Absent either of these assumptions, the behavior of the energy storage and the VSS would differ.

This behavior follows from an analysis of the hour-t terms in (1), which are:

$$\alpha_t \cdot (d_t - c_t) + \sum_{\omega \in \Omega} \phi^\omega \alpha_t^\omega \cdot (\Delta d_t^\omega - \Delta c_t^\omega) = \alpha_t \cdot (d_t - c_t) + \mathbb{E}\left[\alpha_t^\omega \cdot (\Delta d_t^\omega - \Delta c_t^\omega)\right].$$

This expression shows that when the β_t's and β_t^ω's are all zero, the profits that are earned in the day-ahead market are completely independent of how the energy storage is scheduled in real time. Moreover, because $\gamma = 1$, there are no constraints that link the day-ahead and real-time schedules.

We can see the energy storage behaving as an arbitrageur in the day-ahead market more clearly by relaxing (2)–(5). When we relax these constraints, we remove any physical restrictions in the day-ahead operation of the energy storage, so long as all power-capacity and SOC-limit constraints are observed in its real-time schedule. When we relax these constraints, the model consisting of (1), (6)–(9) is unbounded, except in the knife-edge case in which the day-ahead price equals exactly the expected real-time price in each hour. We do not advocate relaxing (2)–(5) in operational modeling of energy storage. Rather, a model in which these constraints are relaxed illustrates further energy storage behaving as a financial arbitrageur in the day-ahead market under the assumptions of prices that are insensitive to its operation and full operational flexibility.

Decreasing γ impacts the device's operation in two ways. First, when $\gamma < 1$ the day-ahead schedule tends to follow the real-time schedule more closely (relative to the $\gamma = 1$ case). Second, Figure 2 shows that when the energy storage has complete flexibility in adjusting its real-time schedule, its day-ahead schedule operates at the limits that its power- and SOC-capacity constraints allow. If $\gamma < 1$, the day-ahead schedule tends to be further from these bounds to allow more flexibility in adjusting its real-time position.

Table 1 summarizes the VSSs that are obtained from our example with different values of γ and the β_t's and β_t^ω's. As expected from Fig. 2, the VSS is zero if $\gamma = 1$ and the β_t's and β_t^ω's are all zero. This is because in such a case day-ahead decisions are made purely on the basis of differences between day-ahead and expected real-time prices. These differences are captured by (12) in the deterministic model.

Table 1: Value of stochastic solution [%] for the example in Section 3 with different values of the β_t's and β_t^ω's and γ

	β_t's and β_t^ω's			
γ	0.000	0.010	0.025	0.050
1.00	0.00	0.01	0.01	0.13
0.75	0.02	0.02	0.14	0.48
0.50	0.84	0.71	0.79	0.94
0.25	0.88	0.94	1.10	1.00
0.00	0.00	0.00	0.00	0.00

The VSS becomes non-zero if $\gamma < 1$, as the energy storage no longer has complete flexibility to behave as a pure financial arbitrageur day ahead. The one exception to this is if $\gamma = 0$, which means that there is no flexibility to make real-time schedule adjustments. In such an instance, there is no value to representing the second stage (in a deterministic or stochastic model). Table 1 shows that the VSS is non-zero if the β_t's and β_t^ω's are non-zero, even if $\gamma = 1$. This is because if β_t^ω is non-zero, the time-step-t/scenario-ω real-time profit term in (1) is:

$$\phi^\omega \cdot [\alpha_t^\omega + \beta_t^\omega \cdot (c_t + \Delta c_t^\omega - d_t - \Delta d_t^\omega)](\Delta d_t^\omega - \Delta c_t^\omega);$$

which shows that the day-ahead schedule affects the profits that are earned in the real-time market through its impact on real-time prices.

Having non-zero β_t's and β_t^ω's results in the energy storage behaving as a monoposonist during hours in which it charges and as a monopolist when discharging. Another interesting phenomenon that we observe with non-zero β_t's and β_t^ω's is energy being 'wasted' through simultaneous charging and discharging. Normally, wasting energy in this manner is observed only with negative prices, and we observe such behavior in real-time scenarios with negative prices. In some extreme real-time scenarios with many hours with negative prices and positive prices that are near zero, the device may have a strictly positive SOC at the end of hour 20. This is because the device charges excess energy (during negative-price hours) that it cannot discharge during subsequent positive-price hours without suppressing the prices (through the impact of the β_t^ω's) to become negative. Instead, it is preferable to keep energy stored at the end of the optimization horizon.

