

When Energy Storage Reduces Social Welfare

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Abstract

This paper examines the potential welfare effects of storage under different market structures. This includes combinations of perfectly competitive and strategic generation and storage sectors, and standalone and generator-owned storage. We demonstrate that if the generation sector is perfectly competitive and does not own storage, then storage cannot be welfare-diminishing. Otherwise, generator-owned storage or standalone storage in a market with strategic generating firms can reduce welfare compared to the no-storage case. This contradicts conventional wisdom that adding firms to an imperfectly competitive market typically reduces welfare losses.

Key words: Energy storage, market structure, arbitrage, social welfare

JEL: C61, C72, D4, D6, Q4

1. Introduction

Recent developments in the electricity industry have increased interest in energy storage. This includes the introduction of markets that provide prices that signal the value of many of the services that storage can provide, and the ability of storage to ease the integration of renewables into power systems. EPRI (1976) provides an early discussion of storage technologies and their relative performance. Since it is framed by the 1970s, before the introduction of restructured electricity markets, it focuses on storage use by a vertically integrated utility to replace peaking generation capacity. More recent works, including those of EPRI-DOE (2003); Eyer et al. (2004); Eyer and Corey (2010); Denholm et al. (2010), recognize and discuss the broader array of services that storage can provide. This includes generation, transmission, and distribution capacity deferral, ancillary services, ramping, renewable curtailment, and end-user applications.

These discussions of potential storage uses are also supplemented by empirical and other analyses attempting to value these services. One of the most studied storage applications is energy arbitrage—charging storage when energy prices are low and discharging when prices are high. A number of works, including those of Graves et al. (1999); Figueiredo et al. (2006); Sioshansi et al. (2009, 2011), use historical price data and optimization models to estimate the value of storage. Walawalkar et al. (2007) estimate arbitrage value using historical price duration curves. These works find arbitrage values ranging between \$29/kW-year

and \$240/kW-year in the markets examined, with the differences mainly stemming from the mix of generators that are marginal on- and off-peak. These analyses implicitly assume that the storage plant is sufficiently small compared to the market that its charging and discharging decisions do not affect prices. Sioshansi et al. (2009) explore the effects of relaxing this assumption, showing that arbitrage values diminish if prices respond to storage use. This is because off-peak prices rise and on-peak prices fall when storage is used, since it results in greater off-peak and less on-peak generation.

Other works expand on these by examining the effects of storage in a setting with responsive prices, from both a private value and social welfare standpoint. Sioshansi (2010) uses a stylized model, in which the generation sector is perfectly competitive, to explore the effects of ownership on storage use and welfare. He shows that regardless of who owns it (generator-, load-, and standalone-ownership cases are examined), storage is used in a suboptimal manner, inasmuch as the welfare gains are less than what a social planner would achieve. Moreover, in some cases the addition of storage can reduce social welfare compared to the no-storage case. Sioshansi (2011) uses a case study, based on the Texas system, to examine storage and wind together in a market in which generators compete *a la* the supply function equilibrium model proposed by Klemperer and Meyer (1989). His work is motivated by the fact that wind can suppress energy prices by displacing high-cost generation. This price suppression can reduce wind profits and investment incentives since the effect is concentrated during hours with high wind availability. He demonstrates that storage can increase the selling price of wind, by charging storage when wind unduly suppresses prices and discharging during hours with lower wind avail-

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ability. He also shows that this use of storage results in social welfare losses, compared to not having storage in the market. [Schill and Kemfert \(2011\)](#) examine the profit and welfare effects of storage in the German electricity market. Using actual market data, they consider cases with generator-owned and standalone storage, assuming that the players follow Nash-Cournot equilibria. They also find cases in which storage reduces social welfare compared to a no-storage case.

The cases in which storage reduces social welfare can be unexpected, inasmuch as adding firms to an imperfectly competitive market typically improves allocative efficiency. Moreover, these findings are different from those of [Sioshansi et al. \(2009\)](#) who also examine storage use with responsive prices, but do not find welfare losses. This raises the question of what role market structure plays in these welfare losses, since these analyses study storage under different settings. [Sioshansi et al. \(2009\)](#); [Sioshansi \(2010\)](#) assume perfectly competitive generation, whereas [Sioshansi \(2011\)](#); [Schill and Kemfert \(2011\)](#) assume strategic behavior. The latter two analyses differ, however, in the specifics of the market structure considered. [Sioshansi \(2011\)](#) studies a high-wind case, in which strategic conventional generators compete in supply functions and strategic storage competes in quantities. [Schill and Kemfert \(2011\)](#), on the other hand, use the existing generator fleet with relatively little wind and assume that strategic conventional generators and strategic storage compete in quantities. [Sioshansi \(2011\)](#) further assumes storage to be standalone or owned by wind generators, whereas [Schill and Kemfert \(2011\)](#) study storage owned by conventional generators. Understanding what types of market and asset-ownership structures can potentially result in welfare losses is important and can help guide important policy decisions given today's storage renaissance.

The aim of this paper is to study these issues more methodically. We use a stylized model to examine what market and ownership structures can lead to storage having welfare-diminishing effects. We consider cases with different combinations of perfect competition or strategic behavior in the generation and storage sectors, and generator-owned and standalone storage, and arrive at four main findings. First, if the generation sector is perfectly competitive, standalone storage that is not owned by generators cannot be welfare-diminishing. However, strategic storage delivers less welfare benefits than perfectly competitive storage under such a market structure. Second, if generators behave strategically with respect to their production decisions, then storage can be welfare-diminishing. Under such conditions, perfectly competitive storage delivers greater welfare losses than strategic storage. Third, if storage is owned by a monopolist generating firm that makes perfectly competitive generation but strategic storage decisions, there can be welfare losses. If there are, instead, multiple symmetric storage-owning generators that make perfectly competitive generation but strategic storage decisions, the addition of storage cannot

give welfare losses. Finally, we show that storage owned by generating firms making strategic generation and storage decisions can lead to welfare losses with any number of firms.

The case of generating firms that make perfectly competitive generation but strategic storage decisions can appear unrealistic. This structure could arise in some markets, however. For instance, the California ISO places restrictions, based on tested heat rates, on the offers of conventional generators in its wholesale markets. Hydroelectric generators, which have a storage capability inasmuch as water can be withheld in one period to be used in another, are not subjected to such restrictions due to complex watershed constraints on their operations. This could give rise to the final set of cases that we examine.

The remainder of this paper is organized as follows. Section 2 details our generation and storage model and the various cases that we consider. Section 3 studies cases with perfectly competitive generation while Section 4 studies strategic generator cases with standalone (non generation-owned) storage. Section 5 considers cases of generator-owned storage and Section 6 concludes.

2. Basic Model

We study interactions between generation and storage and their effects on prices and welfare using a two-period model. The two periods modeled represent off- and on-peak periods. Demand is assumed to be price-responsive, with period- t demand given by:

$$D_t(p_t) = N_t - \gamma_t p_t,$$

where D_t is measured in MW and p_t in \$/MW. We assume $N_t, \gamma_t > 0$, implying that demand is strictly decreasing in price but positive for some range of prices. We use the convention that period $t = 1$ is the off-peak period and $t = 2$ on-peak. Thus, we assume that:

$$D_2(p) \geq D_1(p), \forall p \text{ such that } D_1(p) \geq 0.$$

These functions can also be inverted, giving the inverse demand functions:

$$P_t(d_t) = \frac{N_t - d_t}{\gamma_t}.$$

We study storage use under two generation market structures. One assumes a perfectly competitive generation market, while the other assumes strategic generators that follow Nash-Cournot equilibria. We assume that the same generator fleet, with the same cost, is available in both the off- and on-peak periods. The total per-period generation cost of the fleet in the perfectly competitive case is given by:

$$c(g_t) = b \cdot g_t + \frac{1}{2}c \cdot g_t^2,$$

where $b, c \geq 0$ and g_t and cost are measured in MW and dollars, respectively. We further assume that:

$$P_t(0) > c'(0), \forall t,$$

or that there are always some strictly positive gains from trade. The per-period marginal generation cost is given by:

$$c'(g_t) = b + c \cdot g_t.$$

Inverting the marginal cost function gives the perfectly competitive supply function:

$$s(p_t) = \frac{p_t - b}{c}.$$

The strategic generation cases assume that the generation fleet is divided between G symmetric firms, each of which has a per-period operating cost of:

$$\hat{c}(g_t) = \hat{b} \cdot g_t + \frac{1}{2} \hat{c} \cdot g_t^2.$$

This implies that each firm has a per-period marginal generation cost of:

$$c'(g_t) = \hat{b} + \hat{c} \cdot g_t.$$

Each generator's perfectly competitive supply function is found by inverting these marginal cost functions, giving:

$$\hat{s}(p_t) = \frac{p_t - \hat{b}}{\hat{c}},$$

which yields the sector-wide perfectly competitive supply function:

$$G \cdot \hat{s}(p_t) = G \frac{p_t - \hat{b}}{\hat{c}}$$

For our cases with perfect and imperfect competition to be comparable, we choose \hat{b} and \hat{c} to have equal sector-wide perfectly competitive supply, or such that $G \cdot \hat{s}(p_t) = s(p_t), \forall p$. This implies that $\hat{b} = b$ and $\hat{c} = G \cdot c$, meaning that each firm's cost function in the strategic generation cases is given by:

$$\hat{c}(g_t) = b \cdot g_t + \frac{1}{2} G \cdot c \cdot g_t^2.$$

We assume that the storage has a roundtrip efficiency $\epsilon \in (0, 1)$. For notational convenience, we define $\eta = 1/\epsilon$. Because we only model a two-period problem, we can abstract away energy constraints on storage. We, instead, assume that storage is potentially charged during the off-peak period and discharged in the subsequent on-peak period, only. Due to the efficiency losses, $\eta\delta$ MWh of energy must be stored off-peak in order to discharge δ MWh on-peak. We assume a total of $\bar{\delta}$ MW of storage is installed in the system, meaning at most $\eta\bar{\delta}$ MW can be charged off-peak and $\bar{\delta}$ discharged on-peak. This storage is assumed to be owned by a single firm, however we consider cases in Section 5 in which the storage assets are divided (evenly) among the G symmetric generating firms.

