What Duality Theory Tells Us About Giving Market Operators the Authority to Dispatch Energy Storage

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ABSTRACT

There is a debate about which entity should have the authority to dispatch energy storage that participates in restructured wholesale electricity markets. Some stakeholders raise concerns that market operators' independence can be threatened if they make operational decisions for energy storage. The rationale that underlies this concern is that operating energy storage can affect the balance of the system and price formation. We demonstrate that having market operators make operational decisions for energy storage does not change the fundamental nature of the optimal-power-flow problem. Using duality theory, we show that if market operators co-optimize the operation of energy storage with that of generators and transmission, the optimal-power-flow problem yields short-run dispatch support and incentive compatibility and long-run efficiency. These findings are analogous to those for having market operators co-optimize transmission use with generator dispatch. Our work suggests that concerns around giving market operators the authority to dispatch energy storage are misplaced.

Keywords: Energy storage, energy policy, electricity-market design, energy pricing, investment, optimal power flow, duality theory

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1. INTRODUCTION

An issue that is raised by integrating energy storage into electricity systems with competitive wholesale markets is the role of market operators (MOs) in determining energy-storage dispatch. Much of the extant literature, which Castillo and Gayme (2014); Sioshansi et al. (2022) survey, focuses on optimizing energy-storage operations from the perspective of the asset owner. However, there are benefits to having the operations of energy storage and other assets co-optimized. Pozo et al. (2014) assess the value of incorporating energy storage into a unit-commitment model, whereby a single entity co-optimizes energy-storage and generator operations. Weibelzahl and Märtz (2018) examine the impacts on zonal pricing of incorporating energy-storage-operations decisions into MOs' market-clearing models. Despite these benefits of co-optimizing the operation of energy storage with other power-system assets, there is a concern that MOs' independence can be threatened if they make energy-storage-operations decisions. Sioshansi et al. (2012); Sioshansi (2017) note that a primary rationale behind this concern is that the operation of energy storage can affect the balance of the power system and wholesale-price formation.

An example of this concern involves Lake Elsinore Advanced Pumping Station (LEAPS). LEAPS's developer, Nevada Hydro Corporation, proposed building the plant to relieve congestion in Southern California. Because of its transmission benefits, Nevada Hydro Corporation sought in its

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filing to Federal Energy Regulatory Commission (FERC) an arrangement whereby California ISO (CAISO) dispatches LEAPS to maximize its transmission-relief benefits.[1] In its decision, FERC concludes that having CAISO dispatch LEAPS is akin to CAISO owning and operating generation, which could threaten the independence that is required of MOs by impacting wholesale-price formation. Indeed, market independence is an explicit objection that CAISO raises in its filings in the LEAPS case.[2] This conclusion stems, in part, from CAISO making financially binding unit-commitment and dispatch decisions for generating units. Sioshansi et al. (2010) contrast the roles of MOs in making these decisions between different markets.

This debate is reminiscent of questions raised during the 1990s around the proper role of MOs optimize use of transmission networks. Hogan (1992); Ruff (1994); Oren (1997) provide formative analyses of this aspect of electricity-market design. There are several benefits to having MOs optimize transmission-network use. First, Hogan (1992) demonstrates that having MOs determine transmission use to maximize social welfare is equivalent to minimizing economic rents on transmission networks. This equivalence means that market solutions yield short-run efficiency, dispatch support, and incentive compatibility in power-system operations. Second, Pérez-Arriaga et al. (1995) analyze the congestion rents that are generated by market models that give control of transmission-network use. They show that when considering a fixed time span, these rents are equal to the cost of transmission investment if the dynamic capacity-expansion plan is optimal over the time span and there are neither economies of scale nor lumpiness in transmission investment. This finding means that MOs determining the use of transmission networks is consistent with social-welfare maximization and long-run efficiency in transmission investment.

In this paper, we examine the incentive and efficiency implications of giving MOs operational control of energy storage. We study this question by adapting and extending the approaches that are taken by Hogan (1992); Pérez-Arriaga et al. (1995) to analyze MOs optimizing transmission-network use. We take a two-prong approach to our analysis, which yields two policy-relevant market-design findings.

First, we examine optimal-power-flow (OPF) models with and without energy-storage-operational decisions embedded within them. Comparing the dual problems of these two OPF models shows that incorporating energy storage into market-clearing models does not change fundamentally the price-formation process. So long as the market model maximizes social welfare or minimizes system cost, energy storage factors into market clearing and price formation analogously to an energy producer when it discharges and analogously to an energy consumer when it charges. Analysis of the dual problem of the OPF model with embedded energy storage shows that the market price is dispatch-supporting and incentive-compatible in the sense that energy storage is incentivized to comply with the market solution. This result stems from the convexity of the OPF model and means that MOs having operational control of energy storage provides the same short-run-efficiency properties that Hogan (1992) demonstrates for MOs determining the use of transmission networks.

Second, we examine a stylized energy-storage-investment model and compare the cost of energy-storage investment to energy-storage rents that are engendered by the solution of an OPF model that has energy-storage-operational decisions embedded within it. We show that if a power system has a socially optimal amount of energy-storage capacity, marginal energy-storage rents equal marginal energy-storage-investment costs. This result means that MOs having operational control of energy storage yields the same long-run investment-efficiency properties that Pérez-Arriaga et al. (1995) demonstrate for MOs determining the use of transmission.

Taken together, our work shows that giving MOs operational control of energy storage pro-

[1] *cf.* FERC docket numbers ER06-278-000 through ER06-278-006 for all of the filings and decisions in this case.

[2] *cf.* page 7 of FERC's Order on Rate Incentives and Compliance Filings in this case in which FERC directs CAISO to address 'whether CAISO can effectively operate [LEAPS] in the context of being an independent system operator.' Pages 24 and 25 provide CAISO's response, in which 'CAISO submits that, based on stakeholder input and its own evaluation of the issues . . . CAISO should not assume operational control of [LEAPS and] that any transfer of control analyzed in [the] proceeding would compromise CAISO's independence as envisioned in [FERC] Order No. 2000.'

vides the same short-run properties (*e.g.*, efficiency, dispatch support, and incentive compatibility) and long-run efficiency that MOs determining the use of transmission provides. On the basis of our findings, we argue that the price-formation and market-independence concerns that are raised in the case of LEAPS and similar proposed energy-storage projects are unwarranted. Indeed, we find that giving MOs operational control of energy storage raises no new market-design issues as compared to MOs determining the use of transmission or making operational decisions for generating units.

The remainder of this paper is organized as follows. Section 2 provides the formulation of the stylized OPF model that we analyze and its dual problem. Section 3 provides our theoretical results. Sections 4 and 5 demonstrate the properties of the stylized OPF model using a simple example and real-world case study, respectively. Section 6 concludes and provides a discussion of the market-design implications of our work.

2. MODEL FORMULATION

This section presents the formulation of a multi-period OPF model, which is assumed to have hourly time-steps, and its dual problem. The model is multi-period because energy storage couples decisions between hours. This model and its dual underlie our analysis of MOs having operational control of energy storage. The model is an idealized example of a perfect market, which is known to be efficient. Mas-Colell et al. (1995) provide a detailed treatment of these efficiency results, which we paraphrase. According to the first fundamental theorem of welfare economics, if preferences are locally non-satiated, then a competitive equilibrium is Pareto optimal. Furthermore, the second fundamental theorem of welfare economics states that if each consumer has convex preferences and each firm has a convex production set, then there is a price vector that gives a competitive equilibrium to support any Pareto-optimal allocation. These theorems have two technical requirements, which underlie our model. First, there must not be any information asymmetry. Second, economic agents must be price-taking. In the context of our work, the OPF model provides a competitive equilibrium in which supply equals demand. The dual problem allows us to demonstrate the dispatch-support and incentive-compatibility properties of the prices that are given by a competitive equilibrium.

We begin our model formulation by defining the following notation.

