

Market Equilibria and Interactions Between Strategic Generation, Wind, and Storage

Ali Shahmohammadi^a, Ramteen Sioshansi^{b,*}, Antonio J. Conejo^{b,c}, Saeed Afsharnia^a

^a*School of Electrical and Computer Engineering, University of Tehran, Tehran 11365-4563, Iran*

^b*Department of Integrated Systems Engineering, The Ohio State University, 1971 Neil Avenue, Columbus, OH 43210, United States of America*

^c*Department of Electrical and Computer Engineering, The Ohio State University, 2015 Neil Avenue, Columbus, Ohio 43210, United States of America*

Abstract

Rising wind penetrations can suppress wholesale energy prices by displacing higher-cost conventional generation from the merit order. Wind suffers disproportionately from this price suppression, because the price is most suppressed when wind availability is high, hindering wind-investment incentives. One way to mitigate this price suppression is by wind exercising market power, which introduces efficiency losses. An alternative is to use energy storage, which allows energy to be stored when wind availability is high. This stored energy is later discharged when wind availability is lower and prices are higher.

This paper proposes a bilevel equilibrium model to study market equilibrium interactions between energy storage and wind and conventional generators. We represent the market interaction using an equilibrium problem with equilibrium constraints. An illustrative case study is used to demonstrate the social welfare and profit benefits of using energy storage in this manner.

Keywords: Electricity market, energy storage, wind energy, equilibrium problem with equilibrium constraints

1. Introduction

The electricity industry has seen rising penetrations of nondispatchable renewable generation. This has historically been driven by policy mandates, such as subsidies or renewable portfolio standards [1–3]. These mandates have resulted in renewable cost reductions, through learning-by-doing and economy-of-scale effects, to the point that some renewables are becoming prudent investments without the mandates [3–5].

Green and Vasilakos [6] study the market impacts of high renewable penetrations. They demonstrate that as its penetration increases, wind has the effect of suppressing wholesale energy prices by displacing higher-priced generation from the merit order. Wind suffers disproportionately (compared to conventional generation) from this price suppression, because the price-suppression is greatest during periods that wind availability is high. Thus, this price suppression can disproportionately reduce wind-investment incentives. Buygi *et al.* [7] further demonstrate that this price-suppression effect can also be attributed to the assumption that renewable generators are small play-

ers that behave nonstrategically, meaning that their generation unduly suppresses energy prices.

Wind could mitigate this price suppression by exercising market power [8]. This involves wind generators offering their supply into the market above cost. This solution typically introduces efficiency losses, however. Efficiency losses arise because wind generators are withholding supply, allowing higher-cost generation that would otherwise (absent the exercise of market power) not clear the market to set the price.

An alternate solution to the price-suppression effect is to use energy storage [9]. Energy is stored when high wind availability would otherwise suppress prices and is later discharged when wind availability is lower and prices are higher. Wind derives this benefit from energy storage regardless of whether it owns the storage or not [9]. Previous market analyses of wind and storage are limited, however, because they often rely on highly stylized models to find an equilibrium between storage, wind, and conventional units. This paper relaxes those restrictive assumptions and examines the market interactions between storage, wind, and conventional units under a variety of market and ownership structures within a bilevel-equilibrium framework.

At a high level, the model that we propose assumes that there is a set of firms, each of which can own some combination of storage, conventional, and wind units. The firms make offers, which consist of price/quantity pairs specify-

*Corresponding author

Email addresses: A.shmohammadi@ut.ac.ir (Ali Shahmohammadi), sioshansi.1@osu.edu (Ramteen Sioshansi), conejonavarro.1@osu.edu (Antonio J. Conejo), safshar@ut.ac.ir (Saeed Afsharnia)

ing the price at which storage is willing to be charged or discharged or the price at which generation is willing to supply energy, to a centrally dispatched market. The bilevel nature of the model arises because there is a lower-level problem embedded within the firms' profit-maximization problems representing the clearing of the market (on the basis of the firms' offers). An equilibrium framework arises because the firms simultaneously determine their offers into the market to maximize profits.

Thus, unlike previous analyses, the modeling framework that we propose allows for a great deal of flexibility in modeling interactions between storage, conventional, and wind units within a market environment. Our modeling framework is somewhat simplified compared to other analyses of renewable energy and storage technologies [10–13] in terms of representing engineering details. Other works may model more complex technical characteristics of the technologies in question, but neglect market equilibria. The model that we propose fills this gap, which is important insofar as how conventional, wind, and storage units interact within a market environment raises important policy and market-design issues. Many other works, conversely, examine storage or renewables from the perspective of a central planner. The model that we propose provides a reasonable balance between engineering fidelity, representation of market interactions, and model tractability.

We use optimality conditions of the lower-level problem to convert the bilevel profit-maximization problem of each firm into a mathematical program with equilibrium constraints (MPEC). We then convert the collection of MPECs into an equilibrium problem with equilibrium constraints (EPEC), which we solve to find Nash equilibria.

We apply our modeling framework to an illustrative case study with a number of market and asset-ownership structures. This includes cases in which energy storage and wind are price-taking or price-making and we find equilibria ranging from extremely collusive to competitive outcomes. We find the same price-suppression effect that Green and Vasilakos [6] and Buygi *et al.* [7] do and demonstrate the efficiency losses of having the wind generator exercise market power. Our results show that energy storage is a preferred solution to the price-suppression effect from the perspective of social welfare and wind- and conventional-generator profits.

The remainder of this paper is organized as follows. Section 2 provides a survey of literature that is related to our work and summarizes the major contributions of our work relative to this literature. Section 3 details our modeling framework and the derivation of the market-equilibrium problem. Sections 4 and 5 provide our case-study data and results, respectively. Section 6 summarizes the results of three sensitivity analyses, in which conventional-generator costs and ramping capabilities and wind availability are varied. Section 7 concludes.

2. Related Literature and Our Contributions

A variety of techniques are used in the literature to model the offering and bidding strategies of players participating in energy markets. Complementarity models are recognized as being a particularly powerful tool to represent such games. This is because complementarity models are able to model the simultaneous optimization of firms competing in a market [14]. Zou *et al.* [15] examine the benefits of energy storage in supporting renewable generation in joint energy and ancillary service markets while Hu *et al.* [16] analyze the impact of demand response in an energy market with demand uncertainty. Both of these works take a complementarity approach to finding Nash-Cournot equilibria. Zou *et al.* [17] employ a complementarity model of a multi-period Nash-Cournot equilibrium to study the evolution of the current Chinese power system to a renewable-dominated design in the future.

MPECs are an extension of the complementarity model that can represent more even more complex market interactions. MPECs can be used to represent sequential market interactions and leader/follower games. This is because the optimization problem of the first player or leader in the game can have embedded within it equilibrium constraints that characterize the optimal decisions of the second player or follower. Thus, this framework captures the first player or leader making decisions that take into account the optimal decisions that are subsequently made. Zhang *et al.* [18, 19] use an MPEC model to optimize the trading strategies of a distribution company with distributed energy resources and with demand-responsive customers, respectively.

An EPEC model further extends the MPEC by having multiple leaders as opposed to only one. Thus, an EPEC can be thought of as consisting of one MPEC for each leader, with variables in the different MPECs being interrelated to one another (because the leaders are all participating in the same equilibrium) [14]. One of the complications of EPECs is that they are often highly non-convex problems. Thus, it can be very challenging to find all possible equilibria. In practice, this issue is overcome by using different objective functions in the overall EPEC problem, which can provide a bounding range of equilibria. For instance, one could solve EPECs with welfare-maximization and generator profit-maximization as the objective functions, which provides a range of most- and least-competitive equilibria. Kazempour and Zareipour [20] use an EPEC model to analyze the impacts of large strategic wind producers in day-ahead and real-time markets. However, they solely use profit-maximization of the generators as the objective function of the EPEC, meaning that they examine only the least-competitive market equilibria. Dai and Qiao [21] employ an EPEC model to obtain equilibria in a market with both strategic and nonstrategic wind generators. They find the same price suppression effect that Green and Vasilakos [6] and Buygi *et al.* [7] do and that higher renewable penetrations decrease prices.

They also observe transmission congestion potentially increasing prices.

Although EPEC models are used to study market equilibria with renewables, interactions between renewables and energy storage are not well studied in the literature. This is particularly true of analyses of interactions between renewables and storage within a market equilibrium. This is because modeling energy storage requires a multi-period model to capture intertemporal constraints related to energy storage. Zou *et al.* [22] develop an equilibrium model to study the market impact of strategic storage firms. However, their treatment of the market equilibrium is not as comprehensive as ours. Furthermore, their modeling approach requires the solution of a nonconvex nonlinear optimization problem, which can create computational issues. Our approach, conversely, uses mixed-integer linear programs (MILPs), which are not prone to the same computational challenges. Their study is focused on comparing strategic behavior of different energy storage systems. Our work examines market interactions between storage and renewables.

Given this state of the literature related to market interactions between energy storage and renewables, our work makes four main contributions in this topic area. First, we develop a novel multi-period equilibrium model that comprehensively captures generator and storage offers in the form of price/quantity blocks. The goal of developing this model is to analyze the price-suppressing effect of renewables and investigating possible means of mitigating it. At the same time, the model framework that we develop relaxes many of the restrictive assumptions employed in the scant analyses of market equilibria involving storage in the literature. Second, we show how to recast the multi-period equilibrium model into an EPEC that is formulated as a MILP, meaning that the EPEC can be efficiently solved using commercial software packages. Third, we use different objectives in the EPEC problem to obtain a bounding range of solutions, which we show to be equilibria using a diagonalization process. Finally, we conduct a comprehensive analysis of the interactions between wind and conventional generators and energy storage units under a variety of market and ownership structures. We use the solutions obtained to determine the impacts on market prices, firm profits, and social welfare of the different equilibria. Although the literature includes analyses of interactions between storage and renewables, many of these take a central-planning or command-and-control approach. Our work, conversely, examines interactions within a market-equilibrium setting.

3. Market Equilibrium Model

We model a Nash equilibrium between a set of competing firms, each of which may own some mixture of storage, conventional, and wind units. To reduce the complexity of the model (and to simplify notation) we assume a deterministic setting, and wind-availability uncertainty is not

modeled. We also do not model transmission constraints. This is a common assumption in equilibrium analyses of electricity markets, which often neglect transmission constraints [7, 20, 22–25]. Instead, these works focus on modeling market interactions and drawing insights from market equilibria. These two assumptions can be relaxed, but at the cost of increasing the complexity of the resulting model.

Figure 1 gives a high-level schematic overview of our proposed modeling framework. The figure is divided into five panes. The top pane shows the bilevel structure of our proposed model. At the top there are individual profit-maximization problems for each firm, each of which can own some combination of conventional, wind, and storage units. Each firm individually determines how to offer its units into a spot energy market, which is the lower-level problem. These offer decisions are made to maximize individual firm profits, and are constrained by whatever limits market rules impose (*e.g.*, monotone offers on generation blocks are a common constraint imposed by market rules). The lower-level market operator’s problem takes the supply offers from the firms as inputs and determines the welfare-maximizing dispatch of the units and the resulting energy prices.

The next pane in Figure 1 shows the step in which each firm’s bilevel problem is converted to an MPEC. This is done by replacing the lower-level market operator’s problem with its necessary and sufficient primal/dual optimality conditions. The third pane of the figure shows the next step, which is to convert the collection of MPECs (*i.e.*, one for each firm) into a single EPEC. This is done by replacing each MPEC with its Karush-Kuhn-Tucker (KKT) conditions, which are the constraints of the EPEC. The EPEC is solved using two different objective functions, which provide a bounding range of possible market equilibria. One objective function yields highly competitive equilibria whereas the other yields highly collusive outcomes.

The EPEC has a number of nonlinearities, complicating its solution. The fourth pane of Figure 1 shows a number of steps that are taken to linearize the EPEC. These steps yield an MILP, which can be solved using off-the-shelf software, such as Gurobi. The final pane of the figure shows the last step of our modeling procedure, which is to ensure that a solution of the EPEC is indeed a Nash equilibrium.

We proceed in the remainder of this section by providing further details on the steps that are illustrated in Figure 1. We first examine the market operator’s problem in Section 3.2. Section 3.3 then gives the firms’ bilevel profit-maximization problems, which have the market operator’s problem at the lower level. Section 3.4 finally outlines the Nash equilibrium concept that we employ in our analysis.

We leave the technical details regarding the conversion of the firm’s bilevel profit-maximization problems into MPECs, formation and linearization of the EPEC, and verification that EPEC solutions are Nash equilibria to

the appendices. More specifically, [Appendix A](#) shows how each firm's bilevel model is converted into an MPEC, by replacing the lower-level market operator's problem with necessary and sufficient primal/dual optimality conditions. Next, [Appendix B](#) shows how the firms' MPECs are combined to form an EPEC. [Appendix C](#) describes the steps that are taken to linearize the EPEC. Finally, [Appendix D](#) discusses how we verify that EPEC solutions are Nash equilibria.

