Market Equilibria with Energy Storage as Flexibility Resources

Weiying Gu and Ramteen Sioshansi, *Fellow, IEEE*

Uncertain and variable real-time availability of renewable generation can increase the need for supply-side flexibility in power systems. Energy storage is a potential source of such flexibility. This paper examines the participation of multiple competing strategic profit-maximizing energy storage in a spot electricity market and its impact on consumers, producers, and market equilibria. To this end, we develop a two-stage stochastic bi-level model that has each energy-storage firm determine its market offers at the upper level to maximize its expected profit. The lower level represents market clearing under scenarios with different flexibility needs. We recast the bi-level model as a single-level optimization. A small illustrative example and larger case study show that energy storage can increase market efficiency and reduce renewable-energy curtailment. We show that energy-storage firms neglecting uncertainty in optimizing their market offers can yield profit losses.

***Index Terms*—Energy storage, power-system economics, power-system markets, power-system operations**

NOMENCLATURE

Indices and Sets

- b index of demand, generator, and energy-storage blocks from set, B
- p index of firms from set, P
- t index of hours from set, T
- x index of generators and energy storage from set, X
- Δ_p^S set of energy storage that are owned by firm p
- ω index of scenarios from set, Ω

Parameters

- $\bar{D}_{\omega,t,b}$ hour-t quantity of demand block b under scenario ω (MW)
- $E_{x,0}$ hour-0 state of energy (SOE) of energy storage x (MWh)
- $\bar{E}_{\omega,x}$ energy-carrying capacity of energy storage x under scenario ω (MWh)
- $G_{x,0,b}$ hour-0 dispatch of block b of generator x (MW)
- $\bar{G}_{\omega,x,t,b}$ hour-t available capacity from block b of generator x under scenario ω (MW)
- $O^G_{x,t,b}$ hour-t offer price for block b of generator x into day-ahead market (\$/MWh)
- $O^{G,-}_{x,t,b}$ hour-t decremental offer price for block b of generator x into real-time market (\$/MWh)
- $O^{G,+}_{x,t,b}$ hour-t incremental offer price for block b of generator x into real-time market (\$/MWh)
- R^D_x ramp-down limit of generator x (MW/h)
- R^U_x ramp-up limit of generator x (MW/h)
- $\bar{S}^C_{\omega,x,b}$ capacity of charging block b of energy storage x under scenario ω (MW)
- $\bar{S}^H_{\omega,x,b}$ capacity of discharging block b of energy storage x under scenario ω (MW)
- $U_{\omega,t,b}$ hour-t utility of demand block b under scenario ω (\$/MW)
- η_x round-trip efficiency of energy storage x (p.u.)
- π_ω probability of scenario ω occurring

Decision Variables

- $D_{\omega,t,b}$ hour-t quantity of demand block b that is satisfied under scenario ω (MW)
- $E_{\omega,x,t}$ ending hour-t SOE of energy storage x under scenario ω (MWh)
- $G_{x,t,b}$ hour-t day-ahead dispatch of block b of generator x (MW)
- $G^-_{\omega,x,t,b}$ hour-t decremental real-time dispatch of block b of generator x under scenario ω (MW)
- $G^+_{\omega,x,t,b}$ hour-t incremental real-time dispatch of block b of generator x under scenario ω (MW)
- $O^C_{x,t,b}$ hour-t offer price for charging block b of energy storage x (\$/MWh)
- $O^H_{x,t,b}$ hour-t offer price for discharging block b of energy storage x (\$/MWh)
- $S^C_{\omega,x,t,b}$ hour-t dispatch of charging block b of energy-storage x under scenario ω (MW)
- $S^H_{\omega,x,t,b}$ hour-t dispatch of discharging block b of energy-storage x under scenario ω (MW)

This work was supported by National Science Foundation grants 1808169 and 1922666. *(Corresponding author: Weiying Gu.)*
W. Gu is with Department of Integrated Systems Engineering, The Ohio State University, Columbus, OH 43210-1271, USA (e-mail: gu.775@buckeyemail.osu.edu).
R. Sioshansi is with Department of Integrated Systems Engineering, Department of Electrical and Computer Engineering, and Center for Automotive Research, The Ohio State University, Columbus, OH 43210-1271, USA (e-mail: sioshansi.1@osu.edu).

I. INTRODUCTION

VARIABLE and uncertain real-time availability of renewable energy can increase the need for operational flexibility of power systems, which can be provided by energy storage [1]–[4]. Denholm *et al*. [5] outline the role of energy storage in power systems with high renewable-energy penetrations. Evans *et al*. [6] assess the suitability of different energy-storage technologies for these roles.

There are different approaches to assessing the renewable-integration benefits of energy storage *vis-à-vis* operational flexibility [7]. One approach takes the perspective of a central planner to determine flexibility and energy-storage needs to achieve high renewable-energy penetrations. Such an analysis can use statistical- [8] or optimization-based approaches [9]–[12] that capture varying levels of technical detail [13]. Bruninx *et al*. [14] propose a framework for a central planner or market operator (MO) to dispatch flexible resources.

Another approach examines these synergies from the perspective of the private owner of a generator, energy storage, or a hybrid (*e.g.*, of energy storage and generation) resource [15]–[23]. These works assume that the private firm is a price-taker (*i.e.*, its does not account for the effect of its decisions on prices) and optimizes the operation of its assets to maximize profit or another objective.

Martinek *et al.* [24] compare these two approaches and find that they give similar operating profiles. This finding is expected, because central planning and self-interested behavior are equivalent in a perfectly competitive market [25], [26]. Thus, a natural question, which we examine in this paper, is how energy storage participates in a market *vis-à-vis* the provision of flexibility services absent the price-taking assumption.

There are few works that analyze energy storage without the price-taking assumption, *i.e.*, assume price-making energy storage. This paucity stems from difficulty computing market equilibria that involve energy storage—tracking the state of energy (SOE) of energy storage couples operating periods. Stylized models [27] or heuristics [28] can simplify equilibrium computation. Complementarity models are another approach to studying price-making energy storage [29]–[34]. This includes bi-level complementarity models, whereby the problem is formulated as a mathematical program with equilibrium constraints [29], [30], [32], [34].

We know of three works [31], [33], [35] that model price-making energy storage with a focus on operational flexibility. Our contribution is to address some limitations of these works. All of these works model a single-stage deterministic spot energy market. Thus, they model energy storage competing against ramp-constrained generation to provide energy. Our work assumes a stochastic two-stage spot market to consider uncertain demand and generation (*e.g.*, from wind resources). The MO clears a day-ahead market against expected system conditions. The day-ahead market is followed by a real-time imbalance market that clears against actual system conditions. Our model structure allows us to examine, in detail, how energy storage behaves in a market based on operational-flexibility needs.

Thus, the key contribution of our paper relative to other complementarity models of price-making energy storage [29]–[34] is that we consider uncertainty explicitly. Structurally, uncertainty impacts both sides of the market. The MO must clear the market day ahead anticipating that it may have to take costly real-time decisions to redeploy demand or supply. Contemporaneously, energy storage must structure its offers not knowing with certainty demands or the supply offers of its rivals. Our model allows us to understand how uncertainty impacts both sides of the market.

The remainder of this paper is organized as follows. Section II provides our model formulations and describes the characteristics of market equilibria. The appendix details our equilibrium-computation method. Section III provides data and computational results for an illustrative example. Section IV does the same for a larger case study. Our example and case study show that adding energy storage can be welfare-enhancing, but that welfare gains can be lost if energy-storage owners behave strategically. We demonstrate also that energy-storage owners neglecting uncertainty can be deleterious to them. Section V concludes.

II. Model Formulations

There is a set, P, of strategic energy-storage firms that determine offers, which is followed by the MO clearing a two-stage stochastic spot market. The market proceeds by having firms submit offers into the market before uncertain demand and supply conditions are realized. Market clearing consists of a two-step process. First, the MO determines hourly day-ahead dispatch of the generators without knowing actual real-time supply and demand. Then, after supply and demand uncertainty are realized, the MO determines hourly real-time incremental and decremental dispatch of the generators, as well as energy-storage dispatch and demand served. Hourly real-time prices are determined from this real-time market-clearing process. We allow different costs for day-ahead and real-time generator dispatch, which could arise from mechanical strains of generator ramping [36] or real-time fuel-supply adjustments [37]. Our model employs a bi-level structure, because the dispatch decisions and prices depend upon the supply offers, which are optimized in the upper level. Stochasticity in the model can include uncertainty around demand and rival firms' supply.

A. Lower-Level Model

The MO's two-stage spot-market model is formulated as:

$$\min \sum_{\omega\in\Omega,t\in T,b\in B} \pi_\omega \cdot \Bigg[\sum_{x\in X}\Big(O^G_{x,t,b}G_{x,t,b} + O^{G,-}_{x,t,b}G^-_{\omega,x,t,b} + O^{G,+}_{x,t,b}G^+_{\omega,x,t,b} - O^C_{x,t,b}S^C_{\omega,x,t,b} + O^H_{x,t,b}S^H_{\omega,x,t,b}\Big) - U_{\omega,t,b}D_{\omega,t,b}\Bigg] \tag{1}$$

$$\sum_{x\in X,b\in B}\Big(G_{x,t,b} + G^+_{\omega,x,t,b} - G^-_{\omega,x,t,b} - S^C_{\omega,x,t,b} + S^H_{\omega,x,t,b}\Big) = \sum_{b\in B} D_{\omega,t,b}; \quad \forall \omega\in\Omega, t\in T \quad (\psi_{\omega,t}) \tag{2}$$

$$0 \le D_{\omega,t,b} \le \bar{D}_{\omega,t,b}; \quad \forall\omega\in\Omega, t\in T, b\in B \quad (\theta^{D,-}_{\omega,t,b}, \theta^{D,+}_{\omega,t,b}) \tag{3}$$

$$0 \le G_{x,t,b} - G^-_{\omega,x,t,b} + G^+_{\omega,x,t,b} \le \bar{G}_{\omega,x,t,b}; \forall\omega\in\Omega, t\in T, x\in X, b\in B \quad (\theta^{G,\sum,-}_{\omega,t,x,b}, \theta^{G,\sum,+}_{\omega,t,x,b}) \tag{4}$$

$$0 \le G^-_{\omega,x,t,b}; \quad \forall\omega\in\Omega, t\in T, x\in X, b\in B \quad (\theta^{G,-}_{\omega,t,x,b}) \tag{5}$$

$$0 \le G^+_{\omega,x,t,b}; \quad \forall\omega\in\Omega, t\in T, x\in X, b\in B \quad (\theta^{G,+}_{\omega,t,x,b}) \tag{6}$$

$$-R^D_x \le \sum_{b\in B}(G_{x,t,b} - G_{x,t-1,b}) \le R^U_x; \forall t\in T, x\in X \quad (\theta^{R,-}_{t,x}, \theta^{R,+}_{t,x}) \tag{7}$$

$$-R^D_x \le \sum_{b\in B}\Big(G_{x,t,b} - G^-_{\omega,x,t,b} + G^+_{\omega,x,t,b} - G_{x,t-1,b} + G^-_{\omega,x,t-1,b} - G^+_{\omega,x,t-1,b}\Big) \le R^U_x; \forall\omega\in\Omega, t\in T,$$

$$x \in X \quad (\theta^{R,\sum,-}_{\omega,t,x}, \theta^{R,\sum,+}_{\omega,t,x}) \tag{8}$$

$$0 \le S^C_{\omega,x,t,b} \le \bar{S}^C_{\omega,x,b}; \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (\theta^{C,-}_{\omega,t,x,b}, \theta^{C,+}_{\omega,t,x,b}) \tag{9}$$

$$0 \le S^H_{\omega,x,t,b} \le \bar{S}^H_{\omega,x,b}; \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (\theta^{H,-}_{\omega,t,x,b}, \theta^{H,+}_{\omega,t,x,b}) \tag{10}$$

$$0 \le E_{\omega,x,t} \le \bar{E}_{\omega,x}; \quad \forall \omega \in \Omega, t \in T, x \in X \quad (\theta^{E,-}_{\omega,t,x}, \theta^{E,+}_{\omega,t,x}) \tag{11}$$

$$E_{\omega,x,t} = E_{\omega,x,t-1} + \sum_{b \in B} \left(\eta_x S^C_{\omega,x,t,b} - S^H_{\omega,x,t,b} \right); \quad \forall \omega \in \Omega, t \in T, x \in X \quad (\theta^E_{\omega,t,x}) \tag{12}$$

$$E_{x,0} = E_{\omega,x,|T|}; \forall \omega \in \Omega, x \in X \quad (\theta^{E,0}_{\omega,x}); \tag{13}$$

where the dual variable that is associated with each constraint is in parentheses to its right. The decision variables of (1)–(13) are $D_{\omega,t,b}$, $\forall \omega \in \Omega, t \in T, b \in B$; $E_{\omega,x,t}$, $\forall \omega \in \Omega, x \in X, t \in T$; $G_{x,t,b}$, $\forall x \in X, t \in T, b \in B$; and $G^-_{\omega,x,t,b}$, $G^+_{\omega,x,t,b}$, $S^C_{\omega,x,t,b}$, and $S^H_{\omega,x,t,b}$, $\forall \omega \in \Omega, x \in X, t \in T, b \in B$.

