Welfare Impacts of Electricity Storage and the Implications of Ownership Structure☆

Ramteen Sioshansi*

Integrated Systems Engineering Department, The Ohio State University, Columbus, Ohio, United States
Phone: +1-614-292-3932

Abstract

Increases in electricity price volatility have raised interest in electricity storage and its potential arbitrage value. Large utility-scale electricity storage can decrease the value of energy arbitrage by smoothing differences in prices on- and off-peak, however this price-smoothing effect can result in significant external welfare gains by reducing consumer energy costs and generator profits. As such, the incentives of merchant storage operators, consumers, and generators may not be properly aligned to ensure socially-optimal storage use. We examine storage use incentives for these different agent types and show that under most reasonable market structures a combination of merchant and consumer ownership of storage maximizes potential welfare gains from storage use.

1. Introduction

The emergence of wholesale electricity markets in many countries and regions of the United States and increases in electricity price volatility have raised interest in electricity storage. Electricity storage can take advantage of differences in on- and off-peak electricity prices by buying and storing energy when prices are low and discharging and reselling it when prices are high. In addition to this energy arbitrage, storage can provide other value such as capacity and ancillary services. A number of authors have considered these types of storage applications and have analyzed the potential economic value of investing in energy storage. This analysis has typically assumed the storage device to be a small 'price-taker' relative to the electricity market as a whole, and used historical price data to estimate the value of storage. Examples of this type of analysis include Graves et al. (1999); Figueiredo et al. (2006); Walawalkar et al. (2007).

Larger utility-scale storage, which can smooth the load pattern by lowering on-peak and increasing off-peak generating loads, will tend to result in a similar smoothing of on- and off-peak price patterns and reduce arbitrage opportunities for a storage device. Despite this effect of reducing the value of arbitrage, this load smoothing can have significant welfare effects—by reducing energy prices for consumers and profits for electricity generators. Jenkin and Weiss (2005) suggested this external welfare effect of large-scale storage and Sioshansi et al. (2009) estimated these effects using data from PJM. Sioshansi et al. (2009) show that the price-smoothing effect of large-scale storage can reduce the arbitrage value of 1 GW of storage by more than 20%, compared to the arbitrage value for a price-taker. Their analysis shows, however, that the load and price shifting can result in annual consumer surplus gains on the order of \$16-35 million, and producer surplus losses in the range of \$14-31 million—resulting in massive wealth transfers and net welfare gains. The analysis in Sioshansi et al. (2009) assumes the storage device is owned by a single merchant storage operator, though, and they surmise that a merchant storage operator may not necessarily have proper incentives to

☆The author would like to thank Paul Denholm, Armin Sorooshian, Shmuel Oren, and four anonymous referees for helpful conversations and suggestions. Anthony Afful-Dadzie provided research assistance.

*Corresponding author
Email address: sioshansi.1@osu.edu (Ramteen Sioshansi)

use the storage device in a welfare-maximizing way, since increased use of storage will reduce arbitrage value and the storage owner will generally not be concerned with the external welfare effects of storage use. Thus, an unanswered question is what effect ownership structure can have on the use of storage and whether storage could be used in a more socially beneficial way if it is owned by different market participants, such as consumers[1] or generators.

This paper analyzes and compares the incentives for merchant storage operators, generators, and consumers to use electricity storage. Our analysis shows that merchant storage operators and generators will generally underuse their storage devices compared to the social optimum, whereas consumers will generally overuse storage, however storage use by all three agent types will approach the social optimum in a perfectly competitive market setting. We use market data from the Electric Reliability Council of Texas (ERCOT) power system to simulate storage use by the three agent types and demonstrate that merchant ownership of storage will generally be socially preferred compared to generator or consumer storage. We then demonstrate that if storage assets can be divided between agent types, doing so may be more beneficial than having all the storage owned by merchants. The remainder of this paper is organized as follows: section 2 describes the model used in our analysis and section 3 presents the main theoretical results of our paper, section 4 discusses the results of our numerical analysis and section 5 considers a case with storage assets owned by multiple agent types. Section 6 concludes.

2. Model

We assume that electricity prices respond to generating loads based on a linear relationship given by $p(l) = c_0 + c_1 l$, with $c_0 \geq 0$ and $c_1 > 0$. The market includes a storage device with roundtrip efficiency, η, which captures the energy losses that occur when energy is taken through the storage cycle. Thus, if x MWh of energy is put into storage then at most ηx MWh can be taken out. We assume that $0 < \eta < 1$, which reflects the fact that there are some energy losses in the storage cycle. We assume that the storage device has a discharge capacity of $\overline{\delta}$ MW, and that its total charging capacity is $\overline{\delta}/\eta$ MW, which allows one hour of charging at full capacity to provide enough energy to discharge at full capacity for one hour.[2]

We assume that the electricity market operates in two hour-long periods, and we let l_1 and l_2 denote the loads in an off-peak (period 1) and on-peak (period 2) hour. These loads are assumed to be price-inelastic because consumers face a time-invariant retail rate of electricity, which is the case in most every power system. In period 1, δ/η MWh of energy can be put into the storage device and in period 2, δ MWh of energy can be discharged, where $0 \leq \delta \leq \overline{\delta}$. Although consumer loads will be independent of storage use, energy prices will increase and decrease in periods 1 and 2, respectively, due to changes in generating loads. Figure 1 illustrates this effect in which the price in period 1 increases from $p(l_1)$ to $p(l_1 + \delta/\eta)$, and the price in period 2 decreases from $p(l_2)$ to $p(l_2 - \delta)$. Similarly, the price of energy sales will change in the two periods, which will affect the producer surplus of generators, and generators will produce more energy in period 1 and less energy in period 2.

These changes in electricity prices and the amount of energy generated in periods 1 and 2 will result in welfare changes for consumers and generators, in addition to the profits earned by the owner of the storage device. Because consumer demand does not change in either period, the change in consumer welfare is driven entirely by the change in the price of electricity, which is:

$$\begin{aligned}\Delta CS(\delta) &= l_1[p(l_1) - p(l_1 + \phi\delta)] + l_2[p(l_2) - p(l_2 - \delta)] \\ &= \delta c_1(l_2 - \phi l_1),\end{aligned}$$

[1] Although it may currently be impractical for individual electricity consumers to own energy storage devices, the term 'consumers' and our analysis of consumer use of storage can be generalized to a load-serving entity that seeks to minimize the cost of energy consumed by its customers.

[2] Greenblatt et al. (2007) note that many modern storage devices, such as batteries and compressed-air energy storage, can easily be designed with different charging and discharging capacities, as we assume.