3.1. Model Notation

We begin by first defining the following model notation. This includes sets and set-related parameters, model parameters, and lower- and upper-level variables.

3.1.1. Sets and Index Parameters

| | |
|--------------|---|
| B | number of blocks for demand, generation, and storage bids and offers. |
| P | set of firms. |
| T | number of hours in model horizon. |
| Δ_p^G | set of conventional units owned by firm p . |
| Δ_p^S | set of storage units owned by firm p . |
| Δ_p^W | set of wind units owned by firm p . |

3.1.2. Model Parameters

| | |
|-------------------|--|
| $C_{x,b}$ | marginal cost of generation block b of conventional unit x . |
| $\bar{D}_{t,b}$ | hour- t maximum demand in demand block b . |
| \bar{E}_x | maximum storage capacity of storage unit x . |
| $\bar{G}_{x,b}$ | capacity of generation block b of conventional unit x . |
| R_x^U | ramp-up limit of conventional unit x . |
| R_x^D | ramp-down limit of conventional unit x . |
| $\bar{S}_{x,b}^C$ | charging capacity of block b of storage unit x . |
| $\bar{S}_{x,b}^H$ | discharging capacity of block b of storage unit x . |
| $U_{t,b}$ | hour- t marginal utility of demand block b . |
| $\bar{W}_{t,x,b}$ | hour- t available generation from block b of wind unit x . |
| η_x^C | charging efficiency of storage unit x . |
| η_x^H | discharging efficiency of storage unit x . |

3.1.3. Lower-Level Variables

| | |
|---------------|--|
| $D_{t,b}$ | hour- t demand of demand block b that is satisfied. |
| $E_{t,x}$ | ending hour- t storage level of storage unit x . |
| $G_{t,x,b}$ | hour- t dispatch of block b of conventional unit x . |
| $S_{t,x,b}^C$ | hour- t energy charged in block b of storage unit x . |
| $S_{t,x,b}^H$ | hour- t energy discharged from block b of storage unit x . |
| $W_{t,x,b}$ | hour- t dispatch of block b of wind unit x . |

3.1.4. Upper-Level Variables

| | |
|---------------|---|
| $O_{t,x,b}^C$ | hour- t bid price for charging block b of storage unit x . |
| $O_{t,x,b}^H$ | hour- t offer price for discharging block b of storage unit x . |

| | |
|---------------|--|
| $O_{t,x,b}^G$ | hour- t offer price for block b of conventional unit x . |
| $O_{t,x,b}^W$ | hour- t offer price for block b of wind unit x . |

3.2. Market Operator's Problem

The market operator takes supply offers from the generation and storage units and demand bids as inputs and determines the system dispatch. This problem is formulated as:

$$\min \sum_{t,x,b} (O_{t,x,b}^G G_{t,x,b} + O_{t,x,b}^W W_{t,x,b} + O_{t,x,b}^H S_{t,x,b}^H - O_{t,x,b}^C S_{t,x,b}^C) - \sum_{t,b} U_{t,b} D_{t,b} \quad (1)$$

$$\text{s.t.} \quad \sum_{x,b} (G_{t,x,b} + W_{t,x,b} + S_{t,x,b}^H - S_{t,x,b}^C) = \sum_b D_{t,b}, \quad (2)$$

$$\forall t \quad (\psi_t)$$

$$0 \leq D_{t,b} \leq \bar{D}_{t,b}, \quad \forall t, b \quad (\theta_{t,b}^{D,-}, \theta_{t,b}^{D,+}) \quad (3)$$

$$0 \leq G_{t,x,b} \leq \bar{G}_{x,b}, \quad \forall t, x, b \quad (\theta_{t,x,b}^{G,-}, \theta_{t,x,b}^{G,+}) \quad (4)$$

$$-R_x^D \leq \sum_b (G_{t,x,b} - G_{t-1,x,b}) \leq R_x^U, \quad (5)$$

$$\forall t, x \quad (\theta_{t,x}^{R,-}, \theta_{t,x}^{R,+})$$

$$0 \leq W_{t,x,b} \leq \bar{W}_{t,x,b}, \quad \forall t, x, b \quad (\theta_{t,x,b}^{W,-}, \theta_{t,x,b}^{W,+}) \quad (6)$$

$$0 \leq S_{t,x,b}^C \leq \bar{S}_{x,b}^C, \quad \forall t, x, b \quad (\theta_{t,x,b}^{C,-}, \theta_{t,x,b}^{C,+}) \quad (7)$$

$$0 \leq S_{t,x,b}^H \leq \bar{S}_{x,b}^H, \quad \forall t, x, b \quad (\theta_{t,x,b}^{H,-}, \theta_{t,x,b}^{H,+}) \quad (8)$$

$$E_{t,x} = E_{t-1,x} + \sum_b (\eta_x^C S_{t,x,b}^C - S_{t,x,b}^H / \eta_x^H), \quad (9)$$

$$\forall t, x \quad (\theta_{t,x}^E)$$

$$0 \leq E_{t,x} \leq \bar{E}_x, \quad \forall t, x \quad (\theta_{t,x}^{E,-}, \theta_{t,x}^{E,+}) \quad (10)$$

$$E_{T,x} = E_{0,x}, \quad \forall x \quad (\theta_x^{E,0}) \quad (11)$$

where the dual variable associated with each constraint appear in parentheses to its right. The decision variables of the market operator's problems are all of the dispatch-related variables— $G_{t,x,b}$, $W_{t,x,b}$, $S_{t,x,b}^H$, $S_{t,x,b}^C$, $E_{t,x}$, and $D_{t,b}$.

Objective function (1) maximizes social welfare on the basis of the willingness to pay of demand and supply offers from the generation and storage units. The objective function is given in minimization form. Constraints (2) are the hourly load-balance restrictions. As is common practice in wholesale electricity markets, we assume that energy transacted in the market is priced using the dual variables, ψ_t , on these load-balance constraints.

Constraints (3) limit the energy served in each block to the size of the block and constraints (4) and (6) impose the same restrictions on the dispatch of conventional and wind units. Constraints (5) impose conventional-unit ramping limits.

Constraints (7)–(11) impose restrictions on the operation of the storage units. Constraints (7) and (8) impose

the size of each block on the amount of energy charged or discharged. Constraints (9) are energy-balance equations defining the ending storage level of each storage unit in each hour. Constraints (10) impose the energy limit on each storage unit and constraints (11) restrict each storage unit to have the same state of charge at the end of the operating period as it started with. These constraints are included because without them each storage unit would be left fully discharged at the end of the operating period [26].

3.3. Firm Profit-Maximization Problem

Each firm determines offers for its generation and storage units to maximize its profits. The profit-maximization problem of firm p is formulated as:

$$\begin{aligned} \min \quad & \sum_{t,x \in \Delta_p^G,b} (C_{x,b} - \psi_t) G_{t,x,b} - \sum_{t,x \in \Delta_p^W,b} \psi_t W_{t,x,b} \quad (12) \\ & - \sum_{t,x \in \Delta_p^S,b} \psi_t \cdot (S_{t,x,b}^H - S_{t,x,b}^C) \end{aligned}$$

$$\text{s.t. } O_{t,x,b}^G \geq O_{t,x,b-1}^G, \quad \forall t, x \in \Delta_p^G, b > 1 \quad (\Phi_{p,t,x,b}^G) \quad (13)$$

$$O_{t,x,b}^W \geq O_{t,x,b-1}^W, \quad \forall t, x \in \Delta_p^W, b > 1 \quad (\Phi_{p,t,x,b}^W) \quad (14)$$

$$O_{t,x,b}^C \geq O_{t,x,b-1}^C, \quad \forall t, x \in \Delta_p^S, b > 1 \quad (\Phi_{p,t,x,b}^C) \quad (15)$$

$$O_{t,x,b}^H \geq O_{t,x,b-1}^H, \quad \forall t, x \in \Delta_p^S, b > 1 \quad (\Phi_{p,t,x,b}^H) \quad (16)$$

$$(1)-(11), \quad (17)$$

where the Lagrange multiplier that is associated with each constraint appears in parentheses to its right.

Objective function (12), which is given in minimization form, maximizes the firm's profit. Constraints (13)–(16) impose monotonicity on the offers, which is a typical market requirement. Constraint (17) gives the bilevel nature of this problem, by embedding the market operator's problem as a lower level problem. The market operator's problem is embedded to represent the relationship between firm offers and prices and dispatch. The decision variables in this problem are the firm offers— $O_{t,x,b}^G$, $O_{t,x,b}^W$, $O_{t,x,b}^C$, and $O_{t,x,b}^H$ —and the dispatch variables and prices from the market operator's problem.

3.4. Market Equilibrium

We define a Nash equilibrium as a set of firm offers (*i.e.*, values of $O_{t,x,b}^G$, $O_{t,x,b}^W$, $O_{t,x,b}^C$, and $O_{t,x,b}^H$ for each firm) and corresponding values of the market operator's dispatch decisions (*i.e.*, values of $G_{t,x,b}$, $W_{t,x,b}$, $S_{t,x,b}^H$, $S_{t,x,b}^C$, $E_{t,x}$, and $D_{t,b}$) and energy prices (*i.e.*, values of ψ_t) that are simultaneously optimal in each firm's profit-maximization problem and the market operator's dispatch problem. That is to say, each firm's offers should be individually profit maximizing, in light of the offers that are submitted by its rival firms. Moreover, the resulting dispatch and energy prices should be optimal in the market operator's primal dispatch problem and its dual problem.

As noted before, we defer the detailed technical steps required to obtain such equilibria to the appendices. One of the difficulties in analyzing non-cooperative games is that they may have many Nash equilibria. As such, we impose two different objective functions on the EPEC from which we obtain Nash equilibria. The first:

$$\begin{aligned} \min \quad & \sum_{t,x,b} [C_{x,b} G_{t,x,b} - \psi_t \cdot (G_{t,x,b} + W_{t,x,b} \\ & + S_{t,x,b}^H - S_{t,x,b}^C)], \end{aligned}$$

which is given in minimization form, maximizes total profits across all of the firms. Equilibria arising from the EPEC with this objective function tend to be the least competitive, and as such we informally term them 'collusive equilibria.' The second objective function:

$$\min \sum_{t,x,b} C_{x,b} G_{t,x,b} - \sum_{t,b} U_{t,b} D_{t,b},$$

which is also given in minimization form, maximizes social welfare. Equilibria arising from the EPEC with this objective function tend to be the most competitive, and as such we informally term them 'quasi-competitive equilibria.' Further details on these two objective functions are given in [Appendix B](#).

The idea behind imposing these two objective functions on the EPEC is that they provide a bounding range of equilibria. That is to say, by examining highly competitive and uncompetitive equilibria, we can examine the potential range of market equilibria and outcomes.

4. Case-Study Data

We illustrate our proposed model by using a small eight-period example with two conventional, one wind, and one storage units. One of the main reasons for using an eight-period example is the time involved in solving larger case studies (*e.g.*, a 24-hour example). Our analysis considers numerous cases with different asset-ownership structures and types of equilibria. Moreover, we conduct a thorough sensitivity analysis in [Section 6](#). Finally, the cases are solved using a laptop as opposed to high-performance computing equipment. For these reasons, we opt to use a case study that can solve relatively quickly. For purpose of comparison, a 24-hour variant of our case study can take between 13 seconds and 27 hours to solve, depending on the assumed market and asset-ownership structure. All of the eight-period cases that we use in our analysis take less than one hour to solve. These findings shows that our modeling framework could be applied to larger-scale problems that consider 24 (or more) hours. However, given the volume of cases and sensitivity analyses that we consider, using a 24-hour case study would not be computationally tractable.

This being said, the eight-period example that we use does capture the important salient features that would be

exhibited in a 24-hour example. For instance, we have on- and off-peak load and wind periods. It is also worth noting that each of the time periods in the example could be used to represent a multi-hour block of time (*e.g.*, three-hour time periods). However, in our case study each period represents a single hour. Finally, it should be noted that our analysis is mostly focused on a qualitative assessment of market equilibria. That is to say, the exact values determined by the model are not as important as understanding how market equilibria compare to one another under different market and asset-ownership structures. Among the 24-hour case studies that we solve (for purposes of determining their solution times), we find that the market equilibria are qualitatively similar to those that are obtained from our eight-period case studies. This further suggests that for purposes of determining the qualitative properties of market equilibria, our eight-period case studies are sufficient. Finally, as discussed above, a more thorough analysis of a specific market could be conducted using more operating periods. Such an analysis would be computationally tractable, especially because it would likely not entail the volume of cases that we consider.

Table 1 summarizes the assumed characteristics of the conventional generators. Each unit is assumed to have two blocks of the same size (*i.e.*, 120 MW and 80 MW, respectively). The storage unit has a charging and discharging power capacity of 200 MW, an energy capacity of 200 MWh, a roundtrip efficiency of 85%, and an initial storage level of 100 MWh. For sake of maintaining a generic modeling framework, we do not consider any specific storage technology. Rather, the storage-related parameters that we assume are meant to represent a large-scale technology, such as pumped hydroelectric or compressed air energy storage [11, 27, 28].