Objective function (1), which is expressed in minimization form, maximizes expected social welfare and contains six terms. The first is the cost of day-ahead generator dispatch and the following two terms give the added expected cost from incremental and decremental real-time adjustments to the day-ahead dispatch. The next two terms give the expected cost of dispatching the energy storage. These terms depend upon the supply offers, $O^C_{x,t,b}$ and $O^H_{x,t,b}$, $\forall p \in P, x \in \Delta^S_p, t \in T, b \in B$, which are determined in the upper-level problem and provide a linkage between the two model levels. The final term in (1) is the expected utility from serving load.

Constraint set (2) ensures hourly load balance under each scenario. Constraint set (3) limits the satisfied quantity of each demand block to be no greater than the volume of demand. Constraint set (4) imposes generator-production limits, accounting for day-ahead and real-time dispatch. Time-variant generator capacities capture supply uncertainty. Constraint sets (5)–(6) ensure non-negative decremental and incremental dispatch, respectively. Constraint sets (7)–(8) enforce, respectively, day-ahead and real-time generator ramping limits.

The MO's model excludes unit-commitment decisions, which some but not all markets consider [38]. Our neglecting unit commitment is a very standard assumption in the existing literature that uses complementarity techniques to model market equilibria [31], [33], [35], [39]–[43]. This assumption is taken because modeling multi-level problems with binary variables in the lower level is computationally challenging [44], [45].

Constraint sets (9)–(11) are, respectively, energy-storage-charging, -discharging, and -SOE limits. Constraint set (12) gives the evolution of energy-storage SOE. We model only energy losses on energy-storage charging, but could include energy losses on energy-storage discharging or a self-discharging rate. Constraint set (13) requires the same ending and beginning SOEs, which is a heuristic approach to avoid myopic energy-storage operations [46].

Some models include binary variables to prevent simultaneous energy-storage charging and discharging [7]. The necessity of such variables depends upon model structure. If the model can create conditions whereby 'wasting energy' is beneficial (*e.g.*, negative prices or oversupply conditions), such variables may be needed [47]. Otherwise, simultaneous charging and discharging is suboptimal, because of the cost of cycling energy through energy storage, *cf.* the fourth and fifth terms in (1), and associated energy losses, *cf.* (12). Indeed, neither our example nor our case study yields results wherein energy storage is charged and discharged simultaneously. Given this property of simultaneous energy-storage charging and discharging, there are many works in the literature that exclude binary variables to prevent it [31], [33], [35], [42], [48]–[54].

B. Upper-Level Problems

Each firm, $p \in P$, solves the problem:

$$\min \sum_{\omega \in \Omega, t \in T, x \in \Delta^S_p, b \in B} \pi_\omega \psi_{\omega,t} \cdot \left(S^C_{\omega,x,t,b} - S^H_{\omega,x,t,b} \right) \tag{14}$$

$$\text{s.t. } O^C_{x,t,b} \ge O^C_{x,t,b-1}; \quad \forall t \in T, x \in \Delta^S_p, b \in B \quad (\delta^C_{p,t,x,b}) \tag{15}$$

$$O^H_{x,t,b} \ge O^H_{x,t,b-1}; \quad \forall t \in T, x \in \Delta^S_p, b \in B \quad (\delta^H_{p,t,x,b}) \tag{16}$$

$$(1)\text{–}(13); \tag{17}$$

where Lagrange multipliers that are associated with (15)–(16) are in parentheses to their right. The decision variables of (14)–(17) are the variables of (1)–(13) and $O^C_{x,t,b}$ and $O^H_{x,t,b}$, $\forall x \in \Delta^S_p, t \in T, b \in B$.

Objective function (14), which is given in minimization form, maximizes firm p's expected profit. We assume that $\forall \omega \in \Omega, t \in T$, $\psi_{\omega,t}$ is the scenario-ω/hour-t energy price. These prices and energy-storage dispatch are determined in the MO's lower-level model, which links the models further. This is why (17) embeds the lower-level spot-market model into firm p's upper-level problem. Constraint sets (15)–(16) ensure monotone offers, which is a common market requirement.

C. Model Structure and Nash Equilibrium

The MO's model is embedded as the lower level of each firm's problem. The model is embedded in this manner, because the MO's model determines dispatch and prices, based on the upper-level offer decisions, which yields an interdependency between the upper- and lower-level models. The overall goal of our model is to compute a Nash equilibrium. A Nash equilibrium has the property that each firm, $p \in P$, determines optimal offers, $O^C_{x,t,b}$ and $O^H_{x,t,b}$, $\forall x \in \Delta^S_p, t \in T, b \in B$, given the offers of its rival firms. The appendix details the approach that we take to compute such equilibria.

III. Example

A. Example Data

We begin illustrating our model with an eight-hour example with three equiprobable scenarios, three conventional and one wind generators, and three firms, each of which owns one energy storage. Eight hours is adequate for our purposes, as we need a sufficient number of operating periods for energy storage to cycle energy. Tables I–II summarize scenario-invariant data for the conventional units, each of which has a 10-MW starting output level. Adjusting the real-time output of units 1 and 3 incurs \$55/MWh and \$60/MWh costs, respectively. Unit 2 has the same offers for day-ahead and real-time dispatch. Table III summarizes data for each energy storage, each of which has a 2-MWh starting SOE and an 85% round-trip efficiency. Fig. 1 summarizes hourly wind availability and demand for each scenario. Wind production is costless. Each hour's demand is divided into two blocks, the second of which is 30 MW greater than the first, with marginal utilities of \$60/MWh and \$55/MWh, respectively.

TABLE I
CONVENTIONAL-GENERATOR-CONSTRAINT PARAMETERS FOR EXAMPLE

	x		
	1	2	3
$\bar{G}_{\omega,x,t,1}$	25	25	20
$\bar{G}_{\omega,x,t,2}$	30	30	25
$R^D_x = R^U_x$	10	15	10

TABLE II
CONVENTIONAL-GENERATOR OFFERS FOR EXAMPLE

	$x=1$		$x=2$		$x=3$	
t	$b=1$	$b=2$	$b=1$	$b=2$	$b=1$	$b=2$
1	20	21	27	28	23	25
2	18	19	25	27	22	23
3	17	18	24	26	20	21
4	18	19	25	27	21	22
5	19	20	26	28	23	26
6	20	21	27	29	24	27
7	19	21	26	27	22	26
8	18	20	25	26	20	23

TABLE III
ENERGY-STORAGE DATA FOR EXAMPLE

	p		
	1	2	3
$\bar{E}_{\omega,x}$	8	20	8
$\bar{S}^C_{\omega,x,b} = \bar{S}^H_{\omega,x,b}$	6	20	6

We analyze equilibria in eight cases with and without wind, with and without energy storage that behaves as a price-taker or -maker, and with or relaxed ramping limits. Price-taking energy storage assumes that $O^C_{x,t,b} = O^H_{x,t,b} = 0$, $\forall x \in \Delta^S_p, t \in T, b \in B$ and (1)–(13) is solved to determine a market outcome. Price-making energy storage is modeled as is outlined in the appendix. Relaxed ramping limits are modeled by relaxing (7)–(8) and adjusting the subsequent derivations.

We consider another four cases, which are modeled using a two-step process, with price-making firms that neglect uncertainty. First, the technique that is outlined in the appendix is

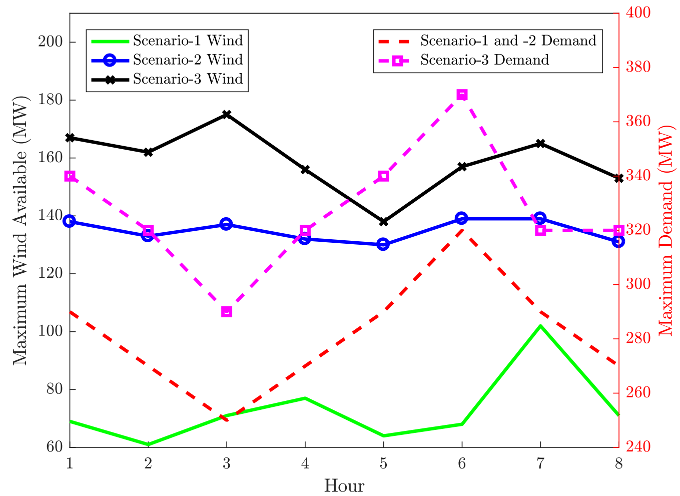

Fig. 1. Hourly wind available and maximum demand under each scenario for example.

applied assuming a single scenario with demand and available generation equal to their expected values, which are:

$$\sum_{\omega \in \Omega} \pi_\omega \bar{D}_{\omega,t,b}; \forall t \in T, b \in B; \tag{18}$$

and:

$$\sum_{\omega \in \Omega} \pi_\omega \bar{G}_{\omega,x,t,b}; \forall x \in X, t \in T, b \in B; \tag{19}$$

respectively. This step optimizes values of $O^C_{x,t,b}$, $O^H_{x,t,b}$, and $G_{x,t,b}$, $\forall x \in X, t \in T, b \in B$ with uncertainty ignored. These values are fixed according to the solution from the first step and the technique that is outlined in the appendix is used with uncertainty represented to model system dispatch.

All four of these cases include wind and strategic energy storage and have or relax ramping limits. Two of the cases use the same scenarios that are summarized in Fig. 1. The other cases use scenarios that are more similar to one another but have the same expected values as are shown in Fig. 1. Specifically, one scenario has demands and available generation set equal to the values that are given by (18)–(19). The other scenarios set these values to be 1% higher and 1% lower than the values that are given by (18)–(19). Contrasting these cases shows the impact of uncertainty on equilibrium behavior.

The example, which has up to 33 157 variables, up to 14 533 of which are binary, and 40 573 equations, is modeled and solved with GAMS v. 33 and Gurobi, using default settings. Finding a candidate equilibrium takes up to 18 minutes. Verifying that a candidate is an equilibrium takes up to 12 minutes.

B. Example Results

Table IV summarizes metrics for the equilibria for the eight cases that capture uncertainty in determining offers. The cases with neither wind nor energy storage yield the lowest expected social welfare. Case 2 yields 8% higher expected generator, consumer, and social welfare compared to Case 1, because relaxed ramping constraints allow more energy consumption and production. Relaxed ramping constraints have similar impacts in the presence of wind or energy storage—expected demand served and expected conventional-generator, consumer, and social welfare increase.

Contrasting Cases 3–4 to 1–2 shows that wind generation increases significantly expected satisfied demand and consumer and social welfare. Wind increases total producer welfare, at a loss to conventional generators, because costless wind generation displaces units 1–3. Relaxing ramping constraints increases conventional-generator production to serve more demand, which translates into higher profits from Case 3 to 4. Wind production increases from Case 3 to 4—1.3% of available wind is curtailed in Case 3 as opposed to no curtailment in the remaining seven cases. Wind-generator profit decreases from Case 3 to 4 because relaxing the ramping constraints suppresses energy prices [55].

Price-taking energy storage under Cases 5–6, which is akin to having energy storage centrally operated or in a perfectly competitive market, yields higher expected producer and social welfare but lower consumer welfare compared to Cases 3–4. These welfare impacts stem from price and quantity effects of energy storage. Energy storage suppresses and increases on- and off-peak energy prices, which increases and decreases on- and off-peak consumption. Energy storage increases the load-weighted-average energy price and energy consumption by 8.58% and 0.90%, respectively, from Case 3 to 5. These increases are 2.93% and 0.98%, respectively, from Case 4 to 6. Consumer welfare is increasing in consumption but decreasing in prices and the price effect outweighs the quantity impact. Generators benefit from energy storage because they sell more energy at a higher average price.