We observe simultaneous charging and discharging with positive prices as well. Simultaneous charging and discharging with positive prices occurs when the device is selling energy in net, if the value of the corresponding β_t or β_t^ω is sufficiently large. This is because simultaneous charging and discharging results in the power system having to produce more energy (as a result of the energy that is wasted by the device). This greater electricity production increases the wholesale energy price, which, in turn, increases the value of the energy that the device sells. The extent to which such a strategy is employed depends on the marginal-price impact of increasing electricity production, which is measured by the corresponding value of β_t or β_t^ω, relative to the value of the energy that is wasted, which is given by energy prices.

As an example, Figure 3 shows this type of behavior in the device's real-time operation in hour 8 of scenario 2. Scenario 2 has relatively low prices, meaning that the marginal value of stored energy is effectively zero. Hour 8 has a day-ahead energy price that is low relative to the expected real-time price. As such, the device schedules 72.69 MW of day-ahead charging (scheduling more would eliminate, through the impact of β_8, the difference between the day-ahead and expected real-time prices). In real time, the energy price is $\alpha_8^2 + \beta_8^2 \cdot (c_8 - d_8) = 0.82$ if there are no real-time adjustments to the day-ahead schedule. Instead, the device adjusts its real-time schedule to charge 81.12 MW while discharging 16.69 MW simultaneously, meaning that it charges 64.43 MW in net. Due to its day-ahead schedule, the device's real-time schedule results in selling 8.26 MW back to the market in net. The simultaneous charging and discharging result in the real-time hour-8 price increasing from $-\$2.81$/MWh to $\$0.41$/MWh, which yields the device an operating profit of \$3.40 in hour 8. If, on the other hand, the energy storage sells 8.26 MW back to the market through its real-time transactions without simultaneously charging and discharging, the hour-8 real-time energy price increases only to \$0.13/MWh, yielding an hour-8 operating profit of \$1.84.

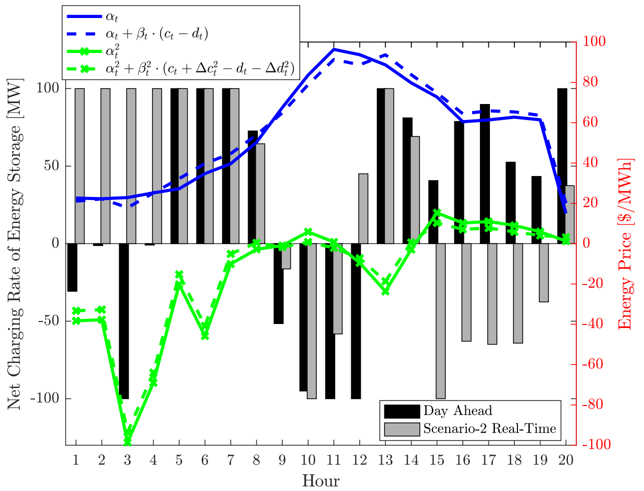

Fig. 3: Day-Ahead and Real-Time Scenario-2 Dispatch of Energy Storage in the Example in Section 3 With $\beta_t = \beta_t^\omega = 0.05, \forall t \in T, \omega \in \Omega$ and $\gamma = 1$

These findings regarding simultaneous charging and discharging are driven by the assumption that market prices depend on the operation of the energy storage. With our assumption of a linear relationship between prices and energy-storage operations, the values of the β_t's and β_t^ω's are critical in determining the profitability of such an operational strategy. If there is a more complex (*e.g.*, nonlinear) relationship between prices and the operation of energy storage, such phenomena may be observed still. The profitability of simultaneous charging and discharging in such a case would be governed by the extent to which prices change with the operation of energy storage.

4 Case Study

This section demonstrates our proposed model using data that correspond to an actual PHS plant that participates in the PJM-operated day-ahead and real-time energy markets. Our case study data are calibrated using historical market and system data from PJM.

4.1 Case Study Data

According to Kim and Powell [25], electricity prices can be highly volatile, non-stationary, and heavy-tailed. We use a three-step process, which is detailed in Appendix A, to generate day-ahead and real-time price data (*i.e.*, values for the α_t's, β_t's, α_t^ω's, and β_t^ω's), with the aim of giving the prices these properties. Figure 4 shows the value of α_t in each hour as well as the maximum and minimum values (across the 100 scenarios that are modeled) of α_t^ω on 15 May, 2012, which is the one-day period that we focus on in our case study results. The values that are shown in the figure are simulated using the technique that is detailed in Appendix A. The 100 scenarios are assumed to have equal probabilities (*i.e.*, $\phi^\omega = 1/100$) and we assume that $S^{\max} = 1000$, $C^{\max} = D^{\max} = 100$, $\eta = 0.75$, and $S_0 = 200$.