Table 1: Conventional-Unit Data

| Unit | $\bar{G}_{x,b}$ | R_x^D | R_x^U | $C_{x,1}$ | $C_{x,2}$ |
|------|-----------------|---------|---------|-----------|-----------|
| 1 | 120 | 180 | 200 | 30 | 40 |
| 2 | 80 | 130 | 150 | 50 | 60 |

Figure 2 shows the profile of hourly wind availability and maximum potential demand. The figure shows that wind availability and load are negatively correlated, which is common in many systems. We consider a case in which total wind availability is about 40% of total maximum demand. This is roughly consistent with the penetration of wind energy in a number of real-world electricity systems, including those of Denmark, Spain, Portugal, and Ireland. Moreover, the assumed structure of the offer behavior in our bilevel modeling framework is roughly consistent with how wholesale electricity markets in these countries are operated. The demand in each hour is assumed to be bid in three blocks. The prices at which the blocks are bid are positively correlated with total demand—demands in hours 4–6 have relatively high willingness to pay compared

to hours 1, 2, and 8.

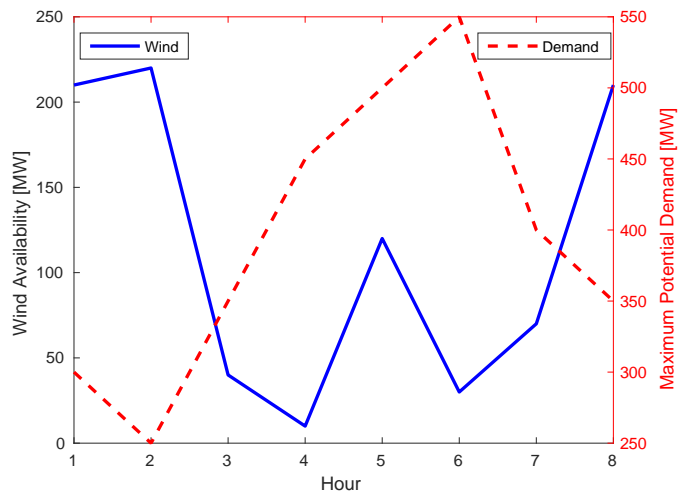


Figure 2: Wind Availability and Maximum Demand in Each Hour

To analyze the interactions between storage, wind, and conventional units, we model equilibria in the nine cases that are listed in Table 2. Case 1 represents a base case without wind or storage units and assumes that the two conventional units are owned by two competing firms. The remaining cases have a third firm, which owns the wind generator. In Cases 2–5 the wind generator behaves as a price-taker, meaning that it offers its generation at its true cost of zero. Cases 6–9 assume that the wind generator behaves as a price-maker, meaning that it follows profit-maximizing generation offers given by its MPEC (and the KKT conditions of the wind generator’s MPEC are included in the EPEC).

Table 2: Cases Examined

| Case | Wind | Storage |
|------|--------------|---------------|
| 1 | None | None |
| 2 | Price-Taking | None |
| 3 | Price-Taking | Price-Taking |
| 4 | Price-Taking | Standalone |
| 5 | Price-Taking | Wind-Operated |
| 6 | Price-Making | None |
| 7 | Price-Making | Price-Taking |
| 8 | Price-Making | Standalone |
| 9 | Price-Making | Wind-Operated |

Cases 2 and 6 assume that there is no storage, while Cases 3–5 and 7–9 assume storage with different types of behavior. Cases 3 and 7 assume price-taking storage, meaning that it offers into the market at its cost of zero. In these cases, the market operator dispatches storage competitively to minimize system-operation costs. Cases 4 and 8 assume that storage is owned by a fourth firm that behaves as a price-maker. Cases 5 and 9 assume that storage is owned by the wind generator, and that the storage

is offered as a price-maker. In Case 5 wind is offered at cost but storage can be offered at a price above its cost of zero.

5. Case-Study Results

Table 3 summarizes the results of the equilibria found for the different cases using the two objective functions for the EPEC. Quasi-competitive equilibria see the highest possible demand levels being met. As such, the firms have very limited opportunity to exercise market power. Conversely, firms successfully restrict output and the amount of demand met to increase market prices in collusive equilibria. As such, Table 3 reports results for all nine collusive cases but only quasi-competitive Cases 1–5. This is because the wind generator is unable to exercise market power in the quasi-competitive equilibria and the results of Cases 6–9 are identical to those of Cases 2–5.

Adding wind to the market has the effect of suppressing prices. In Case 1, which has no wind, the average energy price is \$69.00/MWh and \$82.20/MWh in quasi-competitive and collusive equilibria, respectively. These prices are reduced to \$62.30/MWh and \$69.10/MWh, respectively, when wind is added in Case 2. One way that wind generators can mitigate this price suppression is by withholding their generation from the market. In Case 6, in which wind is price-making, the average energy price increases to \$78.90/MWh in the collusive setting. This comes at the cost of significantly reducing the amount of wind that is used, however. Table 4 shows that in Case 6, the price increase comes with 15.4% of potential wind generation being curtailed. The other means of mitigating the price-suppression effect is using energy storage. In collusive Cases 7–9, the inclusion of the energy storage increases the average energy price to \$78.80/MWh–\$80.50/MWh. As Table 4 shows, there is considerably less wind curtailment in these three cases. It is also worth noting that the case in which the wind generator owns the energy storage minimizes wind curtailment. The other cases that are not listed in Table 4 do not have any wind curtailment, because the wind generator is not able to exercise market power.

Table 4: Total Wind Curtailment in Collusive Market Equilibria

| Case | 6 | 7 | 8 | 9 |
|-----------------|------|-----|-----|-----|
| Curtailment [%] | 15.4 | 6.9 | 6.9 | 1.4 |

Energy storage can also benefit the wind generator in quasi-competitive equilibria, if the wind generator owns the storage (*cf.* Cases 2 and 5). Otherwise, as seen by comparing quasi-competitive Cases 3–4 to 2, energy storage may not benefit the wind generator because in these equilibria storage is used to improve social welfare as opposed to firm profits.

Comparing conventional-generator profits in collusive Cases 3–5 to 2 and 7–9 to 6 shows that conventional generators also benefit from energy storage being in the market. Moreover, conventional generators benefit from storage behaving uncompetitively (either as a standalone or wind-owned profit maximizer). In the quasi-competitive equilibria conventional-generator profits are lower with storage in the market. However, the reductions in conventional-generator profits are reduced if storage is behaving uncompetitively, which is in keeping with the results of the collusive equilibria.

Comparing collusive equilibria in Cases 2 and 6 and quasi-competitive equilibria in Case 2 to the other equilibria shows that energy storage improves social welfare. It is important to stress, however, that these social welfare calculations do not account for the cost of building energy storage, which could yield a net welfare loss. Interestingly, in collusive and quasi-competitive equilibria, and regardless of whether the wind generator is a price taker or price maker, having storage owned by the wind generator is the most desirable from the perspective of maximizing social welfare and total firm profits. This is demonstrated in Figure 3, which shows social welfare and total firm profits in collusive and quasi-competitive equilibria in the different cases examined. Welfare and profits are reported as a percentage of the highest welfare and firm profits among the equilibria found. The figure shows that the equilibria in Cases 5 and 9 result in the highest social welfare and firm profits.

Another question of interest is whether investments in storage units can be justified by the profits that they earn or the increases in social welfare that they engender. This is because the true net value of storage must take into account its capital costs. A complete investment analysis is beyond the scope of this work. Instead, we do a more simplified analysis using an assumed capital charge rate (CCR) [29, 30]. The CCR captures all of the financing and other costs that go into building a storage unit and translates that capital cost into an annualized cost. We use an 11% CCR in our analysis, which is typical for the electric power industry [31]. This means that a storage unit that has a capital cost of $\$Y$ to build must earn an average of $\$(0.11 \cdot Y)$ in operating profits to cover the annualized cost of financing that investment. Conversely, a storage unit that earns an average of $\$Z$ per year in operating profits can have a capital cost of up to $\$(Z/0.11)$ and be able to recover its investment cost from those operating profits.

Our analysis takes this latter approach of translating annual operating profits into the highest capital cost that can be justified, assuming an 11% CCR. We conduct this analysis in three ways. First, we examine the profits earned by the storage unit only (*i.e.*, we use storage profits as the value of Z in the CCR analysis). Second, we examine the increase in the joint profits of the wind and storage units. This is because the focus of our analysis is the use of storage for increasing the profits of wind

Table 3: Results of Market Equilibria

| EPEC Objective Function | Case | Social Welfare [\$] | Demand Met [%] | Firm Profits [\$] | | |
|-------------------------|------|---------------------|----------------|-------------------|-------|---------|
| | | | | Conv. | Wind | Storage |
| Profit | 1 | 89900 | 63.5 | 88700 | n/a | n/a |
| Profit | 2 | 128740 | 76.8 | 65090 | 45850 | n/a |
| Profit | 3 | 136195 | 74.6 | 65900 | 55310 | 3876 |
| Profit | 4 | 136031 | 75.2 | 68611 | 57490 | 629 |
| Profit | 5 | 136204 | 73.4 | 66514 | 57490 | 2900 |
| Profit | 6 | 122450 | 68.3 | 63680 | 53170 | n/a |
| Profit | 7 | 133125 | 71.4 | 65486 | 60706 | 1524 |
| Profit | 8 | 133820 | 71.4 | 66594 | 60926 | 1100 |
| Profit | 9 | 135870 | 73.0 | 66594 | 60064 | 2013 |
| Welfare | 1 | 94950 | 77.8 | 71250 | n/a | n/a |
| Welfare | 2 | 134510 | 89.8 | 59553 | 41937 | n/a |
| Welfare | 3 | 140547 | 88.3 | 40147 | 39170 | 4529 |
| Welfare | 4 | 140547 | 88.3 | 46920 | 39730 | 7697 |
| Welfare | 5 | 140547 | 88.3 | 57507 | 43010 | 7829 |

generators. Finally, we examine the increase in social welfare when storage is in the system, compared to a case without storage. This third case is examined because a policymaker may wish to incentivize investment in energy storage if it gives social-welfare increases that are greater than the profit earned by the storage unit [32].

Because our case study only considers eight hours, we scale the profit and social-welfare values that are found in our case studies by a factor of (8760/8) to convert these values into an annualized value. We then apply the assumed 11% CCR to determine the total capital cost that can be justified by the profit or social-welfare increases. We finally divide this value by the assumed 200000 kW capacity of the storage unit to arrive at a maximum \$/kW capital cost that can be justified by the profit or social-welfare increases. Thus, we compute this maximum capital cost as:

$$\frac{8760}{8} \cdot \frac{1}{0.11} \cdot \frac{1}{200000} \cdot Z, \quad (18)$$

where Z is either profit of the storage unit, increase in the joint profit of wind and storage, or increase in social welfare when the storage unit is added to the system.

Table 5 summarizes the maximum capital cost of storage that can be justified under the different cases and types of equilibria that we examine. Interestingly, we find that in many cases the social value of storage (which is represented by the maximum capital cost justified on the basis of the social-welfare increase that it engenders) is greater than the private value to the storage unit itself. This suggests that incentives for storage investment may be prudent from a societal perspective. In some cases, the value of storage that can be justified through joint profits of wind and storage outweigh the social value. This means that co-owned wind and storage (or a contracting arrangement between wind and storage) can increase storage investment

in some cases. On the other hand, such collaboration between wind and storage can result in overinvestment (from the perspective of social welfare).

The cases that we examine are all formulated using version 24.2.1 of the GAMS mathematical programming software package and solved using Gurobi version 5.6.2. The cases are solved on a computer with a 2.26 GHz Intel Core 2 Duo processor and 2 GB of memory. The solution times of the EPECs for collusive equilibria range between one and 61 minutes, whereas the EPECs solve within two minutes for quasi-competitive equilibria. The diagonalization process takes an additional minute of computing time.

6. Sensitivity Analyses

We present the results of three sensitivity analyses, in which we vary (i) the operating costs of the conventional units, (ii) the ramping capabilities of the conventional units, and (iii) wind availability. The purpose of these analyses is to demonstrate the effects of these parameters on market equilibria and outcomes. Importantly, we show that the qualitative results that are observed in the base case largely carry over to the sensitivity cases. For all three sensitivity cases we only examine the cases that are listed in Table 2 in which the storage firm participates in the market.