3. Effects of Storage with Perfectly Competitive Generation

We analyze the case with perfectly competitive generation by first deriving equilibrium generation and prices in the two periods as a function of storage use. We next derive the equilibrium level of storage use and then examine the welfare effects of this storage use.

3.1. Generation Equilibrium

Since the generation sector is perfectly competitive, quantities and prices during the two periods are given by the intersection of marginal cost and inverse demand. If we let g_t denote period- t generation, these equilibrium conditions are:

$$P_1(g_1 - \eta\delta) = c'(g_1), \quad (1)$$

and

$$P_2(g_2 + \delta) = c'(g_2). \quad (2)$$

The $g_1 - \eta\delta$ and $g_2 + \delta$ terms in (1) and (2) explicitly account for energy that is stored off-peak not being consumed whereas energy that is discharged on-peak is. Substituting the marginal cost and inverse demand functions into (1) and (2) and manipulating them gives the following equilibrium generation levels as a function of storage use:

$$g_1(\delta) = \frac{N_1 - b\gamma_1 + \eta\delta}{1 + c\gamma_1}, \quad (3)$$

and

$$g_2(\delta) = \frac{N_2 - b\gamma_2 - \delta}{1 + c\gamma_2}. \quad (4)$$

Moreover, substituting $g_1(\delta) - \eta\delta$ and $g_2(\delta) + \delta$ into the inverse demand functions gives the following equilibrium prices in each period as a function of storage use:

$$p_1(\delta) = \frac{N_1 c + b + c\eta\delta}{1 + c\gamma_1}, \quad (5)$$

and

$$p_2(\delta) = \frac{N_2 c + b - c\delta}{1 + c\gamma_2}. \quad (6)$$

3.2. Storage Equilibrium

We model two different types of storage equilibria, the first assumes that storage behaves perfectly competitively while the second assumes strategic storage use.

3.2.1. Perfectly Competitive Storage

Perfectly competitive storage is used on the basis of off- and on-peak prices, but does not regard its effect on prices. Thus, perfectly competitive storage stores off-peak energy so long as it costs less than the on-peak price (net of efficiency losses) at which it is sold. This amounts to storage choosing $\delta \in [0, \bar{\delta}]$ such that $p_2(\delta) = \eta p_1(\delta)$, if such a δ exists. The derivatives of price functions (5) and (6) are:

$$p'_1(\delta) = \frac{c\eta}{1 + c\gamma_1} > 0,$$

and:

$$p'_2(\delta) = \frac{-c}{1+c\gamma_2} < 0.$$

Thus, equilibrium storage use, $\delta_{C,C}$,¹ can take on one of three possible values. If $p_2(0) < \eta p_1(0)$ then $\delta_{C,C} = 0$, since the net on-peak energy price is less than the off-peak price. If $p_2(\bar{\delta}) > \eta p_1(\bar{\delta})$ then $\delta_{C,C} = \bar{\delta}$. Since $p_1(\delta)$ and $p_2(\delta)$ are linear in δ , we can write these as first-order Taylor expansions, in which case this latter condition becomes $\delta_{C,C} = \bar{\delta}$ if:

$$p_2(0) + \bar{\delta}p'_2(0) > \eta[p_1(0) + \bar{\delta}p'_1(0)],$$

or if:

$$\frac{p_2(0) - \eta p_1(0)}{\eta p'_1(0) - p'_2(0)} > \bar{\delta}.$$

The third possibility is that $\delta_{C,C}$ is an interior solution (*i.e.*, not equal to either of its bounds) and is given by the unique value that solves:

$$p_2(\delta_{C,C}) = \eta p_1(\delta_{C,C}).$$

Again, we can write $p_1(\delta)$ and $p_2(\delta)$ as their first-order Taylor expansions:

$$p_2(0) + \delta_{C,C} \cdot p'_2(0) = \eta[p_1(0) + \delta_{C,C} \cdot p'_1(0)],$$

which gives the solution:

$$\delta_{C,C} = \frac{p_2(0) - \eta p_1(0)}{\eta p'_1(0) - p'_2(0)}.$$

3.2.2. Strategic Storage

A strategic storage operator determines storage operations from a profit-maximization problem of the form:

$$\begin{aligned} \max_{\delta} \Pi_S(\delta) &= [p_2(\delta) - \eta p_1(\delta)]\delta \\ \text{s.t. } 0 &\leq \delta \leq \bar{\delta} \end{aligned} \quad (7)$$

The Karush-Kuhn-Tucker (KKT) conditions for an optimum are:

$$[\eta p'_1(\delta) - p'_2(\delta)]\delta + \eta p_1(\delta) - p_2(\delta) - \mu^- + \mu^+ = 0,$$

$$0 \leq \delta \perp \mu^- \geq 0,$$

$$\delta \leq \bar{\delta} \perp \mu^+ \geq 0,$$

where μ^- and μ^+ are Lagrange multipliers associated with the inequality constraints. Solving these conditions gives:

$$\delta_{S,C} = \begin{cases} 0, & \text{if } p_2(0) < \eta p_1(0), \\ \bar{\delta}, & \text{if } \frac{p_2(0) - \eta p_1(0)}{2[\eta p'_1(0) - p'_2(0)]} \geq \bar{\delta}, \\ \frac{p_2(0) - \eta p_1(0)}{2[\eta p'_1(0) - p'_2(0)]}, & \text{otherwise.} \end{cases}$$

¹We use the notational convention that δ 's are subscripted by two letters, both either a 'C' or an 'S.' The first indicates perfectly competitive or strategic storage and the second perfectly competitive or strategic generation.

If we compare these conditions to those derived for perfectly competitive storage, it is clear that $\delta_{S,C} \leq \delta_{C,C}$, with strict inequality for some values of N_t and γ_t , which is consistent with the findings of Sioshansi et al. (2009); Sioshansi (2010).

Since we also have that:

$$\frac{\partial^2}{\partial \delta^2} [\Pi_S(\delta)] = -2c \left(\frac{\eta^2}{1+c\gamma_1} + \frac{1}{1+c\gamma_2} \right) < 0,$$

the storage profit function is concave and the KKT conditions are sufficient for a global optimum.

3.3. Welfare Effects of Storage

Storage use has consumer and producer welfare effects, which are illustrated in Figure 1. The figure shows examples of off- and on-peak demand functions without storage, and the effect of storage use on these functions. Specifically, if $\eta\delta$ MW is stored off-peak and discharged on-peak, this shifts the off-peak demand function rightward by $\eta\delta$ and shifts the on-peak function leftward by δ . These demand shifts change off- and on-peak prices, generation, and consumption, giving welfare changes.

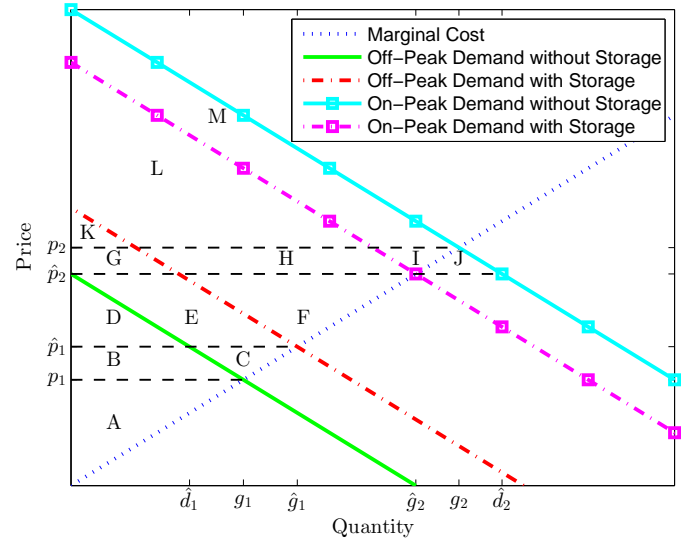


Figure 1: Welfare effects of storage use with a perfectly competitive generation sector.