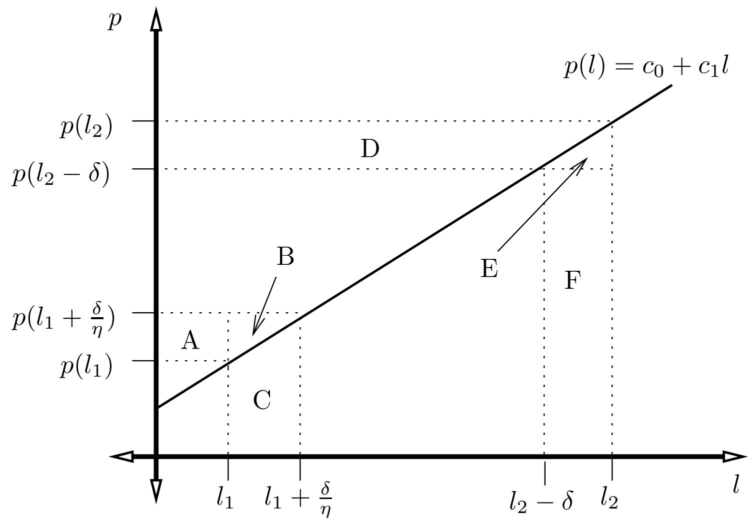

Figure 1: Price-load relationship and the price- and load-shifting effect of storage use in periods 1 and 2.

where we define $\phi = 1/\eta$.[3] This change in consumer surplus is given by the sum of the areas labeled 'D' and 'E' in figure 1 minus the area labeled 'A.' Because the amount of energy generated and energy prices change in both periods, the change in producer surplus is given by:

$$\begin{aligned}\Delta PS(\delta) &= \int_{p(l_1)}^{p(l_1+\phi\delta)} l(p)dp + \int_{p(l_2)}^{p(l_2-\delta)} l(p)dp \\ &= \delta c_1(-l_2 + \phi l_1) + \frac{1}{2}\delta^2 c_1(1+\phi^2).\end{aligned}$$

The change in producer surplus is given by the sum of the areas labeled 'A' and 'B' in figure 1 minus the area labeled 'D.' Finally, the total net profit of the owner(s) of the storage devices will be given by:

$$\begin{aligned}\Pi_{arb}(\delta) &= \delta p(l_2 - \delta) - \phi\delta p(l_1 + \phi\delta) \\ &= \delta[c_0(1-\phi) + c_1(l_2 - \phi l_1)] - \delta^2 c_1(1+\phi^2).\end{aligned}$$

The arbitrage value is given by the area labeled 'F' in figure 1 minus the sum of the areas labeled 'B' and 'C.' Adding these three terms, the social welfare change resulting from usage of the storage device is given by:[4]

$$\begin{aligned}\Delta W(\delta) &= \Delta CS(\delta) + \Delta PS(\delta) + \Pi_{arb}(\delta) \qquad (1) \\ &= \delta[c_0(1-\phi) + c_1(l_2 - \phi l_1)] - \frac{1}{2}\delta^2 c_1(1+\phi^2).\end{aligned}$$

[3]This computation of consumer surplus change may seem confusing, since we assume demand is price-inelastic. Indeed, with a truly price-inelastic demand, consumer surplus is infinite. We assume, however, that there is some fixed upper-bound on the value of electricity, such as a value of lost load (VOLL). As Anderson (1972); Kirschen et al. (2003) discuss, VOLL is commonly used for capacity and reliability planning purposes to capture the fact that consumers would not want to pay an inordinately high price for energy, but cannot express these preferences due to time-invariant rates. Because VOLL is typically assumed to be much higher than marginal generation costs (empirical studies place VOLL in the thousands to ten of thousands of dollars per MWh), demand is effectively inelastic but consumer surplus is bounded and finite.

[4]It may seem counter-intuitive to include arbitrage value in the calculation of social welfare, since arbitrage value may be thought of as a wealth transfer. This view is incorrect, however, because the use of storage provides social value in that higher-cost on-peak generation is replaced with lower-cost off-peak generation, and arbitrage value represents the social value of this load-shifting.

We can find the social-welfare maximizing use of the storage device by maximizing equation 1 subject to the constraint that $0 \le \delta \le \overline{\delta}$. The first-order necessary condition (FONC) for a maximum[5] is given by:

$$\begin{array}{ll} \delta^W = 0, & \text{if } \Delta W'(0) < 0; \\ \delta^W = \overline{\delta}, & \text{if } \Delta W'(\overline{\delta}) > 0; \\ \Delta W'(\delta^W) = 0, & \text{otherwise;} \end{array}$$

which yields the following optimum:

$$\delta^W = \begin{cases} 0, & \text{if } c_0(1-\phi) + c_1(l_2 - \phi l_1) < 0; \\ \overline{\delta}, & \text{if } c_0(1-\phi) + c_1(l_2 - \phi l_1) - \overline{\delta} c_1(1+\phi^2) > 0; \\ \frac{c_0(1-\phi)+c_1(l_2-\phi l_1)}{c_1(1+\phi^2)}, & \text{otherwise.} \end{cases} \tag{2}$$

Thus, δ^W gives the welfare-maximizing use of storage, which is dependent upon the loads in the two periods (which determine the difference in prices in the two periods). We can intuitively tell from figure 1 that the welfare-maximizing use of storage will tend to result in a net consumer surplus gain and a producer surplus loss. The increase in consumer surplus is due to the decrease in prices during the on-peak period, which reduces the amount consumers must pay for energy. Although there is an increase in prices during the off-peak period, this price increase is applied to a smaller load than that in the on-peak period, and as such there is a net surplus gain. Conversely, generators have an increase in profits during the off-peak period due to increased sales and prices, but a decrease in surplus during the on-peak period. Because the price reduction in the on-peak period is applied to a larger volume of sales than the price increase in the off-peak period, there is a net surplus loss. The following proposition, the proof of which is given in the appendix, shows this result holds generally.

Proposition 1. $\Delta CS(\delta^W) \ge 0$ and $\Delta PS(\delta^W) \le 0$.

Proof. See appendix. □

3. Storage Use Incentives

We now examine the incentives for the three different agent types—merchant storage operators, consumers, and generators—to use storage devices, and compare their use of storage to the social optimum. Proposition 1, which shows that welfare-maximizing storage use benefits consumers while reducing producer profits, will result in consumers and producers having vastly different incentives to use storage from one another and from merchant storage owners. This is because the three different agent types will use storage to maximize their net payoffs. In the case of consumers this would consist of the sum of arbitrage value and consumer surplus change, whereas producers would maximize the sum of generation and arbitrage profits. Merchant storage operators, on the other hand, will maximize arbitrage profits only. Because consumer surplus is enhanced by welfare-maximizing storage use, and since consumers that own storage would not consider the impact of storage use on generator profits, they will tend to have an incentive to overuse storage. Conversely, because storage use reduces producer profits, generators will have an incentive to underuse storage.

The propositions in this section, the proofs of which are given in the appendix, formalize these arguments by analyzing the three cases of merchant-, consumer-, and generator-ownership of storage separately. This analysis assumes that the storage assets are divided evenly between N symmetric agents, each of which owns a storage device with roundtrip efficiency η and discharge capacity $\overline{\delta}/N$. We also assume that the agents are competing in a non-cooperative setting and follow pure-strategy Nash equilibrium strategies. The following propositions show that in an oligopoly setting (when N is finite), the incentives of all three agent types are to either overuse or underuse their storage devices relative to the social optimum.

[5]Note that $\Delta W''(\delta) = -c_1\left(1+\phi^2\right)$. Because we assume that $c_1 > 0$ we will have $\Delta W''(\delta) < 0$, meaning the social-welfare maximization problem is strictly convex and the FONC is sufficient for the unique maximum.