6.1. Conventional Unit Cost

Figures 4–6 summarize the range of changes in social welfare, total firm profits, and demand met (respectively) in a set of sensitivity cases in which the costs of conventional units are decreased or increased by 30% relative to their baseline values. The stars in the figures indicate the social welfare, firm profits, and demand met in the

Table 5: Justifiable Capital Cost of the Storage Unit Based on Increases in Profit or Social Welfare [\$/kW]

| Equilibrium Type | Case | Basis of Calculation | | |
|-------------------|------|----------------------|-------------------------|----------------|
| | | Storage Profit | Storage and Wind Profit | Social Welfare |
| Collusive | 3 | 193 | 664 | 371 |
| Collusive | 4 | 31 | 611 | 363 |
| Collusive | 5 | 144 | 724 | 372 |
| Collusive | 7 | 76 | 451 | 531 |
| Collusive | 8 | 55 | 441 | 566 |
| Collusive | 9 | 100 | 443 | 668 |
| Quasi-Competitive | 3 | 225 | 88 | 300 |
| Quasi-Competitive | 4 | 383 | 273 | 300 |
| Quasi-Competitive | 5 | 390 | 443 | 300 |

base case, while the pairs of triangles indicate the ranges of these values obtained in the sensitivity cases. As expected, we find that social welfare decreases across the types of equilibria and market structures as generation cost increases and vice versa.

Changing generation cost has the expected effect on firm profits under collusive equilibria. Decreasing generation costs increases total firm profits while increasing costs has the opposite effect. Interestingly, this result does not necessarily hold under quasi-competitive equilibria. The reason for this is that quasi-competitive equilibria seek to maximize social welfare, which often entails higher levels of demand being served, as opposed to maximizing firm profits. As a result, some quasi-competitive equilibria with decreased generation costs result in more demand being served (to increase social welfare), which gives lower firm profits.

The amount of demand that is met is not adversely affected by increased generation costs under collusive equilibria. This is because both in the base case and with different production costs, generators restrict output to increase their profits under collusive equilibria. On the other hand, the amount of demand met is severely affected by generation costs under quasi-competitive equilibria.

As in the base case, we find that regardless of changes in generation costs, an asset-ownership structure in which wind and storage are co-owned and co-operating yields the greatest social welfare and total firm profits among the structures that are examined. Figure 4 also shows that wind and storage being co-owned and co-operated by the same firm mitigates some of the negative welfare impacts of increasing generation costs. This is because Cases 5 and 9 yield higher social welfare compared to other cases in which storage acts as an independent price-taker or -maker. Figure 4 also shows that having the wind generator behave as a price-maker under collusive equilibria yields social-welfare losses compared to wind behaving as a price-taker. This is keeping with the qualitative welfare results that are observed in the base case. Overall, the social-welfare impacts of storage that are observed in this sensitivity case are qualitatively similar to those that are

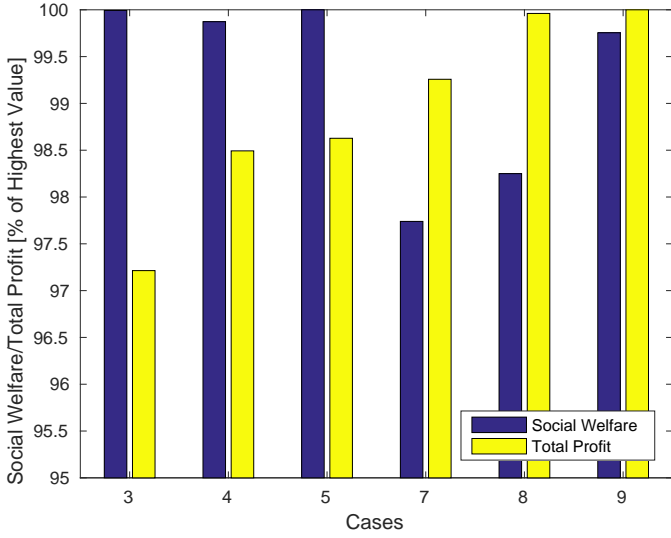
observed in the base case, showing that those results are robust to conventional-generator costs.

6.2. Conventional Unit Ramping

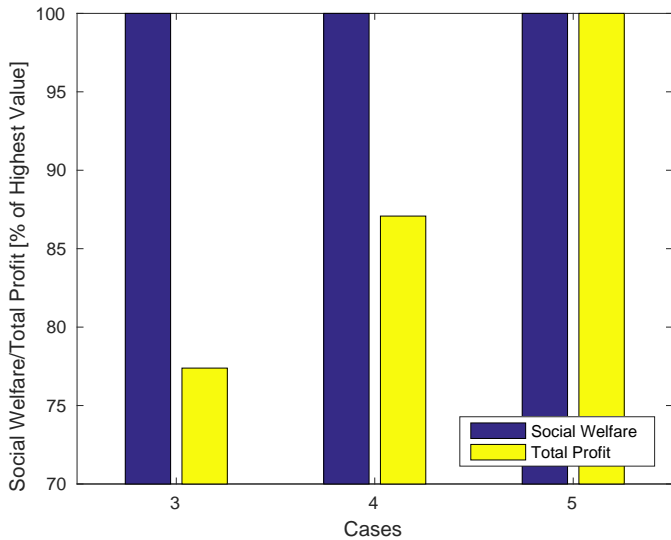
Figures 7–9 summarize the range of changes in social welfare, total firm profits, and demand met (respectively) in a set of sensitivity cases in which the ramping capabilities of conventional units are decreased or increased by 35% relative to their baseline values. Social welfare and firm profits both slightly decrease in most cases if the ramping capabilities of conventional units are decreased. This does not hold, however, for collusive equilibria in which storage is controlled by the market operator (Cases 3 and 7). This is because in these cases the market operator uses storage to increase social welfare while the demand served remains the same, preventing reductions in firm profits.

Social-welfare losses are larger in collusive equilibria than they are under quasi-competitive ones. This is because reducing conventional-unit ramping capabilities restricts the ability of conventional units to withhold generation without affecting production levels in ‘adjacent’ time periods. The amount of demand that is met remains nearly the same with restricted ramping constraints under collusive equilibria whereas slightly more demand is met under quasi-competitive equilibria. This is because under quasi-competitive equilibria more demand is met during off-peak periods to ensure that higher-value demand can be met during on-peak periods.

In cases in which reduced conventional-unit ramping-capabilities yield social welfare losses, storage being co-owned by the wind generator results in smaller social-welfare losses. Greater total firm profits are also achieved with co-owned wind and storage. We also observe the same finding that co-ownership of storage by wind units results in higher social welfare and total firm profits compared to other storage-ownership structures. Thus, we overall find that co-ownership of wind and storage is desirable from a number of perspectives, which is keeping with the base-case results.



(a) Collusive Equilibria



(b) Quasi-Competitive Equilibria

Figure 3: Social Welfare and Total Firm Profits Under Collusive and Quasi-Competitive Equilibria in Different Cases Examined [% of Highest-Value Case]

6.3. Wind Availability

Figures 10–12 summarize the range of changes in social welfare, total firm profits, and demand met (respectively) in a set of sensitivity cases in which wind availability is decreased or increased by 20% relative to its baseline value. As expected, social welfare, total firm profits, and demand met all remain the same or increase with greater wind availability and vice versa. This is because wind provides a zero-cost source of energy.

The cases in which demand met does not increase with greater wind availability are those in which the wind units behave as price makers. This is because price-making wind units curtail wind production in the base case to increase their profits. As such, increasing wind availability does not result in greater wind production or demand being met.

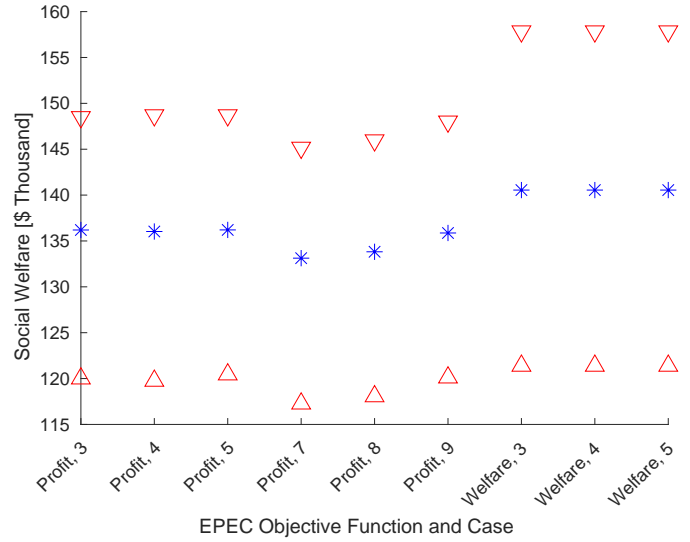


Figure 4: Range of Total Social Welfare Obtained in Equilibria With Between 30% Decrease and 30% Increase in Conventional-Unit Costs Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

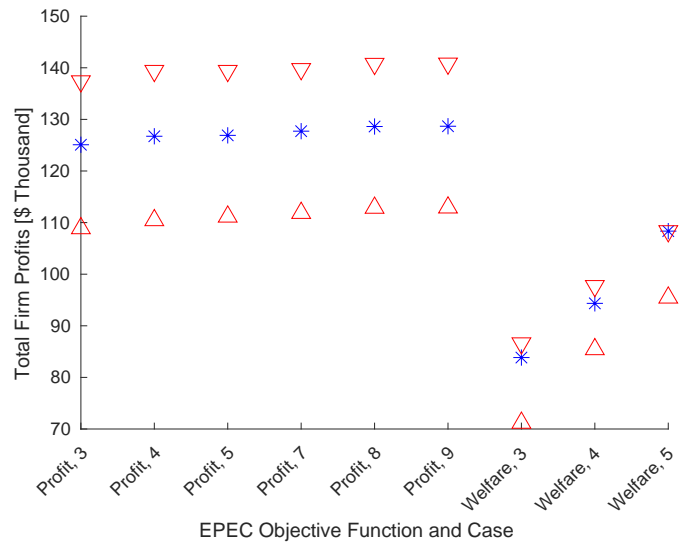


Figure 5: Range of Total Firm Profits Obtained in Equilibria With Between 30% Decrease and 30% Increase in Conventional-Unit Costs Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

Thus, price-making wind results in smaller social-welfare increases arising from greater wind availability compared to cases in which wind behaves as a price taker. However, among the cases in which the wind units behave as price makers, that in which it co-owns storage results in the highest social welfare. As in the base and two other sensitivity cases, co-ownership of storage by the wind units is also desirable from the perspective of maximizing total firm profits.

Overall, this sensitivity analysis shows that the observations in the base case and in the two other sensitivity cases are robust to the assumed case-study data. That is

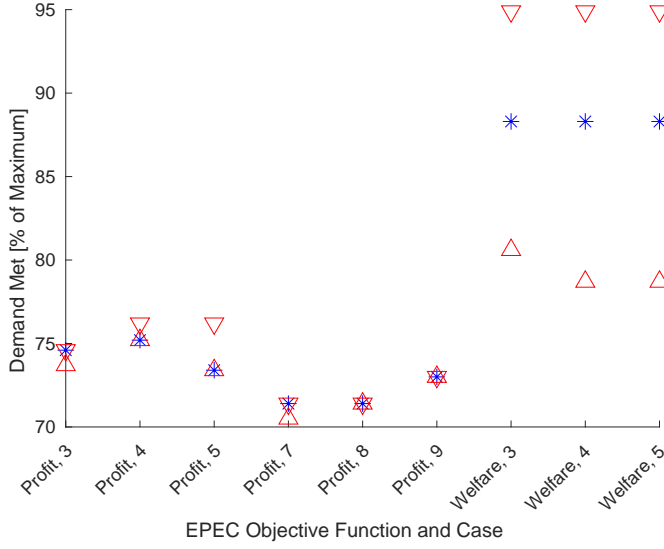


Figure 6: Range of Total Demand Met in Equilibria With Between 30% Decrease and 30% Increase in Conventional-Unit Costs Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

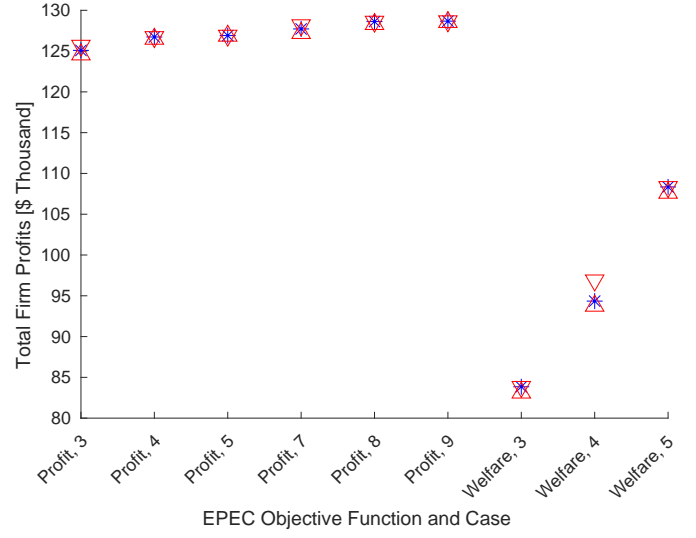


Figure 8: Range of Total Firm Profits Obtained in Equilibria With Between 35% Decrease and 35% Increase in Conventional-Unit Ramping Capabilities Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