Without storage use, the off-peak price is p_1 and consumption and generation are g_1 . When storage is added, off-peak generation increases to \hat{g}_1 , increasing the price to \hat{p}_1 and decreasing consumption to \hat{d}_1 . This gives a consumer surplus loss during the off-peak period—without storage consumer welfare is equal to the sum of the areas denoted 'B' and 'D' whereas with storage it is equal to the area denoted 'D' only. Storage has the opposite effect on-peak. Without storage, the on-peak price is p_2 and consumption and generation are g_2 . When storage is used, on-peak generation decreases to \hat{g}_2 , decreasing the price to \hat{p}_2 and increasing consumption to \hat{d}_2 . These changes yield an

on-peak consumer welfare increase—without storage consumer surplus is equal to the sum of the areas denoted ‘K,’ ‘L,’ and ‘M’ whereas it increases to the sum of the areas denoted ‘G,’ ‘H,’ ‘I,’ ‘J,’ ‘K,’ ‘L,’ and ‘M’ with storage. Thus the net consumer welfare effect of storage is given by the sum of the areas denoted ‘G,’ ‘H,’ ‘I,’ and ‘J,’ less the area denoted ‘B’. We can compute the consumer welfare change in each period as the integral of the difference between the inverse demand function and the price of energy, or as:

$$\begin{aligned} CW(\delta) &= \int_0^{g_1(\delta)-\eta\delta} [P_1(x) - p_1(\delta)]dx \\ &\quad + \int_0^{g_2(\delta)+\delta} [P_2(x) - p_2(\delta)]dx \\ &= \frac{1}{2\gamma_1}[g_1(\delta) - \eta\delta]^2 + \frac{1}{2\gamma_2}[g_2(\delta) + \delta]^2. \end{aligned}$$

These price and generation changes also affect producer (generator) surplus. Off-peak generator profit without storage is given by the area denoted ‘A’ and this increases to the sum of the areas denoted ‘A,’ ‘B,’ and ‘C’ with storage use. Conversely, on-peak generator profit is given by the sum of the areas denoted ‘A,’ ‘B,’ ‘C,’ ‘D,’ ‘E,’ ‘F,’ ‘G,’ ‘H,’ and ‘I,’ without storage and decreases to the sum of the areas denoted ‘A,’ ‘B,’ ‘C,’ ‘D,’ ‘E,’ and ‘F’ with storage. Thus the net producer welfare effect of storage is given by the sum of the areas denoted ‘B’ and ‘C’ less the sum of the areas denoted ‘G,’ ‘H,’ and ‘I.’ We can compute this producer welfare change as the integral of the difference between the price of energy and the marginal cost of generation, or as:

$$\begin{aligned} PW(\delta) &= \int_0^{g_1(\delta)} [p_1(\delta) - c'(x)]dx \\ &\quad + \int_0^{g_2(\delta)} [p_2(\delta) - c'(x)]dx \\ &= \frac{c}{2}[g_1(\delta)^2 + g_2(\delta)^2] \end{aligned}$$

Sioshansi (2010) notes that in addition to the consumer and producer surplus changes, storage profits represent a true welfare effect, and is not merely a wealth transfer to the storage owner. This is because storage displaces high-cost on-peak energy with low-cost off-peak energy. Storage profit measures the net value of this intertemporal generation shifting, including energy lost due to the storage process. Storage profit, $\Pi_S(\delta)$ is defined in (7), and thus total social welfare, as a function of storage use, is given by:

$$W(\delta) = CW(\delta) + PW(\delta) + \Pi_S(\delta). \quad (8)$$

We now show in the following lemmas and corollaries that perfectly competitive storage maximizes social welfare, whereas strategic storage use can result in less welfare than the perfectly competitive case. We also show that standalone storage, whether behaving perfectly competitively or strategically, can never yield net social welfare

losses compared to a case without storage, when generation is perfectly competitive as assumed here.

Lemma 1. *If the generation sector is perfectly competitive and storage is owned by a perfectly competitive standalone firm, the resulting generation and storage use give the unique global welfare-maximizer.*

Proof. Substituting in the terms defining $W(\delta)$ gives:

$$\begin{aligned} W(\delta) &= \delta \left[\frac{N_2c + b - c\delta}{1 + c\gamma_2} - \eta \frac{N_1c + b + c\eta\delta}{1 + c\gamma_1} \right] \\ &\quad + \frac{1}{2} \left\{ \frac{(N_1 - b\gamma_1)^2 + c\gamma_1\eta^2\delta^2}{\gamma_1 \cdot (1 + c\gamma_1)} + \frac{(N_2 - b\gamma_2)^2 + c\gamma_2\delta^2}{\gamma_2 \cdot (1 + c\gamma_2)} \right\}. \end{aligned}$$

Thus we have:

$$\begin{aligned} W'(\delta) &= -\frac{c\delta}{1 + c\gamma_2} + \frac{N_2c + b - c\delta}{1 + c\gamma_2} - \frac{c\eta^2\delta}{1 + c\gamma_1} \\ &\quad - \eta \frac{N_1c + b + c\eta\delta}{1 + c\gamma_1} + \frac{1}{2} \left[\frac{2c\gamma_1\eta^2\delta}{\gamma_1 \cdot (1 + c\gamma_1)} + \frac{2c\gamma_2\delta}{\gamma_2 \cdot (1 + c\gamma_2)} \right] \\ &= p_2(0) - \eta p_1(0) - \delta \cdot [\eta p_1'(0) - p_2'(0)] \end{aligned}$$

The welfare-maximization problem is defined as:

$$\begin{aligned} \max_{\delta} & W(\delta) \\ \text{s.t.} & 0 \leq \delta \leq \bar{\delta}. \end{aligned}$$

Thus, the KKT conditions for a maximum are:

$$\begin{aligned} [\eta p_1'(0) - p_2'(0)]\delta + \eta p_1(0) - p_2(0) - \mu^- + \mu^+ &= 0, \\ 0 \leq \delta \quad \perp \quad \mu^- &\geq 0, \\ \delta \leq \bar{\delta} \quad \perp \quad \mu^+ &\geq 0, \end{aligned}$$

where μ^- and μ^+ are Lagrange multipliers associated with the inequality constraints. Solving these conditions gives:

$$\delta^* = \begin{cases} 0, & \text{if } p_2(0) < \eta p_1(0), \\ \bar{\delta}, & \text{if } \frac{p_2(0) - \eta p_1(0)}{\eta p_1'(0) - p_2'(0)} \geq \bar{\delta}, \\ \frac{p_2(0) - \eta p_1(0)}{\eta p_1'(0) - p_2'(0)}, & \text{otherwise,} \end{cases}$$

which are the same conditions defining $\delta_{C,C}$. Since we also have that:

$$W''(\delta) = - \left(\frac{c}{1 + c\gamma_2} + \frac{c\eta^2}{1 + c\gamma_1} \right) < 0,$$

we know that the welfare-maximization problem is convex and the KKT conditions are sufficient for the unique global maximum. \square

We now show that with perfectly competitive generation, strategic standalone storage yields less social welfare than perfectly competitive storage. It can never, however, yield social welfare losses compared to not having storage in the market.

Corollary 1. *If the generation sector is perfectly competitive and storage is owned by a strategic standalone firm, the resulting generation and storage use yield less social welfare than the perfectly competitive storage case.*

Proof. Since $W(\delta)$ is strictly concave in δ , $W(\delta_{S,C}) < W(\delta_{C,C})$ whenever $\delta_{S,C} \neq \delta_{C,C}$. \square

Corollary 2. *If the generation sector is perfectly competitive, storage use by a standalone storage firm never yields a net social welfare loss compared to not having storage in the market.*

Proof. By definition, $\delta_{C,C}$ cannot result in social welfare losses compared to not having storage, since $\delta_{C,C} = 0$ is feasible and $\delta_{C,C}$ is welfare-maximizing.

To show that strategic storage cannot yield social welfare losses, note that $\delta_{S,C} > 0$ if and only if:

$$p_2(0) - \eta p_1(0) = W'(0) > 0.$$

Moreover, since $\delta_{S,C} \leq \delta_{C,C}$ and $W(\delta)$ is concave in δ , we know that a strategic storage firm uses storage if and only if it is strictly welfare-enhancing. \square

We further show that in the special case in which $\gamma_1 = \gamma_2 = \gamma$, storage use always results in a consumer welfare increase and producer surplus loss.