2.1 Indices, Sets, and Parameters

B	set of transmission buses
B_g	transmission bus at which generator g is located
B_i	transmission bus at which energy storage i is located
b_n	willingness-to-pay for energy of transmission-bus-n customers (\$/MWh)
c_g	operating cost of generator g (\$/MWh)
$D_{n,t}^{\max}$	maximum hour-t demand at transmission bus n (MW)
$F_l^{\max}$	capacity of transmission line l (MW)
g	generator index
G	set of generators
G_n	set of generators that are connected to transmission bus n
H_i	energy-carrying capacity of energy storage i (h)
i	energy storage index
I	set of energy-storage devices
$K_g^{\max}$	production capacity of generator g (MW)
l	transmission-line index
L	set of transmission lines
m, n	transmission-bus indices
$P_i^{\max}$	power capacity of energy storage i (MW)

S_n	set of energy-storage devices that are connected to transmission bus n
t	time index
T	set of hours within model horizon
η_i	round-trip efficiency of energy storage i (p.u.)
$\pi_{n,l}$	transmission-bus-n/transmission-line-l shift factor (p.u.)

2.2 Decision Variables

$e_{n,t}$	hour-t net injection of power from transmission bus n into the transmission network (MW)
$h_{i,t}$	hour-t discharging rate of energy storage i (MW)
$L_{n,t}$	hour-t load at transmission bus n that is served (MW)
$r_{i,t}$	hour-t charging rate of energy storage i (MW)
$s_{i,t}$	ending hour-t state of energy (SOE) of energy storage i (MWh)
$x_{g,t}$	hour-t power output of generator g (MW)

2.3 OPF Model

The OPF model is formulated as:

$$\max \quad \sum_{t \in T} \left(\sum_{n \in B} b_n L_{n,t} - \sum_{g \in G} c_g x_{g,t} \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{g \in G_n} x_{g,t} - e_{n,t} + \sum_{i \in S_n} (h_{i,t} - r_{i,t}) = L_{n,t}; \forall n \in B, t \in T \qquad (\lambda_{n,t}) \tag{2}$$

$$\sum_{n \in B} e_{n,t} = 0; \forall t \in T \qquad (\mu_t) \tag{3}$$

$$0 \leq x_{g,t} \leq K_g^{\max}; \forall g \in G, t \in T \qquad (\alpha_{g,t}^-, \alpha_{g,t}^+) \tag{4}$$

$$-F_l^{\max} \leq \sum_{n \in B} \pi_{n,l} e_{n,t} \leq F_l^{\max}; \forall l \in L, t \in T \qquad (\beta_{l,t}^-, \beta_{l,t}^+) \tag{5}$$

$$0 \leq L_{n,t} \leq D_{n,t}^{\max}; \forall n \in B, t \in T \qquad (\gamma_{n,t}^-, \gamma_{n,t}^+) \tag{6}$$

$$s_{i,t} = s_{i,t-1} + \eta_i r_{i,t} - h_{i,t}; \forall i \in S, t \in T \qquad (\omega_{i,t}) \tag{7}$$

$$0 \leq h_{i,t} \leq P_i^{\max}; \forall i \in S, t \in T \qquad (\tau_{i,t}^-, \tau_{i,t}^+) \tag{8}$$

$$0 \leq r_{i,t} \leq P_i^{\max}; \forall i \in S, t \in T \qquad (\phi_{i,t}^-, \phi_{i,t}^+) \tag{9}$$

$$0 \leq s_{i,t} \leq H_i P_i^{\max}; \forall i \in S, t \in T \qquad (\nu_{i,t}^-, \nu_{i,t}^+); \tag{10}$$

where the dual variable that is associated with each constraint is given in parentheses to its right.

Objective function (1) maximizes social welfare, which is defined as the difference between customers' willingness to pay for energy that they consume and the cost of energy production. Constraints (2) and (3) enforce bus-level and system-wide load balance, respectively. Constraints (4) enforce generator-capacity limits. Constraints (5) impose flow limits on the transmission lines. Constraints (6) limit the amount of load that is served at each bus based on maximum consumer demand.

The remaining constraints pertain to the operation of energy storage. Constraints (7) define the ending hourly SOE of energy storage. These constraints couple operational decisions between hours, which necessitates the use of a multi-period OPF model. Without loss of generality, we assume that the beginning hour-0 SOE of each energy storage is zero. Non-zero starting SOEs would not change our fundamental results. Rather, they would impact boundary conditions in the dual of (1)–(10). Constraints (8) and (9) impose non-negativity and power limits on energy-storage discharging and charging, respectively. Constraints (10) impose SOE bounds on energy storage. We do not impose a constraint on the ending SOE of energy storage. Graves et al. (1999) use such constraints as a heuristic to ascribe value to having stored energy as of the end of the optimization horizon. Including such constraints would not change our fundamental results. Rather, a constraint on ending energy-storage SOEs would impact boundary conditions in the dual of (1)–(10). Unless stored energy has a negative value (*e.g.*, due to over-generation or unit-commitment constraints), the lack of constraints on the ending SOE of energy storage yields solutions typically wherein energy-storage SOE is nil as of the end of the optimization horizon.

The dual of (1)–(10) is:

$$\min \quad \sum_{t \in T} \left[\sum_{g \in G} K_g^{\max} \alpha_{g,t}^+ + \sum_{l \in L} F_l^{\max} \cdot (\beta_{l,t}^+ + \beta_{l,t}^-) + \sum_{n \in B} D_{n,t}^{\max} \gamma_{n,t}^+ \right. \tag{11}$$

$$\left. + \sum_{i \in S} P_i^{\max} \cdot (\tau_{i,t}^+ + \phi_{i,t}^+ + H_i \nu_{i,t}^+) \right]$$

$$\text{s.t.} \quad \alpha_{g,t}^+ \geq \lambda_{B_g,t} - c_g; \forall g \in G, t \in T \tag{12}$$

$$\lambda_{n,t} = \mu_t - \sum_{l \in L} \pi_{n,l} \cdot (\beta_{l,t}^+ - \beta_{l,t}^-); \forall n \in B, t \in T \tag{13}$$

$$\gamma_{n,t}^+ \geq b_n - \lambda_{n,t}; \forall n \in B, t \in T \tag{14}$$

$$\tau_{i,t}^+ \geq \lambda_{B_i,t} - \omega_{i,t}; \forall i \in S, t \in T \tag{15}$$

$$\phi_{i,t}^+ \geq \eta_i \omega_{i,t} - \lambda_{B_i,t}; \forall i \in S, t \in T \tag{16}$$

$$\nu_{i,t}^+ \geq \omega_{i,t+1} - \omega_{i,t}; \forall i \in S, t \in T \tag{17}$$

$$\alpha_{g,t}^+ \geq 0; \forall g \in G, t \in T \tag{18}$$

$$\beta_{l,t}^-, \beta_{l,t}^+ \geq 0; \forall l \in L, t \in T \tag{19}$$

$$\gamma_{n,t}^+ \geq 0; \forall n \in B, t \in T \tag{20}$$

$$\tau_{i,t}^+, \phi_{i,t}^+, \nu_{i,t}^+ \geq 0; \forall i \in S, t \in T. \tag{21}$$

3. PROPERTIES OF GIVING MOS OPERATIONAL CONTROL OF ENERGY STORAGE

We analyze the properties of giving MOs operational control of energy storage in two ways. First, we examine operational decisions and the dispatch-supporting and incentive-compatible nature of prices, by analyzing dual problem (11)–(21). Second, we examine long-run efficiency, by demonstrating that energy-storage rents incentivize a socially optimal amount of energy storage to be built.

3.1 Short-Run Efficiency, Incentive-Compatibility, and Dispatch-Support of Giving MOs Operational Control of Energy Storage

To explore short-run properties, we begin by interpreting the dual variables in (11)–(21), focusing first on a case in which there is no energy storage. Without energy storage, (7)–(10) are eliminated from the OPF problem and the only dual variables are those that are associated with (2)–(6). $\lambda_{n,t}$ is the transmission-bus-n/hour-t locational marginal price (LMP) and μ_t is the hour-t system marginal price of energy. $\alpha_{g,t}^+$ represents the hour-t p.u. rent that is paid to generator g. $\beta_{l,t}^-$ and $\beta_{l,t}^+$ give the hour-t p.u. rents that are paid to transmission line l. $\gamma_{n,t}^+$ is the hour-t p.u. rent that is paid to load at transmission bus n.