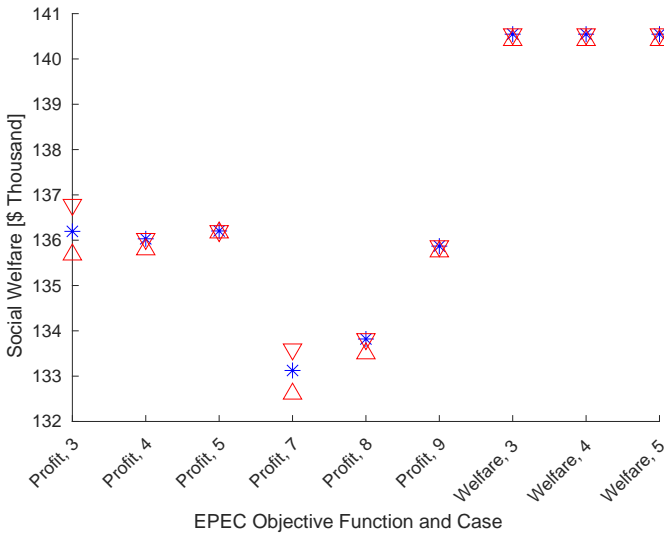


Figure 7: Range of Total Social Welfare Obtained in Equilibria With Between 35% Decrease and 35% Increase in Conventional-Unit Ramping Capabilities Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

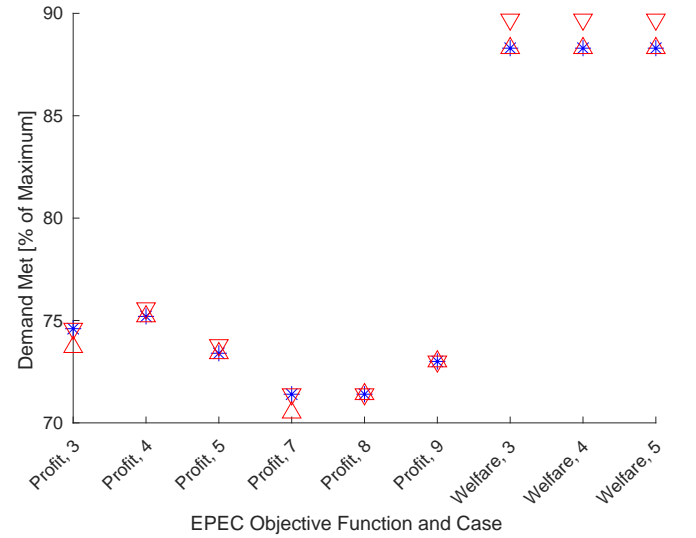


Figure 9: Range of Total Demand Met in Equilibria With Between 35% Decrease and 35% Increase in Conventional-Unit Ramping Capabilities Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

to say, many of the qualitative observations regarding different types of equilibria and asset-ownership structures in the base case carry over to sensitivity cases with different data.

7. Conclusion

This paper provides a framework to analyze market interactions between conventional, wind, and storage units. The resulting model is an EPEC that can be solved as a MILP. We use a case study to examine the interactions between these three types of entities. Our results show that

adding wind to the market significantly depresses market prices [6, 7]. One way that wind generators can address this price suppression is by withholding their generation. This is an inefficient ‘solution,’ however, because zero-cost generation is curtailed. Alternatively, energy storage can be used to store excess energy when prices are depressed that is later discharged when wind availability is lower and prices are higher [9]. We also demonstrate that the profit or social value of storage can justify capital costs of up to more than \$700/kW in some of the cases that are examined. It is notable that this justified cost is comparable to the cost of some large-scale storage technologies that are

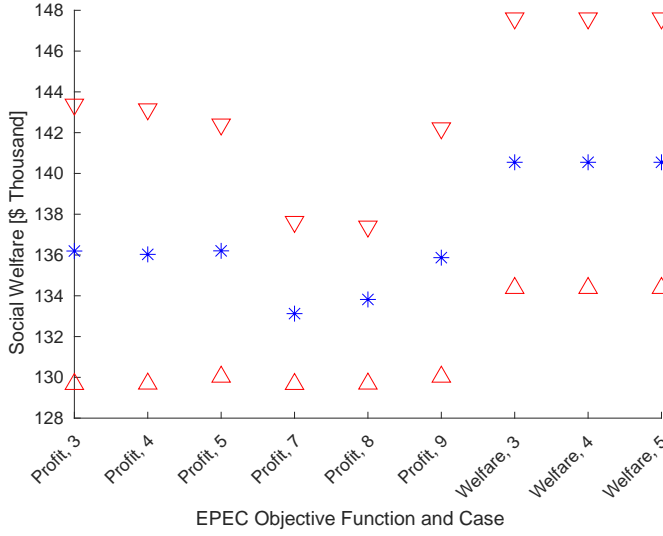


Figure 10: Range of Total Social Welfare Obtained in Equilibria With Between 20% Decrease and 20% Increase in Wind Availability Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

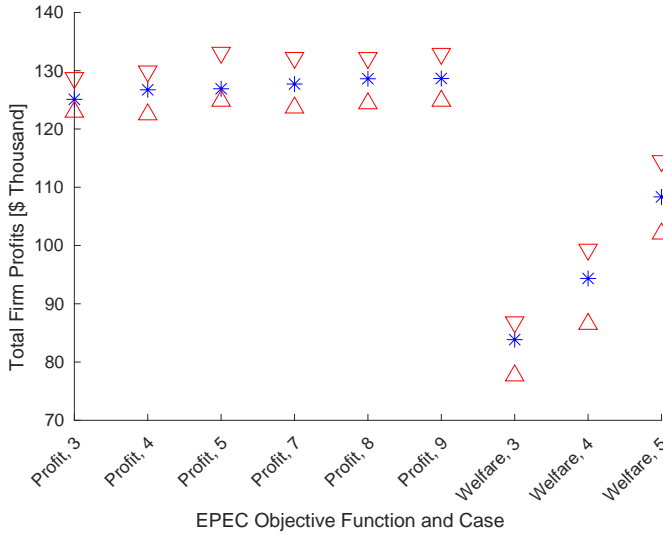


Figure 11: Range of Total Firm Profits Obtained in Equilibria With Between 20% Decrease and 20% Increase in Wind Availability Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

available today [11]. Our sensitivity analyses further show that our results are robust to the parameter values that are assumed in our case study.

Our proposed model can help inform policy makers, market designers, and regulators about the implications of market rules and structures on market outcomes. We demonstrate the inefficiency of allowing wind generators to exercise market power, but that energy storage can mitigate this. From the perspective of maximizing profits and economic incentives for wind, price-making storage and wind-owned storage are preferred in the collusive and quasi-competitive settings, respectively. We also demonstrate that among the different strategies for integrating

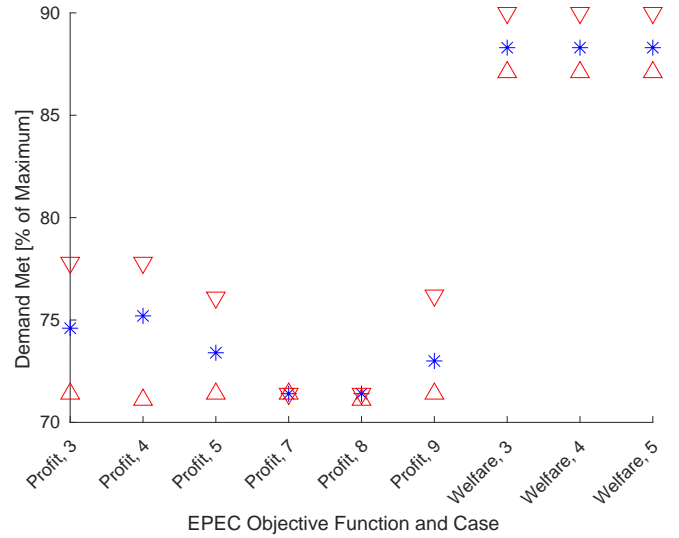


Figure 12: Range of Total Demand Met in Equilibria With Between 20% Decrease and 20% Increase in Wind Availability Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)

storage in the system, wind-owned storage is the best in many respects, such as minimizing wind curtailment, maximizing social welfare and total firm profits, and also recovering storage-investment costs. On the other hand, further numerical testing and more case studies that are tailored to a particular system are likely needed to ensure that the results and the ranking of the different market structures apply in those settings. Our sensitivity analyses do suggest, however, that our qualitative findings are robust to parameter values.

Acknowledgments

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Appendix A. MPEC Form of Profit-Maximization Problem of Firms

Each firm's bilevel profit-maximization problem can be converted to an MPEC, by replacing the lower-level market operator's problem with its primal/dual optimality conditions [33]. Because the objective function and constraints of the market operator's problem are linear in the dispatch variables these conditions are necessary and sufficient for an optimum.

The primal/dual optimality conditions for the market operator's problem in firm p 's profit-maximization prob-

lem (*i.e.*, what we replace constraint (17) with) are:

$$\sum_{x,b} (G_{t,x,b} + W_{t,x,b} + S_{t,x,b}^H - S_{t,x,b}^C) = \sum_b D_{t,b}, \quad (\text{A.1})$$

$$\forall t \quad (\Psi_{p,t})$$

$$0 \leq D_{t,b} \leq \bar{D}_{t,b}, \quad \forall t, b \quad (\Theta_{p,t,b}^{D,-}, \Theta_{p,t,b}^{D,+}) \quad (\text{A.2})$$

$$0 \leq G_{t,x,b} \leq \bar{G}_{x,b}, \quad \forall t, x, b \quad (\Theta_{p,t,x,b}^{G,-}, \Theta_{p,t,x,b}^{G,+}) \quad (\text{A.3})$$

$$-R_x^D \leq \sum_b (G_{t,x,b} - G_{t-1,x,b}) \leq R_x^U, \quad (\text{A.4})$$

$$\forall t, x \quad (\Theta_{p,t,x}^{R,-}, \Theta_{p,t,x}^{R,+})$$

$$0 \leq W_{t,x,b} \leq \bar{W}_{t,x,b}, \quad \forall t, x, b \quad (\Theta_{p,t,x,b}^{W,-}, \Theta_{p,t,x,b}^{W,+}) \quad (\text{A.5})$$

$$0 \leq S_{t,x,b}^C \leq \bar{S}_{x,b}^C, \quad \forall t, x, b \quad (\Theta_{p,t,x,b}^{C,-}, \Theta_{p,t,x,b}^{C,+}) \quad (\text{A.6})$$

$$0 \leq S_{t,x,b}^H \leq \bar{S}_{x,b}^H, \quad \forall t, x, b \quad (\Theta_{p,t,x,b}^{H,-}, \Theta_{p,t,x,b}^{H,+}) \quad (\text{A.7})$$

$$E_{t,x} = E_{t-1,x} + \sum_b (\eta_x^C S_{t,x,b}^C - S_{t,x,b}^H / \eta_x^H), \quad (\text{A.8})$$

$$\forall t, x \quad (\Theta_{p,t,x}^E)$$

$$0 \leq E_{t,x} \leq \bar{E}_x, \quad \forall t, x \quad (\Theta_{p,t,x}^{E,-}, \Theta_{p,t,x}^{E,+}) \quad (\text{A.9})$$

$$E_{T,x} = E_{0,x}, \quad \forall x \quad (\Theta_{p,x}^{E,0}) \quad (\text{A.10})$$

$$U_{t,b} - \psi_t + \theta_{t,b}^{D,-} - \theta_{t,b}^{D,+} = 0, \quad \forall t, b \quad (\omega_{p,t,b}^D) \quad (\text{A.11})$$

$$-O_{t,x,b}^G + \psi_t + \theta_{t,x,b}^{G,-} - \theta_{t,x,b}^{G,+} + \theta_{t,x}^{R,-} - \theta_{t+1,x}^{R,-} \quad (\text{A.12})$$

$$- \theta_{t,x}^{R,+} + \theta_{t+1,x}^{R,+} = 0, \quad \forall t < T, x, b \quad (\omega_{p,t,x,b}^G)$$

$$-O_{T,x,b}^G + \psi_T + \theta_{T,x,b}^{G,-} - \theta_{T,x,b}^{G,+} + \theta_{T,x}^{R,-} - \theta_{T,x}^{R,+} \quad (\text{A.13})$$

$$= 0, \quad \forall x, b \quad (\omega_{p,T,x,b}^G)$$

$$-O_{t,x,b}^W + \psi_t + \theta_{t,x,b}^{W,-} - \theta_{t,x,b}^{W,+} = 0, \quad (\text{A.14})$$

$$\forall t, x, b \quad (\omega_{p,t,x,b}^W)$$

$$O_{t,x,b}^C - \psi_t + \theta_{t,x,b}^{C,-} - \theta_{t,x,b}^{C,+} - \eta_x^C \theta_{t,x}^E = 0, \quad (\text{A.15})$$