Lemma 2. *If the generation sector is perfectly competitive and $\gamma_1 = \gamma_2 = \gamma$, storage use by a standalone storage firm always results in a consumer welfare increase.*

Proof. Substituting in the terms defining $CW(\delta)$ and differentiating gives:

$$\begin{aligned} CW'(0) &= \frac{N_2c - bc\gamma_2}{(1 + c\gamma_2)^2} - \eta \frac{N_1c - bc\gamma_1}{(1 + c\gamma_1)^2} \\ &= \frac{1}{1 + c\gamma_2} [p_2(0) - b] \\ &\quad - \eta \frac{1}{1 + c\gamma_1} [p_1(0) - b], \end{aligned}$$

and

$$CW''(\delta) = \frac{c^2\gamma_1\eta^2}{(1 + c\gamma_1)^2} + \frac{c^2\gamma_2}{(1 + c\gamma_2)^2} > 0.$$

Thus, consumer welfare is convex in δ . If $CW'(0) \geq 0$ whenever storage is used, then storage use is always consumer-welfare-enhancing and $\delta = \bar{\delta}$ maximizes consumer welfare. If we take the special case in which $\gamma_1 = \gamma_2 = \gamma$ we have:

$$CW'(0) = \frac{1}{1 + c\gamma} [p_2(0) - \eta p_1(0) + b \cdot (\eta - 1)]$$

By definition of b and η we have that $b \cdot (\eta - 1) > 0$. Furthermore, we know that perfectly competitive and strategic storage are only used if $p_2(0) - \eta p_1(0) > 0$, thus storage is only used if $CW'(0) > 0$ and always increases consumer

welfare. We further know, due to the convexity of the consumer welfare function that:

$$CW(0) \leq CW(\delta_{S,C}) \leq CW(\delta_{C,C}) \leq CW(\bar{\delta}).$$

\square

Corollary 3. *If the generation sector is perfectly competitive and $\gamma_1 = \gamma_2 = \gamma$, storage use by a standalone storage firm always results in a producer welfare loss.*

Proof. Substituting in the terms defining $PW(\delta)$ and differentiating gives:

$$\begin{aligned} PW'(\delta) &= c \cdot \left[\eta \frac{N_1 - b\gamma_1 + \eta\delta}{(1 + c\gamma_1)^2} - \frac{N_2 - b\gamma_2 - \delta}{(1 + c\gamma_2)^2} \right] \\ &= \frac{\eta}{1 + c\gamma_1} [p_1(0) - b] \\ &\quad - \frac{1}{1 + c\gamma_2} [p_2(0) - b] \\ &\quad + \delta \left[\frac{\eta}{1 + c\gamma_1} p_1'(0) - \frac{1}{1 + c\gamma_2} p_2'(0) \right], \end{aligned}$$

and:

$$PW''(\delta) = \frac{c\eta^2}{(1 + c\gamma_1)^2} + \frac{c}{(1 + c\gamma_2)^2} > 0.$$

We further have that:

$$\begin{aligned} PW'(0) &= \frac{\eta}{1 + c\gamma_1} [p_1(0) - b] \\ &\quad - \frac{1}{1 + c\gamma_2} [p_2(0) - b] \\ &= -CW'(0). \end{aligned}$$

Since we know $CW'(0) > 0$ if storage is used, we have that $PW'(0) < 0$. Since producer welfare is convex in δ , it is possible for storage use to increase generator profits if δ is sufficiently large. A necessary condition for this to occur is that there exists a $\delta \leq \bar{\delta}$ for which $PW'(\delta) \geq 0$. Note, however, that solving $PW'(\delta) = 0$ gives:

$$\delta = \frac{\frac{\eta}{1+c\gamma_1} [p_1(0) - b] - \frac{1}{1+c\gamma_2} [p_2(0) - b]}{\frac{1}{1+c\gamma_2} p_2'(0) - \frac{\eta}{1+c\gamma_1} p_1'(0)},$$

which simplifies to:

$$\delta = \frac{p_2(0) - \eta p_1(0)}{\eta p_1'(0) - p_2'(0)} + b \frac{\eta - 1}{\eta p_1'(0) - p_2'(0)},$$

if $\gamma_1 = \gamma_2 = \gamma$. Since:

$$b \frac{\eta - 1}{\eta p_1'(0) - p_2'(0)} > 0,$$

this quantity is greater than both $\delta_{S,C}$ and $\delta_{C,C}$, implying that storage use always reduces producer welfare. Since $PW'(\delta) < 0$ for $\delta \leq \delta_{C,C}$, we can further conclude that:

$$PW(0) \geq PW(\delta_{S,C}) \geq PW(\delta_{C,C}).$$

\square

4. Effects of Storage with Strategic Generation

We now consider a case in which the generation sector consists of G symmetric strategic firms and a single perfectly competitive or strategic storage firm. We proceed with this case as before, by first deriving equilibrium generation decisions in the two periods, as a function of storage use, and resulting prices. We then derive equilibrium storage use and determine its welfare effects.

4.1. Generation Equilibrium

We assume that the generators follow Nash-Cournot strategies by making generation decisions in the off- and on-peak periods. We let $g_{i,t}$ denote generator i 's period- t production, which gives the following profit-maximization problem for generator i :

$$\begin{aligned} \max_{g_{i,t}} \quad & \Pi_i^G(g) = P_1(g_1^G - \eta\delta) \cdot g_{i,1} - \hat{c}(g_{i,1}) \\ & + P_2(g_2^G + \delta) \cdot g_{i,2} - \hat{c}(g_{i,2}), \end{aligned}$$

where $g_t^G = \sum_{i=1}^G g_{i,t}$ denotes total period- t generation. The KKT conditions for generator i 's problem are:

$$\begin{aligned} P_1(g_1^G - \eta\delta) + P_1'(g_1^G - \eta\delta) \cdot g_{i,1} - \hat{c}'(g_{i,1}) &= 0, \\ P_2(g_2^G + \delta) + P_2'(g_2^G + \delta) \cdot g_{i,2} - \hat{c}'(g_{i,2}) &= 0. \end{aligned}$$

To show that any Nash-Cournot equilibrium is symmetric, we first note that by definition we must have:

$$\frac{\partial}{\partial g_{i,t}} \Pi_i^G(g) - \frac{\partial}{\partial g_{j,t}} \Pi_j^G(g) = 0,$$

which implies that:

$$(g_{j,t} - g_{i,t}) \left(\frac{1}{\gamma_t} + G \cdot c \right) = 0.$$

Since $1/\gamma_t + G \cdot c > 0$ we must have $g_{i,t} = g_{j,t} \forall i, j = 1, 2, \dots, G$ and $t = 1, 2$. Due to the symmetry of the equilibrium, we can rewrite the KKT conditions as:

$$\frac{N_1 - g_1^G + \eta\delta}{\gamma_1} - \frac{g_1^G}{G\gamma_1} - b - c \cdot g_1^G = 0,$$

and:

$$\frac{N_2 - g_2^G - \delta}{\gamma_2} - \frac{g_2^G}{G\gamma_2} - b - c \cdot g_2^G = 0.$$

Manipulating these KKT conditions gives the equilibrium generation levels as:

$$g_1^G(\delta) = \frac{G \cdot (N_1 - b\gamma_1 + \eta\delta)}{1 + G \cdot (1 + c\gamma_1)},$$

and:

$$g_2^G(\delta) = \frac{G \cdot (N_2 - b\gamma_2 - \delta)}{1 + G \cdot (1 + c\gamma_2)}.$$

Substituting $g_1^G(\delta) - \eta\delta$ and $g_2^G(\delta) + \delta$ into the inverse demand functions also gives periods-1 and -2 prices as:

$$p_1^G(\delta) = \frac{(N_1 + \eta\delta)(1 + Gc\gamma_1) + G \cdot b\gamma_1}{\gamma_1 \cdot (1 + G \cdot (1 + c\gamma_1))},$$

and:

$$p_2^G(\delta) = \frac{(N_2 - \delta)(1 + Gc\gamma_2) + Gb\gamma_2}{\gamma_2 \cdot (1 + G \cdot (1 + c\gamma_2))}.$$

Finally, we have that:

$$\nabla^2 \Pi_i^G(g) = \begin{bmatrix} -\frac{2}{\gamma_1} - Gc & 0 \\ 0 & -\frac{2}{\gamma_2} - Gc \end{bmatrix},$$

which is negative definite, meaning that the KKT conditions are sufficient for a unique global maximum to each generator's profit-maximization problem.

4.2. Storage Equilibrium

As before, we model two types of storage equilibria. The first assumes perfectly competitive storage while the second assumes strategic behavior.