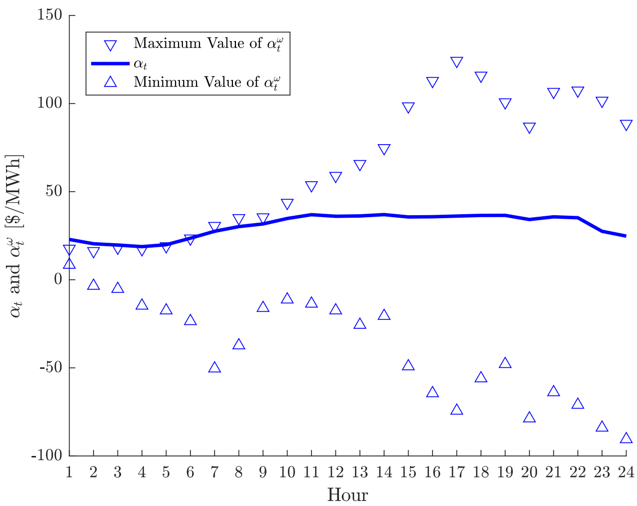

Fig. 4: Values of α_t and Range of Values (Across Scenarios) of α_t^ω in the Case Study in Section 4

4.2 Case Study Results

Table 2 summarizes the optimized value of (1) and the VSS for the case study with different values of γ. As in the example from Section 3, decreasing γ increases the VSS until some threshold value, at which point decreasing γ further results in a lower VSS. The VSS is non-zero even if $\gamma = 1$, due to the impacts of non-zero β_t's and β_t^ω's, which make (1) nonlinear.

Figure 5 summarizes net charging that is scheduled in the day-ahead and real-time markets and the price impacts of the energy storage. The figure shows

Table 2: Optimized value of (1) [$] and value of stochastic solution [%] for the case study in Section 4 with different values of γ

γ	Value of (1)	VSS
1.00	30960	0.17
0.80	29953	0.84
0.70	29288	1.08
0.50	26672	2.05
0.20	17686	1.53
0.00	9264	0.00

day-ahead and scenario-8 real-time prices being suppressed by up to \$1.99/MWh and \$4.13/MWh, respectively. The β_t's range between 0.010 and 0.043 while the β_t^ω's range between 0.002 and 0.056. Figure 5 considers a case with $\gamma = 0.7$, meaning that the device is restricted somewhat in making real-time schedule adjustments. For example, in hours 3–5 energy storage schedules 30 MW of charging day ahead and an additional 70 MW in real time. This is despite the real-time price being lower and is due to limited real-time flexibility. The combined effects of reduced flexibility and non-zero β's and β_t^ω's yield simultaneous charging and discharging of energy storage in hours 6–9 and 15–24.

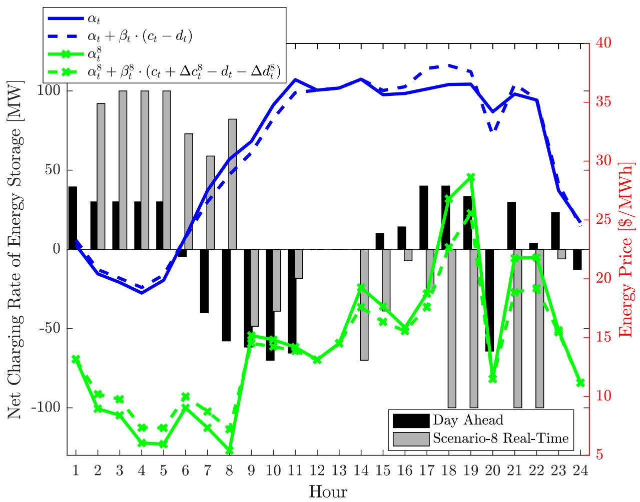

Fig. 5: Day-Ahead and Real-Time Scenario-8 Dispatch of Energy Storage in the Case Study in Section 4 With $\gamma = 0.70$

The example and case study are programmed using GAMS version 24.4.6 and solved using IPOPT version 3.11.9 on a computer with a 2.5-GHz Intel Core i5 processor and 4 GB of memory. The example and case study are all solved in less than one minute of wall clock time.