$$\forall t, x, b \quad (\omega_{p,t,x,b}^C)$$

$$-O_{t,x,b}^H + \psi_t + \theta_{t,x,b}^{H,-} - \theta_{t,x,b}^{H,+} + \theta_{t,x}^E / \eta_x^H = 0, \quad (\text{A.16})$$

$$\forall t, x, b \quad (\omega_{p,t,x,b}^H)$$

$$\theta_{t,x}^E - \theta_{t+1,x}^E + \theta_{t,x}^{E,-} - \theta_{t,x}^{E,+} = 0, \quad (\text{A.17})$$

$$\forall t < T, x \quad (\omega_{p,t,x}^E)$$

$$\theta_{T,x}^E + \theta_{T,x}^{E,-} - \theta_{T,x}^{E,+} + \theta_x^{E,0} = 0, \quad \forall x \quad (\omega_{p,x}^{E,0}) \quad (\text{A.18})$$

$$\theta_{t,b}^{D,-}, \theta_{t,b}^{D,+} \geq 0, \quad \forall t, b \quad (\Omega_{p,t,b}^{D,-}, \Omega_{p,t,b}^{D,+}) \quad (\text{A.19})$$

$$\theta_{t,x,b}^{G,-}, \theta_{t,x,b}^{G,+} \geq 0, \quad \forall t, x, b \quad (\Omega_{p,t,x,b}^{G,-}, \Omega_{p,t,x,b}^{G,+}) \quad (\text{A.20})$$

$$\theta_{t,x}^{R,-}, \theta_{t,x}^{R,+} \geq 0, \quad \forall t, x \quad (\Omega_{p,t,x}^{R,-}, \Omega_{p,t,x}^{R,+}) \quad (\text{A.21})$$

$$\theta_{t,x,b}^{W,-}, \theta_{t,x,b}^{W,+} \geq 0, \quad \forall t, x, b \quad (\Omega_{p,t,x,b}^{W,-}, \Omega_{p,t,x,b}^{W,+}) \quad (\text{A.22})$$

$$\theta_{t,x,b}^{C,-}, \theta_{t,x,b}^{C,+} \geq 0, \quad \forall t, x, b \quad (\Omega_{p,t,x,b}^{C,-}, \Omega_{p,t,x,b}^{C,+}) \quad (\text{A.23})$$

$$\theta_{t,x,b}^{H,-}, \theta_{t,x,b}^{H,+} \geq 0, \quad \forall t, x, b \quad (\Omega_{p,t,x,b}^{H,-}, \Omega_{p,t,x,b}^{H,+}) \quad (\text{A.24})$$

$$\theta_{t,x}^{E,-}, \theta_{t,x}^{E,+} \geq 0, \quad \forall t, x \quad (\Omega_{p,t,x}^{E,-}, \Omega_{p,t,x}^{E,+}) \quad (\text{A.25})$$

$$\sum_{t,x,b} (O_{t,x,b}^G G_{t,x,b} + O_{t,x,b}^W W_{t,x,b} + O_{t,x,b}^H S_{t,x,b}^H) \quad (\text{A.26})$$

$$\begin{aligned} & -O_{t,x,b}^C S_{t,x,b}^C) - \sum_{t,b} U_{t,b} D_{t,b} = - \sum_{t,b} \bar{D}_{t,b} \theta_{t,b}^{D,+} \\ & - \sum_{t,x,b} (\bar{G}_{x,b} \theta_{t,x,b}^{G,+} + R_x^D \theta_{t,x}^{R,-} + R_x^U \theta_{t,x}^{R,+} + \bar{W}_{t,x,b} \theta_{t,x,b}^{W,+}) \\ & + \sum_{x,b} G_{0,x,b} \cdot (\theta_{1,x}^{R,-} - \theta_{1,x}^{R,+}) - \sum_{t,x,b} (\bar{S}_{x,b}^C \theta_{t,x,b}^{C,+} \\ & + \bar{S}_{x,b}^H \theta_{t,x,b}^{H,+}) + \sum_x \left(E_{0,x} \cdot (\theta_{1,x}^E + \theta_x^{E,0}) \right. \\ & \left. - \sum_t \bar{E}_x \theta_{t,x}^{E,+} \right), \quad (\Omega_p^{\text{SD}}) \end{aligned}$$

where the Lagrange multiplier that is associated with each constraint appears in parentheses to its right. Conditions (A.1)–(A.10) are the primal constraints of the market operator's problem, conditions (A.11)–(A.25) are the dual constraints of the market operator's problem, and constraint (A.26) imposes strong duality.

Although conditions (A.1)–(A.26) are common to all of the firms, we index the Lagrange multipliers of each of these conditions by p . This is because these Lagrange multipliers may take on different values in each firm's profit-maximization problem.

With this primal/dual characterization of an optimal solution to the market operator's problem, firm p 's profit-maximization problem can be solved by minimizing (12) subject to constraints (13)–(16) and (A.1)–(A.26).

Appendix B. Multi-Firm Nash Equilibrium

To find a Nash equilibrium, we must find a set of solutions (*i.e.*, offers and resulting primal and dual solutions of the market operator's problem) that are simultaneously optimal in each firm's MPEC. We find such an equilibrium by solving a single EPEC problem, which has as its constraints the Karush-Kuhn-Tucker (KKT) conditions of each firm's MPEC. The KKT conditions for firm p 's MPEC are:

$$- \Phi_{p,t,x,b}^G + \Phi_{p,t,x,b+1}^G + \Omega_p^{\text{SD}} G_{t,x,b} - \omega_{p,t,x,b}^G = 0, \quad (\text{B.1})$$

$$\forall t, x \in \Delta_p^G, b < B$$

$$- \Phi_{p,t,x,B}^G + \Omega_p^{\text{SD}} G_{t,x,B} - \omega_{p,t,x,B}^G = 0, \quad (\text{B.2})$$

$$\forall t, x \in \Delta_p^G$$

$$- \Phi_{p,t,x,b}^W + \Phi_{p,t,x,b+1}^W + \Omega_p^{\text{SD}} W_{t,x,b} - \omega_{p,t,x,b}^W = 0, \quad (\text{B.3})$$

$$\forall t, x \in \Delta_p^W, b < B$$

$$- \Phi_{p,t,x,B}^W + \Omega_p^{\text{SD}} W_{t,x,B} - \omega_{p,t,x,B}^W = 0, \quad (\text{B.4})$$

$$\forall t, x \in \Delta_p^W$$

$$- \Phi_{p,t,x,b}^C + \Phi_{p,t,x,b+1}^C - \Omega_p^{\text{SD}} S_{t,x,b}^C + \omega_{p,t,x,b}^C = 0, \quad (\text{B.5})$$

$$\forall t, x \in \Delta_p^C, b < B$$

$$- \Phi_{p,t,x,B}^C - \Omega_p^{\text{SD}} S_{t,x,B}^C + \omega_{p,t,x,B}^C = 0, \quad (\text{B.6})$$

$$\forall t, x \in \Delta_p^C$$

$$-\Phi_{p,t,x,b}^H + \Phi_{p,t,x,b+1}^H + \Omega_p^{\text{SD}} S_{t,x,b}^H - \omega_{p,t,x,b}^H = 0, \quad (\text{B.7})$$

$$\forall t, x \in \Delta_p^S, b < B$$

$$-\Phi_{p,t,x,B}^H + \Omega_p^{\text{SD}} S_{t,x,B}^H - \omega_{p,t,x,B}^H = 0, \quad (\text{B.8})$$

$$\forall t, x \in \Delta_p^S$$

$$-\Psi_{p,t} + \Theta_{p,t,b}^{D,+} - \Theta_{p,t,b}^{D,-} - \Omega_p^{\text{SD}} U_{t,b} = 0, \quad \forall t, b \quad (\text{B.9})$$

$$C_{x,b} - \psi_t + \Psi_{p,t} + \Theta_{p,t,x,b}^{G,+} - \Theta_{p,t,x,b}^{G,-} + \Theta_{p,t,x}^{R,+} \quad (\text{B.10})$$

$$- \Theta_{p,t+1,x}^{R,+} - \Theta_{p,t,x}^{R,-} + \Theta_{p,t+1,x}^{R,-} + \Omega_p^{\text{SD}} O_{t,x,b}^G = 0,$$

$$\forall t < T, x \in \Delta_p^G, b$$

$$C_{x,b} - \psi_T + \Psi_{p,T} + \Theta_{p,T,x,b}^{G,+} - \Theta_{p,T,x,b}^{G,-} + \Theta_{p,T,x}^{R,+} \quad (\text{B.11})$$

$$- \Theta_{p,T,x}^{R,-} + \Omega_p^{\text{SD}} O_{T,x,b}^G = 0, \quad \forall x \in \Delta_p^G, b$$

$$\Psi_{p',t} + \Theta_{p',t,x,b}^{G,+} - \Theta_{p',t,x,b}^{G,-} + \Theta_{p',t,x}^{R,+} - \Theta_{p',t+1,x}^{R,+} \quad (\text{B.12})$$

$$- \Theta_{p',t,x}^{R,-} + \Theta_{p',t+1,x}^{R,-} + \Omega_{p'}^{\text{SD}} O_{t,x,b}^G = 0,$$

$$\forall p' \neq p, t < T, x \in \Delta_{p'}^G$$

$$\Psi_{p',T} + \Theta_{p',T,x,b}^{G,+} - \Theta_{p',T,x,b}^{G,-} + \Theta_{p',T,x}^{R,+} - \Theta_{p',T,x}^{R,-} \quad (\text{B.13})$$

$$+ \Omega_{p'}^{\text{SD}} O_{T,x,b}^G = 0, \quad \forall p' \neq p, x \in \Delta_{p'}^G$$

$$-\psi_t + \Psi_{p,t} + \Theta_{p,t,x,b}^{W,+} - \Theta_{p,t,x,b}^{W,-} + \Omega_p^{\text{SD}} O_{t,x,b}^W = 0, \quad (\text{B.14})$$

$$\forall t, x \in \Delta_p^W, b$$

$$\Psi_{p',t} + \Theta_{p',t,x,b}^{W,+} - \Theta_{p',t,x,b}^{W,-} + \Omega_{p'}^{\text{SD}} O_{t,x,b}^W = 0, \quad (\text{B.15})$$

$$\forall p' \neq p, t, x \in \Delta_{p'}^W, b$$

$$\psi_t - \Psi_{p,t} - \Theta_{p,t,x,b}^{C,-} + \Theta_{p,t,x,b}^{C,+} - \eta_x^C \Theta_{p,t,x}^E \quad (\text{B.16})$$

$$- \Omega_p^{\text{SD}} O_{t,x,b}^C = 0, \quad \forall t, x \in \Delta_p^S, b$$

$$-\Psi_{p',t} - \Theta_{p',t,x,b}^{C,-} + \Theta_{p',t,x,b}^{C,+} - \eta_x^C \Theta_{p',t,x}^E \quad (\text{B.17})$$

$$- \Omega_{p'}^{\text{SD}} O_{t,x,b}^C = 0, \quad \forall p' \neq p, t, x \in \Delta_{p'}^S, b$$

$$-\psi_t + \Psi_{p,t} - \Theta_{p,t,x,b}^{H,-} + \Theta_{p,t,x,b}^{H,+} + \Theta_{p,t,x}^E / \eta_x^H \quad (\text{B.18})$$

$$+ \Omega_p^{\text{SD}} O_{t,x,b}^H = 0, \quad \forall t, x \in \Delta_p^S, b$$

$$\Psi_{p',t} - \Theta_{p',t,x,b}^{H,-} + \Theta_{p',t,x,b}^{H,+} + \Theta_{p',t,x}^E / \eta_x^H \quad (\text{B.19})$$

$$+ \Omega_{p'}^{\text{SD}} O_{t,x,b}^H = 0, \quad \forall p' \neq p, t, x \in \Delta_{p'}^S, b$$

$$\Theta_{p,t,x}^E - \Theta_{p,t+1,x}^E - \Theta_{p,t,x}^{E,-} + \Theta_{p,t,x}^{E,+} = 0, \quad (\text{B.20})$$

$$\forall t < T, x$$

$$\Theta_{p,T,x}^E - \Theta_{p,T,x}^{E,-} + \Theta_{p,T,x}^{E,+} + \Theta_{p,x}^{E,0} = 0, \quad \forall x \quad (\text{B.21})$$

$$-E_{0,x} \Omega_p^{\text{SD}} + \sum_b (\omega_{p,1,x,b}^H / \eta_x^H - \eta_x^C \omega_{p,1,x,b}^C) \quad (\text{B.22})$$

$$+ \omega_{p,1,x}^E = 0, \quad \forall x$$

$$\sum_b (\omega_{p,t,x,b}^H / \eta_x^H - \eta_x^C \omega_{p,t,x,b}^C) + \omega_{p,t,x}^E \quad (\text{B.23})$$

$$- \omega_{p,t-1,x}^E = 0, \quad \forall t \in \{2, \dots, T-1\}, x$$

$$\sum_b (\omega_{p,T,x,b}^H / \eta_x^H - \eta_x^C \omega_{p,T,x,b}^C) + \omega_{p,x}^{E,0} \quad (\text{B.24})$$