Based on these interpretations, without energy storage, (11) minimizes the total rents that are paid to generators, transmission-line owners, and loads. This interpretation of (11) stems from its first three terms corresponding to these three rents, respectively, whereas the final term in (11) vanishes because the dual variables, $\tau_{i,t}^+$, $\phi_{i,t}^+$, and $\nu_{i,t}^+$, do not exist if there is no energy storage. This interpretation of (11) without energy storage is exactly in-line with the analysis of Hogan (1992), with three differences that are related to the assumptions that underlie our OPF model differing from those that underlie the model that Hogan (1992) examines. Hogan (1992) analyzes a lossy transmission model that includes real and reactive power, whereas we analyze a lossless model with real power only. These two differences yield additional transmission-rent terms in the model that Hogan (1992) analyzes, which correspond to losses and reactive-power flows, which do not appear in (11). The other difference is that we assume an upper-limit on demand with an explicit willingness-to-pay, which gives rise to the third term in (11). Hogan (1992) assumes a generic benefit function from energy consumption with no explicit demand limit, which results in consumer rents being defined implicitly in his analysis.

Constraints (12) and (14) are incentive-compatibility restrictions, which require that the rents that are paid to each generator and to each load, respectively, be at least as large as what they can earn by bypassing the MO and transacting bilaterally in the market. For instance, if $\alpha^+_{g,t}$ is less than $\lambda_{B_g,t} - c_g$ for some $g \in G, t \in T$, generator g can earn greater rents during hour t by selling energy directly to customers at the prevailing LMP, which is given by $\lambda_{B_g,t}$. Constraints (13) give the standard relationship that LMPs are equal to the sum of system marginal price of energy and congestion cost. Constraints (18)–(20) ensure that generator, transmission, and demand rents, respectively, are non-negative. Without energy storage, (15)–(17) and (21) vanish from the dual problem, because they are associated with energy-storage-operational variables in (11)–(21).

We turn our attention now to analyzing problems (1)–(10) and (11)–(21) with energy storage. To do so, we begin by interpreting the dual variables that are associated with (7)–(10) as rents that are paid to energy storage. Because energy storage has constraints on its discharging, charging, and SOE, energy-storage rents have three components. Specifically, $\tau^+_{i,t}$, $\phi^+_{i,t}$, and $\nu^+_{i,t}$ represent hour-t p.u. rents that are paid to energy storage i for its discharging, charging, and energy-carrying capacities, respectively. $\omega_{i,t}$ represents the marginal value of having an additional MWh of energy held in energy storage i as of the end of hour t.

With these definitions, the final term in (11) has an analogous interpretation to the first three terms. Specifically, (11) minimizes total rents that are paid to generators, transmission-line owners, and load, which are the first three terms in (11), and to energy storage, which is the fourth term.

Constraints (15)–(17) impose incentive compatibility on energy-storage rents. The pertinent incentive-compatibility constraint depends on whether energy storage charges or discharges. For instance, (15) ensures that when it discharges the discharging rent that energy storage i receives during hour t is at least what the energy storage could earn by bypassing the MO. If energy storage i bypasses the MO during hour t, discharged energy could be sold at the prevailing LMP, $\lambda_{B_i,t}$, at an opportunity cost of $\omega_{i,t}$. Constraints (16) have an analogous interpretation for determining charging rents. If energy storage i bypasses the MO during hour t, charged energy can be purchased at the prevailing LMP, $\lambda_{B_i,t}$, and provides a net (of energy lost that is through the energy-storage process) opportunity benefit of $\eta_i \omega_{i,t}$. Thus, (15) and (16) treat energy-storage rents in a manner that is analogous to generators when energy storage discharges and analogous to demand when energy storage charges. Constraints (17) determine the rents that are paid for carrying energy from one hour to the next. The difference, $\omega_{i,t+1} - \omega_{i,t}$, is the opportunity cost of doing so and (17) imposes the incentive-compatibility requirement that energy storage i be paid at least this amount to carry energy from hour t to $t + 1$.

Thus, we draw three key conclusions regarding the short-run properties of giving MOs operational control of energy storage. First, the dispatch schedule is short-run efficient, by virtue of the fact that (1) maximizes social welfare. Second, (11) ensures that prices that are obtained from the dispatch schedule minimize rents to the market participants. Third, (12), (14)–(17) ensure that prices support the dispatch, insomuch as generators, consumers, and energy storage do not have incentives to deviate from the dispatch schedule.

We conclude our analysis of (1)–(10) and (11)–(21) by appealing to an alternative interpretation of these problems. Hogan (1992) notes that by solving the OPF problem to optimality, an MO maximizes the value of the transmission network that it operates. This maximization is due to the MO operating the network to extract all spatial price differences, which yields an operating point from which no feasible power-flow deviation can improve power-system operations. Incorporating energy storage into (1)–(10) maximizes the value of the transmission network *and* energy-storage assets. This value maximization is achieved by operating transmission and energy storage to extract all spatial and intertemporal price differences, net of losses. In doing so, the MO attains an operating point from which no feasible power-flow or energy-storage deviation can improve power-system operations. Because we assume a lossless transmission network, a transmission network with sufficient capacity would result in no spatial price differences during any given hour. Conversely, we do model energy-storage losses in (7). As such, even with unlimited energy-storage capacity, there may be intertemporal price differences, if a price difference is too small to yield a non-negative marginal social-welfare/energy-storage-rent change. Section 4 illustrates the elimination of intertemporal price differences net of energy-storage losses.

3.2 Investment Efficiency of Giving MOs Operational Control of Energy Storage

To analyze the investment incentives that (1)–(10) induce, first we prove the following lemma that relates energy-storage rents, as defined in (11), to operating revenues that energy storage earns if its charging and discharging are remunerated with LMPs.

Lemma 1 *For each $i \in S$ we have:*

$$\sum_{t \in T} P_i^{\max} \cdot (\tau_{i,t}^{+*} + \phi_{i,t}^{+*} + H_i \nu_{i,t}^{+*}) = \sum_{t \in T} \lambda_{B_i,t}^{*} \cdot (h_{i,t}^{*} - r_{i,t}^{*}); \tag{22}$$

where the asterisk superscript indicates primal- and dual-optimal variable values.

Proof. Problem (1)–(10) is linear. Thus, as is discussed by Bertsekas (1995); Sioshansi and Conejo (2017), its Karush-Kuhn-Tucker conditions, which are:

$$-b_n + \lambda_{n,t}^{*} - \gamma_{n,t}^{-*} + \gamma_{n,t}^{+*} = 0; \forall n \in B, t \in T \tag{23}$$
$$c_g - \lambda_{B_g,t}^{*} - \alpha_{g,t}^{-*} + \alpha_{g,t}^{+*} = 0; \forall g \in G, t \in T \tag{24}$$
$$\lambda_{n,t}^{*} - \mu_t^{*} + \sum_{l \in L} \pi_{n,l} \cdot (\beta_{l,t}^{+*} - \beta_{l,t}^{-*}) = 0; \forall n \in B, t \in T \tag{25}$$
$$\omega_{i,t}^{*} - \omega_{i,t+1}^{*} - \nu_{i,t}^{-*} + \nu_{i,t}^{+*} = 0; \forall i \in S, t \in T \tag{26}$$
$$-\lambda_{B_i,t}^{*} + \omega_{i,t}^{*} - \tau_{i,t}^{-*} + \tau_{i,t}^{+*} = 0; \forall i \in S, t \in T \tag{27}$$
$$\lambda_{B_i,t}^{*} - \eta_i \omega_{i,t}^{*} - \phi_{i,t}^{-*} + \phi_{i,t}^{+*} = 0; \forall i \in S, t \in T \tag{28}$$
$$0 \leq x_{g,t}^{*} \perp \alpha_{g,t}^{-*} \geq 0; \forall g \in G, t \in T \tag{29}$$
$$x_{g,t}^{*} \leq K_g^{\max} \perp \alpha_{g,t}^{+*} \geq 0; \forall g \in G, t \in T \tag{30}$$
$$-F_l^{\max} \leq \sum_{n \in B} \pi_{n,l} e_{n,t}^{*} \perp \beta_{l,t}^{-*} \geq 0; \forall l \in L, t \in T \tag{31}$$
$$\sum_{n \in B} \pi_{n,l} e_{n,t}^{*} \leq F_l^{\max} \perp \beta_{l,t}^{+*} \geq 0; \forall l \in L, t \in T \tag{32}$$
$$0 \leq L_{n,t}^{*} \perp \gamma_{n,t}^{-*} \geq 0; \forall n \in B, t \in T \tag{33}$$
$$L_{n,t}^{*} \leq D_{n,t}^{\max} \perp \gamma_{n,t}^{+*} \geq 0; \forall n \in B, t \in T \tag{34}$$
$$0 \leq h_{i,t}^{*} \perp \tau_{i,t}^{-*} \geq 0; \forall i \in S, t \in T \tag{35}$$
$$h_{i,t}^{*} \leq P_i^{\max} \perp \tau_{i,t}^{+*} \geq 0; \forall i \in S, t \in T \tag{36}$$
$$0 \leq r_{i,t}^{*} \perp \phi_{i,t}^{-*} \geq 0; \forall i \in S, t \in T \tag{37}$$
$$r_{i,t}^{*} \leq P_i^{\max} \perp \phi_{i,t}^{+*} \geq 0; \forall i \in S, t \in T \tag{38}$$
$$0 \leq s_{i,t}^{*} \perp \nu_{i,t}^{-*} \geq 0; \forall i \in S, t \in T \tag{39}$$

$$s^*_{i,t} \leq H_i P^{\max}_i \perp \nu^{+*}_{i,t} \geq 0; \forall i \in S, t \in T \tag{40}$$

$$(2), (3), (7); \tag{41}$$

are necessary and sufficient for a global optimum. From (36), (38), and (40) we have $\forall i \in S$:

$$\sum_{t \in T} P^{\max}_i \cdot (\tau^{+*}_{i,t} + \phi^{+*}_{i,t} + H_i \nu^{+*}_{i,t}) = \sum_{t \in T} (h^*_{i,t} \tau^{+*}_{i,t} + r^*_{i,t} \phi^{+*}_{i,t} + s^*_{i,t} \nu^{+*}_{i,t}). \tag{42}$$

Substituting (26)–(28) into (42) gives:

$$\sum_{t \in T} P^{\max}_i \cdot (\tau^{+*}_{i,t} + \phi^{+*}_{i,t} + H_i \nu^{+*}_{i,t}) = \sum_{t \in T} \Big[h^*_{i,t} \cdot (\lambda^*_{B_i,t} - \omega^*_{i,t} + \tau^{-*}_{i,t}) \tag{43}$$

$$+ r^*_{i,t} \cdot (-\lambda^*_{B_i,t} + \eta_i \omega^*_{i,t} + \phi^{-*}_{i,t}) + s^*_{i,t} \cdot (-\omega^*_{i,t} + \omega^*_{i,t+1} + \nu^{-*}_{i,t}) \Big];$$

which simplifies to:

$$\sum_{t \in T} P^{\max}_i \cdot (\tau^{+*}_{i,t} + \phi^{+*}_{i,t} + H_i \nu^{+*}_{i,t}) = \tag{44}$$

$$\sum_{t \in T} \Big[\lambda^*_{B_i,t} \cdot (h^*_{i,t} - r^*_{i,t}) - \omega^*_{i,t} h^*_{i,t} + \eta_i \omega^*_{i,t} r^*_{i,t} + s^*_{i,t} \cdot (-\omega^*_{i,t} + \omega^*_{i,t+1}) \Big];$$

due to (35), (37), and (39). Substituting (7) into (44) gives:

$$\sum_{t \in T} P^{\max}_i \cdot (\tau^{+*}_{i,t} + \phi^{+*}_{i,t} + H_i \nu^{+*}_{i,t}) = \sum_{t \in T} \Big[\lambda^*_{B_i,t} \cdot (h^*_{i,t} - r^*_{i,t}) - \omega^*_{i,t} s^*_{i,t-1} + \omega^*_{i,t+1} s^*_{i,t} \Big]; \tag{45}$$

which simplifies to (22), because we assume that $s^*_{i,0} = 0$ and $\omega^*_{i,|T|+1}$ does not exist. ■

We show now that energy-storage rents (which, by Lemma 1, are equivalent to operating revenues) provide the correct incentives for energy-storage investment. Our argument follows the approach that Pérez-Arriaga et al. (1995) use to show a similar result for transmission-network investment. To show our result, we make three simplifying assumptions without loss of generality (*i.e.*, relaxing these assumptions complicates our derivations, which would hold still, and notation without any additional insights).

First, we consider investment in a single energy storage with a p.u. round-trip efficiency of η. We let S denote the total energy-carrying capacity of the energy storage, which is being determined by the investor. We let $\kappa(S)$ denote the convex continuously differentiable function that represents energy-storage-investment cost. Because $\kappa(S)$ is convex and continuously differentiable, we assume implicitly that there are neither economies of scale nor non-convexities in energy-storage investment. This assumption is reasonable for some energy-storage technologies, *e.g.*, electrochemical batteries, but may be more tenuous for others, *e.g.*, pumped-hydroelectric energy storage (PHS).

Second, we assume that energy storage operates for a single charging and discharging cycle during off- and on-peak periods. The off-peak period begins at time 0 and ends at time t_2. During this time the marginal generating unit, which has generating capacity, $K^{\max}_{\text{off}}$, and marginal cost, c_{off}, sets the LMP. Energy storage charges during the off-peak period and we let $P_{\text{off}}(t)$ denote the time-t charging of energy storage. For notational ease, we assume that energy storage charges between times t_1 and $t_1 + H_{\text{off}}(P_{\text{off}}(t))$, where $0 \leq t_1 < t_1 + H_{\text{off}}(P_{\text{off}}(t)) \leq t_2$. Having the duration of the charging window depend on $P_{\text{off}}(t)$ reflects its being related to the charging rate. The on-peak period begins as of time t_2 and continues until time T. During this time the marginal generating unit that sets the LMP has marginal cost, c_{on}, with $c_{\text{off}} < c_{\text{on}}$. Energy storage discharges during the on-peak period and we let $P_{\text{on}}(t)$ denote the time-t discharging of energy storage. Again, for notational ease, we assume that energy storage discharges between times t_3 and $t_3 + H_{\text{on}}(P_{\text{on}}(t))$, where $t_2 \leq t_3 < t_3 + H_{\text{on}}(P_{\text{on}}(t)) \leq T$. Our assumption that c_{off} and c_{on} are fixed implies that the energy storage behaves as a price-taker and that the energy-storage technology allows for infinitesimal capacity additions. We let $L(t)$ denote time-t load. One could allow for multiple charging and discharging cycles, in which case all of the time, charging, discharging, and cost parameters would need to be indexed by the charging and discharging cycle to which they correspond, which is notationally cumbersome.

Finally, we assume that the capacity of energy storage is binding. This means that:

$$\int_{t_1}^{t_1+H_{\text{off}}(P_{\text{off}}(t))} P_{\text{off}}(t)dt = S; \tag{46}$$

or that the full S-MWh energy-carrying capacity of energy storage is exhausted when it charges during the off-peak period. Similarly, we have that:

$$\int_{t_3}^{t_3+H_{\text{on}}(P_{\text{on}}(t))} P_{\text{on}}(t)dt = \eta S; \tag{47}$$

or that all of the charged energy (net of energy that is lost) is discharged during the on-peak period. If the capacity constraints of the energy storage are not exhausted, the dual variables that define energy-storage rents are zero. Such a case yields a trivial and uninteresting result, in which energy-storage rents are zero because there is no marginal benefit to additional energy-storage capacity.