5 Conclusions

This paper develops a two-stage stochastic model with which to make operational decisions for energy storage that can impact market prices through its charging and discharging. Our model allows for imposing flexibility constraints, which limit real-time adjustments to the operating schedule. Such constraints may be imposed by market operators in practice, so as to have day-ahead operating schedules that are somewhat reflective of how the system is operated in real time. We illustrate how historical market data can be used to calibrate the parameters that relate energy prices to the operation of energy storage (*cf.* Appendix A). The aim of estimating these price-related parameters (and of our work) is not to predict the impact of energy storage on prices in a particular market. Rather, our aim is to examine how price-making energy storage behaves in a market in which prices are sensitive to its operations.

We find that using a stochastic model is not valuable (*i.e.*, the VSS is zero) if the market prices are fixed and the energy storage has full flexibility to adjust its day-ahead position in real time. Under these two assumptions the energy storage behaves as a financial arbitrageur. The complexity of a stochastic modeling framework is not needed for such behavior, so long as expected real-time prices are used to determine the day-ahead schedule. Otherwise, if there are restrictions on making real-time adjustments to the day-ahead schedule (*e.g.*, due to γ being less than unity), the VSS can be non-zero. There may be other market-design and operational factors that can make the VSS non-zero. For instance, some markets impose financial costs (*e.g.*, imbalance penalties) on market participants that make sufficiently large changes to their day-ahead positions in the real-time market. An energy-storage owner that is risk- or loss-averse (as opposed to our assumption of a risk-neutral expected-value-maximizer) also may have a non-zero VSS with full flexibility and fixed energy prices.

We observe cases with positive prices in which it is profit-maximizing for energy storage to 'waste' energy by charging and discharging simultaneously. These cases arise due to our assumption that prices can react to the operation of energy storage and depend on whether prices are sufficiently responsive to energy storage operation. In such a case, the implicit opportunity cost of wasting stored energy is outweighed by the pecuniary impact of the wasted energy adjusting the price at which energy is sold.

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A Price Modeling in Case Study

We employ a three-step process to calibrate the price-related parameters from historical market and system data. The first step uses a linear regression model to fit historical day-ahead and real-time wholesale prices to a number of explanatory variables, including temperature and load. Next, we fit seasonal autoregressive integrated moving average (SARIMA) models to historical temperature and load data. Finally, we use the SARIMA model to simulate different sample paths of temperatures and loads, which are input to the regression model to simulate values for the α_t's, β_t's, α_t^ω's, and β_t^ω's. We detail each of these three steps in turn. The technique that we employ to calibrate the values of the α_t's, β_t's, α_t^ω's, and β_t^ω's assumes implicitly that historical data can be used to predict future price-load relationships.

A.1 Price Linear Regression Model

Our first step is to fit historical day-ahead and real-time price data from the PJM market to a set of explanatory variables using a linear regression model. Day-ahead and real-time price data for the Appalachian Power Company (APCO) zone (which is a zone within which a number of PHS plants are located) from between 1 April, 2012 and 30 June, 2012 are used. Specifically, the day-ahead price regression model regresses the hour-t day-ahead price against:

- a constant,
- hour-t load,
- hour-t heating and cooling degrees, which are defined relative to 65° F,
- hour-t month, weekend, and hour dummy variables, and
- interaction terms between:
  - each hour-t month dummy variable and hour-t load,
  - each hour-t month dummy variable and hour-t heating degrees,
  - each hour-t month dummy variable and hour-t cooling degrees,
  - each hour-t month dummy variable and each hour-t weekend dummy variable,
  - each hour-t weekend dummy variable and hour-t load,
  - each hour-t hour dummy variable and hour-t load,
  - each hour-t hour dummy variable and hour-t heating degrees,
  - each hour-t hour dummy variable and hour-t cooling degrees, and
  - each hour-t hour dummy variable and each hour-t weekend dummy variable.

Historical hourly load data for the APCO zone and hourly temperature data for Leesville, Virginia (which is located in the APCO zone) are used to fit the regression model using ordinary least squares (OLS). A separate regression model using real-time prices and the same explanatory variables is estimated also using OLS.