Proposition 2. If the storage assets are owned by $N \geq 1$ symmetric merchant storage operators, which are not consumers and do not own generating assets, storage will be underused compared to the social optimum.

Proof. See appendix. □

Proposition 3. If the storage assets are owned by $N \geq 1$ symmetric generating firms that own all the generating assets, storage will only be used if there are at least two firms, in which case storage will be underused compared to the social optimum.

Proof. See appendix. □

Proposition 4. If the storage assets are owned by $N \geq 1$ symmetric consumers, storage will be overused compared to the social optimum.

Proof. See appendix. □

The three propositions show that if the storage assets are owned by merchant storage operators or generators, then storage will be underused compared to the social optimum. Merchant storage operators underuse storage because they do not internalize the net external welfare gain that results from storage use and are 'overly sensitive' to reducing the price spread between periods 1 and 2—since this price spread yields arbitrage profits. As discussed above, generators similarly underuse storage due to the reduction in producer surplus from storage use. Because of this reduction in generation profits, generators only use storage if the differences in on- and off-peak prices are sufficiently high for arbitrage profits to outweigh the producer surplus loss. The final proposition shows that consumers will generally overuse storage, since they do not internalize the producer surplus loss that results from storage use.

We next show, as a corollary, that in a perfectly competitive setting in which the storage assets are owned by competitive merchant storage operators, generators without market power, or consumers without monopsony power, the socially optimal use of storage will result. This result is driven by the standard competitive argument that as the number of agents increases, the impact that each individual agent has on increasing consumer surplus, decreasing producer surplus, and reducing arbitrage value will become negligible, and each agent will behave as a price-taker.

Corollary 1. A perfectly competitive market with all the storage assets owned by either symmetric merchant storage operators, consumers, or generators will yield the social welfare-maximizing usage of storage.

Proof. See appendix. □

4. Numerical Simulation of Welfare Losses from Suboptimal Storage Use

The results of section 3 show how storage use by the three agent types compares to the social optimum. Although these agents will overuse or underuse storage compared to the social optimum, a more important issue is the extent of social welfare losses from these suboptimal uses of storage. This type of analysis could help to inform policy-makers or regulatory bodies—which may provide incentives for investment in storage technologies—what types of agents those incentives should be targeted towards.

In order to estimate the social welfare impacts of storage being owned by the different agent types, we numerically simulate storage use under different ownership structures using empirical market data from ERCOT in 2005. The generation market in ERCOT was roughly a symmetric duopoly in 2005 with two large generating firms, TXU and Texas Genco. The remaining firms owned substantially less generation and previous analyses of the ERCOT market, such as Sioshansi and Oren (2007), suggest these firms do not behave strategically. As such we model the generation sector as a symmetric duopoly. On the retail side, ERCOT was served by two dominant firms, Reliant Energy Retail Services and TXU Energy Retail. Thus we model the retail sector as having two symmetric consumers.

Storage use is simulated for each day of the year separately. For each day the loads in the two periods, l_1 and l_2, are set to the minimum and maximum hourly load for that day, respectively. We estimate the linear price-load relationship by assuming a competitive generation sector and using linearized marginal generating costs to derive the aggregate supply function. Generating costs are calculated using standard engineering methods—multiplying generator heat rates by spot fuel prices and adding the costs of SO_2 permit prices and variable operations and maintenance costs.

Figure 2 shows the average daily welfare-maximizing use of 1000 MW of storage, and contrasts it with how storage would be used if the assets are owned by two symmetric merchant storage operators, generators, or consumers. The figure shows storage use with device efficiencies above 0.60—below which storage is not used in any of the four cases. The storage use shown is the average total amount of energy discharged each day (*i.e.* the value reported is the average value of δ, and an average of δ/η MWh of energy is put into storage each day). Figure 2 highlights the results of section 3 by showing that merchant storage operators and generators underuse and consumers overuse storage relative to the social optimum.

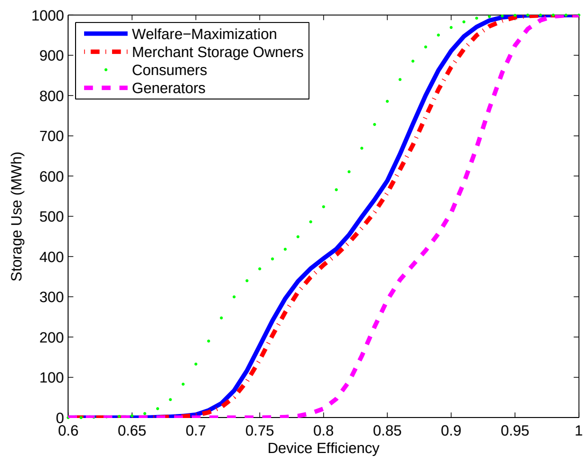

Figure 2: Average total daily use of 1000 MW of storage by a welfare-maximizer and symmetric duopoly merchant storage operators, consumers, and generators. Storage use is given in terms of average energy discharged per day.

Figures 3 through 5 show social welfare losses under merchant, generator, and consumer ownership of storage, respectively, for a range of device capacities and efficiencies. The welfare losses are reported as the percentage of the social welfare gains that are achieved with socially-optimal use of storage. For most device efficiencies social welfare losses are minimized when storage is owned by merchant operators. Moreover, ASCE (1993); Denholm and Kulcinski (2004) note that many utility-scale storage technologies, such as flow batteries and pumped-hydroelectric storage, have efficiencies in the 0.65 to 0.85 range, over which merchant ownership of storage minimizes social welfare losses.

Welfare losses with merchant ownership are also relatively insensitive to device size and efficiency, since losses are always below 12% for the range examined, whereas consumer and generator ownership result in a much wider range of losses. With higher device efficiencies welfare losses from consumer and generator ownership are reduced, for example welfare losses under consumer ownership is less than that under merchant ownership above a device efficiency of 0.92, although these cases are beyond the capabilities of modern storage devices available today. For more realistic device efficiencies, for instance below 0.75, generator ownership of storage results in 100% welfare losses, since the generating firms never use the storage device, however the welfare losses are reduced with higher device efficiencies since generators make some use of storage. We see that efficiency losses from generator ownership are smaller with less than 2 GW of total storage capacity as there is a smaller difference between the socially-optimal use of storage and the generators' underuse with these lower device capacities. Consumer ownership of storage shows similar sensitivity to efficiency and capacity. With an efficiency of 0.74 consumers' overuse of storage results in an average net reduction in total social welfare compared to if storage is not used, since producer surplus losses outweigh consumer welfare gains. These welfare losses can be tapered to some extent by storage capacity—with less than 2000 MW of storage, for instance, the capacity of the storage devices will limit the extent to which consumers

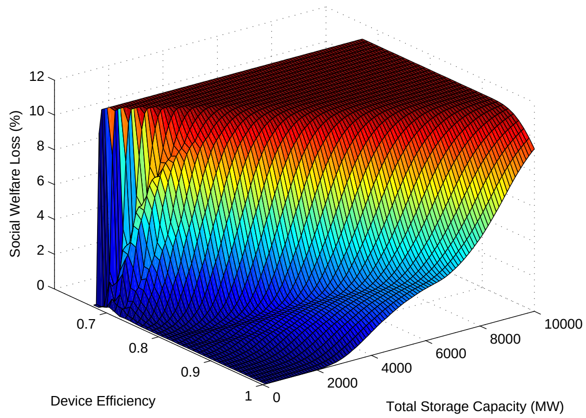

Figure 3: Social welfare losses from suboptimal use of storage by symmetric duopoly merchant storage operators. Welfare losses are given as a percentage of social welfare gains from optimal use of storage.