Theorem 1 *Energy-storage rents, as defined in (11), induce a profit-maximizing investor to build a socially optimal amount of energy-storage capacity.*

Proof. The cost of operating the power system between times 0 and T is:

$$\begin{aligned} O(S) = &\int_0^{t_1} c_{\text{off}} L(t)dt + \int_{t_1}^{t_1+H_{\text{off}}(P_{\text{off}}(t))} c_{\text{off}} \cdot [L(t) + P_{\text{off}}(t)]dt \\ &+ \int_{t_1+H_{\text{off}}}^{t_2} c_{\text{off}} L(t)dt + \int_{t_2}^{t_3} \left\{c_{\text{off}} K_{\text{off}}^{\max} + c_{\text{on}} \cdot \left[L(t) - K_{\text{off}}^{\max}\right]\right\} dt \\ &+ \int_{t_3}^{t_3+H_{\text{on}}(P_{\text{on}}(t))} \left\{c_{\text{off}} K_{\text{off}}^{\max} + c_{\text{on}} \cdot \left[L(t) - K_{\text{off}}^{\max} - P_{\text{on}}(t)\right]\right\} dt \\ &+ \int_{t_3+H_{\text{on}}}^{T} \left\{c_{\text{off}} K_{\text{off}}^{\max} + c_{\text{on}} \cdot \left[L(t) - K_{\text{off}}^{\max}\right]\right\} dt. \end{aligned} \tag{48}$$

The first three terms on the right-hand side of (48) give the operating cost during the off-peak period, during which time the generating unit with cost, c_{off}, is marginal. Energy storage is charged between times t_1 and $t_1 + H_{\text{off}}$. An additional $P_{\text{off}}(t)$ MW must be produced during time t, which is reflected in the second term on the right-hand side of (48). The final three terms on the right-hand side of (48) give the operating cost during the on-peak period, during which time the lower-cost generating unit operates at its capacity and the higher-cost unit serves the residual demand. As a result of energy storage being discharged, the output of this higher-cost unit is reduced between times t_3 and $t_3 + H_{\text{on}}$. This reduced generation is reflected in the fifth term on the right-hand side of (48). Substituting (46) and (47) into (48) gives:

$$O(S) = \int_0^{t_2} c_{\text{off}} L(t)dt + c_{\text{off}} S + \int_{t_2}^{T} \left\{c_{\text{off}} K_{\text{off}}^{\max} + c_{\text{on}} \cdot \left[L(t) - K_{\text{off}}^{\max}\right]\right\} dt - c_{\text{on}} \eta S. \tag{49}$$

Because $O(\cdot)$ is convex in S, a necessary and sufficient condition for socially optimal energy-storage investment is $O'(S) = -\kappa'(S)$. From (49), we have that $O'(S) = c_{\text{off}} - c_{\text{on}}\eta$. Thus, a profit-maximizing investor undertakes socially optimal energy-storage investment if marginal (with respect to S) energy-storage rent is equal to $c_{\text{on}}\eta - c_{\text{off}}$. Energy storage earns $S \cdot (c_{\text{on}}\eta - c_{\text{off}})$ in revenue, which by Lemma 1 is equal to energy-storage rent, as defined by (11). Thus, marginal (with respect to S) energy-storage rent is equal to $c_{\text{on}}\eta - c_{\text{off}}$, which gives the desired result regarding energy-storage capacity that is built by a profit-maximizing investor. ■

4. ILLUSTRATIVE EXAMPLE

This section uses a stylized example to illustrate the impact of co-optimizing the dispatch of generation and energy storage. We assume $|B| = 4$, $|L| = 5$, $|T| = 5$, and that there are two 50-MW generators at transmission buses 1 and 2 with costs $c_1 = 4$ and $c_2 = 8$, respectively. There are loads at three of the four transmission buses, each of which is co-located with energy storage with $\eta_i = 0.8$, $H_i = 1.5$, and $p_i^{\max} = 5$. We label the energy storage so $i = n, \forall i \in S$. The example is programmed in GAMS and solved using Gurobi. We run our model on NEOS Server, which is a cloud-based optimization platform that is described by Czyzyk et al. (1998).

Table 1 summarizes the values of a subset of primal- and dual-optimal variable values that are associated with the energy storage that is located at transmission bus 4. We focus on transmission bus 4 because many of the variables are non-zero, allowing us to draw insights into how primal- and dual-optimal variable values are related to one another. $\tau_{4,4}^* = 5$ means that the optimal value of (1) increases by \$5 if one more MW could be discharged during hour 4 from the energy storage that is located at transmission bus 4. Specifically, there is a \$5 difference between the hour-4 LMP, $\lambda_{4,4}^* = 25$, and the hour-4 marginal value of stored energy, $\omega_{4,4}^* = 20$. Constraint (15) for $i = 4$ and $t = 4$ requires that the rent on energy-storage discharging be at least as great as this difference. Thus, the rent on energy-storage discharging reflects the social-welfare improvement that having additional discharging capacity would provide.

Table 1: Bus-4 Primal- and Dual-Optimal Variable Values in Example from Section 4

	t				
Variable	**1**	**2**	**3**	**4**	**5**
$h_{4,t}^*$	0.000	0.000	2.500	5.000	0.000
$r_{4,t}^*$	5.000	4.375	0.000	0.000	0.000
$s_{4,t}^*$	4.000	7.500	5.000	0.000	0.000
$\tau_{4,t}^*$	0.000	0.000	0.000	5.000	0.000
$\phi_{4,t}^*$	4.000	0.000	0.000	0.000	0.000
$\nu_{4,t}^*$	0.000	5.000	0.000	0.000	0.000
$\lambda_{4,t}^*$	8.000	12.000	20.000	25.000	10.000
$\omega_{4,t}^*$	15.000	15.000	20.000	20.000	10.000

Similarly, $\phi_{4,1}^* = 4$ reflects a \$4 p.u. increase in the optimal value of (1) from the energy storage that is located at transmission bus 4 having additional charging capacity during hour 1. This welfare improvement arises because added charging capacity would allow energy to be stored at a p.u. cost of $\lambda_{4,1}^* = 8$, which yields a (net of losses that are associated with energy-storage use) p.u. benefit of $\eta_4\omega_{4,1}^* = 12$. Constraint (16) for $i = 4$ and $t = 1$ requires that the charging rent be at least as great as the social-welfare gain that is given by this difference. $s_{4,2}^* = 7.5$, meaning that the energy storage that is located at transmission bus 4 reaches its energy-carrying capacity during hour 2. This gives $\nu_{4,2}^* = 5$ a nonzero value, which is defined as the difference between $\omega_{4,3}^* = 20$ and $\omega_{4,2}^* = 15$, as is required by (17). The intuition behind setting $\nu_{4,2}^*$ in this way is that having energy in storage as of the end of hour 3 is more valuable than having it in storage as of the end of hour 2. $\nu_{4,2}^*$ ascribes this value to stored energy through the third component of the energy-storage rent in (11).

Table 2 summarizes LMPs for transmission bus 4 with different values of $P_4^{\max}$, assuming unlimited transmission-line capacities, which yields an uncongested transmission network. The table shows (*cf.* Section 3.1) that energy storage reduces intertemporal price differences. This impact of energy storage is analogous to transmission reducing spatial price differences, and yields our conclusion that energy storage's social value can be maximized by giving MOs operational control of it. With $P_4^{\max} = 30$, energy storage eliminates all economically valuable intertemporal price differences, because with $\eta_4 = 0.8$ there are no marginal social-welfare/energy-storage-rent gains from having additional energy-storage capacity. To see that there are no gains from additional energy-storage capacity with $P_4^{\max} = 30$, consider charging an incremental Δ MW of energy during either of hours 1 or 2 and discharging that incremental stored energy during either of hours 3 or 4. Doing so would yield a marginal social-welfare/energy-storage-rent change of:

$$\Delta \cdot (12\eta_4 - 9.6) = 0;$$

showing that there is no value to incremental energy-storage capacity. This case of $P_4^{\max} = 30$ is analogous to a situation in which there is no transmission congestion, in which case all spatial price differences are eliminated.