Cases 7–8 show that price-making energy storage see total expected profit increase by 81% and 59%, respectively, relative to Cases 5–6. The equilibria for Cases 7–8 are asymmetric between firms 1 and 3, which are identical. There may be other equilibria that yield different profit distributions between the three firms.

Relative to Cases 5–6, increased expected energy-storage profits under Cases 7–8 yield higher expected consumer welfare, lower expected generator welfare, and lower expected social welfare. The consumer- and generator-welfare changes are keeping with other analyses [27], [56]–[58]. Energy storage tends to be charged and discharged during low- and high-load periods, respectively, which has price and quantity impacts on producers and consumers. This energy-storage use tends to increase and decrease energy prices during low- and high-load periods, respectively, due to merit-order effects. Typically, these price changes yield a consumer-welfare gain because the price decrease during the high-load period applies to a larger quantity of consumption than that to which the price increase applies. Generators produce more at a higher price during low-load periods. However, this positive profit impact is outweighed typically by decreased production and prices during high-load periods. These combined welfare changes can yield social-welfare decreases between price-taking and -making cases, as happens between Cases 5–6 and 7–8.

Fig. 2 summarizes price variability among the eight cases by showing probability-weighted standard deviations (across the three scenarios) of hourly energy prices in each case. For a given case, the scenario-ω standard deviation of the hourly energy prices, σ_ω, is:

$$\sigma_\omega = \sqrt{\sum_{t \in T} \left(\psi_{\omega,t} - \bar{\psi}_\omega \right)^2 / |T|}; \forall \omega \in \Omega;$$

where:

$$\bar{\psi}_\omega = \sum_{t \in T} \psi_{\omega,t} / |T|; \forall \omega \in \Omega.$$

The values that are reported in Fig. 2 are:

$$\sum_{\omega \in \Omega} \pi_\omega \sigma_\omega .$$

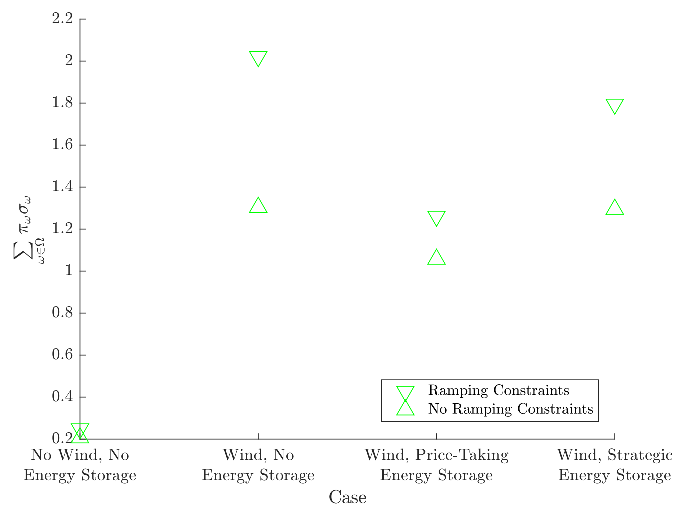

Fig. 2. Probability-weighted standard deviations (across the three scenarios) of hourly energy prices in each case for example with uncertainty captured in determining offers.

The figure shows four price-dispersion properties among the eight cases. First, ramping constraints increase price variability, which is keeping with other analyses [55]. Second, wind generation increases price variability relative to a no-wind case. Price variability increases because wind uncertainty requires real-time dispatch adjustments to maintain supply/demand balance. Third, price-taking or -making energy storage reduces price variability, as there is less need for real-time adjustments to conventional-generator output. Fourth, price variability is higher with price-making as opposed to -taking energy storage. This effect is due to strategic energy storage structuring its offers to maintain larger hourly price differences, which increases energy-storage profit.

Table V summarizes key metrics for the final four cases with price-making energy storage that neglects uncertainty in determining offers. Expected energy-storage profits are negative in Cases 9–10, because energy-storage offers neglect the significant price and revenue variability under the three scenarios. Conversely, expected energy-storage profits are positive under Cases 11–12, because the scenarios are sufficiently similar that neglecting uncertainty has muted impacts on firms' profits. Expected energy-storage profits are lower under Cases 11–12 relative to Cases 7–8 because there is less price variability under the former pair of cases.

TABLE IV
SUMMARY OF RESULTS FOR EXAMPLE WITH UNCERTAINTY CAPTURED IN DETERMINING OFFERS

Case					Expected Welfare ($)						
#	Wind	Energy Storage	Ramping	Expected Demand Served (%)	Conventional Generators	Wind Generators	Firm 1	Firm 2	Firm 3	Consumers	Social
1	No	No	Yes	48.5	25 745	n/a	n/a	n/a	n/a	24 939	50 684
2	No	No	No	52.2	27 702	n/a	n/a	n/a	n/a	26 879	54 581
3	Yes	No	Yes	88.2	19 392	15 322	n/a	n/a	n/a	75 508	110 222
4	Yes	No	No	91.5	20 556	15 012	n/a	n/a	n/a	78 326	113 894
5	Yes	Price-taker	Yes	89.0	21 169	17 053	95	204	95	72 520	111 137
6	Yes	Price-taker	No	92.4	21 235	15 516	78	192	78	77 359	114 458
7	Yes	Strategic	Yes	88.7	20 398	16 243	69	437	206	73 109	110 462
8	Yes	Strategic	No	92.4	20 811	15 034	125	309	115	77 927	114 321

TABLE V
SUMMARY OF RESULTS FOR EXAMPLE WITHOUT UNCERTAINTY CAPTURED IN DETERMINING OFFERS

Case				Expected Welfare ($)						
#	Scenarios	Ramping	Expected Demand Served (%)	Conventional Generators	Wind Generators	Firm 1	Firm 2	Firm 3	Consumers	Social
9	Different	Yes	86.0	67 775	56 122	−396	−3 453	−981	13 882	111 674
10	Different	No	90.0	71 787	54 618	−352	−3 512	−968	16 791	115 278
11	Similar	Yes	90.0	68 421	56 879	127	51	107	12 574	111 566
12	Similar	No	93.4	74 129	56 879	51	140	197	12 574	115 212

Expected social welfare is similar between Cases 7–12, however generators and consumers see expected welfare gains and losses, respectively, under Cases 9–12 relative to Cases 7–8. These differences in the distributions of expected welfare are due to strategic energy storage optimizing offers against expected demand and supply conditions, which yield higher energy prices overall. Case 10 results in 2.08% of available wind energy being curtailed, which yields lower expected wind-generator profits compared to Cases 9 and 11–12, which see no wind curtailment.

IV. CASE STUDY

A. Case-Study Data

We summarize data for a 24-hour case study with two equiprobable scenarios, two conventional generators, one wind unit, and two firms, each of which owns one energy storage. Tables VI–VII summarize scenario-invariant data for the conventional generators, each of which has a 30-MW starting output level. Adjusting the real-time output of unit 1 incurs a \$55/MWh cost. Unit 2 has the same offers for day-ahead and real-time dispatch (*cf.* Table VII). Table VIII summarizes data for each firm's energy storage, each of which has a 10-MWh starting SOE and an 85% round-trip efficiency. Fig. 3 summarizes hourly wind availability and demand for each of the two scenarios. Wind production is costless. The marginal utilities of demand for each scenario are \$60/MWh.

We analyze equilibria for our case study in the same cases that are summarized in Tables IV and V. The case study, which has up to 19 549 variables, of which up to 8 496 are binary, and 23 917 equations, is modeled using the same software that is used for the example. Finding a candidate equilibrium takes up to 31 minutes. Verifying that a candidate is an equilibrium takes up to five minutes.

TABLE VI
CONVENTIONAL-GENERATOR-CONSTRAINT PARAMETERS FOR CASE STUDY

	x = 1	x = 2
$\bar{G}_{\omega,x,t,b}$	60	60
$R^D_x = R^U_x$	20	30

TABLE VII
CONVENTIONAL-GENERATOR OFFERS FOR CASE STUDY

t	x = 1	x = 2	t	x = 1	x = 2	t	x = 1	x = 2
1	20	27	9	20	27	17	20	27
2	18	25	10	18	25	18	18	25
3	17	24	11	17	24	19	17	24
4	18	25	12	18	25	20	18	25
5	19	26	13	19	26	21	19	26
6	20	27	14	20	27	22	20	27
7	19	26	15	19	26	23	19	26
8	18	25	16	18	25	24	18	25

B. Case-Study Results

Table IX summarizes metrics for the equilibria that we find in the eight cases that capture uncertainty in determining offers. The changes in social welfare and energy-storage profit between the cases are keeping with the results from our example that are summarized in Section III-B. Specifically, relaxed ramping constraints yield higher expected social welfare and lower energy-storage profit. Strategic energy storage earns higher profit than price-taking energy storage does.

One key difference between the example and case study

TABLE IX
SUMMARY OF RESULTS FOR CASE STUDY WITH UNCERTAINTY CAPTURED IN DETERMINING OFFERS

Case					Expected Welfare ($)					
#	Wind	Energy Storage	Ramping	Expected Demand Served (%)	Conventional Generators	Wind Generators	Firm 1	Firm 2	Consumers	Social
1	No	No	Yes	87.3	73 230	n/a	n/a	n/a	34 910	108 140
2	No	No	No	87.6	73 530	n/a	n/a	n/a	35 010	108 540
3	Yes	No	Yes	99.3	35 103	12 897	n/a	n/a	96 554	144 554
4	Yes	No	No	99.3	35 996	13 475	n/a	n/a	95 259	144 729
5	Yes	Price-taker	Yes	100.0	31 445	11905	0	0	101 937	145 287
6	Yes	Price-taker	No	100.0	31 573	11 988	0	0	101 810	145 372
7	Yes	Strategic	Yes	100.0	35 457	12 897	244	489	95 468	144 554
8	Yes	Strategic	No	100.0	36 394	13 475	194	449	94 244	144 756

TABLE VIII
ENERGY-STORAGE DATA FOR CASE STUDY

	p = 1	p = 2
$\bar{E}_{\omega,x}$	20	30
$\bar{S}^C_{\omega,x,b} = \bar{S}^H_{\omega,x,b}$	10	20

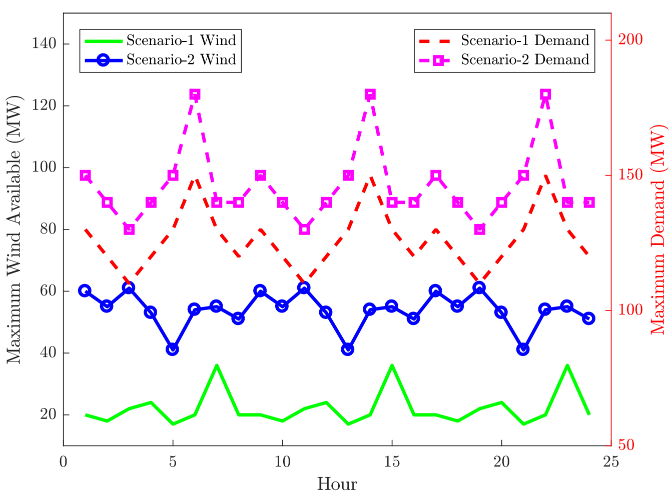

Fig. 3. Hourly wind available and maximum demand under each scenario for case study.

is that in the latter, price-taking energy storage yields higher expected producer and consumer welfare but lower expected conventional and wind-generator welfare compared to a case without energy storage. These welfare changes in the case study are due to price-taking energy storage decreasing the load-weighted-average energy price by 12.6% and 9.9%, respectively, between Cases 3 to 5 and Cases 4 to 6 while increasing conventional generation slightly. Although generator welfare is increasing in production, the price impact of energy storage outweighs this impact, which yields an overall profit decrease. Conversely, consumers benefit from decreased prices, which yields the expected consumer-welfare increase with price-taking energy storage. Another key difference between the example and case study is that in the latter strategic energy storage yields expected consumer-welfare losses relative to price-taking energy storage. This welfare effect is due to strategic energy storage structuring its offers to maintain larger hourly price differences, which increases the load-weighted-average energy price by 11.0% and 8.9%, respectively, from Case 5 to 7 and from Case 6 to 8.

Fig. 4 summarizes price variability among the eight cases by showing probability-weighted standard deviations of hourly energy prices in each case. The figure shows that wind, energy storage, and ramping constraints have similar impacts on price variability in the case study as they do in the example.