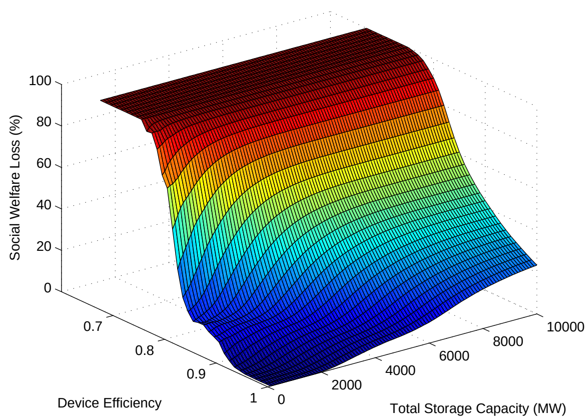

Figure 4: Social welfare losses from suboptimal use of storage by symmetric duopoly generators. Welfare losses are given as a percentage of social welfare gains from optimal use of storage.

can overuse the devices and limit social welfare losses.

5. Welfare Losses with Multiple Agent Types

The analysis of section 4 considered market structures in which storage is owned by either merchants, generators, or consumers only, without allowing for ownership by multiple agent types. We saw that amongst those alternatives, merchant ownership of storage is generally socially preferable to generator or consumer ownership for reasonable device efficiencies. In this section we expand our analysis to allow for ownership of storage by all three agent types simultaneously, to determine if distributing storage assets between the agent types can improve upon storage owned by a single agent type.

5.1. Derivation of Equilibrium with Multiple Agent Types

We begin by expanding our model to allow for all three agent types to simultaneously own and use storage. We assume that the market will have M generators, Z merchants, and N consumers, and that the total capacity of storage owned by the three agent types is $\overline{\delta}^G$, $\overline{\delta}^S$, and $\overline{\delta}^C$. We assume that the generators, merchants, and consumers are symmetric, meaning that each own $\overline{\delta}^G/M$, $\overline{\delta}^S/Z$, and $\overline{\delta}^C/N$ MW of storage capacity and that they simultaneously decide on storage use to maximize their respective profit functions and solve for a Nash equilibrium.

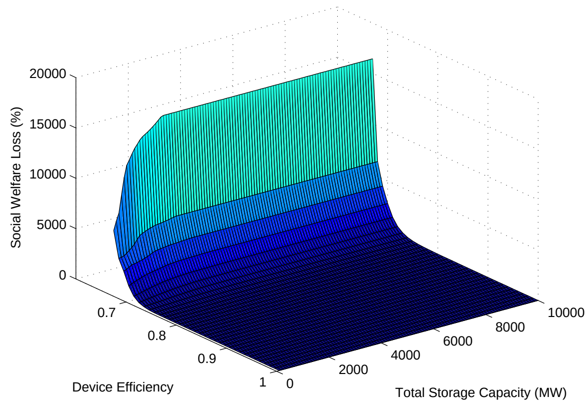

Figure 5: Social welfare losses from suboptimal use of storage by symmetric duopoly consumers. Welfare losses are given as a percentage of social welfare gains from optimal use of storage.

We define δ_i^G, δ_j^S, and δ_k^C to be the amount of storage used by generator i, merchant j, and consumer k; δ_{-i}^G, δ_{-j}^S, and δ_{-k}^C to be the total amount of storage used by everyone except for generator i, merchant j, and consumer k; δ^G, δ^S, and δ^C to be the total amount of storage used by all generators, merchants, and consumers; and δ^T to be the total amount of storage used. We can then formulate and analyze each agent type's maximization problem in turn:

5.1.1. Generators' Optimization Problem

Generators will once again use storage to maximize the sum of arbitrage value and consumer surplus gains. We can write generator i's optimization problem as:

$$\begin{aligned}
\max_{\delta_i^G \in [0, \overline{\delta}^G/M]} \quad \pi_G(\delta_i^G, \delta_{-i}^G) &= \delta_i^G(c_0 + c_1(l_2 - \delta_i^G - \delta_{-i}^G)) - \phi\delta_i^G(c_0 + c_1(l_1 + \phi\delta_i^G + \phi\delta_{-i}^G)) \\
&\quad + \frac{1}{M}\Delta PS(\delta_i^G + \delta_{-i}^G) \\
&= \delta_i^G(c_0 + c_1(l_2 - \delta_i^G - \delta_{-i}^G)) - \phi\delta_i^G(c_0 + c_1(l_1 + \phi\delta_i^G + \phi\delta_{-i}^G)) \\
&\quad + \frac{1}{M}\left((\delta_i^G + \delta_{-i}^G)c_1(-l_2 + \phi l_1) + \frac{1}{2}(\delta_i^G + \delta_{-i}^G)^2 c_1(1 + \phi^2)\right).
\end{aligned}$$

If we let $\lambda_i^G \geq 0$ and $\mu_i^G \geq 0$ be the Lagrange multipliers associated with generator i's lower- and upper-bound constraints, respectively, the FONC[6] for an optimum is given by:

$$-c_1(1+\phi^2)\delta_i^G + c_0 + c_1(l_2 - \delta^T) - \phi(c_0 + c_1(l_1 + \phi\delta^T)) + \frac{1}{M}[c_1(-l_2 + \phi l_1) + \delta^T c_1(1+\phi^2)] + \lambda_i^G - \mu_i^G = 0,$$

Again, it is straightforward to apply the same approach in the proof of Proposition 2 to show that in an equilibrium generators will all use storage identically. Due to this symmetry, the FONC can be rewritten as:

$$\begin{gathered}
c_0(1-\phi) + \tfrac{1}{M}(\delta^S + \delta^C)c_1(1+\phi^2) + \tfrac{M-1}{M}c_1(l_2 - \phi l_1) - c_1\delta^T(1+\phi^2) + \lambda^G - \mu^G = 0 \\
\lambda^G \geq 0 \perp \delta^G \geq 0 \\
\mu^G \geq 0 \perp \delta^G \leq \overline{\delta}^G.
\end{gathered} \tag{3}$$

[6]Note that $\pi_G'' = (1/M - 2)c_1(1+\phi^2)$. Because $M \geq 1$ and $c_1 > 0$, we have $\pi_G'' < 0$, meaning the maximization problem is strictly convex and the FONC is sufficient for the unique maximum.