Table 2: $\lambda_{4,t}^*$ in Example from Section 4 with Different Values of $P_4^{\max}$ and a Transmission Network with Unlimited Capacity

	t				
$P_4^{\max}$	**1**	**2**	**3**	**4**	**5**
5	8.0	8.0	20.0	25.0	10.0
10	8.0	8.0	10.0	25.0	10.0
15	8.0	8.0	10.0	12.0	10.0
20	8.0	9.6	12.0	12.0	10.0
30	9.6	9.6	12.0	12.0	10.0

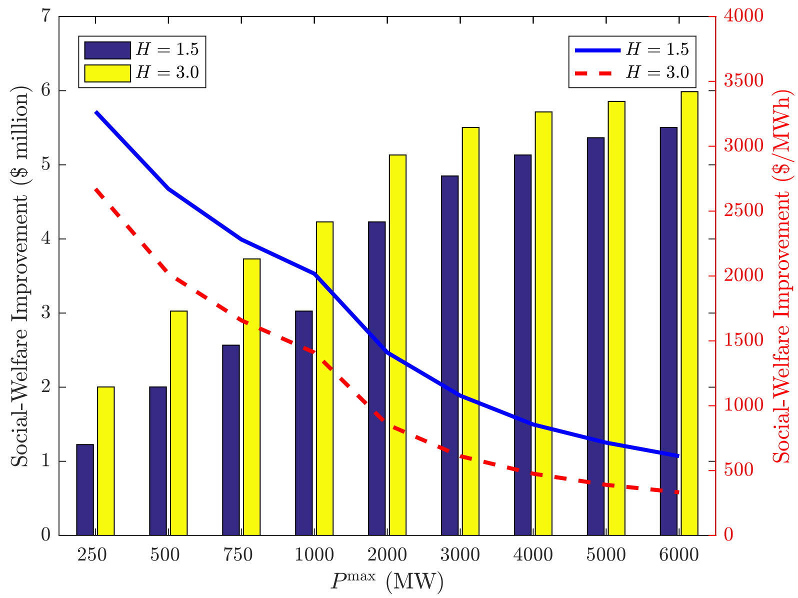
5. CASE STUDY

We examine a larger 283-transmission-bus, 276-generator, 31-day case study that is based on data for ISO New England (ISONE) from August, 2005. Loads are assigned to the eight zones that are in ISONE's market model. We assume that the eight zones have identical energy storage with $\eta_i = 0.8$, $P_i^{\max} = P^{\max}$, and $H_i = H, \forall i \in S$, where $P^{\max}$ and H are varied. The case study is implemented using the same computational resources that are used for the example that is presented in Section 4.

Figure 1 summarizes the total social-welfare improvements in our case study relative to \$984.95 million of social welfare without energy storage. The bars provide absolute welfare improvements and the lines report improvements normalized by the energy-carrying capacity of each energy storage. Figure 1 shows that incremental energy-storage additions deliver social-welfare gains (up until a point at which all intertemporal price differences are eliminated, which occurs if $P^{\max}$ and H are sufficiently high). Figure 1 shows also that the normalized value of energy storage is diminishing in its capacity. The first increment of energy-storage capacity is used to shift loads between periods with large marginal-welfare differences. As more energy storage is added, the incremental capacity is used to shift loads between periods with smaller marginal-welfare differences, giving diminishing marginal-welfare gains.

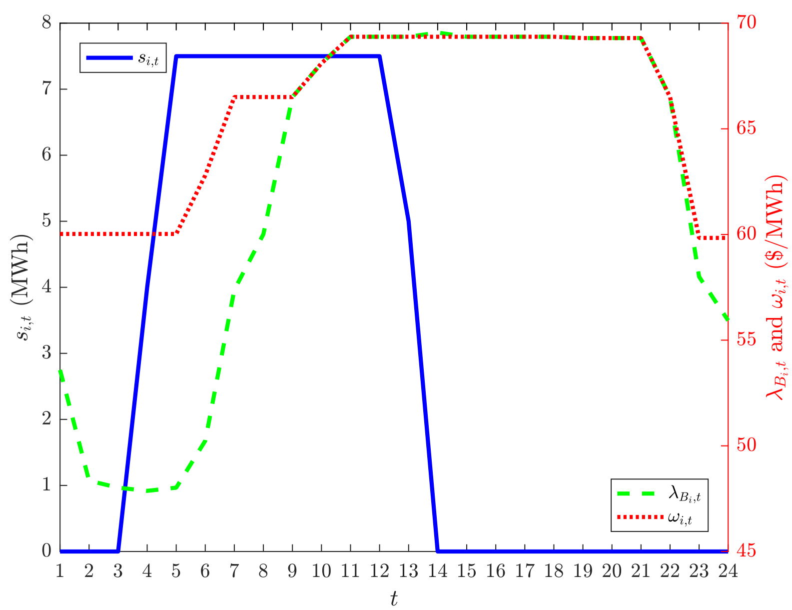
Figure 2 shows the operation during 30 August, 2005 with $H = 1.5$ and $P^{\max} = 5$ of energy storage that is located in Western/Central Massachusetts zone, as well as corresponding dual-variable values. The operating pattern that is shown is typical of most days, with energy being stored during the morning and discharged during the afternoon. The dual variables, $\phi_{i,4}^{+*}$ and $\tau_{i,14}^{+*}$, are non-zero during this day, which reflects the hour-4-charging and hour-14-discharging constraints being binding. These constraints are binding during these two hours because they have the lowest and highest LMPs, respectively. Thus, to the extent possible, the MO seeks to charge energy during hour 4, which is discharged during hour 14 to alleviate the use of a higher-cost generator. Increasing $P^{\max}$

Figure 1: Social-welfare improvement relative to no-energy-storage case (bars are absolute improvements and lines are normalized by the energy-carrying capacity of each energy storage) in case study from Section 5.



to 10 and 20 drives $\phi_{i,4}^{+*}$ and $\tau_{i,14}^{+*}$, respectively, to zero, as the power-capacity constraints become slack. Increasing $P^{\max}$ to 20 decreases the hour-14 LMP to the same value as during hours 11–13 and 15–18.

Figure 2: Operation during 30 **August,** 2005 **with** $H = 1.5$ **and** $P^{\max} = 5$ **of energy storage in Western/Central Massachusetts load zone and corresponding values of** $\omega_{i,t}$ **and LMPs in case study from Section 5.**



Tables 3 and 4 summarize the breakdown of rents to generators, loads, transmission, and energy storage with different values of $P^{\max}$ and for H equal to 1.5 and 3.0, respectively. The table shows that adding energy storage increases total rents, but that the breakdown of rents between the agent types is not monotone. These findings are consistent with the findings of Sioshansi (2010, 2014b). For relatively low values of $P^{\max}$, energy storage profits from high price differences between when it discharges and charges. As $P^{\max}$ increases these price differences decrease because of energy storage's merit-order effect. An alternative interpretation of the rent decrease is that if energy-storage capacity is sufficiently large, energy-storage-capacity constraints become non-binding and their associated dual variables become zero. These dual-variable-value decreases outweigh the countervailing energy-storage-capacity increases. Tables 3 and 4 show that for our case study, $P^{\max} = 2000$ and $P^{\max} = 1000$ with $H = 1.5$ and $H = 3.0$, respectively, are the thresholds beyond which energy-storage rents decrease. Increasing energy-storage capacity beyond the levels that are summarized in Tables 3 and 4 can lead to further energy-storage-rent decreases with the rent becoming zero *in extremis*.