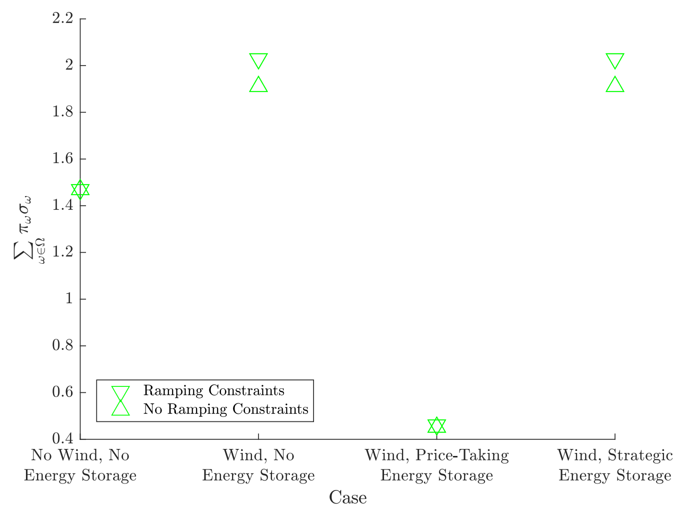

Fig. 4. Probability-weighted standard deviations (across the three scenarios) of hourly energy prices in each case for case study with uncertainty captured in determining offers.

Table X summarizes key metrics for the final four cases, with price-making energy storage that neglects uncertainty in determining offers. As is summarized in Table V, expected energy-storage profits are negative in Cases 9–10, in which the scenarios are very different from one another. Expected energy-storage profits are positive in Case 11–12 but lower than profits that are earned in Cases 7–8.

V. CONCLUSIONS

This paper presents a framework to analyze the role of energy storage in integrating renewable energy into power systems *vis-à-vis* operational flexibility. Our focus is on the impact of strategic energy-storage firms on market efficiency and operations. Thus, our model is structured to represent

TABLE X
SUMMARY OF RESULTS FOR CASE STUDY WITHOUT UNCERTAINTY CAPTURED IN DETERMINING OFFERS

Case				Expected Welfare (\$)					
#	Scenarios	Ramping	Expected Demand Served (%)	Conventional Generators	Wind Generators	Firm 1	Firm 2	Consumers	Social
9	Different	Yes	98.3	71 041	25 994	−504	−3754	51 872	144 649
10	Different	No	100	71 676	26 217	−8	−370	47 100	144 615
11	Similar	Yes	100	70 962	25 994	171	140	47 381	144 649
12	Similar	No	100	72 788	26 949	214	429	44 350	144 729

strategic energy-storage firms. The model could be generalized to capture strategic generators or consumers, which would entail changes to the model structure but the same solution approach that is outlined in the appendix.

Our example demonstrates using the model for drawing market-design and energy-policy conclusions. We find that small amounts of energy storage can yield non-trivial efficiency gains. Our example has 32 MW of energy storage with about 1.1 hours of discharging capacity, which is small relative to 155 MW of conventional-generation capacity, up to 175 MW of wind availability, and up to 2.2 GWh of expected demand served. Energy storage can yield up to 2% expected social-welfare gains relative to not having energy storage.

Some works [27], [57]–[59] find social-welfare losses with strategic energy storage (relative to a no-energy-storage case), which we do not find. Our finding may be specific to our numerical examples and assumed model structure. Having strategic generators or consumers could change the welfare impacts of energy storage. Our modeling framework could be used by policymakers and regulators to explore cases in which strategic energy storage can yield social-welfare losses, to forestall those types of inefficiencies.

Our model is large-scale and computationally challenging. As such, it is well suited for policy, market, or regulatory analysis. There is a growing literature that applies data-driven approaches to market modeling [60]–[67]. Such an approach may be beneficial to our problem. However, data-driven models may not have optimality guarantees and may not be guaranteed to yield Nash equilibria. Nevertheless, such an approach could be of great value for real-time decision support for a market participant.

APPENDIX

In our model, a Nash equilibrium has the property that each firm determines offers that are individually profit-maximizing, while holding the offers of its rival firms fixed. We can compute such an equilibrium by solving (14)–(17), $\forall p \in P$ simultaneously, which we do as follows.

A. Conversion of (14)–(17) to a Single-Level Problem

Problem (1)–(13) is a linear optimization. Thus, (17) can be replaced with the necessary and sufficient primal/dual-optimality conditions [68]:

$$\sum_{x\in X, b\in B}\Big(G_{x,t,b}+G^{+}_{\omega,x,t,b}-G^{-}_{\omega,x,t,b}-S^{C}_{\omega,x,t,b} + S^{H}_{\omega,x,t,b}\Big)=\sum_{b\in B}D_{\omega,t,b}; \quad \forall\omega\in\Omega, t\in T \quad (\phi_{p,\omega,t}) \tag{20}$$

$$0\le D_{\omega,t,b}\le \bar{D}_{\omega,t,b}; \quad \forall\omega\in\Omega, t\in T, b\in B \quad (\vartheta^{D,-}_{p,\omega,t,b},\vartheta^{D,+}_{p,\omega,t,b}) \tag{21}$$

$$0\le G_{x,t,b}-G^{-}_{\omega,x,t,b}+G^{+}_{\omega,x,t,b}\le \bar{G}_{\omega,x,t,b}; \forall\omega\in\Omega, t\in T, x\in X, b\in B \quad (\vartheta^{G,\sum,-}_{p,\omega,t,x,b},\vartheta^{G,\sum,+}_{p,\omega,t,x,b}) \tag{22}$$

$$0\le G^{-}_{\omega,x,t,b}; \quad \forall\omega\in\Omega, t\in T, x\in X, b\in B \quad (\vartheta^{G,-}_{p,\omega,t,x,b}) \tag{23}$$

$$0\le G^{+}_{\omega,x,t,b}; \quad \forall\omega\in\Omega, t\in T, x\in X, b\in B \quad (\vartheta^{G,+}_{p,\omega,t,x,b}) \tag{24}$$

$$-R^{D}_{x}\le\sum_{b\in B}(G_{x,t,b}-G_{x,t-1,b})\le R^{U}_{x}; \forall t\in T, x\in X \quad (\vartheta^{R,-}_{p,t,x},\vartheta^{R,+}_{p,t,x}) \tag{25}$$

$$-R^{D}_{x}\le\sum_{b\in B}\Big(G_{x,t,b}-G^{-}_{\omega,x,t,b}+G^{+}_{\omega,x,t,b}-G_{x,t-1,b} +G^{-}_{\omega,x,t-1,b}-G^{+}_{\omega,x,t-1,b}\Big)\le R^{U}_{x}; \forall\omega\in\Omega, t\in T, x\in X \quad (\vartheta^{R,\sum,-}_{p,\omega,t,x},\vartheta^{R,\sum,+}_{p,\omega,t,x}) \tag{26}$$

$$0\le S^{C}_{\omega,x,t,b}\le \bar{S}^{C}_{\omega,x,b}; \forall\omega\in\Omega, t\in T, x\in X, b\in B \quad (\vartheta^{C,-}_{p,\omega,t,x,b},\vartheta^{C,+}_{p,\omega,t,x,b}) \tag{27}$$

$$0\le S^{H}_{\omega,x,t,b}\le \bar{S}^{H}_{\omega,x,b}; \forall\omega\in\Omega, t\in T, x\in X, b\in B \quad (\vartheta^{H,-}_{p,\omega,t,x,b},\vartheta^{H,+}_{p,\omega,t,x,b}) \tag{28}$$

$$0\le E_{\omega,x,t}\le \bar{E}_{\omega,x}; \quad \forall\omega\in\Omega, t\in T, x\in X \quad (\vartheta^{E,-}_{p,\omega,t,x},\vartheta^{E,+}_{p,\omega,t,x}) \tag{29}$$

$$E_{\omega,x,t}=E_{\omega,x,t-1}+\sum_{b\in B}\left(\eta_x S^{C}_{\omega,x,t,b}-S^{H}_{\omega,x,t,b}\right); \quad \forall\omega\in\Omega, t\in T, x\in X \quad (\vartheta^{E}_{p,\omega,t,x}) \tag{30}$$

$$E_{x,0}=E_{\omega,x,|T|}; \forall\omega\in\Omega, x\in X \quad (\vartheta^{E,0}_{p,\omega,x}) \tag{31}$$

$$\pi_\omega U_{\omega,t,b}-\psi_{\omega,t}+\theta^{D,-}_{\omega,t,b}-\theta^{D,+}_{\omega,t,b}=0; \quad \forall\omega\in\Omega, t\in T, b\in B \quad (\xi^{D}_{p,\omega,t,b}) \tag{32}$$

$$-O^{G}_{x,t,b}+\theta^{R,-}_{t,x}-\theta^{R,-}_{t+1,x}-\theta^{R,+}_{t,x}+\theta^{R,+}_{t+1,x}+\sum_{\omega\in\Omega}\Big(\psi_{\omega,t} +\theta^{G,\sum,-}_{\omega,t,x,b}-\theta^{G,\sum,+}_{\omega,t,x,b}+\theta^{R,\sum,-}_{\omega,t,x}-\theta^{R,\sum,-}_{\omega,t+1,x}-\theta^{R,\sum,+}_{\omega,t,x} +\theta^{R,\sum,+}_{\omega,t+1,x}\Big)=0; \forall t\in T, x\in X, b\in B \quad (\xi^{G}_{p,t,x,b}) \tag{33}$$

$$-\pi_\omega O^{G,-}_{x,t,b}-\psi_{\omega,t}-\theta^{G,\sum,-}_{\omega,t,x,b}+\theta^{G,\sum,+}_{\omega,t,x,b}+\theta^{G,-}_{\omega,t,x,b}-\theta^{R,\sum,-}_{\omega,t,x}$$

$$+ \theta^{R,\sum,-}_{\omega,t+1,x} + \theta^{R,\sum,+}_{\omega,t,x} - \theta^{R,\sum,+}_{\omega,t+1,x} = 0;$$
$$\forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (\xi^{G,-}_{p,\omega,t,x,b}) \tag{34}$$

$$- \pi_\omega O^{G,+}_{x,t,b} + \psi_{\omega,t} + \theta^{G,\sum,-}_{\omega,t,x,b} - \theta^{G,\sum,+}_{\omega,t,x,b} + \theta^{G,+}_{\omega,t,x,b} + \theta^{R,\sum,-}_{\omega,t,x}$$
$$- \theta^{R,\sum,-}_{\omega,t+1,x} - \theta^{R,\sum,+}_{\omega,t,x} + \theta^{R,\sum,+}_{\omega,t+1,x} = 0;$$
$$\forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (\xi^{G,+}_{p,\omega,t,x,b}) \tag{35}$$

$$\pi_\omega O^{C}_{x,t,b} - \psi_{\omega,t} + \theta^{C,-}_{\omega,t,x,b} - \theta^{C,+}_{\omega,t,x,b} - \eta_x \theta^{E}_{\omega,t,x} = 0;$$
$$\forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (\xi^{C}_{p,\omega,t,x,b}) \tag{36}$$

$$- \pi_\omega O^{H}_{x,t,b} + \psi_{\omega,t} + \theta^{H,-}_{\omega,t,x,b} - \theta^{H,+}_{\omega,t,x,b} + \theta^{E}_{\omega,t,x} = 0;$$
$$\forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (\xi^{H}_{p,\omega,t,x,b}) \tag{37}$$

$$\theta^{E,-}_{\omega,t,x} - \theta^{E,+}_{\omega,t,x} + \theta^{E}_{\omega,t,x} - \theta^{E}_{\omega,t+1,x} = 0;$$
$$\forall \omega \in \Omega, t \in T, t < |T|, x \in X \quad (\xi^{E}_{p,\omega,t,x}) \tag{38}$$

$$\theta^{E,-}_{\omega,|T|,x} - \theta^{E,+}_{\omega,|T|,x} + \theta^{E}_{\omega,t,x} - \theta^{E,0}_{\omega,x} = 0;$$
$$\forall \omega \in \Omega, x \in X \quad (\xi^{E}_{p,\omega,|T|,x}) \tag{39}$$

$$\theta^{D,-}_{\omega,t,b}, \theta^{D,+}_{\omega,t,b} \geq 0;$$
$$\forall \omega \in \Omega, t \in T, b \in B \quad (\mu^{D,-}_{p,\omega,t,b}, \mu^{D,+}_{p,\omega,t,b}) \tag{40}$$