5.1.2. Merchants' Optimization Problem

Merchants will maximize arbitrage value. We can write merchant j's optimization problem as:

$$\max_{\delta_j^S \in [0, \overline{\delta}^S/Z]} \pi_S(\delta_j^S, \delta_{-j}^S) = \delta_j^S(c_0 + c_1(l_2 - \delta_j^S - \delta_{-j}^S)) - \phi\delta_j^S(c_0 + c_1(l_1 + \phi\delta_j^S + \phi\delta_{-j}^S)).$$

If we let $\lambda_j^S \geq 0$ and $\mu_j^S \geq 0$ be the Lagrange multipliers associated with merchant j's lower- and upper-bound constraints, respectively, the FONC[7] for an optimum is given by:

$$-c_1(1+\phi^2)\delta_j^S + c_0 + c_1(l_2 - \delta^T) - \phi(c_0 + c_1(l_1 + \phi\delta^T)) + \lambda_j^S - \mu_j^S = 0.$$

We can again use the approach in the proof of Proposition 2 to show that in an equilibrium merchants will all use storage identically, in which case the FONC can be rewritten as:

$$\begin{array}{c} c_0(1-\phi) - (\delta^G + \delta^C)c_1(1+\phi^2) + c_1(l_2 - \phi l_1) - \frac{Z+1}{Z}c_1\delta^S(1+\phi^2) + \lambda^S - \mu^S = 0 \\ \lambda^S \geq 0 \perp \delta^S \geq 0 \\ \mu^S \geq 0 \perp \delta^S \leq \overline{\delta}^S. \end{array} \tag{4}$$

5.1.3. Consumers' Optimization Problem

Consumers will use storage to maximize the sum of arbitrage value and the change in consumer surplus. We can write consumer k's optimization problem as:

$$\begin{aligned} \max_{\delta_k^C \in [0, \overline{\delta}^C/N]} \pi_C(\delta_k^C, \delta_{-k}^C) &= \delta_k^C(c_0 + c_1(l_2 - \delta_k^C - \delta_{-k}^C)) - \phi\delta_k^C(c_0 + c_1(l_1 + \phi\delta_k^C + \phi\delta_{-k}^C)) \\ &\quad + \frac{1}{N}\Delta CS(\delta_k^C + \delta_{-k}^C) \\ &= \delta_k^C(c_0 + c_1(l_2 - \delta_k^C - \delta_{-k}^C)) - \phi\delta_k^C(c_0 + c_1(l_1 + \phi\delta_k^C + \phi\delta_{-k}^C)) \\ &\quad + \frac{1}{N}(\delta_k^C + \delta_{-k}^C)c_1(l_2 - \phi l_1). \end{aligned}$$

If we let $\lambda_k^C \geq 0$ and $\mu_k^C \geq 0$ be the Lagrange multipliers associated with consumer k's lower- and upper-bound constraints, respectively, the FONC[8] for an optimum is given by:

$$-c_1(1+\phi^2)\delta_k^C + c_0 + c_1(l_2 - \delta^T) - \phi(c_0 + c_1(l_1 + \phi\delta^T)) + \frac{1}{N}c_1(l_2 - \phi l_1) + \lambda_k^C - \mu_k^C = 0.$$

We can again use the approach in the proof of Proposition 2 to show that in an equilibrium consumers will all use storage identically, in which case the FONC can be rewritten as:

$$\begin{array}{c} c_0(1-\phi) - (\delta^G + \delta^S)c_1(1+\phi^2) + \frac{N+1}{N}c_1(l_2 - \phi l_1) - \frac{N+1}{N}c_1\delta^C(1+\phi^2) + \lambda^C - \mu^C = 0 \\ \lambda^C \geq 0 \perp \delta^C \geq 0 \\ \mu^C \geq 0 \perp \delta^C \leq \overline{\delta}^C. \end{array} \tag{5}$$

5.2. Numerical Simulation with Multiple Agent Types

A Nash equilibrium will consist of a set of storage use variables, δ^G, δ^M, and δ^C, which simultaneously solve the equilibrium conditions given in equations 3 through 5. These conditions define a complementarity problem, which we solved using the PATH solver developed by Dirkse and Ferris (1995). Figure 6 shows welfare losses with 1000 MW of storage with an efficiency of 0.89 being divided between generators, merchants,

[7] Again, $\pi_S'' = -2c_1(1+\phi^2)$. Because $c_1 > 0$, this gives $\pi_S'' < 0$, meaning the profit-maximization problem is strictly convex and the FONC is sufficient for the unique maximum.

[8] $\pi_C'' = -2c_1(1+\phi^2)$. Because $c_1 > 0$, $\pi_C'' < 0$, meaning the maximization problem is strictly convex and the FONC is sufficient for the unique maximum.

and consumers. The figure assumes the same market structure as before with two symmetric generators, merchants, and consumers in the market. The total capacity of the generators' storage can be deduced from the figure, since the 1000 MW are distributed amongst the three agent types. Contrasting figure 6 with the results of section 4 shows that when multiple agent types can own storage simultaneously, consumer ownership of storage can be beneficial since the overuse of storage by consumers can compensate for the underuse of storage by merchants. In fact, in this example welfare losses are minimized when 865 MW of storage are owned by the two merchants and the remaining 135 MW are owned by the two consumers.

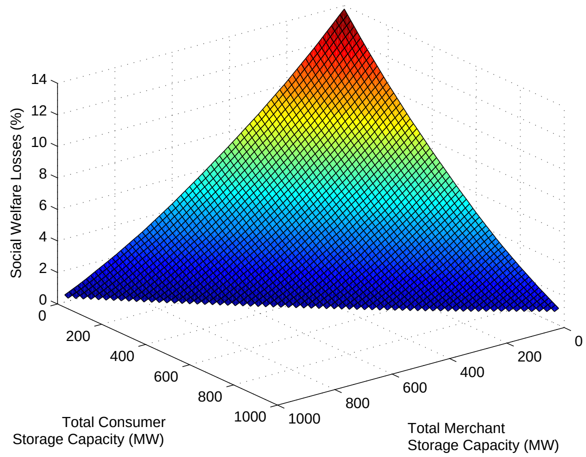

Figure 6: Social welfare losses from suboptimal use of 1000 MW of storage with efficiency of 0.89 divided between symmetric duopoly merchant storage operators, generators, and consumers. Welfare losses are given as a percentage of social welfare gains from optimal use of storage.

Figure 7 summarizes the welfare-maximizing distribution of 1000 MW storage between merchants, generators, and consumers for a range of different device efficiencies. It once again assumes the market has two symmetric merchants, generators, and consumers. The figure shows that for most device efficiencies, merchant ownership is welfare maximizing, however for higher device efficiencies some consumer ownership is also beneficial. Figure 8 considers a more competitive market that consists of five symmetric generators, five symmetric consumers, and two symmetric storage merchants. In this more competitive setting we see that consumer ownership of storage is even more beneficial—with more storage being allocated to consumers and with lower device efficiencies—but that generator ownership of storage is still suboptimal, since generators still significantly underuse storage even with five generators in the market.