4.2.1. Perfectly Competitive Storage

Perfectly competitive storage takes prices as given and chooses δ as a function of prices. In equilibrium, prices adjust such that $p_2^G(\delta) = \eta p_1^G(\delta)$, if such a δ exists. As in the perfectly competitive generation case, we have that:

$$p_1^{G'}(\delta) = \frac{\eta \cdot (1 + Gc\gamma_1)}{\gamma_1 \cdot (1 + G \cdot (1 + c\gamma_1))} > 0,$$

and

$$p_2^{G'}(\delta) = -\frac{1 + Gc\gamma_2}{\gamma_2 \cdot (1 + G \cdot (1 + c\gamma_2))} < 0.$$

Therefore, equilibrium storage use, $\delta_{C,S}$, can take on one of three possible values. If $p_2^G(0) < \eta p_1^G(0)$ then $\delta_{C,S} = 0$ whereas if $p_2^G(\bar{\delta}) > \eta p_1^G(\bar{\delta})$ then $\delta_{C,S} = \bar{\delta}$. Otherwise:

$$\delta_{C,S} = \frac{p_2^G(0) - \eta p_1^G(0)}{\eta p_1^{G'}(0) - p_2^{G'}(0)}.$$

4.2.2. Strategic Storage

The storage profit-maximization problem is defined as:

$$\begin{aligned} \max_{\delta} \quad & \Pi_S^G(\delta) = [p_2^G(\delta) - \eta p_1^G(\delta)]\delta \\ \text{s.t.} \quad & 0 \leq \delta \leq \bar{\delta}. \end{aligned} \quad (9)$$

The KKT conditions for an optimum are:

$$\begin{aligned} [\eta p_1^{G'}(\delta) - p_2^{G'}(\delta)]\delta + \eta p_1^G(\delta) - p_2^G(\delta) - \mu^- + \mu^+ &= 0, \\ \delta \geq 0 \quad \perp \quad \mu^- &\geq 0, \\ \delta \leq \bar{\delta} \quad \perp \quad \mu^+ &\geq 0, \end{aligned}$$

where μ^- and μ^+ are Lagrange multipliers associated with the two inequality constraints. Solving these conditions gives:

$$\delta_{S,S} = \begin{cases} 0, & \text{if } p_2^G(0) < \eta p_1^G(0), \\ \bar{\delta}, & \text{if } \frac{p_2^G(0) - \eta p_1^G(0)}{2[\eta p_1^{G'}(0) - p_2^{G'}(0)]} \geq \bar{\delta}, \\ \frac{p_2^G(0) - \eta p_1^G(0)}{2[\eta p_1^{G'}(0) - p_2^{G'}(0)]}, & \text{otherwise.} \end{cases}$$

We also note that since:

$$\Pi_S^{G''}(\delta) = 2[p_2^{G'}(\delta) - \eta p_1^{G'}(\delta)] < 0,$$

the KKT conditions are sufficient for a unique global maximum of the storage firm's problem.

4.3. Welfare Effects of Storage

As in the perfectly competitive generation case, storage use has three welfare impacts. Consumer welfare is defined by the integral of the difference between the inverse demand functions and energy prices in the two periods:

$$\begin{aligned} CW^G(\delta) &= \int_0^{g_1^G(\delta) - \eta\delta} [P_1(x) - p_1^G(\delta)] dx \\ &\quad + \int_0^{g_2^G(\delta) + \delta} [P_2(x) - p_2^G(\delta)] dx \\ &= \frac{1}{2\gamma_1} [g_1^G(\delta) - \eta\delta]^2 + \frac{1}{2\gamma_2} [g_2^G(\delta) + \delta]^2. \end{aligned}$$

Producer surplus is defined by the integral of the difference between the energy prices and marginal generation costs in the two periods:

$$\begin{aligned} PW^G(\delta) &= \int_0^{g_1^G(\delta)} [p_1^G(\delta) - c'(x)] dx \\ &\quad + \int_0^{g_2^G(\delta)} [p_2^G(\delta) - c'(x)] dx \\ &= \frac{2 + Gc\gamma_1}{2G\gamma_1} g_1^G(\delta)^2 + \frac{2 + Gc\gamma_2}{2G\gamma_2} g_2^G(\delta)^2. \end{aligned}$$

Storage profit is defined in (9).

Taking the special case in which $\gamma_1 = \gamma_2 = \gamma$, we now show that with strategic generation it is generally possible for storage to be used when it is social welfare-diminishing.

Lemma 3. *If the generation sector consists of G symmetric firms following Nash-Cournot equilibrium strategies and $\gamma_1 = \gamma_2 = \gamma$, storage use can yield a social welfare loss compared to not having storage in the market.*

Proof. See appendix. \square

5. Effects of Generator-Owned Storage

We finally consider a case in which storage is owned by the generators themselves, as opposed to the standalone case considered thus far. As in Section 4, we assume that

there are G symmetric generating firms, and that generator i 's period- t cost is given by $\hat{c}(g_{i,t})$. We assume the total discharging capacity of all storage assets in the market to be $\bar{\delta}$ MW and that these are evenly divided among the G generating firms. This means each generating firm owns $\bar{\delta}/G$ MW of discharging capacity.

We examine two different firm-behavior cases. The first assumes that the firms behave perfectly competitively with respect to their generation decisions, but use storage strategically. The second assumes that the firms use both generation and storage strategically.

5.1. Perfectly Competitive-Generation Equilibrium

In the perfectly competitive-generation case, equilibrium energy prices and total generation, as functions of total storage use, are given by equations (3) through (6). Each generator determines how much energy to store off-peak and discharge on-peak to maximize total firm profits from generation and storage use. We assume that in doing so the generators follow Nash-Cournot equilibrium strategies. If we let δ_i denote the amount of energy discharged on-peak by generator i (implying $\eta\delta_i$ is stored off-peak), then generator i 's profit-maximization problem is defined as:

$$\begin{aligned} \max_{\delta_i} \Pi_i^{C,O}(\delta) &= p_1(\delta) \frac{g_1(\delta)}{G} + p_2(\delta) \frac{g_2(\delta)}{G} \\ &\quad + \delta_i [p_2(\delta) - \eta p_1(\delta)] \\ \text{s.t. } &0 \leq \delta_i \leq \bar{\delta}/G, \end{aligned}$$

where $\delta = \sum_{i=1}^G \delta_i$ is total storage use and the added 'O' superscript on Π denotes the fact that storage is owned by generator firms that make perfectly competitive generation decisions. The KKT conditions for generator i 's problem are:

$$\begin{aligned} -p_1(\delta) \frac{g_1'(\delta)}{G} - p_1'(\delta) \frac{g_1(\delta)}{G} - p_2(\delta) \frac{g_2'(\delta)}{G} \\ - p_2'(\delta) \frac{g_2(\delta)}{G} + \delta_i [\eta p_1'(\delta) - p_2'(\delta)] \\ + \eta p_1(\delta) - p_2(\delta) - \mu_i^- + \mu_i^+ = 0, \end{aligned} \quad (10)$$

$$\begin{aligned} \delta_i \geq 0 &\quad \perp \quad \mu_i^- \geq 0, \\ \delta_i \leq \bar{\delta}/G &\quad \perp \quad \mu_i^+ \geq 0, \end{aligned}$$

where μ_i^- and μ_i^+ are the Lagrange multipliers associated with the inequality constraints in generator i 's problem. We next show, in the following lemma, that the equilibrium must be symmetric.

Lemma 4. *If the market consists of G symmetric storage-owning generating firms that use generation perfectly competitively and storage strategically, any storage-use Nash-Cournot equilibrium is symmetric.*

Proof. See appendix. \square

Since the equilibrium is symmetric, the KKT conditions can be rewritten as:

$$-p_1(\delta)\frac{g_1'(\delta)}{G} - p_1'(\delta)\frac{g_1(\delta)}{G} - p_2(\delta)\frac{g_2'(\delta)}{G} - p_2'(\delta)\frac{g_2(\delta)}{G} + \frac{\delta}{G}[\eta p_1'(\delta) - p_2'(\delta)] + \eta p_1(\delta) - p_2(\delta) - \mu^- + \mu^+ = 0,$$

where μ^- and μ^+ are Lagrange multipliers associated with the constraints on total storage use. The KKT conditions imply that $\delta_C^O = 0$ if:

$$p_2(0)[1 - 1/\phi_2] + \frac{p_2'(0)g_2(0)}{G} < \eta p_1(0)[1 - 1/\phi_1] - \frac{p_1'(0)g_1(0)}{G}, \quad (11)$$

where $\phi_t = G \cdot (1 + c\gamma_t)$, $\delta_C^O = \bar{\delta}$ if:

$$\frac{p_2(0)[1 - \frac{1}{\phi_2}] - \eta p_1(0)[1 - \frac{1}{\phi_1}] + \frac{p_1'(0)g_1(0) + p_2'(0)g_2(0)}{G}}{\eta p_1'(0)[1 + \frac{c\gamma_1 - 1}{\phi_1}] - p_2'(0)[1 + \frac{c\gamma_2 - 1}{\phi_2}]} > \bar{\delta},$$

and that:

$$\delta_C^O = \frac{p_2(0)[1 - \frac{1}{\phi_2}] - \eta p_1(0)[1 - \frac{1}{\phi_1}] + \frac{p_1'(0)g_1(0) + p_2'(0)g_2(0)}{G}}{\eta p_1'(0)[1 + \frac{c\gamma_1 - 1}{\phi_1}] - p_2'(0)[1 + \frac{c\gamma_2 - 1}{\phi_2}]}$$

otherwise. We further have that:

$$\Pi_i^{C,O''}(\delta) = \frac{2c\eta^2 \cdot (1 - \phi_1)}{\phi_1 \cdot (1 + c\gamma_1)} + \frac{2c \cdot (1 - \phi_2)}{\phi_2 \cdot (1 + c\gamma_2)} < 0.$$

Thus, $\Pi_i^{C,O}(\delta)$ is strictly concave in δ and the KKT conditions are sufficient for a unique global maximum.