$$\theta^{G,\sum,-}_{\omega,t,x,b}, \theta^{G,\sum,+}_{\omega,t,x,b} \geq 0; \forall \omega \in \Omega, t \in T, x \in X, b \in B$$
$$(\mu^{G,\sum,-}_{p,\omega,t,x,b}, \mu^{G,\sum,+}_{p,\omega,t,x,b}) \tag{41}$$

$$\theta^{G,-}_{\omega,t,x,b}, \theta^{G,+}_{\omega,t,x,b} \geq 0; \forall \omega \in \Omega, t \in T, x \in X, b \in B$$
$$(\mu^{G,-}_{p,\omega,t,x,b}, \mu^{G,+}_{p,\omega,t,x,b}) \tag{42}$$

$$\theta^{R,-}_{t,x}, \theta^{R,+}_{t,x} \geq 0; \forall t \in T, x \in X \quad (\mu^{R,-}_{p,t,x}, \mu^{R,+}_{p,t,x}) \tag{43}$$

$$\theta^{R,\sum,-}_{\omega,t,x}, \theta^{R,\sum,+}_{\omega,t,x} \geq 0;$$
$$\forall \omega \in \Omega, t \in T, x \in X \quad (\mu^{R,\sum,-}_{p,\omega,t,x}, \mu^{R,\sum,+}_{p,\omega,t,x}) \tag{44}$$

$$\theta^{C,-}_{\omega,t,x,b}, \theta^{C,+}_{\omega,t,x,b} \geq 0; \forall \omega \in \Omega, t \in T, x \in X, b \in B$$
$$(\mu^{C,-}_{p,\omega,t,x,b}, \mu^{C,+}_{p,\omega,t,x,b}) \tag{45}$$

$$\theta^{H,-}_{\omega,t,x,b}, \theta^{H,+}_{\omega,t,x,b} \geq 0; \forall \omega \in \Omega, t \in T, x \in X, b \in B$$
$$(\mu^{H,-}_{p,\omega,t,x,b}, \mu^{H,+}_{p,\omega,t,x,b}) \tag{46}$$

$$\theta^{E,-}_{\omega,t,x}, \theta^{E,+}_{\omega,t,x} \geq 0;$$
$$\forall \omega \in \Omega, t \in T, x \in X \quad (\mu^{E,-}_{p,\omega,t,x}, \mu^{E,+}_{p,\omega,t,x}) \tag{47}$$

$$\sum_{\omega \in \Omega, t \in T, b \in B} \pi_\omega \cdot \Bigg[\sum_{x \in X} \Big(O^{G}_{x,t,b} G_{x,t,b} + O^{G,-}_{x,t,b} G^{-}_{\omega,x,t,b}$$
$$+ O^{G,+}_{x,t,b} G^{+}_{\omega,x,t,b} - O^{C}_{x,t,b} S^{C}_{\omega,x,t,b} + O^{H}_{x,t,b} S^{H}_{\omega,x,t,b} \Big)$$
$$- U_{\omega,t,b} D_{\omega,t,b} \Bigg] = - \sum_{\omega \in \Omega, t \in T, b \in B} \Bigg[\bar{D}_{\omega,t,b} \theta^{D,+}_{\omega,t,b}$$
$$+ \sum_{x \in X} \Big(\bar{G}_{\omega,x,t,b} \theta^{G,\sum,+}_{\omega,t,x,b} + \bar{S}^{C}_{\omega,x,b} \theta^{C,+}_{\omega,t,x,b} +$$
$$\bar{S}^{H}_{\omega,x,b} \theta^{H,+}_{\omega,t,x,b} \Big) \Bigg] - \sum_{t \in T, x \in X} \Bigg[R^{D}_{x} \theta^{R,-}_{t,x} + R^{U}_{x} \theta^{R,+}_{t,x}$$
$$+ \sum_{\omega \in \Omega} \Big(R^{D}_{x} \theta^{R,\sum,-}_{\omega,t,x} + R^{U}_{x} \theta^{R,\sum,+}_{\omega,t,x} + \bar{E}_{\omega,x} \theta^{E,+}_{\omega,t,x} \Big) \Bigg]$$
$$+ \sum_{x \in X, b \in B} \Bigg[G_{x,0,b} \cdot \Bigg(\theta^{R,-}_{1,x} - \theta^{R,+}_{1,x} + \sum_{\omega \in \Omega} \Big(\theta^{R,\sum,-}_{\omega,1,x}$$
$$- \theta^{R,\sum,+}_{\omega,1,x} \Big) \Bigg) \Bigg] + \sum_{\omega \in \Omega, x \in X} E_{x,0} \cdot \big(\theta^{E}_{\omega,1,x}$$
$$+ \theta^{E,0}_{\omega,x} \big). \quad (\zeta_p) \tag{48}$$

The variable set of (14)–(16), (20)–(48) is expanded to include the dual variables of (1)–(13). The Lagrange multiplier that is associated with each of (20)–(48) is in parentheses to its right.

B. Candidate Equilibria

A market equilibrium solves (14)–(16), (20)–(48), $\forall p \in P$ simultaneously. We find candidate equilibria by solving simultaneously the Karush-Kuhn-Tucker (KKT) conditions of (14)–(16), (20)–(48), $\forall p \in P$, which are:

$$- \delta^{C}_{p,t,x,b} + \delta^{C}_{p,t,x,b+1} + \sum_{\omega \in \Omega} \pi_\omega \cdot \big(\xi^{C}_{p,\omega,t,x,b} - S^{C}_{\omega,x,t,b} \zeta_p \big)$$
$$= 0; \forall t \in T, x \in \Delta^{S}_{p}, b \in B \tag{49}$$

$$- \delta^{H}_{p,t,x,b} + \delta^{H}_{p,t,x,b+1} - \sum_{\omega \in \Omega} \pi_\omega \cdot \big(\xi^{H}_{p,\omega,t,x,b} - S^{H}_{\omega,x,t,b} \zeta_p \big)$$
$$= 0; \forall t \in T, x \in \Delta^{S}_{p}, b \in B \tag{50}$$

$$- \phi_{p,\omega,t} - \vartheta^{D,-}_{p,\omega,t,b} + \vartheta^{D,+}_{p,\omega,t,b} - \pi_\omega U_{\omega,t,b} \zeta_p = 0;$$
$$\forall \omega \in \Omega, t \in T, b \in B \tag{51}$$

$$\sum_{\omega \in \Omega} \Big(\phi_{p,\omega,t} - \vartheta^{G,\sum,-}_{p,\omega,t,x,b} + \vartheta^{G,\sum,+}_{p,\omega,t,x,b} - \vartheta^{R,\sum,-}_{p,\omega,t,x} + \vartheta^{R,\sum,-}_{p,\omega,t+1,x}$$
$$+ \vartheta^{R,\sum,+}_{p,\omega,t,x} - \vartheta^{R,\sum,+}_{p,\omega,t+1,x} \Big) - \vartheta^{R,-}_{p,t,x} + \vartheta^{R,-}_{p,t+1,x} + \vartheta^{R,+}_{p,t,x}$$
$$- \vartheta^{R,+}_{p,t+1,x} + O^{G}_{x,t,b} \zeta_p = 0; \forall t \in T, x \in X, b \in B \tag{52}$$

$$- \phi_{p,\omega,t} + \vartheta^{G,\sum,-}_{p,\omega,t,x,b} - \vartheta^{G,\sum,+}_{p,\omega,t,x,b} - \vartheta^{G,-}_{p,\omega,t,x,b} + \vartheta^{R,\sum,-}_{p,\omega,t,x}$$
$$- \vartheta^{R,\sum,-}_{p,\omega,t+1,x} - \vartheta^{R,\sum,+}_{p,\omega,t,x} + \vartheta^{R,\sum,+}_{p,\omega,t+1,x} + \pi_\omega O^{G,-}_{x,t,b} \zeta_p = 0;$$
$$\forall \omega \in \Omega, t \in T, x \in X, b \in B \tag{53}$$

$$\phi_{p,\omega,t} - \vartheta^{G,\sum,-}_{p,\omega,t,x,b} + \vartheta^{G,\sum,+}_{p,\omega,t,x,b} - \vartheta^{G,+}_{p,\omega,t,x,b} - \vartheta^{R,\sum,-}_{p,\omega,t,x}$$
$$+ \vartheta^{R,\sum,-}_{p,\omega,t+1,x} + \vartheta^{R,\sum,+}_{p,\omega,t,x} - \vartheta^{R,\sum,+}_{p,\omega,t+1,x} + \pi_\omega O^{G,+}_{x,t,b} \zeta_p = 0;$$
$$\forall \omega \in \Omega, t \in T, x \in X, b \in B \tag{54}$$

$$\pi_\omega \psi_{\omega,t} - \phi_{p,\omega,t} - \vartheta^{C,-}_{p,\omega,t,x,b} + \vartheta^{C,+}_{p,\omega,t,x,b} - \eta_x \vartheta^{E}_{p,\omega,t,x}$$
$$- \pi_\omega O^{C}_{x,t,b} \zeta_p = 0; \forall \omega \in \Omega, t \in T, x \in X, b \in B \tag{55}$$

$$- \pi_\omega \psi_{\omega,t} + \phi_{p,\omega,t} - \vartheta^{H,-}_{p,\omega,t,x,b} + \vartheta^{H,+}_{p,\omega,t,x,b} + \vartheta^{E}_{p,\omega,t,x}$$
$$+ \pi_\omega O^{H}_{x,t,b} \zeta_p = 0; \forall \omega \in \Omega, t \in T, x \in X, b \in B \tag{56}$$

$$- \vartheta^{E,-}_{p,\omega,t,x} + \vartheta^{E,+}_{p,\omega,t,x} + \vartheta^{E}_{p,\omega,t,x} - \vartheta^{E}_{p,\omega,t+1,x} = 0;$$
$$\forall \omega \in \Omega, t \in T, t < |T|, x \in X, b \in B \tag{57}$$

$$- \vartheta^{E,-}_{p,\omega,|T|,x} + \vartheta^{E,+}_{p,\omega,|T|,x} + \vartheta^{E}_{p,\omega,|T|,x} - \vartheta^{E,0}_{p,\omega,x} = 0;$$
$$\forall \omega \in \Omega, x \in X, b \in B \tag{58}$$

$$\sum_{b \in B} \Bigg\{ - \xi^{D}_{p,\omega,t,b} + \sum_{x \in X} \Big[\pi_\omega \cdot \big(S^{C}_{\omega,x,t,b} - S^{H}_{\omega,x,t,b} \big) + \xi^{G}_{p,t,x,b}$$
$$- \xi^{G,-}_{p,\omega,t,x,b} + \xi^{G,+}_{p,\omega,t,x,b} - \xi^{C}_{p,\omega,t,x,b} + \xi^{H}_{p,\omega,t,x,b} \Big] \Bigg\} = 0;$$
$$\forall \omega \in \Omega, t \in T \tag{59}$$

$$\xi^{D}_{p,\omega,t,b} - \mu^{D,-}_{p,\omega,t,b} = 0; \forall \omega \in \Omega, t \in T, b \in B \tag{60}$$

$$- \xi^{D}_{p,\omega,t,b} - \mu^{D,+}_{p,\omega,t,b} + \bar{D}_{\omega,t,b} \zeta_p = 0;$$

$$\forall \omega \in \Omega, t \in T, b \in B \quad (61)$$

$$\xi^{G}_{p,t,x,b} - \xi^{G,-}_{p,\omega,t,x,b} + \xi^{G,+}_{p,\omega,t,x,b} - \mu^{G,\sum,-}_{p,\omega,t,x,b} = 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (62)$$

$$-\xi^{G}_{p,t,x,b} + \xi^{G,-}_{p,\omega,t,x,b} - \xi^{G,+}_{p,\omega,t,x,b} - \mu^{G,\sum,+}_{p,\omega,t,x,b} + \bar{G}_{\omega,x,t,b}\zeta_p = 0; \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (63)$$

$$\xi^{G,-}_{p,\omega,t,x,b} - \mu^{G,-}_{p,\omega,t,x,b} = 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (64)$$

$$\xi^{G,+}_{p,\omega,t,x,b} - \mu^{G,+}_{p,\omega,t,x,b} = 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (65)$$