$$- \omega_{p,T-1,x}^E = 0, \quad \forall x$$

$$-E_{0,x} \Omega_p^{\text{SD}} + \omega_{p,x}^{E,0} = 0, \quad \forall x \quad (\text{B.25})$$

$$- \sum_{x \in \Delta_p^G, b} G_{t,x,b} - \sum_{x \in \Delta_p^W, b} W_{t,x,b} \quad (\text{B.26})$$

$$- \sum_{x \in \Delta_p^S, b} (S_{t,x,b}^H - S_{t,x,b}^C) - \sum_b \omega_{p,t,b}^D + \sum_{x,b} (\omega_{p,t,x,b}^G$$

$$+ \omega_{p,t,x,b}^W + \omega_{p,t,x,b}^H - \omega_{p,t,x,b}^C) = 0, \quad \forall t$$

$$\omega_{p,t,b}^D - \Omega_{p,t,b}^{D,-} = 0, \quad \forall t, b \quad (\text{B.27})$$

$$\bar{D}_{t,b} \Omega_p^{\text{SD}} - \omega_{p,t,b}^D - \Omega_{p,t,b}^{D,+} = 0, \quad \forall t, b \quad (\text{B.28})$$

$$\omega_{p,t,x,b}^G - \Omega_{p,t,x,b}^{G,-} = 0, \quad \forall t, x, b \quad (\text{B.29})$$

$$\bar{G}_{x,b} \Omega_p^{\text{SD}} - \omega_{p,t,x,b}^G - \Omega_{p,t,x,b}^{G,+} = 0, \quad \forall t, x, b \quad (\text{B.30})$$

$$\omega_{p,t,x,b}^W - \Omega_{p,t,x,b}^{W,-} = 0, \quad \forall t, x, b \quad (\text{B.31})$$

$$\bar{W}_{t,x,b} \Omega_p^{\text{SD}} - \omega_{p,t,x,b}^W - \Omega_{p,t,x,b}^{W,+} = 0, \quad \forall t, x, b \quad (\text{B.32})$$

$$R_x^D \Omega_p^{\text{SD}} - \sum_b (G_{0,x,b} \Omega_p^{\text{SD}} - \omega_{p,1,x,b}^G) - \Omega_{p,1,x}^{R,-} \quad (\text{B.33})$$

$$= 0, \quad \forall x$$

$$R_x^D \Omega_p^{\text{SD}} + \sum_b (\omega_{p,t,x,b}^G - \omega_{p,t-1,x,b}^G) - \Omega_{p,t,x}^{R,-} = 0, \quad (\text{B.34})$$

$$\forall t > 1, x$$

$$R_x^U \Omega_p^{\text{SD}} + \sum_b (G_{0,x,b} \Omega_p^{\text{SD}} - \omega_{p,1,x,b}^G) - \Omega_{p,1,x}^{R,+} = 0, \quad (\text{B.35})$$

$$\forall x$$

$$R_x^U \Omega_p^{\text{SD}} - \sum_b (\omega_{p,t,x,b}^G - \omega_{p,t-1,x,b}^G) - \Omega_{p,t,x}^{R,+} = 0, \quad (\text{B.36})$$

$$\forall t > 1, x$$

$$\omega_{p,t,x,b}^C - \Omega_{p,t,x,b}^{C,-} = 0, \quad \forall t, x, b \quad (\text{B.37})$$

$$\bar{S}_{x,b}^C \Omega_p^{\text{SD}} - \omega_{p,t,x,b}^C - \Omega_{p,t,x,b}^{C,+} = 0, \quad \forall t, x, b \quad (\text{B.38})$$

$$\omega_{p,t,x,b}^H - \Omega_{p,t,x,b}^{H,-} = 0, \quad \forall t, x, b \quad (\text{B.39})$$

$$\bar{S}_{x,b}^H \Omega_p^{\text{SD}} - \omega_{p,t,x,b}^H - \Omega_{p,t,x,b}^{H,+} = 0, \quad \forall t, x, b \quad (\text{B.40})$$

$$\omega_{p,t,x}^E - \Omega_{p,t,x}^{E,-} = 0, \quad \forall t < T, x, b \quad (\text{B.41})$$

$$\omega_{p,x}^{E,0} - \Omega_{p,T,x}^{E,-} = 0, \quad \forall x, b \quad (\text{B.42})$$

$$\bar{E}_x \Omega_p^{\text{SD}} - \omega_{p,t,x}^E - \Omega_{p,t,x}^{E,+} = 0, \quad \forall t < T, x, b \quad (\text{B.43})$$

$$\bar{E}_x \Omega_p^{\text{SD}} - \omega_{p,x}^{E,0} - \Omega_{p,T,x}^{E,+} = 0, \quad \forall x, b \quad (\text{B.44})$$

$$(\text{A.1}), (\text{A.8}), (\text{A.10})-(\text{A.18}), (\text{A.26}) \quad (\text{B.45})$$

$$0 \leq O_{t,x,b}^G - O_{t,x,b-1}^G \perp \Phi_{p,t,x,b}^G \geq 0, \quad (\text{B.46})$$

$$\forall t, x \in \Delta_p^G, b > 1$$

$$0 \leq O_{t,x,b}^W - O_{t,x,b-1}^W \perp \Phi_{p,t,x,b}^W \geq 0, \quad (\text{B.47})$$

$$\forall t, x \in \Delta_p^W, b > 1$$

$$0 \leq O_{t,x,b}^C - O_{t,x,b-1}^C \perp \Phi_{p,t,x,b}^C \geq 0, \quad (\text{B.48})$$

$$\forall t, x \in \Delta_p^S, b > 1$$

$$0 \leq O_{t,x,b}^H - O_{t,x,b-1}^H \perp \Phi_{p,t,x,b}^H \geq 0, \quad (\text{B.49})$$

$$\forall t, x \in \Delta_p^S, b > 1$$

$$0 \leq D_{t,b} \perp \Theta_{p,t,b}^{D,-} \geq 0, \quad \forall t, b \quad (\text{B.50})$$

$$0 \leq \bar{D}_{t,b} - D_{t,b} \perp \Theta_{p,t,b}^{D,+} \geq 0, \quad \forall t, b \quad (\text{B.51})$$

$$0 \leq G_{t,x,b} \perp \Theta_{p,t,x,b}^{G,-} \geq 0, \quad \forall t, x, b \quad (\text{B.52})$$

$$0 \leq \bar{G}_{x,b} - G_{t,x,b} \perp \Theta_{p,t,x,b}^{G,+} \geq 0, \quad \forall t, x, b \quad (\text{B.53})$$

$$0 \leq \sum_b (G_{t,x,b} - G_{t-1,x,b}) + R_x^D \perp \Theta_{p,t,x}^{R,-} \geq 0, \quad (\text{B.54})$$

$$\forall t, x$$

$$0 \leq R_x^U - \sum_b (G_{t,x,b} - G_{t-1,x,b}) \perp \Theta_{p,t,x}^{R,+} \geq 0, \quad (\text{B.55})$$

$$\forall t, x$$

$$0 \leq W_{t,x,b} \perp \Theta_{p,t,x,b}^{W,-} \geq 0, \quad \forall t, x, b \quad (\text{B.56})$$

$$0 \leq \bar{W}_{t,x,b} - W_{t,x,b} \perp \Theta_{p,t,x,b}^{W,+} \geq 0, \quad \forall t, x, b \quad (\text{B.57})$$

$$0 \leq S_{t,x,b}^C \perp \Theta_{p,t,x,b}^{C,-} \geq 0, \quad \forall t, x, b \quad (\text{B.58})$$

$$0 \leq \bar{S}_{x,b}^C - S_{t,x,b}^C \perp \Theta_{p,t,x,b}^{C,+} \geq 0, \quad \forall t, x, b \quad (\text{B.59})$$

$$0 \leq S_{t,x,b}^H \perp \Theta_{p,t,x,b}^{H,-} \geq 0, \quad \forall t, x, b \quad (\text{B.60})$$

$$0 \leq \bar{S}_{x,b}^H - S_{t,x,b}^H \perp \Theta_{p,t,x,b}^{H,+} \geq 0, \quad \forall t, x, b \quad (\text{B.61})$$

$$0 \leq E_{t,x} \perp \Theta_{p,t,x}^{E,-} \geq 0, \quad \forall t, x \quad (\text{B.62})$$

$$0 \leq \bar{E}_x - E_{t,x} \perp \Theta_{p,t,x}^{E,+} \geq 0, \quad \forall t, x \quad (\text{B.63})$$

$$0 \leq \theta_{t,b}^{D,-} \perp \Omega_{p,t,b}^{D,-} \geq 0, \quad \forall t, b \quad (\text{B.64})$$

$$0 \leq \theta_{t,b}^{D,+} \perp \Omega_{p,t,b}^{D,+} \geq 0, \quad \forall t, b \quad (\text{B.65})$$

$$0 \leq \theta_{t,x,b}^{G,-} \perp \Omega_{p,t,x,b}^{G,-} \geq 0, \quad \forall t, x, b \quad (\text{B.66})$$

$$0 \leq \theta_{t,x,b}^{G,+} \perp \Omega_{p,t,x,b}^{G,+} \geq 0, \quad \forall t, x, b \quad (\text{B.67})$$

$$0 \leq \theta_{t,x}^{R,-} \perp \Omega_{p,t,x}^{R,-} \geq 0, \quad \forall t, x \quad (\text{B.68})$$

$$0 \leq \theta_{t,x}^{R,+} \perp \Omega_{p,t,x}^{R,+} \geq 0, \quad \forall t, x \quad (\text{B.69})$$

$$0 \leq \theta_{t,x,b}^{W,-} \perp \Omega_{p,t,x,b}^{W,-} \geq 0, \quad \forall t, x, b \quad (\text{B.70})$$

$$0 \leq \theta_{t,x,b}^{W,+} \perp \Omega_{p,t,x,b}^{W,+} \geq 0, \quad \forall t, x, b \quad (\text{B.71})$$

$$0 \leq \theta_{t,x,b}^{C,-} \perp \Omega_{p,t,x,b}^{C,-} \geq 0, \quad \forall t, x, b \quad (\text{B.72})$$

$$0 \leq \theta_{t,x,b}^{C,+} \perp \Omega_{p,t,x,b}^{C,+} \geq 0, \quad \forall t, x, b \quad (\text{B.73})$$

$$0 \leq \theta_{t,x,b}^{H,-} \perp \Omega_{p,t,x,b}^{H,-} \geq 0, \quad \forall t, x, b \quad (\text{B.74})$$

$$0 \leq \theta_{t,x,b}^{H,+} \perp \Omega_{p,t,x,b}^{H,+} \geq 0, \quad \forall t, x, b \quad (\text{B.75})$$

$$0 \leq \theta_{t,x}^{E,-} \perp \Omega_{p,t,x}^{E,-} \geq 0, \quad \forall t, x \quad (\text{B.76})$$

$$0 \leq \theta_{t,x}^{E,+} \perp \Omega_{p,t,x}^{E,+} \geq 0, \quad \forall t, x \quad (\text{B.77})$$

where ‘ \perp ’ denotes complementary slackness between an inequality constraint in firm p ’s MPEC and the corresponding non-negativity constraint on the corresponding Lagrange multiplier.

The EPEC is formed by combining Constraints (B.1)–(B.77) for all $p \in P$ [33]. An EPEC can have many solutions, corresponding to different possible Nash equilibria. To help find equilibria that are economically meaningful, we impose two objective functions on the EPEC [33]. The first:

$$\min \sum_{t,x,b} [C_{x,b} G_{t,x,b} - \psi_t \cdot (G_{t,x,b} + W_{t,x,b} + S_{t,x,b}^H - S_{t,x,b}^C)], \quad (\text{B.78})$$

maximizes total profits across all of the firms. The second:

$$\min \sum_{t,x,b} C_{x,b} G_{t,x,b} - \sum_{t,b} U_{t,b} D_{t,b}, \quad (\text{B.79})$$

maximizes total social welfare (both objective functions are given in minimization form). Thus, the EPEC that maximizes firm profits is solved by minimizing (B.78) subject to (B.1)–(B.77) $\forall p \in P$, while the one that maximizes social welfare is solved by minimizing (B.79) subject to (B.1)–(B.77) $\forall p \in P$.

We include objective functions (B.78) and (B.79) in the EPEC so we can find and evaluate a range of possible market outcomes. Nash equilibria given by objective functions (B.78) tend to be the least competitive, because firms are colluding to maximize their joint profits (we refer to these, informally, as collusive equilibria). It should be noted that these so-called equilibria are not truly collusive (*e.g.*, they will not be the behavior of an explicit cartel), because they must ultimately be Nash equilibria that are unilateral-deviation-proof.