Table 3: Breakdown of Rents ($ million) in Case Study from Section 5 with $H = 1.5$

$P^{\max}$	Generator	Load	Transmission	Energy Storage	Total
0	505.6	453.8	25.6	0.0	984.9
250	502.9	456.6	25.7	1.0	986.2
500	501.7	460.6	23.3	1.3	986.9
750	500.0	463.3	22.7	1.5	987.5
1000	498.9	465.9	21.5	1.7	988.0
2000	496.9	470.7	19.8	1.8	989.2
3000	494.8	474.8	19.1	1.1	989.8
4000	495.4	476.5	17.1	1.0	990.1
5000	493.2	479.0	17.3	0.8	990.3
6000	494.4	477.9	17.4	0.8	990.4

Table 4: Breakdown of Rents ($ million) in Case Study from Section 5 with $H = 3.0$

$P^{\max}$	Generator	Load	Transmission	Energy Storage	Total
0	505.6	453.8	25.6	0.0	984.9
250	501.7	460.6	23.3	1.3	986.9
500	498.9	465.9	21.5	1.7	988.0
750	497.2	469.2	20.6	1.7	988.7
1000	496.9	470.7	19.8	1.8	989.2
2000	495.4	476.5	17.1	1.0	990.1
3000	494.4	477.9	17.4	0.8	990.4
4000	493.1	478.8	18.1	0.7	990.7
5000	492.2	479.7	18.2	0.7	990.8
6000	493.0	477.1	20.3	0.5	990.9

Tables 3 and 4 show also that increasing energy-storage capacity tends to decrease generator rents, increase load rents, and decrease transmission-network congestion and rents. Thus, the addition of energy storage to a power system impacts generation- and transmission-investment incentives.

We conclude our analysis of our case study by examining energy-storage-cost recovery and Theorem 1 using a two-step process. First, we solve the following auxiliary problem:

$$\max \quad \sum_{t \in T} \left(\sum_{n \in B} b_n L_{n,t} - \sum_{g \in G} c_g x_{g,t} \right) - \sum_{i \in S} \xi_i H_i P_i^{\max} \tag{50}$$

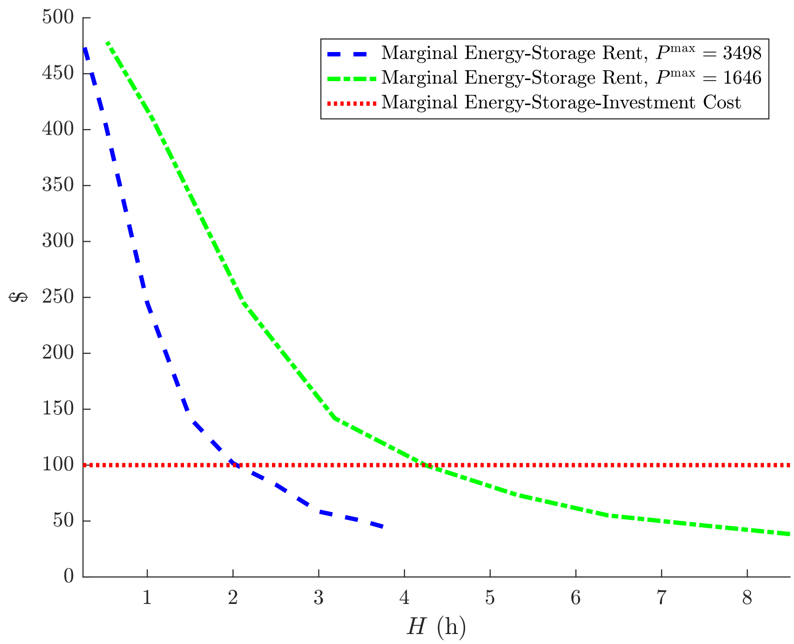
$$\text{s.t.} \quad (2)\text{–}(10) \tag{51}$$

$$P_i^{\max} \geq 0; \forall i \in S; \tag{52}$$

where H_i is held fixed, $P_i^{\max}$ is a decision variable, and ξ_i is the per-MWh investment cost of energy storage i. Problem (50)–(52) determines a socially optimal amount of energy storage to build, given the tradeoff between its investment cost and increased operating-stage welfare. Next, given an optimal set of capacities, $P_i^{\max *}, \forall i \in S$, we solve (1)–(10) with different values of H_i and compare energy-storage rents to the associated investment costs.

We conduct this analysis in two cases, assuming a single energy storage in the Connecticut zone with $\eta = 0.8$ and a per-MWh investment cost of \$100. We solve (50)–(52) with $H = 2.00$ and $H = 4.25$. In the former case, $P^{\max} = 3498$ is optimal and in the latter we have $P^{\max} = 1646$ as optimal. Figure 3 summarizes marginal energy-storage rents and marginal energy-storage-investment costs if we fix $P^{\max}$ to these two values and vary H. As expected from Theorem 1, if $P^{\max} = 3498$ we have that marginal energy-storage rent and marginal investment cost co-incide with $H = 2.00$. If H is lower than 2.00, marginal energy-storage rent is greater than marginal investment cost and the excess marginal rent incentivizes added energy-storage investment. We have the opposite result if H is greater than 2.00. The case wherein we fix $P^{\max} = 1646$ gives an analogous result, except that $H = 4.25$ is the threshold value at which marginal rent and investment cost are equal.

Figure 3: Marginal energy-storage rents and marginal energy-storage-investment costs as a function of H in case study from Section 5 with $P^{\max}$ = 3498 MW and $P^{\max}$ = 1646 MW.



Finally, we use the case with $P^{\max} = 1646$ and $H = 4.25$ to elucidate some nuances of the price-taking and convexity assumptions that underlie Theorem 1. Theorem 1 assumes that the marginal costs of energy during the on- and off-peak periods, which are denoted c_{on} and c_{off}, respectively, are fixed. This assumption does not mean that marginal costs and LMPs are not affected by the amount of energy storage that is built or how energy storage is operated. For instance, Table 2 shows the LMPs for transmission bus 4 changing with $P_4^{\max}$. Rather, the assumption that c_{on} and c_{off} are fixed means that energy storage behaves as a price-taker and does not behave strategically *vis-à-vis* its investment decision to manipulate prices and increase its energy-storage rent. Instead, energy storage invests so long as marginal energy-storage rent outweighs marginal energy-storage-investment cost. For instance, without any energy storage added, the load-weighted average LMP for Connecticut zone between hours 11 and 22 of 1 August, 2005 is \$68.23/MWh whereas the load-weighted average LMP for the remaining hours of the day is \$51.21/MWh. Thus, during this day, there is a load-weighted-average difference of \$17.02/MWh between energy prices during on-peak midday hours and the remaining off-peak hours. If energy storage with $P^{\max} = 1646$ and $H = 4.25$ is added to Connecticut zone, the load-weighted average price between hours 11 and 22 decreases to \$66.59/MWh and the average price for the remaining hours increases to \$52.64/MWh. As expected, adding energy storage diminishes the load-weighted-average difference between on- and off-peak prices to \$13.95/MWh, which reduces energy-storage rent. $P^{\max} = 1646$ and $H = 4.25$ is a threshold at which marginal energy-storage rent and marginal energy-storage investment equal one another, which co-incides with socially optimal energy-storage investment.

Next, we consider the potential impact of relaxing the assumption of convex energy-storage investment. For our case study, social welfare increases from \$984.95 million without energy storage to \$986.94 million if energy storage with $P^{\max} = 1646$ and $H = 4.25$ is added to Connecticut zone, which is a \$1.99 million welfare increase. If we fix $P^{\max} = 1646$ and assume that infinitesimal investments are possible with a per-MWh cost of \$100, marginal energy-storage-investment cost is \$164 600/h. Figure 3 shows that if $H = 4.25$, marginal energy-storage rent is \$164 600/h and equal to marginal energy-storage-investment cost. Thus, under the assumptions of our case study, the total cost of building energy storage with $P^{\max} = 1646$ and $H = 4.25$ is \$699 550, which is equal to total revenue/energy-storage rent.