$$\sum_{b \in B} \left(\xi^{G}_{p,t,x,b} - \xi^{G}_{p,t-1,x,b}\right) - \mu^{R,-}_{p,t,x} + R^{D}_{x}\zeta_p = 0; \quad \forall t \in T, t > 1, x \in X \quad (66)$$

$$\sum_{b \in B} \left(\xi^{G}_{p,1,x,b} - G_{x,0,b}\zeta_p\right) - \mu^{R,-}_{p,1,x} + R^{D}_{x}\zeta_p = 0; \quad \forall x \in X \quad (67)$$

$$\sum_{b \in B} \left(\xi^{G}_{p,t-1,x,b} - \xi^{G}_{p,t,x,b}\right) - \mu^{R,+}_{p,t,x} + R^{U}_{x}\zeta_p = 0; \quad \forall t \in T, t > 1, x \in X \quad (68)$$

$$\sum_{b \in B} \left(G_{x,0,b}\zeta_p - \xi^{G}_{p,1,x,b}\right) - \mu^{R,+}_{p,1,x} + R^{U}_{x}\zeta_p = 0; \quad \forall x \in X \quad (69)$$

$$\sum_{b \in B} \Big(\xi^{G}_{p,t,x,b} - \xi^{G}_{p,t-1,x,b} - \xi^{G,-}_{p,\omega,t,x,b} + \xi^{G,-}_{p,\omega,t-1,x,b} + \xi^{G,+}_{p,\omega,t,x,b} - \xi^{G,+}_{p,\omega,t-1,x,b}\Big) - \mu^{R,\sum,-}_{p,\omega,t,x} + R^{D}_{x}\zeta_p = 0; \quad \forall \omega \in \Omega, t \in T, t > 1, x \in X \quad (70)$$

$$\sum_{b \in B} \Big(\xi^{G}_{p,1,x,b} - \xi^{G,-}_{p,\omega,1,x,b} + \xi^{G,+}_{p,\omega,1,x,b} - G_{x,0,b}\zeta_p\Big) - \mu^{R,\sum,-}_{p,\omega,1,x} + R^{D}_{x}\zeta_p = 0; \forall \omega \in \Omega, x \in X \quad (71)$$

$$\sum_{b \in B} \Big(\xi^{G}_{p,t-1,x,b} - \xi^{G}_{p,t,x,b} - \xi^{G,-}_{p,\omega,t-1,x,b} + \xi^{G,-}_{p,\omega,t,x,b} + \xi^{G,+}_{p,\omega,t-1,x,b} - \xi^{G,+}_{p,\omega,t,x,b}\Big) - \mu^{R,\sum,+}_{p,\omega,t,x} + R^{U}_{x}\zeta_p = 0; \quad \forall \omega \in \Omega, t \in T, t > 1, x \in X \quad (72)$$

$$\sum_{b \in B} \Big(G_{x,0,b}\zeta_p - \xi^{G}_{p,1,x,b} + \xi^{G,-}_{p,\omega,1,x,b} - \xi^{G,+}_{p,\omega,1,x,b}\Big) - \mu^{R,\sum,+}_{p,\omega,1,x} + R^{U}_{x}\zeta_p = 0; \forall \omega \in \Omega, x \in X \quad (73)$$

$$\xi^{C}_{p,\omega,t,x,b} - \mu^{C,-}_{p,\omega,t,x,b} = 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (74)$$

$$-\xi^{C}_{p,\omega,t,x,b} - \mu^{C,+}_{p,\omega,t,x,b} + \bar{S}^{C}_{\omega,x,b}\zeta_p = 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (75)$$

$$\xi^{H}_{p,\omega,t,x,b} - \mu^{H,-}_{p,\omega,t,x,b} = 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (76)$$

$$-\xi^{H}_{p,\omega,t,x,b} - \mu^{H,+}_{p,\omega,t,x,b} + \bar{S}^{H}_{\omega,x,b}\zeta_p = 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (77)$$

$$\xi^{E}_{p,\omega,t,x} - \mu^{E,-}_{p,\omega,t,x} = 0; \forall \omega \in \Omega, t \in T, x \in X \quad (78)$$

$$-\xi^{E}_{p,\omega,t,x} - \mu^{E,+}_{p,\omega,t,x} + \bar{E}_{\omega,x}\zeta_p = 0; \quad \forall \omega \in \Omega, t \in T, x \in X \quad (79)$$

$$-\eta_x\xi^{C}_{p,\omega,t,x,b} + \xi^{H}_{p,\omega,t,x,b} + \xi^{E}_{p,\omega,t,x} - \xi^{E}_{p,\omega,t-1,x} = 0; \quad \forall \omega \in \Omega, t \in T, t > 1, x \in X \quad (80)$$

$$-\eta_x\xi^{C}_{p,\omega,1,x,b} + \xi^{H}_{p,\omega,1,x,b} + \xi^{E}_{p,\omega,1,x} - E_{x,0}\zeta_p = 0; \quad \forall \omega \in \Omega, x \in X \quad (81)$$

$$-\xi^{E}_{p,\omega,|T|,x} - E_{x,0}\zeta_p = 0; \forall \omega \in \Omega, x \in X \quad (82)$$

(20), (30)–(39), (48) (83)

$$O^{C}_{x,t,b} \geq O^{C}_{x,t,b-1} \perp \delta^{C}_{p,t,x,b} \geq 0; \quad \forall t \in T, x \in \Delta^{S}_{p}, b \in B \quad (84)$$

$$O^{H}_{x,t,b} \geq O^{H}_{x,t,b-1} \perp \delta^{H}_{p,t,x,b} \geq 0; \quad \forall t \in T, x \in \Delta^{S}_{p}, b \in B \quad (85)$$

$$0 \leq D_{\omega,t,b} \perp \vartheta^{D,-}_{p,\omega,t,b} \geq 0; \forall \omega \in \Omega, t \in T, b \in B \quad (86)$$

$$D_{\omega,t,b} \leq \bar{D}_{\omega,t,b} \perp \vartheta^{D,+}_{p,\omega,t,b} \geq 0; \forall \omega \in \Omega, t \in T, b \in B \quad (87)$$

$$0 \leq G_{x,t,b} - G^{-}_{\omega,x,t,b} + G^{+}_{\omega,x,t,b} \perp \vartheta^{G,\sum,-}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (88)$$

$$G_{x,t,b} - G^{-}_{\omega,x,t,b} + G^{+}_{\omega,x,t,b} \leq \bar{G}_{\omega,x,t,b} \perp \vartheta^{G,\sum,+}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (89)$$

$$0 \leq G^{-}_{\omega,x,t,b} \perp \vartheta^{G,-}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (90)$$

$$0 \leq G^{+}_{\omega,x,t,b} \perp \vartheta^{G,+}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (91)$$

$$-R^{D}_{x} \leq \sum_{b \in B}(G_{x,t,b} - G_{x,t-1,b}) \perp \vartheta^{R,-}_{p,t,x} \geq 0; \quad \forall t \in T, x \in X \quad (92)$$

$$\sum_{b \in B}(G_{x,t,b} - G_{x,t-1,b}) \leq R^{U}_{x} \perp \vartheta^{R,+}_{p,t,x} \geq 0; \quad \forall t \in T, x \in X \quad (93)$$

$$-R^{D}_{x} \leq \sum_{b \in B} \Big(G_{x,t,b} - G^{-}_{\omega,x,t,b} + G^{+}_{\omega,x,t,b} - G_{x,t-1,b} + G^{-}_{\omega,x,t-1,b} - G^{+}_{\omega,x,t-1,b}\Big) \perp \vartheta^{R,\sum,-}_{p,\omega,t,x} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X \quad (94)$$

$$\sum_{b \in B} \Big(G_{x,t,b} - G^{-}_{\omega,x,t,b} + G^{+}_{\omega,x,t,b} - G_{x,t-1,b} + G^{-}_{\omega,x,t-1,b} - G^{+}_{\omega,x,t-1,b}\Big) \leq R^{U}_{x} \perp \vartheta^{R,\sum,+}_{p,\omega,t,x} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X \quad (95)$$

$$0 \leq S^{C}_{\omega,x,t,b} \perp \vartheta^{C,-}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (96)$$

$$S^{C}_{\omega,x,t,b} \leq \bar{S}^{C}_{\omega,x,b} \perp \vartheta^{C,+}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (97)$$

$$0 \leq S^{H}_{\omega,x,t,b} \perp \vartheta^{H,-}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (98)$$

$$S^{H}_{\omega,x,t,b} \leq \bar{S}^{H}_{\omega,x,b} \perp \vartheta^{H,+}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (99)$$

$$0 \leq E_{\omega,x,t} \perp \vartheta^{E,-}_{p,\omega,t,x} \geq 0; \forall \omega \in \Omega, t \in T, x \in X \quad (100)$$

$$E_{\omega,x,t} \leq \bar{E}_{\omega,x} \perp \vartheta^{E,+}_{p,\omega,t,x} \geq 0; \forall \omega \in \Omega, t \in T, x \in X \quad (101)$$
$$\theta^{D,-}_{\omega,t,b} \geq 0 \perp \mu^{D,-}_{p,\omega,t,b} \geq 0; \forall \omega \in \Omega, t \in T, b \in B \quad (102)$$
$$\theta^{D,+}_{\omega,t,b} \geq 0 \perp \mu^{D,+}_{p,\omega,t,b} \geq 0; \forall \omega \in \Omega, t \in T, b \in B \quad (103)$$
$$\theta^{G,\sum,-}_{\omega,t,x,b} \geq 0 \perp \mu^{G,\sum,-}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (104)$$
$$\theta^{G,\sum,+}_{\omega,t,x,b} \geq 0 \perp \mu^{G,\sum,+}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (105)$$
$$\theta^{G,-}_{\omega,t,x,b} \geq 0 \perp \mu^{G,-}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (106)$$
$$\theta^{G,+}_{\omega,t,x,b} \geq 0 \perp \mu^{G,+}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (107)$$
$$\theta^{R,-}_{t,x} \geq 0 \perp \mu^{R,-}_{p,t,x} \geq 0; \forall t \in T, x \in X \quad (108)$$
$$\theta^{R,+}_{t,x} \geq 0 \perp \mu^{R,+}_{p,t,x} \geq 0; \forall t \in T, x \in X \quad (109)$$
$$\theta^{R,\sum,-}_{\omega,t,x} \geq 0 \perp \mu^{R,\sum,-}_{p,\omega,t,x} \geq 0; \forall \omega \in \Omega, t \in T, x \in X \quad (110)$$
$$\theta^{R,\sum,+}_{\omega,t,x} \geq 0 \perp \mu^{R,\sum,+}_{p,\omega,t,x} \geq 0; \forall \omega \in \Omega, t \in T, x \in X \quad (111)$$
$$\theta^{C,-}_{\omega,t,x,b} \geq 0 \perp \mu^{C,-}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (112)$$
$$\theta^{C,+}_{\omega,t,x,b} \geq 0 \perp \mu^{C,+}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (113)$$
$$\theta^{H,-}_{\omega,t,x,b} \geq 0 \perp \mu^{H,-}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (114)$$
$$\theta^{H,+}_{\omega,t,x,b} \geq 0 \perp \mu^{H,+}_{p,\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (115)$$
$$\theta^{E,-}_{\omega,t,x} \geq 0 \perp \mu^{E,-}_{p,\omega,t,x} \geq 0; \forall \omega \in \Omega, t \in T, x \in X \quad (116)$$
$$\theta^{E,+}_{\omega,t,x} \geq 0 \perp \mu^{E,+}_{p,\omega,t,x} \geq 0; \forall \omega \in \Omega, t \in T, x \in X; \quad (117)$$

where the decision variables are all of the primal and dual variables of (1)–(13) and all of the primal variables and Lagrange multipliers of (14)–(16), (20)–(48).

C. Linearizing KKT Conditions

KKT conditions (49)–(117) have nonlinearities, which complicate their solution. We address this difficulty as follows.