6. Conclusions

We have examined incentives for the use of energy storage by different agents in an electricity market. We found that consumers will generally overuse storage compared to the social optimum, whereas merchants and generators will tend to underuse their assets. As the market becomes increasingly competitive with more agents, storage use will approach the social optimum, however with a finite number of agents welfare losses can be substantial. Our numerical example showed that for most reasonable storage device efficiencies merchant ownership of storage is welfare-maximizing compared to the alternatives of consumer or generator ownership. This result is due to generators underusing storage significantly because of the resulting producer surplus losses and consumers overusing storage since they neglect the producer surplus losses associated with their storage use. When storage assets can be divided amongst agent types the socially optimal allocation of storage favors merchants, although some consumer ownership of storage can be beneficial since their overuse of storage can compensate for underuse by merchants. Importantly, our results suggest that regulatory and government authorities should be wary of encouraging consumer or generator investment in storage, since merchant storage tends to minimize welfare losses.

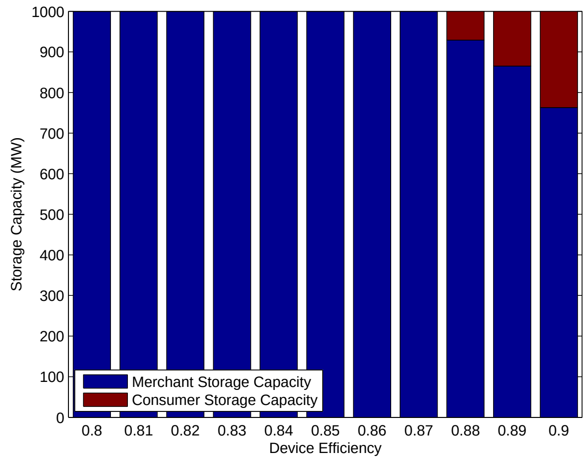

Figure 7: Welfare-maximizing distribution of 1000 MW of storage with different device efficiencies between symmetric duopoly merchant storage operators, generators, and consumers.

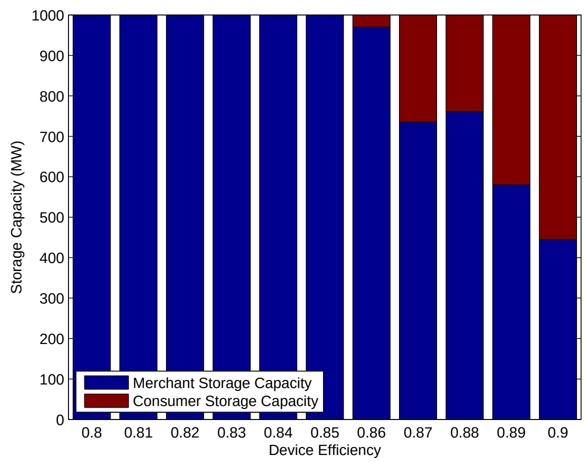

Figure 8: Welfare-maximizing distribution of 1000 MW of storage with different device efficiencies between symmetric duopoly merchant storage operators, and five symmetric generators and consumers.

The inclusion of other agent types can change our results regarding optimal storage ownership structure, since these agents would ostensibly have different incentives to use storage than the three cases considered here. Storage owned by a municipality, cooperative, or integrated utility, which owns generation assets and serves native loads, may result in more socially optimal storage use since these entities would be concerned with both producer and consumer surplus changes. As an extreme case, a perfectly-regulated integrated utility may be expected to duplicate the optimum, since such an entity will be charged with maximizing welfare. Similarly, the design of market mechanisms or contracts that compensate storage owners for the external effects of their storage use could also improve storage use incentives relative to the cases analyzed here.

It is worth noting that our analysis is illustrative since prices and loads may not have a linear relationship. This is due to non-convex generation costs, such as startups, and the use of different generating technologies. These cost structures can result in non-linearities and steps in the price-load relationship. While our model is a simplification of actual power system operations, our results are nonetheless illustrative of the incentives that different agent types will generally have in using storage and the resulting welfare implications.

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A. Proof of Propositions

We present the proofs of the propositions in the paper.

Proposition 1. $\Delta CS(\delta^W) \geq 0$ and $\Delta PS(\delta^W) \leq 0$.

Proof. It should be clear that when $\delta^W = 0$ the change in consumer and producer surplus are zero, thus we only analyze the case in which $\delta^W > 0$.

Note that from equation 2 if $\delta^W > 0$ then we must have $c_0(1-\phi) + c_1(l_2 - \phi l_1) \geq 0$, which can be rewritten $c_1(l_2 - \phi l_1) \geq -c_0(1-\phi)$. The change in consumer surplus is given by $\Delta CS(\delta) = \delta c_1(l_2 - \phi l_1)$. Combining these two gives:

$$\Delta CS(\delta) \geq -\delta c_0(1-\phi),$$

which is non-negative, since $c_0(1-\phi) < 0$.

To show that the producer surplus is always non-positive, first note that the change in producer surplus can be written as:

$$\Delta PS(\delta) = -\delta \left[c_1(l_2 - \phi l_1) - \frac{1}{2}\delta c_1(1+\phi^2) \right].$$

If the bound constraints on δ^W are non-binding, then substituting in for the welfare-maximizing use of storage, δ^W, gives:

$$\Delta PS(\delta) = -\frac{1}{2}\delta [c_1(l_2 - \phi l_1) - c_0(1-\phi)].$$

From above, we know that $c_0(1-\phi) < 0$ and that we must have $c_1(l_2 - \phi l_1) \geq -c_0(1-\phi) > 0$ for the non-negativity constraint to be non-binding. Thus, we must have $\Delta PS(\delta) \leq 0$.

Similarly, if $\delta^W = \overline{\delta}$ then we must have $c_0(1-\phi) + c_1(l_2 - \phi l_1) > \overline{\delta} c_1(1+\phi^2)$, which gives:

$$\Delta PS(\overline{\delta}) = -\overline{\delta} \left[c_1(l_2 - \phi l_1) - \frac{1}{2}\overline{\delta} c_1(1+\phi^2) \right] < -\frac{1}{2}\overline{\delta}[c_1(l_2 - \phi l_1) - c_0(1-\phi)],$$

which was shown to be non-positive above. □

Proposition 2. If the storage assets are owned by $N \geq 1$ symmetric merchant storage operators, which are not consumers and do not own generating assets, storage will be underused compared to the social optimum.

Proof. Because the firms are merchant storage owners, they are concerned solely with maximizing the arbitrage value of their storage devices. Moreover, because the firms are symmetric we can express firm i's use of its storage device, δ_i, in terms of the total use of its rival firms, δ_{-i}:

$$\max_{\delta_i \in [0,\overline{\delta}/N]} \quad \pi_S(\delta_i, \delta_{-i}) = \delta_i(c_0 + c_1(l_2 - \delta_i - \delta_{-i})) - \phi\delta_i(c_0 + c_1(l_1 + \phi\delta_i + \phi\delta_{-i})).$$

If we let $\lambda_i \geq 0$ and $\mu_i \geq 0$ be the Lagrange multipliers associated with firm i's lower- and upper-bound constraints, respectively, the FONC[9] for an optimum is given by:

$$-c_1(1+\phi^2)\delta_i + c_0 + c_1(l_2 - \delta^S) - \phi(c_0 + c_1(l_1 + \phi\delta^S)) + \lambda_i - \mu_i = 0, \tag{6}$$

where $\delta^S = \sum_{n=1}^{N} \delta_n$, is the total usage of storage devices by all the firms.