5.2. Strategic-Generation Equilibrium

In this case generators are assumed to co-optimize their generation quantity and storage decisions to maximize total firm profits, while accounting for the effect of their decisions on prices. We, again, assume that in doing so they follow Nash-Cournot equilibrium strategies. Letting $g_{i,t}$ denote firm i 's period- t generation and δ_i the amount of energy discharged on-peak, generator i 's profit-maximization problem is defined as:

$$\begin{aligned} \max_{g_{i,t}, \delta_i} \Pi_i^{S,O}(g, \delta) &= P_1(g_1^G - \eta\delta) \cdot (g_{i,1} - \eta\delta_i) - \hat{c}(g_{i,1}) \\ &\quad + P_2(g_2^G + \delta) \cdot (g_{i,2} + \delta_i) - \hat{c}(g_{i,2}) \\ \text{s.t. } 0 &\leq \delta_i \leq \bar{\delta}/G, \end{aligned}$$

where $g_t^G = \sum_{i=1}^G g_{i,t}$ is total period- t generation and $\delta = \sum_{i=1}^G \delta_i$ is total storage use. The KKT conditions for an optimum to generator i 's problem are:

$$-P_1(g_1^G - \eta\delta) - (g_{i,1} - \eta\delta_i) \cdot P_1'(g_1^G - \eta\delta) + \hat{c}'(g_{i,1}) = 0, \quad (12)$$

$$-P_2(g_2^G + \delta) - (g_{i,2} + \delta_i) \cdot P_2'(g_2^G + \delta) + \hat{c}'(g_{i,2}) = 0, \quad (13)$$

$$\begin{aligned} \eta P_1(g_1^G - \eta\delta) + \eta \cdot (g_{i,1} - \eta\delta_i) \cdot P_1'(g_1^G - \eta\delta) - P_2(g_2^G + \delta) \\ - (g_{i,2} + \delta_i) \cdot P_2'(g_2^G + \delta) - \mu_i^- + \mu_i^+ = 0, \quad (14) \\ \delta_i \geq 0 \quad \perp \quad \mu_i^- \geq 0, \\ \delta_i \leq \bar{\delta}/G \quad \perp \quad \mu_i^+ \geq 0, \end{aligned}$$

where μ_i^- and μ_i^+ are Lagrange multipliers associated with the inequality constraints in generator i 's problem. We next show, in the following lemma, that the equilibrium must be symmetric.

Lemma 5. *If the market consists of G symmetric storage-owning generating firms that make strategic generation and storage decisions, any generation and storage-use Nash-Cournot equilibrium is symmetric.*

Proof. See appendix. \square

Due to the symmetry of the equilibrium, the KKT stationarity conditions can be rewritten as:

$$\begin{aligned} -P_1(g_1^G - \eta\delta) - \frac{g_1^G - \eta\delta}{G} P_1'(g_1^G - \eta\delta) + \hat{c}'(g_1^G/G) = 0, \\ -P_2(g_2^G + \delta) - \frac{g_2^G + \delta}{G} P_2'(g_2^G + \delta) + \hat{c}'(g_2^G/G) = 0, \\ \eta P_1(g_1^G - \eta\delta) + \eta \frac{g_1^G - \eta\delta}{G} P_1'(g_1^G - \eta\delta) - P_2(g_2^G + \delta) \\ - \frac{g_2^G + \delta}{G} P_2'(g_2^G + \delta) - \mu^- + \mu^+ = 0, \end{aligned}$$

where μ^- and μ^+ are Lagrange multipliers associated with the inequality constraints on total storage use. The first two stationarity conditions define total period- t generation as:

$$g_1^{S,O}(\delta) = \frac{G \cdot (N_1 - b\gamma_1) + (G + 1)\eta\delta}{1 + G \cdot (1 + c\gamma_1)},$$

and

$$g_2^{S,O}(\delta) = \frac{G \cdot (N_2 - b\gamma_2) - (G + 1)\delta}{1 + G \cdot (1 + c\gamma_2)}.$$

Substituting $g_1^{S,O}(\delta) - \eta\delta$ and $g_2^{S,O}(\delta) + \delta$ into the inverse demand functions gives equilibrium energy prices:

$$p_1^{S,O}(\delta) = \frac{(N_1 + \eta\delta)(1 + Gc\gamma_1) + Gb\gamma_1 - \eta\delta}{\gamma_1 \cdot (1 + G \cdot (1 + c\gamma_1))},$$

and

$$p_2^{S,O}(\delta) = \frac{(N_2 - \delta)(1 + Gc\gamma_2) + Gb\gamma_2 + \delta}{\gamma_2 \cdot (1 + G \cdot (1 + c\gamma_2))},$$

as a function of storage use. We can also substitute the first two stationarity conditions into the third, which shows that equilibrium storage use is $\delta_S^O = 0$ if:

$$Gp_2^{S,O}(0) - g_2^{S,O}(0)/\gamma_2 < \eta Gp_1^{S,O}(0) - \eta g_1^{S,O}(0)/\gamma_1,$$

$\delta_S^O = \bar{\delta}$ if:

$$\frac{G[p_2^{S,O}(0) - \eta p_1^{S,O}(0)] + \eta g_1^{S,O}(0)/\gamma_1 - g_2^{S,O}(0)/\gamma_2}{\eta \cdot (G + 1)p_1^{S,O'}(0) - (G + 1)p_2^{S,O'}(0)} > \bar{\delta},$$

and that:

$$\delta_S^O = \frac{G[p_2^{S,O}(0) - \eta p_1^{S,O}(0)] + \eta g_1^{S,O}(0)/\gamma_1 - g_2^{S,O}(0)/\gamma_2}{\eta \cdot (G+1)p_1^{S,O'}(0) - (G+1)p_2^{S,O'}(0)}$$

otherwise.

We also have that:

$$\nabla^2 \Pi_i^{S,O}(g, \delta) = \begin{bmatrix} -\frac{2}{\gamma_1} - Gc & 0 & \frac{2\eta}{\gamma_1} \\ 0 & -\frac{2}{\gamma_2} - Gc & -\frac{\gamma_1}{\gamma_2} \\ \frac{2\eta}{\gamma_1} & -\frac{2}{\gamma_1} & -\frac{2\eta^2}{\gamma_1} - \frac{2}{\gamma_2} \end{bmatrix}.$$

Pre- and post-multiplying this by $(g_{i,1}, g_{i,2}, \delta_i)$ gives:

$$-Gc \cdot (g_{i,1}^2 + g_{i,2}^2) - \frac{2}{\gamma_1}(g_{i,1} - \eta\delta)^2 - \frac{2}{\gamma_2}(g_{i,2} + \delta)^2,$$

which is non-positive. Thus, generator i 's objective is concave and the KKT conditions are sufficient for a global optimum.

5.3. Welfare Effects of Storage

Taking the special case in which $\gamma_1 = \gamma_2 = \gamma$, we next show conditions under which generator-owned storage can reduce social welfare. We first examine the case in which the generating firms behave perfectly competitively with respect to generation but strategically with respect to storage decisions. Note that because equilibrium generation and prices (as a function of storage) in this case are given by the same expressions derived in the perfectly competitive-generation case considered in Section 3, social welfare is given by (8).

Lemma 6. *If the market consists of G symmetric storage-owning generating firms that behave perfectly competitively with respect to generation and follow a Nash-Cournot storage equilibrium and $\gamma_1 = \gamma_2 = \gamma$, storage use can yield a social welfare loss compared to not having storage in the market if $G = 1$. Otherwise, if $G \geq 2$, storage use cannot yield a social welfare loss.*

Proof. Differentiating welfare function (8) gives:

$$W'(0) = p_2(0) - \eta p_1(0)$$

and:

$$W''(\delta) = -\left(\frac{c}{1+c\gamma_2} + \frac{c\eta^2}{1+c\gamma_1}\right) < 0.$$

Thus, if $\delta_C^O > 0$ when $p_2(0) - \eta p_1(0) < 0$, having storage in the market is strictly welfare-decreasing. If $\gamma_1 = \gamma_2 = \gamma$ we have:

$$p_2(0) - \eta p_1(0) = \frac{N_2c + b - \eta \cdot (N_1c + b)}{1 + c\gamma},$$

which is negative if and only if $N_2c + b - \eta \cdot (N_1c + b) < 0$. We further know from (11) that $\delta_C^O > 0$ if and only if:

$$\begin{aligned} p_2(0)[1 - 1/\phi_2] + \frac{p_2'(0)g_2(0)}{G} \\ > \eta p_1(0)[1 - 1/\phi_1] - \frac{p_1'(0)g_1(0)}{G}, \end{aligned}$$

which becomes:

$$\begin{aligned} [G \cdot (1 + c\gamma) - 2][N_2c + b - \eta \cdot (N_1c + b)] \\ + b \cdot (1 - \eta)(1 + c\gamma) > 0. \end{aligned} \quad (15)$$