1) Bi-linear Terms in (48)

Equality (48) contains bi-linear terms. Because (1)–(13) is linear, (48) can be replaced in (83) by the equivalent complementary-slackness conditions [40], [69]:

$$0 \leq D_{\omega,t,b} \perp \theta^{D,-}_{\omega,t,b} \geq 0; \forall \omega \in \Omega, t \in T, b \in B \quad (118)$$
$$D_{\omega,t,b} \leq \bar{D}_{\omega,t,b} \perp \theta^{D,+}_{\omega,t,b} \geq 0; \forall \omega \in \Omega, t \in T, b \in B \quad (119)$$
$$0 \leq G_{x,t,b} - G^-_{\omega,x,t,b} + G^+_{\omega,x,t,b} \perp \theta^{G,\sum,-}_{\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (120)$$
$$G_{x,t,b} - G^-_{\omega,x,t,b} + G^+_{\omega,x,t,b} \leq \bar{G}_{\omega,x,t,b} \perp \theta^{G,\sum,+}_{\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (121)$$
$$0 \leq G^-_{\omega,x,t,b} \perp \theta^{G,-}_{\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (122)$$
$$0 \leq G^+_{\omega,x,t,b} \perp \theta^{G,+}_{\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (123)$$
$$-R^D_x \leq \sum_{b \in B} (G_{x,t,b} - G_{x,t-1,b}) \perp \theta^{R,-}_{t,x} \geq 0; \quad \forall t \in T, x \in X \quad (124)$$
$$\sum_{b \in B} (G_{x,t,b} - G_{x,t-1,b}) \leq R^U_x \perp \theta^{R,+}_{t,x} \geq 0; \quad \forall t \in T, x \in X \quad (125)$$
$$-R^D_x \leq \sum_{b \in B} \Big(G_{x,t,b} - G^-_{\omega,x,t,b} + G^+_{\omega,x,t,b} - G_{x,t-1,b} + G^-_{\omega,x,t-1,b} - G^+_{\omega,x,t-1,b}\Big) \perp \theta^{R,\sum,-}_{\omega,t,x} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X \quad (126)$$
$$\sum_{b \in B} \Big(G_{x,t,b} - G^-_{\omega,x,t,b} + G^+_{\omega,x,t,b} - G_{x,t-1,b} + G^-_{\omega,x,t-1,b} - G^+_{\omega,x,t-1,b}\Big) \leq R^U_x \perp \theta^{R,\sum,+}_{\omega,t,x} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X \quad (127)$$
$$0 \leq S^C_{\omega,x,t,b} \perp \theta^{C,-}_{\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (128)$$
$$S^C_{\omega,x,t,b} \leq \bar{S}^C_{\omega,x,b} \perp \theta^{C,+}_{\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (129)$$
$$0 \leq S^H_{\omega,x,t,b} \perp \theta^{H,-}_{\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (130)$$
$$S^H_{\omega,x,t,b} \leq \bar{S}^H_{\omega,x,b} \perp \theta^{H,+}_{\omega,t,x,b} \geq 0; \quad \forall \omega \in \Omega, t \in T, x \in X, b \in B \quad (131)$$
$$0 \leq E_{\omega,x,t} \perp \theta^{E,-}_{\omega,t,x} \geq 0; \forall \omega \in \Omega, t \in T, x \in X \quad (132)$$
$$E_{\omega,x,t} \leq \bar{E}_{\omega,x} \perp \theta^{E,+}_{\omega,t,x} \geq 0; \forall \omega \in \Omega, t \in T, x \in X. \quad (133)$$

2) Other Bi-linear Terms

Conditions (49)–(50) and (55)–(56) contain bi-linear terms, in which ζ_p multiplies another variable. We address this non-linearity by parameterizing and fixing the value of ζ_p, which is a standard approach that is used in the literature [33], [35], [70].

3) Complementary Slackness

Conditions (84)–(133) are non-linear, because a generic complementary-slackness condition of the form:

$$h(y) \leq 0 \perp \rho \geq 0; \quad (134)$$

is equivalent to:

$$h(y) \leq 0 \quad (135)$$
$$\rho \geq 0 \quad (136)$$
$$h(y)\rho = 0. \quad (137)$$

We address this non-linearity using the technique of Fortuny-Amat and McCarl [71], which requires the introduction of an auxiliary binary or special-ordered-set variable, which we denote as β, and an arbitrarily large constant, which we denote as M. With this auxiliary variable and constant, (134), or, equivalently, (135)–(137), is equivalent to:

$$-M\beta \leq h(y) \leq 0$$
$$M \cdot (1-\beta) \geq \rho \geq 0.$$

We linearize each of (84)–(133) using this technique, which requires introducing one auxiliary variable for each condition that is linearized.

D. Verifying Nash Equilibria

If ζ_p is fixed and (84)–(133) are linearized, (20), (30)–(39), (49)–(82), (84)–(133), $\forall p \in P$ is a mixed-integer linear program (MILP). An MILP solution is a candidate Nash equilibrium because KKT conditions (49)–(117) are necessary but not sufficient for an optimum of each firm's expected-profit maximization (each firm's problem satisfies Slater conditions [72]). $\forall p \in P$ we let $\bar{\Lambda}_p$ denote the value of (14) for firm p from the MILP solution.

We verify that an MILP solution is a Nash equilibrium using diagonalization, which is a standard approach [33], [35], [40]. Diagonalization involves solving (14)–(16), (20)–(48), $\forall p \in P$ while holding the offers of all firms except for p equal to the MILP solution and letting $\tilde{\Lambda}_p$ equal the value of (14) for firm p. If $\tilde{\Lambda}_p \leq \bar{\Lambda}_p$, $\forall p \in P$, then the MILP solution has the Nash property of no firm having a profitable deviation. Otherwise, the MILP solution is not a Nash equilibrium.

In the course of examining our example and case study we do not find any MILP solutions that fail the diagonalization test. However, such an outcome can occur. In such a case, one could re-solve the MILP by providing the solver a different starting solution (*i.e.*, the MILP may have multiple optimal solutions) or by changing the value of ζ_p, $\forall p \in P$. Subject to some mild assumptions, all economic games, such as the one that we model, are guaranteed to have at least one Nash equilibrium [73]. Thus, we know that the MILP should yield a Nash equilibrium. Without much stronger assumptions, there is no guarantee that a Nash equilibrium is unique [7]. Thus, it is possible that our example and case study may have other equilibria that yield different market and welfare outcomes.

ACKNOWLEDGMENT

The authors thank A. Shahmohammadi for motivating this work and the editors, four anonymous reviewers, and A. Sorooshian for useful conversations, suggestions, and comments. Any errors or omissions are attributable to the authors alone.

REFERENCES

[1] T. Mai, D. Sandor, R. Wiser, and T. R. Schneider, "Renewable Electricity Futures Study: Executive Summary," National Renewable Energy Laboratory, Golden, CO, Tech. Rep. NREL/TP-6A20-52409-ES, 2012.

[2] A. Bloom, A. Townsend, D. Palchak, J. Novacheck, J. King, C. Barrows, E. Ibanez, M. O'Connell, G. Jordan, B. Roberts, C. Draxl, and K. Gruchalla, "Eastern Renewable Generation Integration Study," National Renewable Energy Laboratory, Golden, CO, Tech. Rep. NREL/TP-6A20-64472, August 2016.

[3] J. E. Bistline, "Economic and technical challenges of flexible operations under large-scale variable renewable deployment," *Energy Economics*, vol. 64, pp. 363–372, May 2017.

[4] C. Zöphel, S. Schreiber, T. Müller, and D. Möst, "Which Flexibility Options Facilitate the Integration of Intermittent Renewable Energy Sources in Electricity Systems?" *Current Sustainable/Renewable Energy Reports*, vol. 5, pp. 37–44, March 2018.

[5] P. Denholm, E. Ela, B. Kirby, and M. R. Milligan, "The Role of Energy Storage with Renewable Electricity Generation," National Renewable Energy Laboratory, Golden, CO, Tech. Rep. NREL/TP-6A2-47187, January 2010.

[6] A. Evans, V. Strezov, and T. J. Evans, "Assessment of utility energy storage options for increased renewable energy penetration," *Renewable and Sustainable Energy Reviews*, vol. 16, pp. 4141–4147, August 2012.

[7] R. Sioshansi, P. Denholm, J. Arteaga, S. Awara, S. Bhattacharjee, A. Botterud, W. Cole, A. Cortés, A. de Queiroz, J. DeCarolis, Z. Ding, N. DiOrio, Y. Dvorkin, U. Helman, J. X. Johnson, I. Konstantelos, T. Mai, H. Pandžić, D. Sodano, G. Stephen, A. Svoboda, H. Zareipour, and Z. Zhang, "Energy-Storage Modeling: State-of-the-Art and Future Research Directions," *IEEE Transactions on Power Systems*, vol. 37, pp. 860–875, March 2022.

[8] J. V. Paatero and P. D. Lund, "Effect of energy storage on variations in wind power," *Wind Energy*, vol. 8, pp. 421–441, October/December 2005.

[9] P. Denholm and M. Hand, "Grid flexibility and storage required to achieve very high penetration of variable renewable electricity," *Energy Policy*, vol. 39, pp. 1817–1830, March 2011.

[10] A. Tuohy and M. O'Malley, "Pumped storage in systems with very high wind penetration," *Energy Policy*, vol. 39, pp. 1965–1974, April 2011.

[11] A. A. Solomon, D. M. Kammen, and D. Callaway, "The role of large-scale energy storage design and dispatch in the power grid: A study of very high grid penetration of variable renewable resources," *Applied Energy*, vol. 134, pp. 75–89, 1 December 2014.

[12] C. O'Dwyer and D. Flynn, "Using Energy Storage to Manage High Net Load Variability at Sub-Hourly Time-Scales," *IEEE Transactions on Power Systems*, vol. 30, pp. 2139–2148, July 2015.

[13] D. J. Swider, "Compressed Air Energy Storage in an Electricity System With Significant Wind Power Generation," *IEEE Transactions on Energy Conversion*, vol. 22, pp. 95–102, March 2007.

[14] K. Bruninx, Y. Dvorkin, E. Delarue, H. Pandžić, W. D'haeseleer, and D. S. Kirschen, "Coupling Pumped Hydro Energy Storage With Unit Commitment," *IEEE Transactions on Sustainable Energy*, vol. 7, pp. 786–796, April 2016.

[15] G. N. Bathurst and G. Strbac, "Value of combining energy storage and wind in short-term energy and balancing markets," *Electric Power Systems Research*, vol. 67, pp. 1–8, October 2003.

[16] M. Korpaas, A. T. Holen, and R. Hildrum, "Operation and sizing of energy storage for wind power plants in a market system," *International Journal of Electrical Power and Energy Systems*, vol. 25, pp. 599–606, October 2003.

[17] E. D. Castronuovo and J. A. P. Lopes, "On the Optimization of the Daily Operation of a Wind-Hydro Power Plant," *IEEE Transactions on Power Systems*, vol. 19, pp. 1599–1606, August 2004.

[18] P. Denholm, "Improving the technical, environmental and social performance of wind energy systems using biomass-based energy storage," *Renewable Energy*, vol. 31, pp. 1355–1370, July 2006.

[19] J. B. Greenblatt, S. Succar, D. C. Denkenberger, R. H. Williams, and R. H. Socolow, "Baseload wind energy: modeling the competition between gas turbines and compressed air energy storage for supplemental generation," *Energy Policy*, vol. 35, pp. 1474–1492, March 2007.

[20] E. Hittinger, J. F. Whitacre, and J. Apt, "Compensating for wind variability using co-located natural gas generation and energy storage," *Energy Systems*, vol. 1, pp. 417–439, December 2010.

[21] S. H. Madaeni, R. Sioshansi, and P. Denholm, "How Thermal Energy Storage Enhances the Economic Viability of Concentrating Solar Power," *Proceedings of the IEEE*, vol. 100, pp. 335–347, February 2012.

[22] H. Ding, Z. Hu, and Y. Song, "Stochastic optimization of the daily operation of wind farm and pumped-hydro-storage plant," *Renewable Energy*, vol. 48, pp. 571–578, December 2012.

[23] J. Chen and H. E. Garcia, "Economic optimization of operations for hybrid energy systems under variable markets," *Applied Energy*, vol. 177, pp. 11–24, 1 September 2016.

[24] J. Martinek, J. Jorgenson, M. Mehos, and P. Denholm, "A comparison of price-taker and production cost models for determining system value, revenue, and scheduling of concentrating solar power plants," *Applied Energy*, vol. 231, pp. 854–865, 1 December 2018.

[25] S. Chen, A. J. Conejo, R. Sioshansi, and Z. Wei, "Operational Equilibria of Electric and Natural Gas Systems with Limited Information Interchange," *IEEE Transactions on Power Systems*, vol. 35, pp. 662–671, January 2020.

[26] Y. Jiang and R. Sioshansi, "What Duality Theory Tells Us About Giving Market Operators the Authority to Dispatch Energy Storage," *The Energy Journal*, 2022, in press.