We next show that a Nash equilibrium must be symmetric by subtracting firm j's equation 6 from that of firm i, which gives:

$$(1+\phi^2)c_1(\delta_j - \delta_i) + \lambda_i - \lambda_j - \mu_i + \mu_j = 0. \tag{7}$$

We can now examine the following three cases to show that $\delta_i = \delta_j$ in an equilibrium:

- Case 1: If $\delta_i = 0$ then $\lambda_i \geq 0$ and $\mu_i = 0$. In this case equation 7 becomes:

  $$(1+\phi^2)c_1\delta_j + \lambda_i - \lambda_j + \mu_j = 0.$$

  Clearly we cannot have $\delta_j > 0$ because then this equation becomes:

  $$(1+\phi^2)c_1\delta_j + \lambda_i + \mu_j = 0,$$

  which cannot hold.

- Case 2: If $0 < \delta_i < \overline{\delta}/N$ then $\lambda_i = 0$ and $\mu_i = 0$. In this case equation 7 becomes:

  $$(1+\phi^2)c_1(\delta_j - \delta_i) - \lambda_j + \mu_j = 0.$$

  If $\delta_j = 0$ then this equation becomes:

  $$-(1+\phi^2)c_1\delta_i - \lambda_j = 0,$$

  which cannot hold, and similarly if $\delta_j = \overline{\delta}/N$ this equation becomes:

  $$(1+\phi^2)c_1(\overline{\delta}/N - \delta_i) + \mu_j = 0,$$

  which cannot hold either. Thus we must have $\lambda_j = \mu_j = 0$ in which case equation 7 becomes:

  $$(1+\phi^2)c_1(\delta_j - \delta_i) = 0,$$

  which requires $\delta_j = \delta_i$.

- Case 3: If $\delta_i = \overline{\delta}/N$ then $\lambda_i = 0$ and $\mu_i \geq 0$. In this case equation 7 becomes:

  $$(1+\phi^2)c_1(\delta_j - \overline{\delta}/N) - \lambda_j - \mu_i + \mu_j = 0.$$

  If $\delta_j < \overline{\delta}/N$ then this equation becomes:

  $$(1+\phi^2)c_1(\delta_j - \overline{\delta}/N) - \lambda_j - \mu_i = 0,$$

  which cannot hold.

[9] Again, because $c_1 > 0$, $\pi_S'' = -2c_1(1+\phi^2) < 0$, meaning the profit-maximization problem is strictly convex and the FONC is sufficient for the unique maximum.

Since we know the equilibrium is symmetric, we can rewrite the FONC as:

$$-c_1(1+\phi^2)\delta^S/N + c_0 + c_1(l_2 - \delta^S) - \phi(c_0 + c_1(l_1 + \phi\delta^S)) + \lambda - \mu = 0,$$

which yields the following equilibrium:

$$\delta^S = \begin{cases} 0, & \text{if } c_0(1-\phi) + c_1(l_2 - \phi l_1) < 0; \\ \overline{\delta}, & \text{if } c_0(1-\phi) + c_1(l_2 - \phi l_1) - \frac{N+1}{N}\overline{\delta}c_1(1+\phi^2) > 0; \\ \frac{N}{N+1}\frac{c_0(1-\phi)+c_1(l_2-\phi l_1)}{c_1(1+\phi^2)}, & \text{otherwise.} \end{cases} \tag{8}$$

Comparing the terms in equations 2 and 8 we see that $\delta^S = 0$ whenever $\delta^W = 0$ and that $\delta^S = \frac{N}{N+1}\delta^W$ whenever the bound constraints are not binding. Moreover, because:

$$-\overline{\delta}c_1(1+\phi^2) > -\frac{N+1}{N}\overline{\delta}c_1(1+\phi^2),$$

there are values of l_1 and l_2 for which $\delta^S < \overline{\delta}$ while $\delta^W = \overline{\delta}$. Thus δ^S is always less than or equal to δ^W. □

Proposition 3. If the storage assets are owned by $N \geq 1$ symmetric generating firms that own all the generating assets, storage will only be used if there are at least two firms, in which case storage will be underused compared to the social optimum.

Proof. Because the storage owners are also generators, they will use the storage device to maximize the sum of arbitrage value and the change in their producer surplus. Moreover, because the firms are symmetric we can express generator i's use of its storage device, δ_i, in terms of the total use of its rival generators, δ_{-i}:

$$\begin{aligned} \max_{\delta_i \in [0,\overline{\delta}/N]} \pi_G(\delta_i, \delta_{-i}) &= \delta_i(c_0 + c_1(l_2 - \delta_i - \delta_{-i})) - \phi\delta_i(c_0 + c_1(l_1 + \phi\delta_i + \phi\delta_{-i})) + \frac{1}{N}\Delta PS(\delta_i + \delta_{-i}) \\ &= \delta_i(c_0 + c_1(l_2 - \delta_i - \delta_{-i})) - \phi\delta_i(c_0 + c_1(l_1 + \phi\delta_i + \phi\delta_{-i})) \\ &\quad + \frac{1}{N}\left((\delta_i + \delta_{-i})c_1(-l_2 + \phi l_1) + \frac{1}{2}(\delta_i + \delta_{-i})^2 c_1(1+\phi^2)\right). \end{aligned}$$

If we let $\lambda_i \geq 0$ and $\mu_i \geq 0$ be the Lagrange multipliers associated with generator i's lower- and upper-bound constraints, respectively, the FONC[10] for an optimum is given by:

$$-c_1(1+\phi^2)\delta_i + c_0 + c_1(l_2 - \delta^G) - \phi(c_0 + c_1(l_1 + \phi\delta^G)) + \frac{1}{N}[c_1(-l_2 + \phi l_1) + \delta^G c_1(1+\phi^2)] + \lambda_i - \mu_i = 0,$$

where $\delta^G = \sum_{n=1}^N \delta_n$, is the total usage of storage devices by all of the generating firms. It is straightforward to show that the equilibrium is symmetric using the same approach in Proposition 2, which we do not repeat for brevity. Since the equilibrium is symmetric, we can rewrite the FONC as:

$$-c_1(1+\phi^2)\delta^G/N + c_0 + c_1(l_2 - \delta^G) - \phi(c_0 + c_1(l_1 + \phi\delta^G)) + \frac{1}{N}[c_1(-l_2 + \phi l_1) + \delta^G c_1(1+\phi^2)] + \lambda - \mu = 0,$$

which yields the following equilibrium:

$$\delta^G = \begin{cases} 0, & \text{if } c_0(1-\phi) + \frac{N-1}{N}c_1(l_2 - \phi l_1) < 0; \\ \overline{\delta}, & \text{if } c_0(1-\phi) + \frac{N-1}{N}c_1(l_2 - \phi l_1) - \overline{\delta}c_1(1+\phi^2) > 0; \\ \frac{c_0(1-\phi)+\frac{N-1}{N}c_1(l_2-\phi l_1)}{c_1(1+\phi^2)}, & \text{otherwise.} \end{cases} \tag{9}$$

Note that if $N = 1$, then the first condition in equation 9 becomes $c_0(1-\phi) < 0$, which is necessarily true since $c_0 > 0$ and $\phi > 1$. Thus, with $N = 1$ the storage device is never used. If $N \geq 2$ then we can compare the following three cases:

[10]Because $N \geq 1$ and $c_1 > 0$, $\pi_G'' = (1/N - 2)c_1(1+\phi^2) < 0$, meaning the maximization problem is strictly convex and the FONC is sufficient for the unique maximum.