A.2 Temperature and Load Seasonal Autoregressive Integrated Moving Average Models

SARIMA models are a generalization of autoregressive integrated moving average (ARIMA) models, which capture seasonality in time series [26, 27]. ARIMA and other time series models are used commonly for temperature, load, and electricity-price modeling [28, 29]. Historical hourly temperature data from between 1 April, 2012 and 30 June, 2012 for Leesville, Virginia are fit to a $(2,1,0)\times(0,1,1)_{24}$ SARIMA model. Hourly load data for the APCO zone from the same time period are fit to a different $(1,1,0)\times(0,1,1)_{24}$ SARIMA model.

A.3 Generating α_t's, β_t's, α_t^ω's, and β_t^ω's

The α_t's, β_t's, α_t^ω's, and β_t^ω's represent the day-ahead and real-time electricity prices as depending on the amount that the energy storage is charged or discharged. We capture these impacts by using the coefficients multiplying the hour-t load in the linear regression models that are described in Section A.1. Specifically, we define $\mathbb{L}$ as the set of terms on the right-hand side of the regression model that have load as an explanatory variable. Load itself appears as an explanatory variable on the right-hand side of the regression model. However, there are also terms in which load is interacted with dummy variables. All of these terms are in the set, $\mathbb{L}$. We can write each of the regression models (for day-ahead and real-time prices) as:

$$y_t = l_t \sum_{i\in\mathbb{L}} \hat{\zeta}_i x_{i,t} + \sum_{i\notin\mathbb{L}} \hat{\zeta}_i x_{i,t} + \epsilon_t, \tag{23}$$

where y_t and l_t represent the hour-t price and load, respectively, $\hat{\zeta}_i$ is the OLS estimate of the coefficient on each term, and ϵ_t is the hour-t error term. For terms that are in the set, $\mathbb{L}$, $x_{i,t}$ is defined as any other right-hand side variable multiplying load (*e.g.*, for each of the terms in which each hour-t month dummy variable is interacted with hour-t load, $x_{i,t}$ would be defined as the month dummy variable, whereas we have $x_{i,t} = 1$ for the term in which hour-t is not interacted). For terms that are not in the set, $\mathbb{L}$, $x_{i,t}$ is defined as the right-hand side variable in that term.

Next, we simulate a sample path of hourly prices and loads using the two SARIMA models that are described in Section A.2, by simulating randomly the white noise processes (using the white noise variances, which are obtained when fitting the two SARIMA models to the historical data). We let $\{\hat{L}_t\}_{t\in T}$ denote the sample path of loads. Substituting the sample path of loads into (23) we obtain:

$$y_t = \hat{L}_t \sum_{i\in\mathbb{L}} \hat{\zeta}_i x_{i,t} + \sum_{i\notin\mathbb{L}} \hat{\zeta}_i x_{i,t} + \Delta L_t \sum_{i\in\mathbb{L}} \hat{\zeta}_i x_{i,t} + \epsilon_t, \tag{24}$$

where ΔL_t is meant to represent any changes in APCO-zone load that occur from charging or discharging energy storage, and where we use the sample path of temperatures to fix the heating and cooling degree right-hand side variables. Based on (24) we define:

$$\alpha_t = \hat{L}_t \sum_{i\in\mathbb{L}} \hat{\zeta}_i x_{i,t} + \sum_{i\notin\mathbb{L}} \hat{\zeta}_i x_{i,t},$$

and:

$$\beta_t = \sum_{i\in\mathbb{L}} \hat{\zeta}_i x_{i,t},$$

using the coefficients that are estimated from the model that uses day-ahead prices, and:

$$\alpha_t^\omega = \hat{L}_t \sum_{i\in\mathbb{L}} \hat{\zeta}_i x_{i,t} + \sum_{i\notin\mathbb{L}} \hat{\zeta}_i x_{i,t},$$

and:

$$\beta_t^\omega = \sum_{i\in\mathbb{L}} \hat{\zeta}_i x_{i,t},$$

using the coefficients that are estimated from the model that uses real-time prices.

This process allows us to simulate multiple scenarios of real-time prices by generating multiple sample paths of hourly loads and prices.

In practice, day-ahead and real-time energy prices often are correlated, because they are driven by similar underlying dynamics (*e.g.*, load and temperature). Our regression models do not account explicitly for such correlations. However, day-ahead and real-time prices that are generated using our technique do exhibit implicit correlation. This is because the loads and temperatures that impact prices in (24) are correlated to one another. Thus, our price-simulation technique provides a balance between model complexity and fidelity.