Since:

$$b \cdot (1 - \eta)(1 + c\gamma) < 0,$$

condition (15) only holds if:

$$[G \cdot (1 + c\gamma) - 2][N_2c + b - \eta \cdot (N_1c + b)] > 0.$$

Thus, for storage to be used when $p_2(0) - \eta p_1(0) < 0$, we must have:

$$G \cdot (1 + c\gamma) - 2 < 0.$$

If $G = 1$, then:

$$G \cdot (1 + c\gamma) - 2 = c\gamma - 1,$$

which can be negative, depending on the magnitude of $c\gamma$. On the other hand, if $G \geq 2$, then:

$$G \cdot (1 + c\gamma) - 2 \geq Gc\gamma > 0.$$

This, thus, shows that if $G = 1$ it possible for storage to be used when it reduces social welfare, whereas this is impossible if $G \geq 2$. \square

We finally examine the case in which the generators both strategically determine their generation levels and storage use. Analogously to the case examined in Section 4, consumer welfare is computed as:

$$\begin{aligned} CW^{S,O}(\delta) &= \int_0^{g_1^{S,O}(\delta) - \eta\delta} [P_1(x) - p_1^{S,O}(\delta)] dx \\ &\quad + \int_0^{g_2^{S,O}(\delta) + \delta} [P_2(x) - p_2^{S,O}(\delta)] dx \\ &= \frac{1}{2\gamma_1} [g_1^{S,O}(\delta) - \eta\delta]^2 + \frac{1}{2\gamma_2} [g_2^{S,O}(\delta) + \delta]^2, \end{aligned}$$

producer surplus as:

$$\begin{aligned} PW^{S,O}(\delta) &= \int_0^{g_1^{S,O}(\delta)} [p_1^{S,O}(\delta) - c'(x)] dx \\ &\quad + \int_0^{g_2^{S,O}(\delta)} [p_2^{S,O}(\delta) - c'(x)] dx \\ &= \frac{2 + Gc\gamma_1}{2G\gamma_1} g_1^{S,O}(\delta)^2 + \frac{2 + Gc\gamma_2}{2G\gamma_2} g_2^{S,O}(\delta)^2, \end{aligned}$$

and storage profits are given by:

$$\Pi_S^{S,O}(\delta) = \delta[p_2^{S,O}(\delta) - \eta p_1^{S,O}(\delta)].$$

We now show, in the following lemma, that generator-owned storage can result in welfare losses, regardless of the number of firms, when they strategically make generation and storage decisions.

Lemma 7. *If the market consists of G symmetric storage-owning generating firms that follow a Nash-Cournot generation and storage use equilibrium and $\gamma_1 = \gamma_2 = \gamma$, storage use can yield a social welfare loss compared to not having storage in the market.*

Proof. The KKT conditions show that $\delta_S^O > 0$ if and only if:

$$G[p_2^{S,O}(0) - \eta p_1^{S,O}(0)] + \eta g_1^{S,O}(0)/\gamma_1 - g_2^{S,O}(0)/\gamma_2 > 0.$$

If $\gamma_1 = \gamma_2 = \gamma$ this condition becomes:

$$Gc\gamma \cdot (N_2 - \eta N_1) + b\gamma \cdot (G + 1)(1 - \eta) > 0. \quad (16)$$

Since $b\gamma \cdot (G + 1)(1 - \eta) < 0$, (16) holds if and only if $Gc\gamma \cdot (N_2 - \eta N_1)$ is positive and sufficiently large in magnitude compared to $b\gamma \cdot (G + 1)(1 - \eta)$.

Differentiating the welfare function gives:

$$\begin{aligned} W^{S,O'}(0) &= p_2^{S,O}(0) - \eta p_1^{S,O}(0) \\ &+ \frac{1}{G\gamma} \cdot \left(1 + \frac{G + 1}{1 + G \cdot (1 + c\gamma)}\right) \cdot (\eta g_1^{S,O}(0) - g_2^{S,O}(0)) \\ &= \frac{Gc\gamma \cdot (N_2 - \eta N_1) + b\gamma(G + 1)(1 - \eta)}{\gamma \cdot (1 + G \cdot (1 + c\gamma))} \\ &+ (G + 1) \cdot \frac{b\gamma \cdot (1 - \eta) - N_2 + \eta N_1}{\gamma \cdot (1 + G \cdot (1 + c\gamma))^2}. \end{aligned} \quad (17)$$

If $\delta_S^O > 0$ then (16) implies that the first term in (17):

$$\frac{Gc\gamma \cdot (N_2 - \eta N_1) + b\gamma(G + 1)(1 - \eta)}{\gamma \cdot (1 + G \cdot (1 + c\gamma))},$$

is positive. Condition (16) also implies that $-N_2 + \eta N_1 < 0$ and by assumption we have that $b\gamma \cdot (1 - \eta) < 0$. Thus, the second term in (17) is negative, and if this term is sufficiently large in magnitude compared to the first, storage may be used when it is strictly welfare-diminishing.

Since $W^{S,O''}(\delta)$ is not necessarily non-positive, it may be the case that with sufficiently great storage capacity, storage use is welfare enhancing. We have, however, found numerical cases in which storage use is welfare-diminishing under this market structure. \square

6. Conclusions

This paper examines the welfare effects of storage under a multitude of market structures. This includes combinations of generators behaving perfectly competitively or as Cournot oligopolists, storage behaving perfectly competitively or *a la* Cournot, and standalone and generator-owned storage. We demonstrate that under some conditions, storage reduces allocative efficiency relative to not having storage in the market. This is noteworthy, since adding firms to an imperfectly competitive market typically reduces the extent to which the exercise of market power results in welfare losses. Importantly, we show that

under most market structures, market power in the generation sector is necessary for storage to be welfare diminishing. These findings have important implications for storage development and how storage-related policy should be targeted to maximize its social value.

It is important to note, however, that we examine a highly stylized model that assumes a simple two-period interaction among the generation and storage firms. Further study, either using similar stylized models or numerical case studies, is needed to fully explore the welfare implications of storage. Important questions that must be addressed include whether these findings hold with different strategic interactions between the firms, such as competition in supply functions or prices, and the implications of the generation mix on storage and welfare. Moreover, most existing storage studies are short-run analyses, which take the generation and storage mix as fixed. An equally important question is what incentives generating and standalone storage firms have to invest in storage, and the associated social welfare implications. Indeed, although storage cannot yield welfare losses in the perfectly competitive-generation case, this only considers short-run impacts and does not account for the potentially large investment cost in most storage technologies. Thus, long-term incentives for storage investment and subsequent use is an important topic needing further research.

A. Proofs of Lemmas

Proof of Lemma 3. Social welfare, as a function of storage use, is defined as:

$$\begin{aligned} W^G(\delta) &= CW^G(\delta) + PW^G(\delta) + \Pi_S^G(\delta) \\ &= \frac{1}{2\gamma_1}[g_1^G(\delta) - \eta\delta]^2 + \frac{1}{2\gamma_2}[g_2^G(\delta) + \delta]^2 \\ &\quad + \frac{2 + Gc\gamma_1}{2G\gamma_1}g_1^G(\delta)^2 \\ &\quad + \frac{2 + Gc\gamma_2}{2G\gamma_2}g_2^G(\delta)^2 \\ &\quad + \delta[p_2^G(\delta) - \eta p_1^G(\delta)]. \end{aligned}$$

Noting that by definition we have:

$$g_1^G(\delta) = N_1 - \gamma_1 p_1^G(\delta) + \eta\delta,$$

and

$$g_2^G(\delta) = N_2 - \gamma_2 p_2^G(\delta) - \delta,$$

the welfare function can be rewritten as:

$$\begin{aligned} W^G(\delta) &= \frac{1}{2\gamma_1}[N_1 - \gamma_1 p_1^G(\delta)]^2 + \frac{1}{2\gamma_2}[N_2 - \gamma_2 p_2^G(\delta)]^2 \\ &\quad + \frac{2 + Gc\gamma_1}{2G\gamma_1}[N_1 - \gamma_1 p_1^G(\delta) + \eta\delta]^2 \\ &\quad + \frac{2 + Gc\gamma_2}{2G\gamma_2}[N_2 - \gamma_2 p_2^G(\delta) - \delta]^2 \\ &\quad + \delta[p_2^G(\delta) - \eta p_1^G(\delta)]. \end{aligned}$$