[27] R. Sioshansi, "Increasing the Value of Wind with Energy Storage," *The Energy Journal*, vol. 32, pp. 1–30, 2011.

[28] H. Ding, P. Pinson, Z. Hu, J. Wang, and Y. Song, "Optimal Offering and Operating Strategy for a Large Wind-Storage System as a Price

Maker," *IEEE Transactions on Power Systems*, vol. 32, pp. 4904–4913, November 2017.

[29] H. Mohsenian-Rad, "Coordinated Price-Maker Operation of Large Energy Storage Units in Nodal Energy Markets," *IEEE Transactions on Power Systems*, vol. 31, pp. 786–797, January 2016.

[30] K. Hartwig and I. Kockar, "Impact of Strategic Behavior and Ownership of Energy Storage on Provision of Flexibility," *IEEE Transactions on Sustainable Energy*, vol. 7, pp. 744–754, April 2016.

[31] P. Zou, Q. Chen, Q. Xia, G. He, C. Kang, and A. J. Conejo, "Pool equilibria including strategic storage," *Applied Energy*, vol. 177, pp. 260–270, 1 September 2016.

[32] E. Nasrolahpour, S. J. Kazempour, H. Zareipour, and W. D. Rosehart, "A Bilevel Model for Participation of a Storage System in Energy and Reserve Markets," *IEEE Transactions on Sustainable Energy*, vol. 9, pp. 582–598, April 2018.

[33] A. Shahmohammadi, R. Sioshansi, A. J. Conejo, and S. Afsharnia, "Market Equilibria and Interactions Between Strategic Generation, Wind, and Storage," *Applied Energy*, vol. 220, pp. 876–892, 15 June 2018.

[34] S. Bhattacharjee, R. Sioshansi, and H. Zareipour, "Benefits of Strategically Sizing Wind-Integrated Energy Storage and Transmission," *IEEE Transactions on Power Systems*, vol. 36, pp. 1141–1151, March 2021.

[35] A. Shahmohammadi, R. Sioshansi, A. J. Conejo, and S. Afsharnia, "The Role of Energy Storage in Mitigating Ramping Inefficiencies Caused by Variable Renewable Generation," *Energy Conversion and Management*, vol. 162, pp. 307–320, 15 April 2018.

[36] R. Sioshansi and S. S. Oren, "How good are supply function equilibrium models: an empirical analysis of the ERCOT balancing market," *Journal of Regulatory Economics*, vol. 31, pp. 1–35, February 2007.

[37] A. Alabdulwahab, A. Abusorrah, X. Zhang, and M. Shahidehpour, "Stochastic Security-Constrained Scheduling of Coordinated Electricity and Natural Gas Infrastructures," *IEEE Systems Journal*, vol. 11, pp. 1674–1683, September 2017.

[38] J. E. Duggan, Jr. and R. Sioshansi, "Another Step Towards Equilibrium Offers in Unit Commitment Auctions with Nonconvex Costs: Multi-Firm Oligopolies," *The Energy Journal*, vol. 40, pp. 249–281, November 2019.

[39] D. Pozo and J. Contreras, "Finding Multiple Nash Equilibria in Pool-Based Markets: A Stochastic EPEC Approach," *IEEE Transactions on Power Systems*, vol. 26, pp. 1744–1752, August 2011.

[40] C. Ruiz, A. J. Conejo, and Y. Smeers, "Equilibria in an Oligopolistic Electricity Pool With Stepwise Offer Curves," *IEEE Transactions on Power Systems*, vol. 27, pp. 752–761, May 2012.

[41] S. J. Kazempour and H. Zareipour, "Equilibria in an Oligopolistic Market With Wind Power Production," *IEEE Transactions on Power Systems*, vol. 29, pp. 686–697, March 2014.

[42] W. Wei, F. Liu, and S. Mei, "Energy Pricing and Dispatch for Smart Grid Retailers Under Demand Response and Market Price Uncertainty," *IEEE Transactions on Smart Grid*, vol. 6, pp. 1364–1374, May 2015.

[43] T. Dai and W. Qiao, "Finding Equilibria in the Pool-Based Electricity Market With Strategic Wind Power Producers and Network Constraints," *IEEE Transactions on Power Systems*, vol. 32, pp. 389–399, January 2017.

[44] H. Ye, Y. Ge, M. Shahidehpour, and Z. Li, "Uncertainty Marginal Price, Transmission Reserve, and Day-Ahead Market Clearing With Robust Unit Commitment," *IEEE Transactions on Power Systems*, vol. 32, pp. 1782–1795, May 2017.

[45] D. Huppmann and S. A. Siddiqui, "An exact solution method for binary equilibrium problems with compensation and the power market uplift problem," *European Journal of Operational Research*, vol. 266, pp. 622–638, 16 April 2018.

[46] F. Graves, T. Jenkin, and D. Murphy, "Opportunities for Electricity Storage in Deregulating Markets," *The Electricity Journal*, vol. 12, pp. 46–56, October 1999.

[47] Y. H. Zhou, A. Scheller-Wolf, N. Secomandi, and S. Smith, "Electricity Trading and Negative Prices: Storage vs. Disposal," *Management Science*, vol. 62, pp. 880–898, March 2016.

[48] Y. Luo, L. Shi, and G. Tu, "Optimal sizing and control strategy of isolated grid with wind power and energy storage system," *Energy Conversion and Management*, vol. 80, pp. 407–415, April 2014.

[49] A. Fleischhacker, H. Auer, G. Lettner, and A. Botterud, "Sharing Solar PV and Energy Storage in Apartment Buildings: Resource Allocation and Pricing," *IEEE Transactions on Smart Grid*, vol. 10, pp. 3963–3973, July 2019.

[50] X. Kong, J. Xiao, C. Wang, K. Cui, Q. Jin, and D. Kong, "Bi-level multi-time scale scheduling method based on bidding for multi-operator virtual power plant," *Applied Energy*, vol. 249, pp. 178–189, 1 September 2019.

[51] G. De Vivero-Serrano, K. Bruninx, and E. Delarue, "Implications of bid structures on the offering strategies of merchant energy storage systems," *Applied Energy*, vol. 251, p. 113375, 1 October 2019.

[52] Q. Huang, Y. Xu, and C. Courcoubetis, "Stackelberg competition between merchant and regulated storage investment in wholesale electricity markets," *Applied Energy*, vol. 264, p. 114669, 15 April 2020.

[53] H. Rashidizadeh-Kermani, M. Vahedipour-Dahraie, M. Shafie-Khah, and P. Siano, "A Regret-Based Stochastic Bi-Level Framework for Scheduling of DR Aggregator Under Uncertainties," *IEEE Transactions on Smart Grid*, vol. 11, pp. 3171–3184, July 2020.

[54] V. A. Evangelopoulos, T. P. Kontopoulos, and P. S. Georgilakis, "Heterogeneous aggregators competing in a local flexibility market for active distribution system management: A bi-level programming approach," *International Journal of Electrical Power & Energy Systems*, vol. 136, p. 107639, March 2022.

[55] R. Sioshansi, S. S. Oren, and R. O'Neill, "Three-Part Auctions versus Self-Commitment in Day-ahead Electricity Markets," *Utilities Policy*, vol. 18, pp. 165–173, December 2010.

[56] R. Sioshansi, "Welfare Impacts of Electricity Storage and the Implications of Ownership Structure," *The Energy Journal*, vol. 31, pp. 173–198, 2010.

[57] W.-P. Schill and C. Kemfert, "Modeling Strategic Electricity Storage: The Case of Pumped Hydro Storage in Germany," *The Energy Journal*, vol. 32, pp. 59–88, 2011.

[58] R. Sioshansi, "When Energy Storage Reduces Social Welfare," *Energy Economics*, vol. 41, pp. 106–116, January 2014.

[59] A. S. Siddiqui, R. Sioshansi, and A. J. Conejo, "Merchant Storage Investment in a Restructured Electricity Industry," *The Energy Journal*, vol. 40, pp. 129–163, 2019.

[60] D. Kalathil, C. Wu, K. Poolla, and P. Varaiya, "The Sharing Economy for the Electricity Storage," *IEEE Transactions on Smart Grid*, vol. 10, pp. 556–567, January 2019.

[61] R. Chen, I. C. Paschalidis, M. C. Caramanis, and P. Andrianesis, "Learning From Past Bids to Participate Strategically in Day-Ahead Electricity Markets," *IEEE Transactions on Smart Grid*, vol. 10, pp. 5794–5806, September 2019.

[62] B. Li, X. Wang, M. Shahidehpour, C. Jiang, and Z. Li, "DER Aggregator's Data-Driven Bidding Strategy Using the Information Gap Decision Theory in a Non-Cooperative Electricity Market," *IEEE Transactions on Smart Grid*, vol. 10, pp. 6756–6767, November 2019.

[63] H. Guo, Q. Chen, Y. Gu, M. Shahidehpour, Q. Xia, and C. Kang, "A Data-Driven Pattern Extraction Method for Analyzing Bidding Behaviors in Power Markets," *IEEE Transactions on Smart Grid*, vol. 11, pp. 3509–3521, July 2020.

[64] Y. Du, F. Li, H. Zandi, and Y. Xue, "Approximating Nash Equilibrium in Day-ahead Electricity Market Bidding with Multi-agent Deep Reinforcement Learning," *Journal of Modern Power Systems and Clean Energy*, vol. 9, pp. 534–544, May 2021.

[65] J.-F. Toubeau, J. Bottieau, Z. D. Grève, F. Vallée, and K. Bruninx, "Data-Driven Scheduling of Energy Storage in Day-Ahead Energy and Reserve Markets With Probabilistic Guarantees on Real-Time Delivery," *IEEE Transactions on Power Systems*, vol. 36, pp. 2815–2828, July 2021.

[66] Y. Li, X. Wang, C. Cheng, B. Liu, and G. Li, "A Data-Driven Bilevel Model for Estimating Operational Information of a Neighboring Rival's Reservoir in a Competitive Context," *IEEE Access*, vol. 9, pp. 159 640–159 651, 2021.

[67] X. Li, C. Li, G. Chen, and Z. Y. Dong, "A Data-driven Joint Chance-constrained Game for Renewable Energy Aggregators in the Local Market," *IEEE Transactions on Smart Grid*, 2022, in press.

[68] R. Sioshansi and A. J. Conejo, *Optimization in Engineering: Models and Algorithms*, ser. Springer Optimization and Its Applications. Gewerbestraße 11, 6330 Cham, Switzerland: Springer Nature, 2017, vol. 120.

[69] S. Wogrin, J. Barquín, and E. Centeno, "Capacity Expansion Equilibria in Liberalized Electricity Markets: An EPEC Approach," *IEEE Transactions on Power Systems*, vol. 28, pp. 1531–1539, May 2013.

[70] S. Chen, G. Sun, Z. Wei, and D. Wang, "Dynamic pricing in electricity and natural gas distribution networks: An EPEC model," *Energy*, vol. 207, p. 118138, 15 September 2020.

[71] J. Fortuny-Amat and B. McCarl, "A Representation and Economic Interpretation of a Two-Level Programming Problem," *The Journal of the Operational Research Society*, vol. 32, pp. 783–792, September 1981.

[72] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed., ser. optimization and computation. Belmont, Massachusetts: Athena Scientific, 1995.

[73] J. F. Nash, Jr., "Equilibrium points in n-person games," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 36, pp. 48–49, 1 January 1950.

Weiying Gu received her B.S. degree in electrical engineering and automation from Wuhan University of Technology, Wuhan, China and an M.S. in electrical and computer engineering from The Ohio State University, Columbus, OH, USA, where she is pursuing a Ph.D. degree in Department of Integrated Systems Engineering.

Her research interests are in analysis of energy markets and renewable and sustainable energy systems.

Ramteen Sioshansi (M'11–SM'12–F'21) holds the B.A. degree in economics and applied mathematics and the M.S. and Ph.D. degrees in industrial engineering and operations research from University of California, Berkeley, Berkeley, CA, USA and an M.Sc. in econometrics and mathematical economics from London School of Economics and Political Science, London, U.K.

He is a professor in Department of Integrated Systems Engineering and Department of Electrical and Computer Engineering and an associate fellow in Center for Automotive Research at The Ohio State University, Columbus, OH, USA. His research focuses on analyzing renewable and sustainable energy systems and the design of restructured competitive electricity markets.