Conversely, equilibria that are given by objective function (B.79) tend to be the most competitive, because firms are behaving in a manner to maximize social welfare (we refer to these, informally, as quasi-competitive equilibria). It should be stressed, again, that these equilibria are not perfectly competitive, because each firm is assumed to be individually maximizing its profits through its offer strategy. This range of equilibria can be helpful for regulators, market operators, or policy makers to understand the possible outcomes in a market that is dispatched on the basis of supply offers from competing generating firms.

Appendix C. Linearization of EPEC

There are several nonlinearities in the objective function and constraints of the EPECs. We outline the steps taken to linearize these.

Appendix C.1. Strong-Duality Property

Constraint (B.45) includes the strong-duality equality, which is given by (A.26). The strong-duality property has numerous nonlinear terms in which offer variables are multiplied by dispatch variables.

Because the objective function and constraints of the market operator’s problem are linear in the market operator’s decision variables, the strong-duality property is equivalent to the complementary-slackness conditions on the inequality constraints of the market operator’s problem and the non-negativity constraints on the associated dual variables [33, 34]. As such, we can replace (A.26) in (B.45) with the following equivalent conditions:

$$0 \leq D_{t,b} \perp \theta_{t,b}^{D,-} \geq 0, \quad \forall t, b \quad (\text{C.1})$$

$$0 \leq \bar{D}_{t,b} - D_{t,b} \perp \theta_{t,b}^{D,+} \geq 0, \quad \forall t, b \quad (\text{C.2})$$

$$0 \leq G_{t,x,b} \perp \theta_{t,x,b}^{G,-} \geq 0, \quad \forall t, x, b \quad (\text{C.3})$$

$$0 \leq \bar{G}_{x,b} - G_{t,x,b} \perp \theta_{t,x,b}^{G,+} \geq 0, \quad \forall t, x, b \quad (\text{C.4})$$

$$0 \leq \sum_b (G_{t,x,b} - G_{t-1,x,b}) + R_x^D \perp \theta_{t,x}^{R,-} \geq 0, \quad (\text{C.5})$$

$\forall t, x$

$$0 \leq R_x^U - \sum_b (G_{t,x,b} - G_{t-1,x,b}) \perp \theta_{t,x}^{R,+} \geq 0, \quad (\text{C.6})$$

$\forall t, x$

$$0 \leq W_{t,x,b} \perp \theta_{t,x,b}^{W,-} \geq 0, \quad \forall t, x, b \quad (\text{C.7})$$

$$0 \leq \bar{W}_{t,x,b} - W_{t,x,b} \perp \theta_{t,x,b}^{W,+} \geq 0, \quad \forall t, x, b \quad (\text{C.8})$$

$$0 \leq S_{t,x,b}^C \perp \theta_{t,x,b}^{C,-} \geq 0, \quad \forall t, x, b \quad (\text{C.9})$$

$$0 \leq \bar{S}_{t,x,b}^C - S_{t,x,b}^C \perp \theta_{t,x,b}^{C,+} \geq 0, \quad \forall t, x, b \quad (\text{C.10})$$

$$0 \leq S_{t,x,b}^H \perp \theta_{t,x,b}^{H,-} \geq 0, \quad \forall t, x, b \quad (\text{C.11})$$

$$0 \leq \bar{S}_{t,x,b}^H - S_{t,x,b}^H \perp \theta_{t,x,b}^{H,+} \geq 0, \quad \forall t, x, b \quad (\text{C.12})$$

$$0 \leq E_{t,x} \perp \theta_{t,x}^{E,-} \geq 0, \quad \forall t, x \quad (\text{C.13})$$

$$0 \leq \bar{E}_x - E_{t,x} \perp \theta_{t,x}^{E,+} \geq 0. \quad \forall t, x \quad (\text{C.14})$$

Appendix C.2. Complementary Slackness Conditions

Constraints (B.46)–(B.77) and (C.1)–(C.14) are complementary slackness conditions, which are inherently nonlinear. This is because a complementary slackness condition of the form:

$$0 \leq f(z) \perp \zeta \geq 0, \quad (\text{C.15})$$

can be written as:

$$\begin{aligned} 0 &\leq f(z) \\ \zeta &\geq 0 \\ \zeta f(z) &= 0. \end{aligned}$$

Complementary slackness condition (C.15) can be linearized by introducing a new binary variable, which we denote π , and a sufficiently large constant, which we denote M . We then replace (C.15) with [35]:

$$\begin{aligned} 0 &\leq f(z) \leq M\pi \\ 0 &\leq \zeta \leq M \cdot (1 - \pi) \\ \pi &\in \{0, 1\}. \end{aligned}$$

We linearize all of the complementary slackness conditions in this way.

Appendix C.3. Product of Ω_p^{SD}

There are terms in constraints (B.1)–(B.8) and (B.10)–(B.19) in which Ω_p^{SD} multiplies a dispatch or offer variable. Because Ω_p^{SD} is the common nonlinearity to all of these constraints, we parameterize the solutions of the EPEC by fixing the values of this variable, which is a commonly used solution to linearizing such a nonlinearity [33, 36]. Once the value of Ω_p^{SD} is fixed, these terms are no longer nonlinear.

Appendix C.4. Objective Function (B.78)

Objective function (B.78) is nonlinear because of bilinear terms in which ψ_t multiplies the dispatch variables. We eliminate these nonlinearities by approximating the function using binary expansion [37]. To do this, we first rewrite objective function (B.78) as:

$$\min \sum_{t,x,b} C_{x,b} G_{t,x,b} - \sum_t \psi_t \nu_t,$$

where we let the variable ν_t , which is defined as:

$$\nu_t = \sum_{x,b} (G_{t,x,b} + W_{t,x,b} + S_{t,x,b}^H - S_{t,x,b}^C),$$

denote total hour- t net generation. After rewriting the objective function, the $\psi_t \nu_t$ terms are the only nonlinearities that remain in this objective function.

To eliminate these nonlinearities, we approximate ν_t as taking on one of a fixed set of values, which we denote as $\bar{\nu}_{t,1}, \dots, \bar{\nu}_{t,\Xi}$. We assume that these values are equally spaced, meaning that $\bar{\nu}_{t,2} - \bar{\nu}_{t,1} = \dots = \bar{\nu}_{t,\Xi} - \bar{\nu}_{t,\Xi-1} = \bar{\nu}_t^\Delta$. We next introduce a set of continuous and binary variables, which we denote as $\gamma_{t,1}, \dots, \gamma_{t,\Xi}$ and $\chi_{t,1}, \dots, \chi_{t,\Xi}$, respectively.

We then replace objective function (B.78) with:

$$\min \sum_{t,x,b} C_{x,b} G_{t,x,b} - \sum_{t,\xi} \gamma_{t,\xi} \bar{\nu}_{t,\xi},$$

and add the constraints:

$$\begin{aligned} \nu_t &= \sum_{x,b} (G_{t,x,b} + W_{t,x,b} + S_{t,x,b}^H - S_{t,x,b}^C), \quad \forall t \\ \nu_t - \bar{\nu}_t^\Delta &\leq \sum_{\xi} \bar{\nu}_{t,\xi} \chi_{t,\xi} \leq \nu_t, \quad \forall t \end{aligned} \quad (\text{C.16})$$

$$\sum_{\xi} \chi_{t,\xi} = 1, \quad \forall t \quad (\text{C.17})$$

$$0 \leq \psi_t - \gamma_{t,\xi} \leq M \cdot (1 - \chi_{t,\xi}), \quad \forall t, \xi \quad (\text{C.18})$$

$$0 \leq \gamma_{t,\xi} \leq M \cdot \chi_{t,\xi}, \quad \forall t, \xi \quad (\text{C.19})$$

$$\chi_{t,\xi} \in \{0, 1\}, \quad \forall t, \xi \quad (\text{C.20})$$

to the EPEC. Constraints (C.16), (C.17), and (C.20) force the variable $\chi_{t,\xi}$ that has a corresponding value of $\bar{\nu}_{t,\xi}$ closest to ν_t to equal 1, while the other $\chi_{t,\xi}$'s are forced to equal 0. Constraints (C.18) force the value of $\gamma_{t,\xi}$ corresponding to the $\chi_{t,\xi}$ that is equal to 1 to equal ψ_t , while constraints (C.19) force the other $\gamma_{t,\xi}$'s to equal zero. Thus:

$$\sum_{t,\xi} \gamma_{t,\xi} \bar{\nu}_{t,\xi},$$

represents the product between ψ_t and the value of $\bar{\nu}_{t,\xi}$ that is closest to ν_t .

Appendix D. Verification of Equilibria

The linearizations that are outlined in [Appendix C](#) allow us to convert the EPECs into MILPs. However, a solution to an EPEC is not necessarily a Nash equilibrium. Rather, it is a point that satisfies the KKT conditions of each firm's MPEC. To verify whether an EPEC solution is a Nash equilibrium, we use a diagonalization technique [36]. This method works by sequentially solving each firm's MPEC, holding the offers of its rival firms fixed. If the EPEC solution is optimal in each firm's MPEC, this means that no firm has a profitable unilateral deviation, and that the EPEC solution is a Nash equilibrium. Otherwise, the EPEC solution is not a Nash equilibrium.

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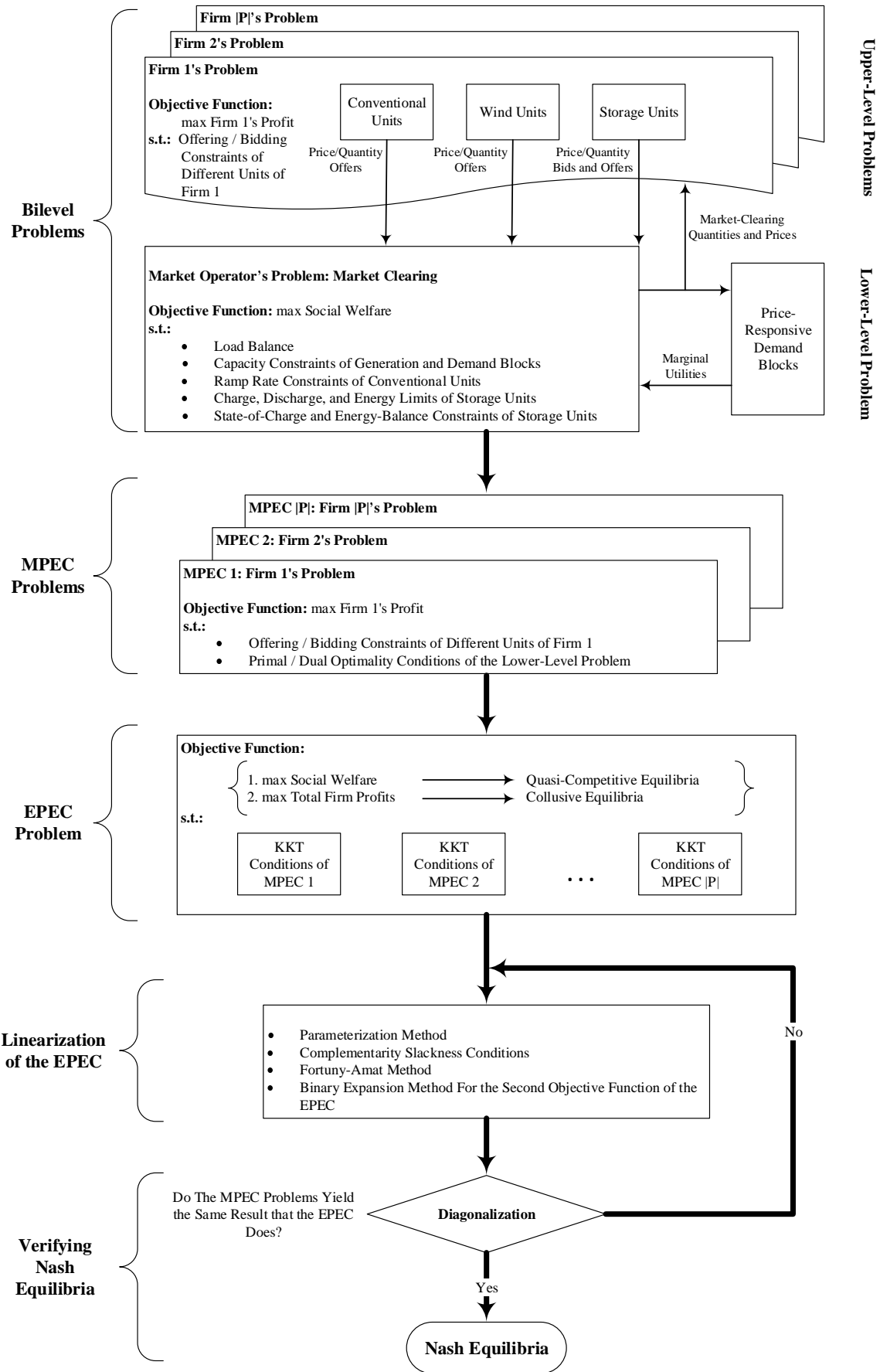


Figure 1: Overview of Equilibrium Modeling Approach