Differentiating the welfare function gives:

$$\begin{aligned} W^{G'}(\delta) &= -p_1^{G'}(\delta)[N_1 - \gamma_1 p_1^G(\delta)] - p_2^{G'}(\delta)[N_2 - \gamma_2 p_2^G(\delta)] \\ &\quad + \frac{2 + Gc\gamma_1}{G\gamma_1}[\eta - \gamma_1 p_1^{G'}(\delta)][N_1 - \gamma_1 p_1^G(\delta) + \eta\delta] \\ &\quad - \frac{2 + Gc\gamma_2}{G\gamma_2}[1 + \gamma_2 p_2^{G'}(\delta)][N_2 - \gamma_2 p_2^G(\delta) - \delta] \\ &\quad + \delta[p_2^{G'}(\delta) - \eta p_1^{G'}(\delta)] + p_2^G(\delta) - \eta p_1^G(\delta). \end{aligned}$$

If we let $\gamma_1 = \gamma_2 = \gamma$ and fix $\delta = 0$ this gives:

$$\begin{aligned} W^{G'}(0) &= p_2^G(0) - \eta p_1^G(0) \\ &\quad + [N_1 - \gamma p_1^G(0)] \left[\frac{2 + Gc\gamma}{G\gamma} [\eta - \gamma p_1^{G'}(0)] - p_1^{G'}(0) \right] \\ &\quad - [N_2 - \gamma p_2^G(0)] \left[\frac{2 + Gc\gamma}{G\gamma} [1 + \gamma p_2^{G'}(0)] + p_2^{G'}(0) \right] \\ &= \frac{\eta N_1 - N_2 + [2 + G \cdot (1 + c\gamma)][\gamma p_2^G(0) - \eta \gamma p_1^G(0)]}{\gamma \cdot (1 + G \cdot (1 + c\gamma))} \\ &= \frac{(1 + G \cdot (1 + c\gamma))[Gc\gamma \cdot (N_2 - \eta N_1) + Gb\gamma \cdot (1 - \eta)]}{\gamma \cdot (1 + G \cdot (1 + c\gamma))^2} \\ &\quad + \frac{(1 + Gc\gamma)(N_2 - \eta N_1) + Gb\gamma \cdot (1 - \eta)}{\gamma \cdot (1 + G \cdot (1 + c\gamma))^2}. \end{aligned}$$

We then have that $W^{G'}(0) \leq 0$ if and only if:

$$\left(\frac{1}{2 + G \cdot (1 + c\gamma)} + Gc\gamma \right) (N_2 - \eta N_1) + Gb\gamma \cdot (1 - \eta) < 0. \quad (18)$$

Storage is used, in both the perfectly competitive and strategic cases, if and only if $p_2^G(0) - \eta p_1^G(0) > 0$. This condition can be rewritten as:

$$\frac{N_2 \cdot (1 + Gc\gamma) + Gb\gamma}{\gamma \cdot (1 + G \cdot (1 + c\gamma))} - \eta \frac{N_1 \cdot (1 + Gc\gamma) + Gb\gamma}{\gamma \cdot (1 + G \cdot (1 + c\gamma))} > 0,$$

or as:

$$(1 + Gc\gamma)(N_2 - \eta N_1) + Gb\gamma \cdot (1 - \eta) > 0. \quad (19)$$

Since $1 - \eta < 0$, condition (19) can only be true if $N_2 - \eta N_1 > 0$. Since:

$$\frac{1}{2 + G \cdot (1 + c\gamma)} \leq \frac{1}{3},$$

the left-hand side of (18) is less than that of (19), meaning that storage can be used when $W^{G'}(0) \leq 0$. We further have that:

$$\begin{aligned} W^{G''}(\delta) &= \gamma p_1^{G'}(\delta)^2 + \gamma p_2^{G'}(\delta)^2 \\ &\quad + \frac{2 + Gc\gamma}{G\gamma} [\eta - \gamma p_1^{G'}(\delta)]^2 + \frac{2 + Gc\gamma}{G\gamma} [1 + \gamma p_2^{G'}(\delta)]^2 \\ &\quad + 2[p_2^{G'}(\delta) - \eta p_1^{G'}(\delta)] \\ &= \eta^2 \frac{G - (1 + Gc\gamma)(1 + G \cdot (1 + c\gamma))}{\gamma(1 + G \cdot (1 + c\gamma))^2} \\ &\quad + \frac{G - (1 + Gc\gamma)(1 + G \cdot (1 + c\gamma))}{\gamma(1 + G \cdot (1 + c\gamma))^2}. \end{aligned}$$

Since $1 + G \cdot (1 + c\gamma) > G$ and $1 + Gc\gamma > 1$, we have that $W^{G''}(\delta) < 0$ and W^G is concave in δ .

Thus, if (18) holds when $p_2^G(0) - \eta p_1^G(0) > 0$, the concavity of the welfare function implies that storage use reduces welfare compared to not having storage. Moreover, since $\delta_{S,S} \leq \delta_{C,S}$ we can further conclude that:

$$W^G(0) \geq W^G(\delta_{S,S}) \geq W^G(\delta_{C,S}),$$

meaning that perfectly competitive storage yields greater welfare losses than strategic storage does. \square

Proof of Lemma 4. Subtracting condition (10j) from (10i) gives:

$$\begin{aligned} (\delta_i - \delta_j) \left[\frac{c\eta^2}{1 + c\gamma_1} + \frac{c}{1 + c\gamma_2} \right] \\ - \mu_i^- + \mu_i^+ + \mu_j^- - \mu_j^+ = 0. \end{aligned} \quad (20)$$

Note that if $\delta_i > \delta_j$ (20) becomes:

$$(\delta_i - \delta_j) \left[\frac{c\eta^2}{1 + c\gamma_1} + \frac{c}{1 + c\gamma_2} \right] + \mu_i^+ + \mu_j^- = 0,$$

which cannot hold, since $(\delta_i - \delta_j) > 0$ and $\mu_i^+, \mu_j^- \geq 0$. Conversely, if $\delta_i < \delta_j$, then (20) becomes:

$$(\delta_i - \delta_j) \left[\frac{c\eta^2}{1 + c\gamma_1} + \frac{c}{1 + c\gamma_2} \right] - \mu_i^- - \mu_j^+ = 0,$$

which also cannot hold, since $(\delta_i - \delta_j) < 0$ and $\mu_i^-, \mu_j^+ \geq 0$. \square

Proof of Lemma 5. Subtracting (12j) and (13j) from (12i) and (13i) gives:

$$(g_{i,1}^G - g_{j,1}^G) \left(\frac{1}{\gamma_1} + Gc \right) = \frac{\eta}{\gamma_1} (\delta_i - \delta_j), \quad (21)$$

$$(g_{i,2}^G - g_{j,2}^G) \left(\frac{1}{\gamma_2} + Gc \right) = \frac{1}{\gamma_2} (\delta_j - \delta_i). \quad (22)$$

We also subtract (14j) from (14i), and use (21) and (22) to substitute for the $\delta_i - \delta_j$ terms, which gives:

$$\begin{aligned} \eta Gc \cdot (g_{i,1}^G - g_{j,1}^G) - Gc \cdot (g_{i,2}^G - g_{j,2}^G) \\ + \mu_i^+ - \mu_j^+ - \mu_i^- + \mu_j^- = 0. \end{aligned} \quad (23)$$

We first demonstrate that $g_{i,1}^G = g_{j,1}^G$ and $g_{i,2}^G = g_{j,2}^G$ for all i and j by examining all of the cases in which they are not equal and arriving at contradictions.

Suppose first that $g_{i,1}^G > g_{j,1}^G$, (21) then implies that $\delta_i > \delta_j$, which combined with (22) implies that $g_{i,2}^G < g_{j,2}^G$. Substituting these into (23) then implies that at least one of μ_j^+ or μ_i^- is positive, contradicting $\delta_i > \delta_j$.

Suppose next that $g_{i,1}^G < g_{j,1}^G$, (21) implies that $\delta_i < \delta_j$, which combined with (22) implies that $g_{i,2}^G > g_{j,2}^G$. Substituting these into (23) finally implies that at least one of μ_i^+ or μ_j^- is positive, which contradicts $\delta_i < \delta_j$.

If $g_{i,2}^G > g_{j,2}^G$, then (22) implies that $\delta_j > \delta_i$. Equation (21) then gives $g_{i,1}^G < g_{j,1}^G$, which when substituted into (23) forces at least one of μ_i^+ or μ_j^- to be positive, contradicting $\delta_j > \delta_i$.

Finally, if $g_{i,2}^G < g_{j,2}^G$ then (22) gives $\delta_j < \delta_i$, and (21) implies that $g_{i,1}^G > g_{j,1}^G$. Equation (23) then forces at least one of μ_i^+ or μ_j^- to be positive, which contradicts $\delta_j < \delta_i$.

This, thus, shows that $g_{i,1}^G = g_{j,1}^G = g_1^G/G$ and $g_{i,2}^G = g_{j,2}^G = g_2^G/G$ for all i and j . Substituting these into (21) gives:

$$0 = \frac{\eta}{\gamma_1}(\delta_i - \delta_j),$$

which implies that $\delta_i = \delta_j = \delta/G$ for all i and j . \square

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