- Case 1: If $\delta^G = 0$ then we must have $\frac{N-1}{N}c_1(l_2 - \phi l_1) < -c_0(1-\phi)$. If $\frac{N-1}{N}c_1(l_2 - \phi l_1) \leq 0$ then $c_1(l_2 - \phi l_1) \leq 0$ and $\delta^W = 0$. If $\frac{N-1}{N}c_1(l_2 - \phi l_1) > 0$, however, $c_1(l_2 - \phi l_1)$ might be greater than $-c_0(1-\phi)$ in which case we would have $\delta^W > 0$ while $\delta^G = 0$.

- Case 2: If $\delta^W = \overline{\delta}$ then we must have:

$$c_1(l_2 - \phi l_1) > \overline{\delta} c_1(1+\phi^2) - c_0(1-\phi).$$

For us to have $\delta^G = \overline{\delta}$ we must have:

$$\frac{N-1}{N}c_1(l_2 - \phi l_1) > \overline{\delta} c_1(1+\phi^2) - c_0(1-\phi).$$

Note that because $\overline{\delta} c_1(1+\phi^2) - c_0(1-\phi) > 0$ there are values of l_1 and l_2 for which the first condition holds, but the second does not. In such a case, we would have $\delta^G < \overline{\delta}$ while $\delta^W = \overline{\delta}$.

- Remaining Case: If the bound constraints on δ^W and δ^G are non-binding, then:

$$\delta^W - \delta^G = \frac{l_2 - \phi l_1}{N(1+\phi^2)}.$$

We know from above that for the non-negativity constraint to be non-binding we must have $l_2 - \phi l_1 > 0$, thus the difference $\delta^W - \delta^G$ is positive.

Thus, we have shown that when $N \geq 2$, δ^G is always less than or equal to δ^W. □

Proposition 4. If the storage assets are owned by $N \geq 1$ symmetric consumers, storage will be overused compared to the social optimum.

Proof. Consumers will be concerned with maximizing the sum of arbitrage value and the change in their consumer surplus. Because consumers are symmetric we can express consumer i's use of its storage device, δ_i, in terms of the total use by other consumers, δ_{-i}:

$$\begin{aligned}
\max_{\delta_i \in [0, \overline{\delta}/N]} \pi_C(\delta_i, \delta_{-i}) &= \delta_i(c_0 + c_1(l_2 - \delta_i - \delta_{-i})) - \phi\delta_i(c_0 + c_1(l_1 + \phi\delta_i + \phi\delta_{-i})) + \frac{1}{N}\Delta CS(\delta_i + \delta_{-i}) \\
&= \delta_i(c_0 + c_1(l_2 - \delta_i - \delta_{-i})) - \phi\delta_i(c_0 + c_1(l_1 + \phi\delta_i + \phi\delta_{-i})) \\
&\quad + \frac{1}{N}(\delta_i + \delta_{-i})c_1(l_2 - \phi l_1).
\end{aligned}$$

If we let $\lambda_i \geq 0$ and $\mu_i \geq 0$ be the Lagrange multipliers associated with consumer i's lower- and upper-bound constraints, respectively, the FONC[11] for an optimum is given by:

$$-c_1(1+\phi^2)\delta_i + c_0 + c_1(l_2 - \delta^C) - \phi(c_0 + c_1(l_1 + \phi\delta^C)) + \frac{1}{N}c_1(l_2 - \phi l_1) + \lambda_i - \mu_i = 0,$$

where $\delta^C = \sum_{n=1}^N \delta_n$, is the total usage of storage devices by all consumers. It is straightforward to show that the equilibrium is symmetric using the same approach in Proposition 2, which we do not repeat. Since the equilibrium is symmetric, we can rewrite the FONC as:

$$-c_1(1+\phi^2)\delta^C/N + c_0 + c_1(l_2 - \delta^C) - \phi(c_0 + c_1(l_1 + \phi\delta^C)) + \frac{1}{N}c_1(l_2 - \phi l_1) + \lambda - \mu = 0,$$

which yields the following equilibrium:

$$\delta^C = \begin{cases} 0, & \text{if } c_0(1-\phi) + \frac{N+1}{N}c_1(l_2 - \phi l_1) < 0; \\ \overline{\delta}, & \text{if } c_0(1-\phi) + \frac{N+1}{N}c_1(l_2 - \phi l_1) - \frac{N+1}{N}\overline{\delta} c_1(1+\phi^2) > 0; \\ \frac{\frac{N}{N+1}c_0(1-\phi) + c_1(l_2 - \phi l_1)}{c_1(1+\phi^2)}, & \text{otherwise.} \end{cases} \tag{10}$$

We now examine the three possible cases for δ^W and δ^C separately.

[11] Because $c_1 > 0$, $\pi''_C = -2c_1(1+\phi^2) < 0$, meaning the maximization problem is strictly convex and the FONC is sufficient for the unique maximum.

- Case 1: If $\delta^W = 0$ then we must have $c_1(l_2 - \phi l_1) < -c_0(1-\phi)$. If $c_1(l_2 - \phi l_1) \leq 0$ then $\frac{N+1}{N} c_1(l_2 - \phi l_1) \leq 0$ and $\delta^C = 0$. If $c_1(l_2 - \phi l_1) > 0$ then $\frac{N+1}{N} c_1(l_2 - \phi l_1)$ might greater than $-c_0(1-\phi)$ in which case $\delta^C > 0$ while $\delta^W = 0$.

- Case 2: If $\delta^C = \overline{\delta}$ then we must have:

$$\frac{N+1}{N} c_1(l_2 - \phi l_1) - \frac{N+1}{N} \overline{\delta} c_1(1+\phi^2) > -c_0(1-\phi).$$

  For us to have $\delta^W = \overline{\delta}$ we must have:

$$c_1(l_2 - \phi l_1) - \overline{\delta} c_1(1+\phi^2) > -c_0(1-\phi).$$

  Note that because $-c_0(1-\phi) > 0$ there are values of l_1 and l_2 for which the first condition holds but the second does not, in which case we would have $\delta^W < \overline{\delta}$ while $\delta^C = \overline{\delta}$.

- Remaining case: If the bound constraints on δ^W and δ^C are not binding, then:

$$\delta^W - \delta^C = \frac{c_0(1-\phi)}{(N+1)c_1(1+\phi^2)} < 0.$$

Thus, the usage of the storage device by consumers will always be weakly greater than the social optimum.

□

Corollary 1. A perfectly competitive market with all the storage assets owned by either symmetric merchant storage operators, consumers, or generators will yield the social welfare-maximizing usage of storage.

Proof. A perfectly competitive storage market will be a storage market in which there are an infinite number of infinitesimally small storage operators (*i.e.* in which $N \to +\infty$). From equations 8, 9, and 10 we see that δ^S, δ^G and $\delta^C \to \delta^W$ as $N \to +\infty$. □