Now, consider a case in which energy-storage investment is a binary or lumpy decision whereby either no energy storage or energy storage with $P^{\max} = 1646$ and $H = 4.25$ must be built. In this case, investment incentives can be misaligned between a social planner and a private investor. Specifically, if the cost of building the energy storage is less than or equal to \$699 550 or greater than \$1.99 million, their incentives are aligned (energy storage is built by both entities in the former case and not built in the latter). However, if the cost of building the energy storage is between \$699 550 and \$1.99 million, a social planner would opt to build the energy storage (the social-welfare gain outweighs the investment cost) whereas a private investor would not (investment cost outweighs energy-storage rent).[3] Joskow and Tirole (2005) demonstrate an analogous result for the case of lumpy transmission investment.

6. CONCLUSION, DISCUSSION, AND MARKET-DESIGN IMPLICATIONS

Policymakers, regulators, and industry participants increasingly are interested in the use and deployment of energy storage. This interest raises concerns regarding how energy storage should be incorporated into power-system operations. Some stakeholders claim that MOs' independence is harmed if they make energy-storage-operations decisions, because these decisions can affect other units and price formation. These concerns are raised explicitly by CAISO in regards to LEAPS and are a determining factor in FERC's ultimate decision regarding the regulatory treatment of LEAPS.

We demonstrate that the concerns surrounding MO independence are unfounded. We find that giving MOs operational control over energy storage raises no novel market-design issues compared to their making operational decisions for generators and transmission networks. Our work extends the analyses of Hogan (1992); Pérez-Arriaga et al. (1995), which show that allowing MOs to determine transmission-network is consistent with social-network maximization and efficient transmission investment. Analogous results apply if MOs are given operational authority over energy storage. Thus, market designers and policymakers should not be concerned about the purported issues of MO independence that are raised in cases that involve energy storage. We show that energy storage impacts price formation in a manner that is analogous to generation and demand when it is discharged and charged, respectively. Moreover, MOs maximize the social value of energy storage

[3] A potentially important nuance in comparing the incentives of a social planner and a private investor is the cost of capital, especially for the former. Laffont and Tirole (1993) discuss cases in which a social planner may not undertake a socially beneficial project if the shadow cost or deadweight loss that is associated with raising the necessary funds outweigh the social benefits of the project. Such considerations are beyond the scope of our work, but worth noting when contrasting incentives for energy-storage investment.

by using it to minimize intertemporal LMP differences. This is analogous to the finding of Hogan (1992) that MOs maximize the social value of transmission by using it to minimize spatial LMP differences.

We focus on the use of energy storage for energy shifting. Sioshansi et al. (2012) discuss the use of energy storage for the provision of ancillary services. Kim et al. (2022) discuss the evolution of capacity markets to use energy storage for resource-adequacy purposes. So long as market models for the provision of these and other services are convex, our results regarding the short- and long-run properties of giving MOs control of energy storage should extend to the provision of these services.

Problem (1)–(10) treats energy storage as a public or shared asset. This treatment of energy storage stems from the MO having flexibility to operate energy storage and there not being direct costs on energy-storage use in (1). Rather, the implicit cost of energy-storage use is based on the cost of charging energy. Such treatment of energy storage is akin to the treatment of transmission in most restructured markets today, *i.e.*, there is no direct cost on using the network in (1). This treatment of energy storage differs from current practice in some markets, which assume that energy storage is a private asset. Indeed, some stakeholders advocate for a market-participation model that sees energy storage self-scheduling or submitting price-responsive charging and discharging offers into the wholesale market. We do not advocate for one model over another. Rather, our aim in this paper is to dispel misconceptions regarding MOs having operational control of energy storage. In doing so, we give market designers and policymakers a broader range of options for designing efficient market-participation models for energy storage.

Model (1)–(10) is stylized in a number of ways. However, our main results regarding the properties of MO control of energy storage are not dependent on our simplifying assumptions, which are made to ease notation and the analysis. We employ a lossless linearized power-flow model, whereas Hogan (1992) uses a convex power-flow model that accounts for transmission losses. Linearized power-flow models are relatively common in market models that are used by MOs. We could incorporate transmission losses into our model and Rau (2003); Stott et al. (2009); Frank and Rebennack (2016) survey the modeling of transmission losses. We exclude transmission losses from our model to simplify our analysis of the dual problem and the rent terms in (11) in particular. Nevertheless, there may be interesting insights that would be gleaned from a model that includes transmission losses. For instance, Bustos et al. (2018) find that, depending upon power-system configuration, energy storage can act as a substitute or complement to transmission.

Another simplifying assumption is constant marginal-generation costs and consumer willingnesses to pay. These assumptions yield a linear objective function, which allows for an explicit derivation of the dual problem. These assumption could be relaxed to allow for more general convex generation-cost or concave willingness-to-pay functions and the efficiency properties would hold still. However, with general cost and willingness-to-pay functions we would need to define the rents in the dual objective function implicitly. A reasonable compromise could be to approximate convex generation-cost and concave willingness-to-pay functions as convex and concave piecewise-linear functions. Doing so would maintain the same basic structures of (1) and (11). However, such a model would be more notationally cumbersome, as the costs and willingnesses to pay must be indexed by the segments of the piecewise-linear functions.

We assume an energy-storage technology without any direct operating cost. As such, there is no cost on energy-storage use in (1). Some technologies have operating costs, however, *e.g.*, diabatic compressed-air energy storage combusts natural gas in the discharging cycle and electrochemical batteries suffer cycle-life degradation. These and other types of costs could be incorporated into (1) without impacting the properties that we show. Barnes et al. (2015); Xi and Sioshansi (2016) propose approaches to modeling such costs. Such costs would impact energy-storage operation by requiring a larger difference between the marginal-welfare impact of discharging and charging energy, just as the efficiency factors, η_i, do (*cf.* Table 2). Thus, these costs would impact the operation of energy storage much as transmission losses do in a lossy OPF model.

We assume also a constant efficiency factor and power and energy constraints and a linear relationship between SOE, charging, and discharging of energy storage. These assumptions are standard for market-modeling of energy storage. The works of Walawalkar et al. (2007); Sioshansi et al. (2009); Kazemi et al. (2017) are among numerous examples that employ such assumptions. Nevertheless, non-linearities can apply to modeling energy-storage technologies, and Wang et al. (2013); Qiu et al. (2014); Ortega-Vazquez (2014); Pandžić and Bobanac (2019); Padmanabhan et al. (2020); Sioshansi et al. (2022) survey examples of these. So long as they are represented in a manner that maintains convexity of (1)–(10), such non-linearities can be incorporated in an energy-storage model and our results remain.

On the other hand, some energy-storage technologies do exhibit important non-convexities, which can impact our results. Larsen and Sauma (2021) examine energy-storage investment in Chile by a central planner. They find different incentives to invest in electrochemical batteries compared to PHS. Indeed, a technology such as PHS can raise two important types of non-convexities. One is that the operation of PHS may yield non-convexities that are akin to unit-commitment constraints (*i.e.*, there may be binary decisions of whether the turbine or pump is operated and they may have non-zero minimum-load requirements). There is a vast literature, which includes the works of Scarf (1990, 1994); Pérez-Arriaga and Meseguer (1997); O'Neill et al. (2005); Sioshansi (2014a), that examines the impact of these types of constraints on producing prices that are incentive-compatible and dispatch-supporting. Another complication is that PHS is an energy-storage technology that may exhibit economies of scale or lumpiness in its investment (*e.g.*, economies of scale in building a dam once a suitable site for a PHS plant is selected). Thus, the misaligned incentives between a social planner and a private investor that are discussed in Section 5 may arise. As such, our results should be viewed through the same lens that is applied to other works that examine the properties of spot markets for electricity services. We demonstrate desirable results in a stylized case, but policymakers, market designers, and other stakeholders should be cognizant of the limitations of the stylized model.

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